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Topological Shape Optimization Scheme Based on the Artificial Bee Colony Algorithm

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We propose a new topological shape optimization scheme based on the artificial bee colony algorithm (ABCA). Since the level set method (LSM) and phase field method (PFM) in topological shape optimization have been developed, one of any algorithms in this field has not yet been proposed. To perform the topological shape optimization based on the ABCA, a variable called the "Boundary Element Indicator (BEI)," is introduced, which serves to define the boundary elements whenever a temporary candidate solution is found in the employed and onlooker bee phases. Numerical examples are provided to verify the performance of the suggested ABCA compared with the discrete LSM and the ABCA for topology optimization. The numerical examples showed that holes in the structure are naturally created in the ABCA for topological shape optimization. Moreover, the objective function of the suggested ABCA is lower than that of the ABCA for topology optimization, and is similar to that of the discrete LSM. The convergence rate of the suggested ABCA is the fastest among the comparison methods. Therefore, it can be verified that the suggested topological shape optimization scheme, based on the ABCA, is the most effective among the comparison methods.

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1. Introduction

Structural optimization is usually employed to determine which structure performs best under prescribed conditions, such as volume, stress, and/or deformation constraints. The methods for structural optimization are generally classified as size, shape, and topology optimizations. Size optimization is used when geometric features, such as the thickness, diameter, width, and height of the structure are optimized while maintaining the same shape. Shape optimization is employed to obtain an optimized shape, based on design variables which define the shape of the structure. Topology optimization is a method to determine the optimal layout in a defined design domain for the given boundary/loading conditions and constraints, without having any information about the initial topology. Topology optimization is used to obtain appropriate topology at the initial design step, with shape or size optimization being subsequently performed in order to obtain a refined optimized structure.

Topological shape optimization is a more efficient structural optimization method, as it performs topology and shape optimization simultaneously. Sethian and Wiegmann first proposed a topological shape optimization method.[1] They presented a combined level set method (LSM) and a finite difference technique for structural optimization, based on the von Mises stress. Challis[2] used a discrete LSM for topological shape optimization, applying the method to compliance minimization problems of statically loaded structures. However, since the use of intermediate densities was avoided, this method was only able to produce clear boundaries of the structure at each iteration in the optimization process. In addition, the shape and topological sensitivities were employed for use with the Hamilton-Jacobi equation. The LSM has been successfully applied to various problems, such as thermo-elastic structures,[3] stress-based problems,[4] and compliant mechanisms,[5] and its performance and applicability has been well verified.

The phase field method (PFM) is another topological shape optimization that was conceptualized by Bourdin and Chambolle.[6] The structure in the PFM is represented as a subset of the reference domain, and the complement of the subset is made of two other phases, the void and a fictitious liquid that exerts a pressure force on its interface with a solid structure. The PFM has been successfully applied to various issues, such as: local stress constraint problems,[7] multimaterial topology optimization,[8] and the eigenfrequency maximization problem.[9] The main difference between the PFM and the LSM is the governing equation for each method. Typically, the PFM uses the Allen-Cahn equation[10] or the Cahn-Hilliard equation,[11] whereas the LSM uses the Hamilton-Jacobi equation.[12] In spite of this difference, however, the optimized results using both methods are very similar.

Although researchers found that the proposed LSMs[1-5] are very effective for topological shape optimization and various methods for new holes creation[13] such as topological derivatives and front-tracking algorithm etc., have been proposed, they do have some difficulties. First, the optimized structures are highly dependent on the number of initial holes in the direct LSM,[5] weighting factor for topological sensitivity[2] or hole creation methods.[13] The proper number of initial holes or the ideal values of the weighting factor should be empirically selected in order to obtain an optimized shape. Also, it is difficult to determine when it would be more optimal to create a hole or continue with boundary updates in the front-tracking algorithm. Furthermore, the computational cost is significant, because the LSM is based on a mathematical gradient approach and can only evolve from the existing boundaries.[14]

Recent research has focused on diverse technologies, inspired by natural phenomena and scientific principles of living things. Namely, swarm intelligence algorithms have been developed for global optimization. These algorithms are inspired by colonies of animals, such as ants, bees, bats, and wolves.[15-20] In particular, Karaboga and Basturk[16-18] verified that the artificial bee colony algorithm (ABCA), one of the swarm intelligence algorithms that was inspired by the behavior of honey bees looking for food sources, has shown outstanding performances when compared with genetic algorithm (GA),[21] differential evolution (DE),[22] particle swarm optimization (PSO),[23] and evolutionary algorithms (EA).[24]

The ABCA was applied to various structural optimization problems. Sonmez[25] employed the ABCA to solve structural size optimization problems attempting to determine the optimal cross sectional areas of truss structures. The ABCA was also employed for structural topology optimization problems, such as linear static and dynamic structural problems, by Park and Han.[26,27] The aforementioned research proved that the ABCA is a powerful and effective search algorithm for structural topology optimization.

Similar to the LSM and PFM, topological shape optimization is attractive, as it performs topology and shape optimization simultaneously. Topology optimization should be the first step, in order to determine the topology at the initial design stage, followed by shape optimization, which allows researchers to subsequently obtain the optimized shape; therefore, the LSM and the PFM are arguably more effective than any of the other topology optimization methods.

