INTERNATIONAL JOURNAL OF PRECISION ENGINEERING AND MANUFACTURING Vol. 18, No. 2, pp. 163-173 FEBRUARY 2017 / 163
DOI: 10.1007/s12541-017-0022-7 ISSN 2234-7593 (Print) / ISSN 2005-4602 (Online)

Adaptive Tracking Control of a Quadrotor Unmanned Vehicle

To Xuan Dinh[1] and **Kyoung Kwan Ahn**[2,#]

1 Graduate School of Mechanical Engineering, University of Ulsan, 93, Daehak-ro, Namgu, Ulsan, 44610, South Korea
2 School of Mechanical Engineering, University of Ulsan, 93, Daehak-ro, Namgu, Ulsan, 44610, South Korea
Corresponding Author / E-mail: kkahn@ulsan.ac.kr, TEL: +82-52-259-2282, FAX: +82-52-259-1680

KEYWORDS: Quadrotor helicopter, Adaptive sliding mode, Adaptive backstepping

The design of a flight control system for an under-actuated quadrotor aircraft in presence of parametric uncertainties and external disturbance is quite challenging. In this study, we propose an adaptive trajectory tracking control base on sliding mode approach, an adaptive command filtered backstepping technique to stabilize the attitude of the quadrotor and using online estimators to estimate unknown aerodynamic parameters and external disturbances. First of all, the mathematical model of a quadrotor unmanned aerial vehicle (UAV) is presented. The adaptive tracking trajectory position and attitude control scheme are then formulated and the perturbations in quadrotor system are compensated by employing special Lyapunov functions. Simulation results are given to demonstrate the validity and effectiveness of the proposed algorithm on a quadrotor model under different conditions.

Manuscript received: May 12, 2015 / Revised: September 1, 2016 / Accepted: September 7, 2016

1. Introduction

In recent years, quadrotor unmanned aerial vehicles have increasingly been developed by many universities and centers for research in a number of different fields as real time system, robotics, flight control, etc. Quadrotor helicopters have several advantages over traditional helicopters. One advantage is the using of four rotors with their smaller diameter help them to save more kinetic energy than comparably-scaled traditional helicopters during flight and reduce the damage caused when a rotor hits something. In addition to this, quadcopters don't need mechanisms to change the rotor blade pitch angle as they spin, and this makes them simpler to design and easier to maintain. Their ability to take off and land easily in limited spaces also makes them very interesting.

In this paper, a four-rotors aircraft is considered for flight control algorithms. The quadrotor UAV has six degrees of freedom (DOF) with only four control inputs consisting of thrust and the three rotational torque inputs, therefore the UAV is an under-actuated and nonlinear coupled system which presents great control challenges.[1] Since the quadrotor helicopter was first invented, control techniques have been introduced for various tasks in many recent researches. Linear control has been successfully presented to stabilize the quadrotor in stable operating environments.[2-4] Nonlinear control techniques[5-9] have been employed recently to broaden the capabilities of the quadrotor as well. The nonlinear control schemes have focused on three main techniques: backstepping control, sliding mode technique, and adaptive control scheme. Madani and Benallegue[10] designed a full state backstepping approach to track the desired position and yaw angle while keeping the pitch and roll angles stable. In addition, saturation functions based on backstepping technique are applied in stabilization control algorithm.[11] Then, F. Kendou[12] proposed a nest-saturation-based controller in quadrotor UAV stabilization with bounded inputs. Furthermore, a command-filtered backstepping technique and a linear tracking differentiator are employed in trajectory tracking control task.[13] These control schemes have been implemented to control the quadrotor under several conditions. However, these control strategies[2-13] tested on the quadrotor helicopters have not considered the aerodynamic effects and parametric uncertainties of the model. Considering the unmodelled dynamics and aerodynamic interaction, many extended sliding mode techniques have been employed in various trajectory tracking controller design for quadrotor. Benallegue[14] introduced a sliding mode observer to estimate and compensate the effect of external disturbance in quadrotor flight. On the other hand, the dynamical model of a quadcopter was taken to test the performance of a proposed sliding mode controller in stabilization a class of underactuated systems which were in cascaded form.[15] Then, a method based on second order sliding mode controller with the nonlinear coefficients of the sliding manifold obtained by using Hurwitz stability analysis is presented.[16] However, this kind of robust control design is conservative in case of the details of the uncertainty upper-bound cannot be exactly obtained. Raffo et al.[17] proposed a nonlinear H-infinity control law for path tracking task of a quadrotor system with coupling between longitudinal and lateral movements with roll and pitch motions. Nonetheless, the presentation of a tilt angle leads to certain loss of control authority. The works[1,18] utilized neural networks (NNs) to learn the dynamics of the UAV online and offline. However, the offline data collection is expensive and the online estimation is impratical due to the limited onboard computational resource. Trajectory tracking approaches with online estimators were presented to handle with uncertainties in quadrotor helicopter dynamics.[19,20] To deal with the input constraint, nested saturation technique,[21] command filtered backstepping approach[22] have been applied to control design of quadrotor UAV. These nonlinear controller syntheses, the design parameter selection and the stability analysis were quite intricate. Moreover, in the most of existing literatures and research efforts on the control of the quadrotor UAVs, few publications consider the adaptation of aerodynamic effects and parametric uncertainties in quadrotor dynamics and compensate them in tracking control design. In this paper, the control methodology employed a cascaded control structure, which included an adaptive sliding mode approach for trajectory tracking task and an adaptive command filtered backstepping for attitude command tracking design. These proposed control algorithms have following properties: (1) online estimation to the parametric uncertainty and the external disturbance; (2) global asymptotic stability of trajectory tracking and attitude command tracking design; (3) less dependency on the mathematical model by introducing command filtered compensation in backstepping; (4) avoiding spending time and intrication in computation of differentiation by employing tracking differentiator.