In this paper, a new topological shape optimization algorithm based on the ABCA is suggested, since it has already been confirmed that the ABCA is an effective algorithm to apply to the structural optimization problems.[25-27] The proposed ABCA for topological shape optimization overcomes the limitations of previous algorithms, as it can naturally create holes in a topology without initial holes or topological sensitivity. Therefore, it can be expected that topological shape optimization based on the ABCA would be much more effective than the LSM, since it can freely create holes in the design domain and optimize the boundaries of topology by suitably defining the boundary elements. In addition, when topological shape optimization, based on the ABCA is performed, the searching domain for topological shape optimization is much narrower than that for topology optimization. This means that the convergence rate of the topological shape optimization algorithm should be faster than that of the original topology optimization algorithm.

This paper is composed as follows: Formulation for topological shape optimization including the problem statement, sensitivity number based on the waggle index update rule and definition of the boundary elements is described in section 2. Two methods, the discrete LSM[2] and the ABCA for topological shape optimization are explained in section 3. Numerical examples are provided to examine the performance of the ABCA for topological shape optimization comparing to the discrete LSM[2] and the ABCA for topology optimization.[26,27] in section 4. Discussion is also descri-bed in this section. Conclusions are followed in section 5.

2. Formulation for Topological Shape Optimization

2.1 Problem statement

In this study, topological shape optimizations for static stiffness problems are performed. The aim of the topological shape optimizations is to obtain an optimized structure with maximal stiffness, while satisfying the given constraints, such as the volume fraction constraint. Therefore, the total strain energy is considered to be the objective function for the maximal stiffness of the structures. The topological shape optimization problem can be stated as follows:

$$\begin{aligned} &Minimize : U = \frac{1}{2} f^T u \\ &Subject\ to : Ku = f \\ &V^* - \sum_{i=1}^{N} V_i \chi_i = 0 \end{aligned} \tag{1}$$

where U is the total strain energy, f is the applied load vector, u is the nodal displacement vector, K is the global stiffness matrix, T denotes the transpose, and V^* and V_i denote the prescribed volume constraint and the volume of each element, respectively. N is the total number of finite elements in the discretized domain, and χ_i is the binary design variable indicating a solid or void element.

2.2 Sensitivity number based on the waggle index update rule

In this research, traces of honey bees searching for food sources are used as the intermediated variables, which are inspired by the ant colony optimization (ACO) algorithm.[28] The traces are defined using the waggle index update rule:[26,27]

$$I_i^k = \delta \times I_i^{k-1} + (1-\delta) \times e_i^k \tag{2}$$

where I_i is the waggle index, e_i is the employed bee presence/absence, δ is the coefficient of the waggle index update (it is generally set between 0.5 and 0.8), and k is the present iteration number.

To obtain a solid-void design, the material interpolation scheme,[29] using the waggle index update rule, should be used. This scheme assumes that Young's modulus is a function of the waggle index I_i, as follows:

$$E(I_i) = I_i^p E^1$$

$$K = \sum_{i=1}^{N} I_i^p K_i^1 \tag{3}$$

where E is Young's modulus, p is the penalty factor, E^1 is Young's modulus of a solid element, and K_i^1 is the elemental stiffness matrix of a solid element. K is constructed by assembling the waggle index matrix and the elemental stiffness matrix.

The sensitivity number can be determined as follows:[26]

$$f_i = \phi U_i = \frac{1}{2} I_i^{p-1} u_i^T K_i^1 u_i \quad (0 < I_{min} \le I_i \le 1) \tag{4}$$

where f_i is the sensitivity number of ith element, u_i the displacement vector of the ith element, and $I_{\min}$ is typically employed as a sufficiently small value (i.e. 0.001), to avoid singularity.

2.3 Definition of the boundary elements

In this research, the ABCA is employed for topological shape optimization. The fitness value is used to obtain information about a measure, to determine which element is more efficient in the ABCA. The temporary fitness value $temp\_fit_i$ is obtained using Eq. (5) since the sensitivity number of the ith element, f_i is always positive.

$$temp\_fit_i = \frac{1}{1+f_i} \tag{5}$$

However, the $temp\_fit_i$ provides the information about the fitness value in the overall design domain. Therefore, the $temp\_fit_i$ should be divided, showing the boundary region and the other regions, as topological shape optimization focuses on the boundary region of the structure using predetermined variables by creating the holes in the topology. In order to determine the boundary region of a structure, the boundary elements are defined (Fig. 1). As seen in Fig. 1, the black and white elements represent the solid and void elements, respectively, and the grey elements are defined as the boundary elements. The boundary line is determined by the interface of the solid and void elements. Both the solid and void elements, which occur on each layer at both sides of the boundary line, are defined as the boundary elements.

The boundary elements are defined based on a variable called the "Boundary Element Indicator (BEI)," which is determined as follows:

$$BEI_i = \begin{cases} +1 & \text{(if it is a boundary element)} \\ -1 & \text{(if it is not a boundary element)} \end{cases} \tag{6}$$

where BEI_i is the boundary element indicator of the ith element. By applying the BEI_i to the $temp\_fit_i$, the fitness value for the topological shape optimization fit_i can be obtained as follows:

$$fit_i = BEI_i \times temp\_fit_i \tag{7}$$

Using this equation, it is possible to verify whether the fit_i is positive, which would make the ith element a boundary element.

If the BEI_i is updated once at each iteration, this method can only search the solutions in the boundary elements of the structure; therefore, holes in the structure cannot be created. In order to create holes naturally and to optimize the boundary elements simultaneously in topological shape optimization, the BEI_i should be updated continuously whenever v_i (v_i'), which is the location of a temporary candidate solution in each iteration is found (the v_i (v_i') is further explained in Section 3.2).

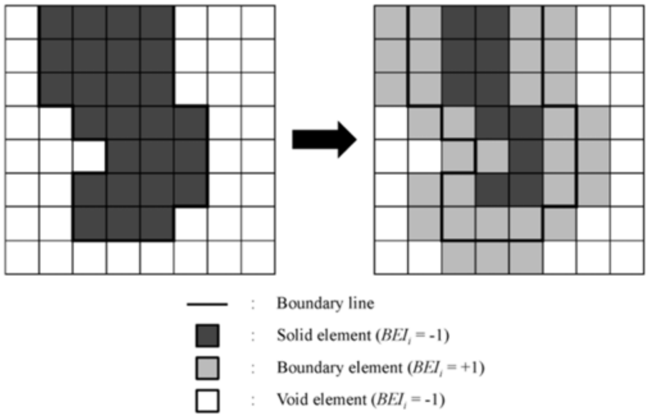


Fig. 1 Definition of the boundary elements

Through the aforementioned process, the solid elements are distributed to efficient regions in the overall design domain, based on fitness values, although only the boundary elements are optimized. When the BEI_i is updated continuously, whenever v_i (v_i') is found, the locations of the defined boundary elements can be moved sequentially, from the locations of the boundary elements in the initial topology to the locations of the efficient elements in the overall design domain, in each iteration. This results in a continual process of topology optimization, although only a shape optimization based on the ABCA is performed.

After an approximate structure is created, the locations of the defined boundary elements are almost fixed in each iteration, because it is impossible to create any subsequent new holes in the structure. Hence, shape optimization occurs after the convergence of an approximate topology of the structure. In addition, as the number of elements to be searched decreases from that of the overall design domain to only that of the boundary elements of a structure, the convergence rate becomes faster as optimization progresses.

In summary, if the locations of the boundary elements are defined continuously using the BEI_i in each iteration, holes can be created naturally and an approximate topology can be defined. Once an approximate topology is created, shape optimization can occur, as new holes in the structure cannot be created. Through the aforementioned procedure, the topology and shape optimizations can be performed simultaneously.

3. Methods for Topological Shape Optimization

3.1 Discrete LSM

Before describing the method proposed in this study, it is necessary to provide a brief description of the discrete LSM suggested by Challis.[2] This method is governed by the Hamilton-Jacobi equation, as follows:

$$\frac{\partial \phi}{\partial t} = -v|\nabla\phi| - wg \tag{8}$$

where ϕ is the level set function and v is the normal velocity, chosen based on the shape sensitivity of the objective function. The forcing term, g, which creates holes in the structure, is chosen based on the topological sensitivity and is influenced by the weighting factor, w, which should be between 1 and 4.[2] However, it is difficult to determine the proper value of w, as the optimized design is highly dependent on its value.

3.2 ABCA for topological shape optimization

The ABCA consists of main phases for searching for food sources: the employed bee, the onlooker bee, and the scout bee phases. The detailed explanations can be found in Refs. 16-18.

The procedure of topological shape optimization based on the ABCA is as follows:[26,27]

Step 1: Establish the design domain using rectangular elements and the initial parameters.

Step 2: Perform a finite element analysis (FEA) for the initial design and calculate the sensitivity number f_i using Eq. (4).

Step 3: Calculate the temporary fitness value $temp\_fit_i$ using Eq. (5).

Step 4: Perform a modified employed bee phase for topological shape optimization. In this step, the boundary element indicator, BEI_i, should be updated continuously whenever the location of a temporary candidate solution is found in the topological shape optimization using Eq. (6). If the BEI_i is updated, fit_i can be automatically updated using Eq. (7). Based on the updated BEI_i and fit_i, x_i, x_k (the locations of the initial solutions) and v_i (the location of the temporary candidate solution) which only have a fit_i that is a positive value, should be searched in the modified employed bee phase for topological shape optimization using Eq. (9).

$$\begin{aligned} &v_i = x_i + int[rand[0,1]\times abs[x_i - x_k]] \\ &x_i = int[rand[0,1]\times sum[BEI_i > 0]] \\ &x_k = int[rand[0,1]\times sum[BEI_i > 0]] \\ &x_i, x_k, \text{ and } v_i \text{ for } fit_i > 0 \end{aligned} \tag{9}$$

where $int[x]$ is the integer number of x, abs[x] is the absolute value of x, $rand[a,b]$ is a random number between a and b, and $sum[x]$ is the summation of x.