The remainder of this paper is organised as follows. In Section 2, the mathematical model of a quadrotor helicopter is given. The hierarchical control design which included position tracking and attitude command tracking control algorithms is presented in Section 3 and Section 4. In Section 5, numerical simulation results of a quadrotor trajectory tracking are compared and discussed. Finally, we conclude our work in Section 6.

2. Modeling of Quadrotor

2.1 System Description

The autonomous aerial vehicle used in this paper is a four rotor helicopter. The quadrotor is propelled by four rotors. The motion of this vehicle is controlled by changing the speed of rotation of the four rotors. The quadrotor is a typical under-actuated, non-linear coupled system, and Table 1 shows the quadrotor concept motion description. In order to obtain forward motion, the speed of rotation of the rear rotor must be increased and, simultaneously, the front rotor velocity must be decreased. The lateral motion is achieved with the same strategy but using the right and left motors. Yaw rotation is results from the difference in the counter-torque between each pair of propellers, i.e., accelerating the two clockwise turning rotors while decelerating the counterclockwise turning rotors, and vice-versa.

Table 1 Quadrotor helicopter motion concept

	ω_1	ω_2	ω_3	ω_4
Up	⇑	⇑	⇑	⇑
Roll	–	⇓	–	⇑
Pitch	⇓	–	⇑	–
Yaw	⇑	⇓	⇑	⇓

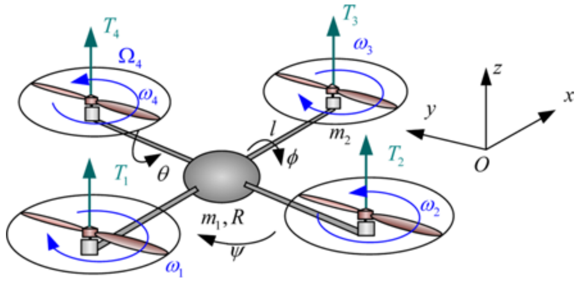

Fig. 1 Practical configuration of a quadrotor

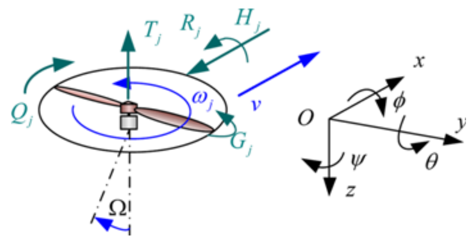

Fig. 2 Moments and forces on a free rotating propeller

The acting forces and moments applied onto a rotating propeller can be described as shown in Fig. 2. The object coordinates system is related to the translational positions (x, y, z) and the attitude described by three angular Cartesian axes (ϕ, θ, ψ), respectively. We assume that the propeller j^{th} rotates in the xy plane, the rotating speed is ω_j, the propeller is moving sideward through the air with velocity v, and the propeller rotates about a particular axis of angular velocity $\dot{\alpha}$. Thrust force (upward) is the result of drag torque Q_j (inverse direction with ω_j) on the rotor shaft, which is generated by the power applied to each motor. Sideward movement with velocity v generates hub force H_j (inverse direction with v) and hub moment R_j (unbalanced lift between advancing and retreating blades). The gyroscopic effect of a rotary propeller produces a gyroscopic moment G_j corresponding to angular velocity Ω.