Step 5: Perform the modified onlooker bee phase for topological shape optimization. In this step, the BEI_i and fit_i should be also defined using Eqs. (6) and (7) whenever the location of a temporary candidate solution is found, as in Step 4. In this step, holes in the topology can be created naturally. The onlooker bee phase is performed using the following equation:

$$\begin{aligned} &v_i' = x_i' + int[rand[0,1]\times abs[x_i' - x_k']]\ for\ rand[0,1] < p_i \\ &x_i' = int[rand[0,1]\times sum[BEI_i > 0]] \\ &x_k' = int[rand[0,1]\times sum[BEI_i > 0]] \\ &x_i', x_k',\ and\ v_i'\ for\ fit_i > 0 \\ &p_i = \frac{fit_i}{\sum_{i=1}^{N} fit_i} \end{aligned} \tag{10}$$

where x_i' and x_k' are the new locations of the initial solutions, v_i' is the new location of the temporary candidate solution, and p_i is the probability value of the ith element.

Step 6: Perform the scout bee phase. Specify the elements occupied by the employed bees (solid elements) and the elements abandoned by the employed bees (void elements) based on the fitness value of each element. Next, the locations of the solid and void elements should be determined using the prescribed volume constraint from the first iteration, because the number of bees in each colony remains constant during the optimization process in a standard ABCA.[16-18] Subsequently, a simple averaging scheme, based on a $limit\_value$,[26,27] is carried out using Eq. (11). This scheme helps to avoid the local minima.

$$fit_i^k = \begin{cases} \dfrac{fit_i^k + fit_i^{k-1}}{2} & (\text{if } k \neq limit\_value \text{ step}) \\ fit_i^k & (\text{if } k = limit\_value \text{ step}) \end{cases} \tag{11}$$
$$limit\_value \text{ step} = limit\_value \times \text{step}(=1,2,3)$$

where the $limit\_value$ is the predetermined value. Although the value is a number, chosen between 10 and 15, for topology optimization,[26,27] it is determined to be either 5 or 10 for topological shape optimization. The reason for this decision is that the solid elements can move in the defined boundary region, which slows down the convergence rate. Therefore, the scheme should be performed for topological shape optimization more often than for topology optimization.[26,27]

Step 7: Specify an optimized topological shape solution based on the results of Steps 4 to 6, after which the waggle index update rule is performed using Eq. (2). When k is between 15 and 20, δ in the Eq. (2) should become 0, since the waggle index matrix I_i (that is, the trace of previous locations of the elements occupied by the employed bees) encourages the employed bee elements to try to move to the no boundary region. Therefore, the employed bee matrix e_i (the present location of the elements occupied by the employed bees) is only used to calculate the sensitivity numbers from the prescribed iteration number.

Step 8: Calculate the objective function using the FEA and obtain the elemental sensitivity number based on the waggle index matrix I_i. In this study, the mesh-independency filter scheme[30] using Eq. (12) is employed to avoid a checkerboard pattern.

$$f_i = \frac{\sum_{j=1}^{M} w(r_{ij}) f_j^n}{\sum_{j=1}^{M} w(r_{ij})},\ w(r_{ij}) = r_{min} - r_{ij} \quad (j = 1, 2, \ldots, M) \tag{12}$$

where M is the total number of nodes in the sub-domain, $w(r_{ij})$ is the linear weight factor, f_j^n is the jth nodal sensitivity number, r_{ij} is the distance between the center of the ith element and the jth node, and $r_{\min}$ is the length scale parameter.

Step 9: Check whether the updated design has converged or not using Eq. (13). If the design has not converged, it is necessary to go back to Step 3 and repeat the subsequent steps until this criterion is satisfied.[26,27,30] The schematic diagram of the ABCA is shown in Fig. 2.

$$error = \frac{\left|\sum_{i=1}^{N'} (U_{k-i+1} - U_{k-N'-i+1})\right|}{\sum_{i=1}^{N'} U_{k-i+1}} \le \tau \tag{13}$$

where U is the total strain energy of the structure, τ is the allowable convergence tolerance, and N' is an integer number resulting in a converged objective function. In this paper, N' is given a value to 5, so as to make the change in the objective function sufficiently small for the final 10 iterations.