The dynamic model of the system is obtained under several reasonable assumptions. First, the vehicle is a rigid body in the space and therefore, Newton-Euler equations can be used to describe its dynamics. Second, by reason of linear the velocities of the quadrotor ($\dot{x}$, $\dot{y}$, $\dot{z}$) are relatively low, the effects of hub force and moment are assumed to be negligible. Third, the quadrotor helicopter is symmetrical with respect to the x, y, and z axes.

2.2 Quadrotor Helicopter Kinematics

Let $\{E\} = \{O_e x_e y_e z_e\}$ denote an earth-fixed inertial frame and $\{B\} =$

{*Oxyz*} a body-fixed frame whose origin *O* is at the centre of mass of the quadrotor, as shown in Fig. 1. The absolute position of the quadrotor is defined by $P = (x, y, z)$ and the attitude by three Euler angles $\Theta = (\phi, \theta, \psi)$. Using Euler angles parameterization, the airframe orientation in space is given by a rotation matrix R from the inertial frame to the body frame, where $R \in SO(3)$ is an orthogonal matrix.

The quadrotor is a 6DOF rigid body described by three translations $v = (v_x, v_y, v_z)^T$ and three rotations $\Omega = (p, q, r)^T$. Using Newton-Euler formalism, the multi-rotors kinematic equations can be described as follows:

$$\dot{P} = v \tag{1}$$

$$\dot{v} = -gz_e + \frac{T}{m} Rz_e \tag{2}$$

$$R = \begin{bmatrix} c\theta c\psi & s\theta c\psi s\phi - s\psi c\phi & s\theta c\psi c\phi - s\psi s\phi \\ c\theta s\psi & s\theta s\psi s\phi + c\psi c\phi & s\theta s\psi c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{3}$$

$$\dot{R} = RS(\Omega) \tag{4}$$

$$I_f \Omega = -\Omega \times I_f \Omega - G + \tau \tag{5}$$

where $s(.) \triangleq \sin(.)$, $c(.) \triangleq \cos(.)$, and $S(\Omega)$ is a skew-symmetric and defined as follows:

$$S(\Omega) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \tag{6}$$

and m denotes the quadrotor mass, $I_f \in R^{3\times3}$ the total inertial matrix and a symmetric positive definite constant matrix express in frame {*B*}, g the gravity acceleration, $z_e = [0, 0, 1]^T$ the unit vector expressed in the frame {*E*}, $T \in R$ and $\tau = (\tau^1, \tau^2, t\tau^3) \in R^3$ the total thrust and the total torque produced by four rotors in free air.

The vector G denotes the gyroscopic torque which is given by:

$$G = \sum_{i=1}^{4} I_{TP} \left(\Omega \times z_e \right) \left(-1 \right)^{i+1} \omega_i \tag{7}$$

where I_{TP} and ω_i denote the total rotational moment of inertial around the propeller (rotor) axis and the speed of the rotor i.

After simple algebraic calculation, Eq. (3) can be rewritten as

$$\dot{\Theta} = W\Omega \tag{8}$$

where

$$W = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi \sec\theta & \cos\phi \sec\theta \end{bmatrix} \tag{9}$$

and $\det(W) = \sec\theta$. Thus, the matrix W is invertible when the pitch angle satisfies $\theta \neq \pm(2k+1)(\pi)/2$, $k \in \mathbb{Z}$.

The previous model is a simplification of complex dynamic interactions. To make the quadcopter dynamic more realistic, drag force generated by the air resistance is included. The translational and rotational dynamic equations can be expressed as

$$\dot{v} = gz_e - \frac{T}{m} Rz_e + \frac{1}{m}\left(A_t + d_t \right) \tag{10}$$

$$\dot{\Omega} = I_f^{-1}\left(\Omega \times I_f \Omega \right) + I_f^{-1}\tau - I_f^{-1} G + I_f^{-1}\left(A_r + d_r \right) \tag{11}$$

in which, A_t and A_r denote the drag force and torque coefficients for velocities and angular velocities of the inertial frame, d_t and d_r the external disturbances.