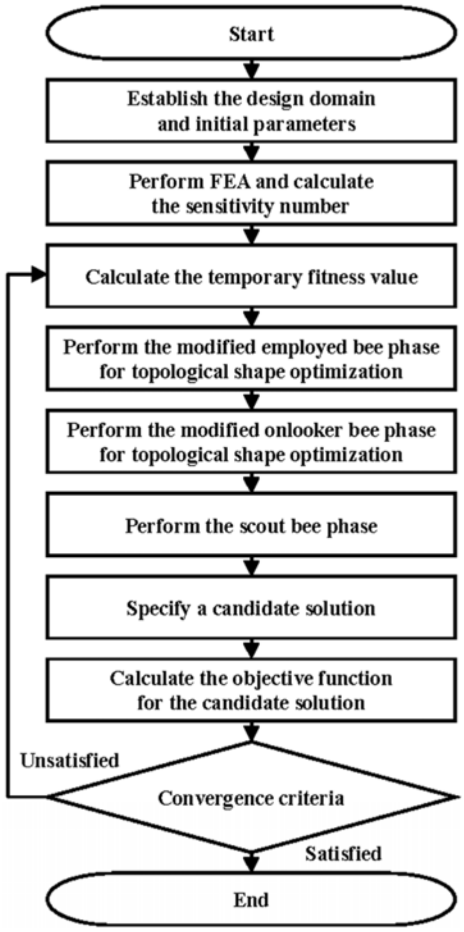


Fig. 2 Schematic diagram of the suggested ABCA

4. Numerical Examples

In order to examine the performance of the proposed method, numerical examples are proposed that could be compared to the discrete LSM,[2] because the optimized structure that uses the ABCA for topological shape optimization is also represented discretely. No comparison with the PFM is done, as its methodology is very similar to the LSM. In addition, the ABCA for topology optimization[26,27] is compared. No comparison is made of the shape optimization based on the ABCA, because it cannot create holes in the structure naturally. Since the ABCA is a stochastic method, each example using the ABCA for both the topology and topological shape optimizations is carried out for 10 runs. The objective functions using the ABCA comprise the mean values of the 10 runs, as well as the standard deviations.

4.1 Cantilever beam

A cantilever beam measuring 0.8 m × 0.6 m × 0.001 m is subjected to 100 kN at the free end (Fig. 3). The design domain is divided into 80 × 60 by rectangular finite elements. The material properties are assumed to have a Young's modulus of 100 GPa and a Poisson's ratio of 0.3. The *limit_value* is set to 5, the allowable convergence tolerance τ is set to 0.001, and $r_{\min}$ is set to 1.5. The overall objective is to determine the topological shape design for a minimum value of the total strain energy, while satisfying the volume constraint of 40%. The

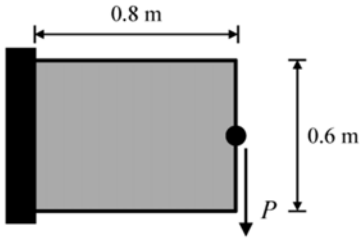


Fig. 3 Problem definition of the cantilever beam

Table 1 Comparison of the optimized designs from the discrete LSM and the ABCA for the cantilever beam

Method	Optimized designs along with iteration				
	1	15	30	…	Converged (Iteration)
Discrete LSM[2]				…	 (48)
ABCA for topology[26,27]				…	 (49)
ABCA for topological shape			…	…	 (28)

w for the discrete LSM is set to 4.

The optimized designs of the discrete LSM,[2] the ABCA for the topology optimization,[26,27] and the ABCA for topological shape optimization are shown in Table 1. These results show that the optimal designs based on the discrete LSM, the ABCA for topology optimization, and the ABCA for topological shape optimization are almost identical, with the convergence rate of the ABCA for topological shape optimization being the fastest among all of the methods. The total strain energy of the optimized design is 3.0874 J for the discrete LSM, 3.1229 J for the ABCA for topology optimization, and 3.0881 J for the ABCA for topological shape optimization. The standard deviation is 0.0078 for the ABCA for topology optimization and 0.0032 for the ABCA for topological shape optimization. Fig. 4 shows the convergence histories of the total strain energy for the cantilever beam problem; as seen in this figure, the values of the suggested method converge stably.

4.2 Bridge

A bridge measuring 0.8 m × 0.4 m × 0.001 m is subjected to 100 kN at the bottom (Fig. 5). The design domain is divided into a grid of 80 × 40 by rectangular finite elements. It is assumed that the material properties have a Young's modulus of 100 GPa and a Poisson's ratio of 0.3. The *limit_value* is set to 5, the allowable convergence tolerance τ is set to 0.001, and $r_{\min}$ is set to 1.5. The objective of this problem is to determine the topological shape design that would have a minimum value of the total strain energy while satisfying the volume constraint of 20%. The *w* for the discrete LSM is set to 2.5.

The optimized designs of the discrete LSM,[2] the ABCA for topology optimization,[26,27] and the ABCA for topological shape optimization are shown in Table 2. These results show that the optimal designs based on

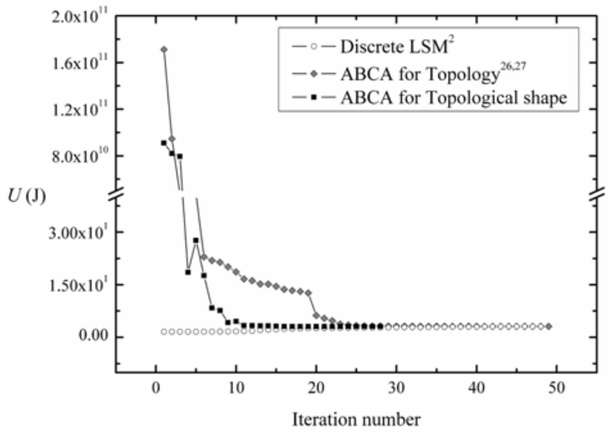


Fig. 4 Convergence histories of the cantilever beam

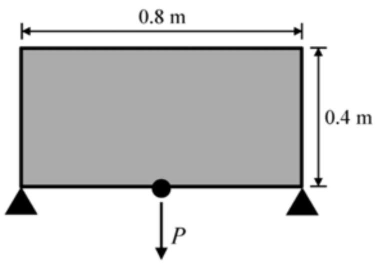
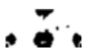