The rotor system can be modeled with the following differential equation:

$$\dot{\omega}_i = -\frac{K_E K_M}{R_{DC} I_{TP}} \omega_i - \frac{d}{I_{TP}} \omega_i^2 + \frac{K_M}{R_{DC} I_{TP}} v_i \tag{12}$$

where d denote drag factor, R_{DC} the DC motor resistance, K_E electric motor constant, K_M mechanic motor constant and v_i the voltage input of DC motor i.

The total thrust generated by the multi propellers is calculated as

$$T = \sum_{i=1}^{4} f_i = b \sum_{i=1}^{4} \omega_i^2 \tag{13}$$

where parameters b is related to the density of air, the shape of the blades, the number of blades, the chord length of the blades, the pitch angle of the blade airfoil and the drag constant.

Let the distance from the rotors to the center of mass be denoted by l, the control torques generated by the four rotors as

$$\tau = \begin{bmatrix} \tau^1 \\ \tau^2 \\ \tau^3 \end{bmatrix} = \begin{bmatrix} bl\left(\omega_4^2 - \omega_2^2 \right) \\ bl\left(\omega_3^2 - \omega_1^2 \right) \\ bl\left(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2 \right) \end{bmatrix} \tag{14}$$

In order to facilitate the computation of the real control inputs, that is $\omega_i (i = 1 \sim 4)$, Eqs. (12) and (13) are put together:

$$\begin{bmatrix} T \\ \tau^1 \\ \tau^2 \\ \tau^3 \end{bmatrix} = \begin{bmatrix} b\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 \right) \\ bl\left(\omega_4^2 - \omega_2^2 \right) \\ bl\left(\omega_3^2 - \omega_1^2 \right) \\ bl\left(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2 \right) \end{bmatrix} M\bar{\omega} \tag{15}$$

3. Control Design Methodology

The control problem presented in this work is to preform approaching position and attitude tracking of the quadrotor helicopter by trajectory tracking controllers based on an adaptive sliding mode controller and attitude stabilization based on adaptive backstepping controller.

Based on the dynamical model in Eqs. (1), (7), (9), and (10), the control system is divided into two subsystems: position subsystem and attitude subsystem. The quadcopter model developed in the previous system can be rewritten as the following.

Position Subsystem

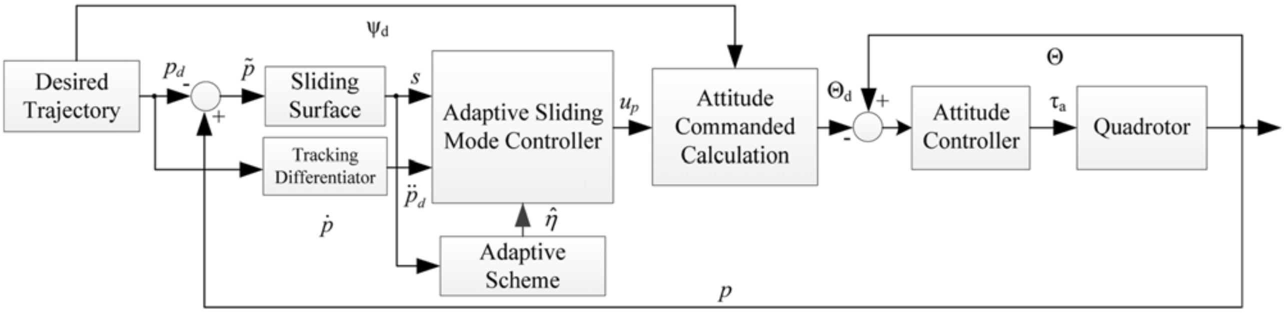

Fig. 3 Adaptive sliding mode controller

$$\begin{cases}\dot{\mathrm{P}}(\mathrm{t}) = v(\mathrm{t}) \\ \dot{v}(\mathrm{t}) = u(\mathrm{t}) + \eta(\Theta, v, t)\end{cases} \tag{16}$$

Attitude Subsystem

$$\begin{cases}\dot{\Theta}(\mathrm{t}) = W\Omega(\mathrm{t}) \\ \dot{\Omega} = \Gamma(\Omega) + \bar{u}(\mathrm{t}) + \zeta\left(\Theta, \Omega, t\right)\end{cases} \tag{17}$$

where $\eta(\Theta, v, t) = \frac{1}{m}(A_t + d_t + T(R - R_d)z_e)$; $\Gamma(\Omega) = I_f^{-1}(\Omega \times I_f\Omega)$ and $\zeta(\Theta,\ \Omega,\ t) = I_f^{-1}(A_r + d_r - G)$ denote the uncertainties in the quadrotor helicopter dynamics; $u(t)$ $gz_e - \frac{1}{m}TR_d z_e = [u_1, u_2, u_3]^T$ and $\bar{u}(t) = I_f^{-1}\tau$ $[\bar{u}_1, \bar{u}_2, \bar{u}_3]^T$ the virtual control signals of position subsystem and attitude subsystem, respectively. The 'output' of the quadrotor helicopter system is selected as $P(t) = [x(t), y(t), z(t)]^T$.

The hierarchical design scheme divides the quadrotor control problem into outer-loop position tracking control and inner-loop attitude tracking control designs which is shown in Fig. 3. First, the flight tracking control design is applied to track the quadrotor to desired trajectory and obtain the equivalent control thrust force as well an attitude commanded signal. In the outer-loop control scheme, an adaptive sliding mode technique for position control is proposed and a tracking differentiator is employed to get the derivative of the desired signal without differentiation. The desired attitude commanded signal was calculated basing on virtual control signal u and then was sent to the attitude subsystem in the inner-loop. The subsequent attitude tracking control scheme focuses on tracking the desired attitude signal to satisfy the requirement of position control and also synthesizes the control torque input to the quadrotor system. An adaptive command filtered backstepping is proposed for attitude tracking control task. Finally, the control thrust force and control torques which calculated from virtual control signals u and $\bar{u}$ became the real control inputs of the four-rotors quadcopter system.

3.1 Trajectory tracking controller design

Let $\hat{\eta}$ denote the estimated value of η; $\tilde{\eta} = \eta - \hat{\eta}$ and $\tilde{P} = P - P_d$ be estimation errors.

The design of SMC as follows:

$$s = \dot{\tilde{\mathrm{P}}} + \lambda\tilde{\mathrm{P}} \tag{18}$$

where λ is strictly positive constant.

Consider a Lyapunov function:

$$V = \frac{1}{2}s^T s + \frac{1}{2}\tilde{\eta}^T T_\eta^{-1}\tilde{\eta} \tag{19}$$

where T_σ is the positive definite constant diagonal matrix.

$$\begin{aligned}\dot{V} &= s^T\dot{s} - \tilde{\eta}^T T_\eta^{-1}\dot{\hat{\eta}} \\ &= s^T\left(\ddot{\tilde{\mathrm{P}}} + \lambda\dot{\tilde{\mathrm{P}}}\right) - \tilde{\eta}^T T_\eta^{-1}\dot{\hat{\eta}} \\ &= s^T\left(\ddot{\mathrm{P}} - \ddot{\mathrm{P}}_d + \lambda\dot{\tilde{\mathrm{P}}}\right) - \tilde{\eta}^T T_\eta^{-1}\dot{\hat{\eta}} \\ &= s^T\left(\eta + u - \ddot{\mathrm{P}}_d + \lambda\dot{\tilde{\mathrm{P}}}\right) - \tilde{\eta}^T T_\eta^{-1}\dot{\hat{\eta}}\end{aligned} \tag{20}$$

Choosing control law as:

$$u = -K\mathrm{sign}(s) - \hat{\eta} + \ddot{\mathrm{P}}_d - \lambda\dot{\tilde{\mathrm{P}}} \tag{21}$$

where K is the positive definite constant diagonal matrix. Then, the time derivative of Lyapunov function is calculated is the following.

$$\begin{aligned}\dot{V} &= s^T\left(\tilde{\eta} - K\mathrm{sign}(s)\right) - \tilde{\eta}^T T_\eta^{-1}\dot{\hat{\eta}} \\ &= -s^T K\mathrm{sign}(s) + \tilde{\eta}^T\left(s - T_\eta^{-1}\dot{\hat{\eta}}\right)\end{aligned} \tag{22}$$

The update law can be chosen so that the inequality holds:

$$\tilde{\eta}^T\left(s - T_\eta^{-1}\dot{\hat{\eta}}\right) \le 0 \tag{23}$$

So, the adaptation law for uncertain term is designed as follows:

$$\dot{\hat{\eta}} = T_\eta s \tag{24}$$

From Eqs. (20) and (22), we got $\dot{V} \le -s^T K\mathrm{sign}(s)$. Thus, the position variables $[x, y, z]$ converge to the desired values $[x_d, y_d, z_d]$

In order to avoid taking time and intrication in computation exact values of $\ddot{P}_d$ and $\dot{P}_d$, a tracking differentiator proposed[23] is utilized.