Fig. 5 Problem definition of the bridge

Table 2 Comparison of the optimized designs from the discrete LSM and the ABCA for the bridge

Method	Optimized designs along with iteration				
	1	15	30	…	Converged (Iteration)
Discrete LSM[2]				…	 (66)
ABCA for topology[26,27]				…	 (43)
ABCA for topological shape				…	 (35)

the discrete LSM, the ABCA for topology optimization, and the ABCA for topological shape optimization are almost identical, with the convergence rate of the ABCA for topological shape optimization being the fastest among the methods. The total strain energy of the optimized design is 1.9645 J for the discrete LSM, 2.0050 J for the ABCA for topology optimization, and 1.9878 J for the ABCA for topological shape optimization. The standard deviation is 0.0101 for the ABCA for topology optimization and 0.0098 for the ABCA for topological shape optimization. As seen in Fig. 6, which shows the convergence histories of the total strain energy for the bridge problem, the values of the suggested method converge stably.

4.3 L-bracket

An L-bracket measuring 0.6 m × 0.6 m × 0.001 m is subjected to

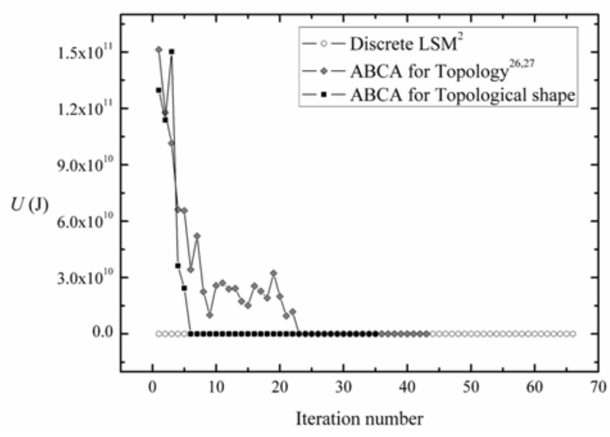


Fig. 6 Convergence histories of the bridge

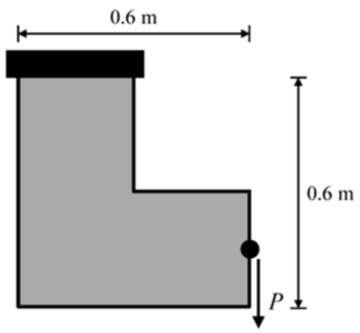
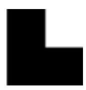
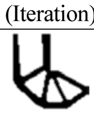


Fig. 7 Problem definition of the L-bracket

Table 3 Comparison of the optimized designs from the discrete LSM and the ABCA for the L-bracket

Method	Optimized designs along with iteration				
	1	20	40	…	Converged (Iteration)
Discrete LSM[2]				…	 (83)
ABCA for topology[26,27]				…	 (62)
ABCA for topological shape			…	…	 (38)

100 kN at the bottom (Fig. 7). The design domain is divided into 60 × 60 by rectangular finite elements. The material properties are assumed, with a Young's modulus of 100 GPa and a Poisson's ratio of 0.3. The *limit_value* is set to 5, the allowable convergence tolerance τ is set to 0.001, and r_{min} is set to 3.0. The objective of this problem is to determine the topological shape design that has the minimum value of total strain energy while satisfying the volume constraint of 40%. The w for the discrete LSM is set to 7.

The optimized designs of the discrete LSM,[2] the ABCA for topology

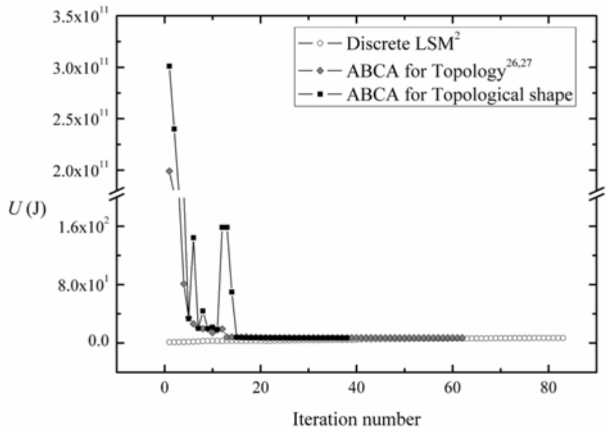


Fig. 8 Convergence histories of the L-bracket

optimization,[26,27] and the ABCA for topological shape optimization are shown in Table 3. The optimal designs, based on the discrete LSM, the ABCA for topology optimization, and the ABCA for topological shape optimization, are almost identical, and the convergence rate of the ABCA for topological shape optimization is the fastest among all of the methods. The total strain energy of the optimized design is 6.8820 J for the discrete LSM, 7.1255 J for the ABCA for topology optimization, and 6.9087 J for the ABCA for topological shape optimization. The standard deviation is obtained as 0.0069 for the ABCA for topology optimization and 0.0084 for the ABCA for topological shape optimization. In terms of the convergence histories of the total strain energy for the L-bracket problem, the values of the suggested method converge stably (Fig. 8).