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -r\,\mathrm{sign}\left(x_1 - \omega(\mathrm{t}) + \dfrac{x_2|x_2|}{2r}\right)\end{cases} \tag{25}$$

where $\omega(t)$ is the input signal need to be differentiated, x_1 tracks $\omega(t)$, x_2 tracks $\dot{\omega}(t)$ and r determines the tracking speed. In this case, P_d is the input of the first tracking differentiator and then $\dot{P}_d$ is calculated without differentiation. Then the output of the first tracking differentiator is the input of second tracking differentiator, which compute the approximated value of $\ddot{P}_d$.

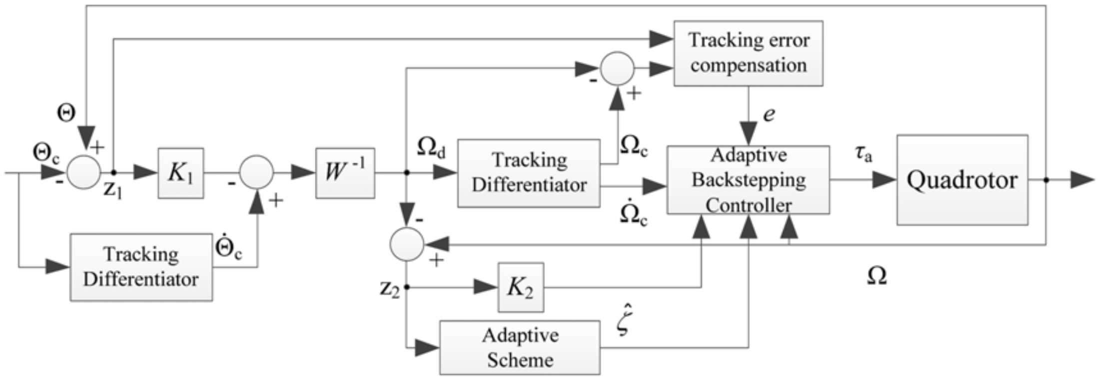

Fig. 4 Adaptive command filtered backstepping controller

3.2 Reference attitude calculation

As a result of the under-actuated mechanical system of the quadrotor helicopter, the desired attitude signals were calculated basing on the virtual signal u and need to be tracking followed by an attitude control scheme. The purpose of this section is to find a desired orientation $R_d \in SO(3)$ that satisfies the following.

$$u = gz_e - \frac{1}{m} TR_d z_e \tag{26}$$

The Eq. (26) can be rewritten as the following:

$$TR_d = m\left(gz_e - u\right) \tag{27}$$

Squaring both sides of the Eq. (27), we got:

$$\begin{gathered} TR_d R_d^T T = m^2\left(gz_e - u\right)^T\left(gz_e - u\right) \\ T^2 = m^2\left(gz_e - u\right)^T\left(gz_e - u\right) \\ T = m\sqrt{\left(gz_e - u\right)^T\left(gz_e - u\right)} = m\left\|gz_e - u\right\| \end{gathered} \tag{28}$$

The Eq. (26) is also rewritten to the following system of equations

$$\begin{cases} \sin\theta_d \cos\phi_d \cos\psi_d + \sin\phi_d \sin\psi_d = \dfrac{m}{T} u_1 \\ \sin\theta_d \cos\phi_d \sin\psi_d - \sin\phi_d \cos\psi_d = \dfrac{m}{T} u_2 \\ \cos\theta_d \cos\phi_d = \dfrac{m}{T}\left(g - u_3\right) \end{cases} \tag{29}$$

With given virtual control signal $u = [u_1, u_2, u_3]^T$ and desired yaw angle ψ_d, the reference roll ϕ_d and pitch angle θ_d are expressed as the following formula:

$$\phi_d = \arcsin\left(\frac{u_1 \sin\psi_d - u_2 \cos\psi_d}{\sqrt{u_1^2 + u_2^2 + \left(u_3 - g\right)^2}}\right) \tag{30}$$

$$\theta_d = \arctan\left(\frac{u_1 \cos\psi_d + u_2 \sin\psi_d}{g - u_3}\right) \tag{31}$$