4.4 Discussion

The results show that the convergence rate of the topological shape optimization based on ABCA is the fastest of the three methods. This may be due to the fact that the searching region is defined continuously; therefore, the searching region is much narrower than that of the ABCA for topology optimization. In addition, its convergence rate may be faster than that of the discrete LSM, because the discrete LSM optimizes the structures using the Hamilton-Jacobi equation, which can only evolve the geometry from the existing boundary region, based on the shape sensitivity, and cannot create new holes in the structure without topological sensitivity. The objective function values of the discrete LSM and the ABCA for topological shape optimization are lower (better) than those of the ABCA for topology optimization. The results show that the discrete LSM and the ABCA for topological shape optimization reflect the effect of shape optimization, in comparison to the ABCA for topology optimization. Furthermore, the objective function values of the discrete LSM are slightly better than those of the ABCA for topological shape optimization. The ABCA for topological shape optimization uses only the shape sensitivity number of each element. However, the discrete LSM uses the normal vector of the structural boundary in addition to the shape sensitivity information. The normal vector of the structural boundary in the discrete LSM helps to find a better solution than the ABCA in terms of topological shape optimization. Even though the objective function values of the discrete LSM are lower than those of the proposed ABCA, the differences are very slight.

Although the discrete LSM can search for a solution more properly than the ABCA for topological shape optimization, this method has one minor drawback, as it cannot create holes in the structure naturally without the topological sensitivity. In order to employ topological sensitivity in the discrete LSM, an appropriate value of the weighting factor controlling the effect of topological sensitivity needs to be empirically specified, because the optimized design from the discrete LSM is influenced significantly by the weighting factor.[2] However, the proposed ABCA does not have such a control parameter, since this method can create the holes in the structure naturally by defining the boundary elements continuously in each iteration.

In addition to the weighting factor, various control parameters, including $r_{\min}$, should also be predetermined for a discrete LSM, namely: the number of Courant-Friedrichs-Lewy (CFL) time steps, the number of iterations of the algorithm before a level set re-initialization is performed, and two types of Lagrangian multipliers for a prescribed volume constraint.[2] To obtain an optimized design using the discrete LSM, it is necessary to determine suitable predetermined values for the control parameters. Even though appropriate ranges of the control parameters for the discrete LSM may be suggested, inaccurate values may lead to different optimized solutions.[2] In the ABCA for the topological shape optimization, there are only three types of control parameters: the *limit_value*, the coefficient of waggle index update δ, and $r_{\min}$. Although suitable values for these parameters should also be predetermined in the ABCA for topological shape optimization, the recommended ranges for the parameters do not influence the optimized design to the extent that the parameters for the discrete LSM do.

In addition, in order to examine the stability and robustness of this method, the convergence histories and mean values of the objective functions are provided for 10 runs. The proposed ABCA converges stably and always converges in such a way so as to show similar results.

5. Conclusions

In this paper, a new topological shape optimization based on the ABCA is proposed. To perform the topological shape optimization, the fitness values based on the "Boundary Element Indicator (*BEI*)," defining the boundary elements should be defined continuously during the employed and onlooker bee phases. From the results of our analysis, the following conclusions can be made:

(1) The proposed method is able to perform topological shape optimization using only shape optimization.

(2) The proposed method is more effective than the discrete LSM and topology optimization scheme based on the ABCA in the viewpoint of convergence rate and the weighting factor for topological sensitivity, respectively. Furthermore, the recommended ranges for the parameters in the suggested method do not influence the optimized design as much as the parameters do with the discrete LSM.

(3) The objective function value of the ABCA for topological shape optimization is lower than that of that of the ABCA for topology optimization, but similar to that of the discrete LSM.

(4) The convergence rate of the suggested method improved as much

as 40% over the discrete LSM, and 20% over the ABCA for topology optimization.

(5) The proposed method can be expanded to include nonlinear topological shape optimization problems.

REFERENCES

1. Sethian, J. A. and Wiegmann, A., “Structural Boundary Design via Level Set and Immersed Interface Methods,” Journal of Computational Physics, Vol. 163, No. 2, pp. 489-528, 2000.
2. Challis, V. J., “A Discrete Level-set Topology Optimization Code Written in Matlab,” Structural and Multidisciplinary Optimization, Vol. 41, No. 3, pp. 453-464, 2010.
3. Xia, Q. and Wang, M. Y., “Topology Optimization of Thermoelastic Structures Using Level Set Method,” Computational Mechanics, Vol. 42, No. 6, pp. 837-857, 2008.
4. Allaire, G. and Jouve, F., “Minimum Stress Optimal Design with the Level Set Method,” Engineering Analysis with Boundary Elements, Vol. 32, No. 11, pp. 909-918, 2008.
5. Allaire, G., Jouve, F., and Toader, A.-M., “Structural Optimization Using Sensitivity Analysis and a Level-Set Method,” Journal of Computational Physics, Vol. 194, No. 1, pp. 363-393, 2004.
6. Bourdin, B. and Chambolle, A., “Design-Dependent Loads in Topology Optimization,” ESAIM: Control, Optimisation and Calculus of Variations, Vol. 9, pp. 19-48, 2003.
7. Burger, M. and Stainko, R., “Phase-Field Relaxation of Topology Optimization with Local Stress Constraints,” SIAM Journal on Control and Optimization, Vol. 45, No. 4, pp. 1447-1466, 2006.
8. Zhou, S. and Wang, M. Y., “Multimaterial Structural Topology Optimization with a Generalized Cahn-Hilliard Model of Multiphase Transition,” Structural and Multidisciplinary Optimization, Vol. 33, No. 2, pp. 89-111, 2007.
9. Takezawa, A., Nishiwaki, S., and Kitamura, M., “Shape and Topology Optimization Based on the Phase Field Method and Sensitivity Analysis,” Journal of Computational Physics, Vol. 229, No. 7, pp. 2697-2718, 2010.
10. Allen, S. M. and Cahn, J. W., “A Microscopic Theory for Antiphase Boundary Motion and Its Application to Antiphase Domain Coarsening,” Acta Metallurgica, Vol. 27, No. 6, pp. 1085-1095, 1979.
11. Cahn, J. W. and Hilliard, J. E., “Free Energy of a Nonuniform System. I. Interfacial Free Energy,” The Journal of Chemical Physics, Vol. 28, No. 2, pp. 258-267, 1958.
12. Osher, S. and Sethian, J. A., “Fronts Propagating with Curvature-Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations,” Journal of Computational Physics, Vol. 79, No. 1, pp. 12-49, 1988.
13. Dunning, P. D. and Kim, H. A., “A New Hole Insertion Method for Level Set Based Structural Topology Optimization,” International Journal for Numerical Methods in Engineering, Vol. 93, No. 1, pp. 118-134, 2013.
14. Sigmund, O. and Maute, K., “Topology Optimization Approaches,” Structural and Multidisciplinary Optimization, Vol. 48, No. 6, pp. 1031-1055, 2013.
15. Colorni, A., Dorigo, M., and Maniezzo, V., “Distributed Optimization by Ant Colonies,” Proc. of the 1st European Conference on Artificial Life, pp. 134-142, 1991.
16. Karaboga, D., “An Idea Based on Honey Bee Swarm for Numerical Optimization,” Erciyes University, Technical Report-TR06, 2005.
17. Karaboga, D. and Basturk, B., “A Powerful and Efficient Algorithm for Numerical Function Optimization: Artifical Bee Colony (ABC) Algorithm,” Journal of Global Optimization, Vol. 39, No. 3, pp. 459-471, 2007.
18. Karaboga, D. and Basturk, B., “On the Performance of Artificial Bee Colony (ABC) Algorithm,” Applied Soft Computing, Vol. 8, No. 1, pp. 687-697, 2008.
19. Mirjalili, S., Mirjalili, S. M., and Lewis, A., “Grey Wolf Optimizer,” Advances in Engineering Software, Vol. 69, pp. 46-61, 2014.
20. Yang, X.-S., “A New Metaheuristic Bat-inspired Algorithm,” in: Nature Inspired Cooperative Strategies for Optimization (NICSO 2010), González J. R., Pelta D. A., Cruz C., Terrazas G., Krasnogor N., (Eds.), Springer, Vol. 284, pp. 65-74, 2010.
21. Holland, J. H., “Adaptation in Natural and Artificial Systems,” University of Michigan Press, 1975.
22. Storn, R. and Price, K., “Differential Evolution – A Simple and Efficient Heuristic for Global Optimization Over Continuous Spaces,” Journal of Global Optimization, Vol. 11, No. 4, pp. 341-359, 1997.
23. Kennedy, J., “Particle Swarm Optimization,” in: Encyclopedia of Machine Learning, Sammut, C., Webb, G. I., (Eds.), Springer, pp. 760-766, 2010.
24. Bäck, T., “Evolutionary Algorithms in Theory and Practice,” Oxford University Press, 1996.
25. Sonmez, M., “Artificial Bee Colony Algorithm for Optimization of Truss Structures,” Applied Soft Computing, Vol. 11, No. 2, pp. 2406-2418, 2011.
26. Park, J.-Y. and Han, S.-Y., “Swarm Intelligence Topology Optimization Based on Artificial Bee Colony Algorithm,” Int. J. Precis. Eng. Manuf., Vol. 14, No. 1, pp. 115-121, 2013.
27. Park, J.-Y. and Han, S.-Y., “Application of Artificial Bee Colony Algorithm to Topology Optimization for Dynamic Stiffness Problems,” Computers and Mathematics with Applications, Vol. 66, No. 10, pp. 1879-1891, 2013.
28. Kaveh, A., Hassani, B., Shojaee, S. and Tavakkoli, S. M., “Structural Topology Optimization Using Ant Colony Methodology,” Engineering Structures, Vol. 30, No. 9, pp. 2559-2565, 2008.

29. Bendsøe, M. P. and Sigmund, O., "Topology Optimization: Theory, Methods and Applications," Springer Science & Business Media, pp. 1-69, 2003.

30. Huang, X. and Xie, Y., "Convergent and Mesh-Independent Solutions for the Bi-Directional Evolutionary Structural Optimization Method," Finite Elements in Analysis and Design, Vol. 43, No. 14, pp. 1039-1049, 2007.