In addition, the commanded total thrust force produced by four rotors is given as follows

$$T = m\sqrt{\left(u_1^2 + u_2^2 + \left(u_3 - g\right)^2\right)} \tag{32}$$

3.3 Attitude stabilization control

In order to track the desired speed command proposed in the above subsection, an adaptive backstepping controller is developed for the system of Eq. (17). As shown in Fig. 4, the desired angular velocity Ω_d is obtained base on Lyapunov approach. Then a command filterer Ω_c of the desired angular velocity is calculated using tracking differentiator. The error between Ω_c and Ω_d is compensated by the tracking error compensation. Two tracking differentiator block are employed to calculate the derivative of signals without differentiation. Finally, a backstepping controller with adaptive scheme is proposed to obtain the required control torque signal for quadcopter. The procedure for designing the controller is described as follows:

Step 1: Let the attitude error be defined as

$$z_1 = \Theta - \Theta_\mathrm{d} \tag{33}$$

where Θ_d denotes reference attitude signal.

Then the time derivative term of attitude error is determined as

$$\dot{z}_1 = \dot{\Theta} - \dot{\Theta}_\mathrm{d} \tag{34}$$

Consider a Lyapunov function

$$V_1 = \frac{1}{2} z_1^T z_1 \tag{35}$$

then

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T\left(W\Omega - \dot{\Theta}_\mathrm{d}\right) \tag{36}$$

With $\theta \neq (2k-1)\pi/2$, the virtual control signal for the system in Eq. (36) to satisfy $\dot{V}_1 < 0$ is presented as

$$\Omega_\mathrm{d} = W^{-1}\left(\dot{\Theta}_\mathrm{d} - K_1 z_1\right) \tag{37}$$

where Ω_d is desired command angular velocity and K_1 is a positive definite matrix.

Table 2 Quadrotor helicopter model parameters

Parameter	m	g	l	I_r	I_x	I_y	I_z	b	d	K_E	K_M	R_{DC}
Value	0.85	9.81	0.24	104e-6	8.1e-3	8.1e-3	14.2e-3	54.2e-6	1.1e-6	6.3e-3	6.3e-3	0.6
Units	kg	m/s^2	m	kg.m^2	kg.m^2	kg.m^2	kg.m^2	N.s^2/rad^2	Nm.s^2/rad^2	Vs/rad	Vs/rad	Ω

Due to the analytic computation of $\dot{\Omega}_d$, a command filtered version of Ω_d is introduced as Ω_c and a tracking error is defined as follows.

$$z_2 = \Omega - \Omega_c \tag{38}$$

Moreover, a compensated tracking error signal e is defined as

$$e = z_1 - \alpha \tag{39}$$

where α is a vector is introduced to compensate the tracking error and its derivative is given by

$$\dot{\alpha} = -K_1\alpha + W\left(\Omega_c - \Omega_d\right) \tag{40}$$

Consider another Lyapunov function defined as follows.

$$V_2 = \frac{1}{2}e^T e \tag{41}$$

Then, the time derivative of the above Lyapunov function is calculated as

$$\dot{V}_2 = e^T\dot{e} = e^T\left(-K_1 e + Wz_2\right) = -e^T K_1 e + e^T W z_2 \tag{42}$$

It is obviously that if the angular velocity error z_2 is close to zero, the tracking attitude error e will converge to zero. Thus, the next step is to keep z_2 as small as possible.

Step 2: This step aims to determine the actual control law for the torque input of quadrotor system and the adaptive law for update uncertainties value of the system.

Consider the candidate Lyapunov function

$$V_3 = V_2 + \frac{1}{2}z_2^T z_2 \tag{43}$$

then,

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + z_2^T\dot{z}_2 = -e^T K_1 e + e^T W z_2 + z_2^T\dot{z}_2\\ &= -e^T K_1 e + z_2^T\left(W^T e + \dot{z}_2\right)\\ &= -e^T K_1 e + z_2^T\left(W^T e + \Gamma(\Omega) + \zeta\left(\Theta,\Omega,t\right) + \bar{u}(\mathrm{t}) - \dot{\Omega}_{\mathrm{c}}\right)\end{aligned} \tag{44}$$

Let $\hat{\zeta}$ be the estimation of ζ, and the estimation error is defined as

$$\tilde{\zeta} = \zeta - \hat{\zeta} \tag{45}$$

The adaptation scheme for uncertainty is built by utilizing the following candidate Lyapunov function.

$$V_{\mathrm{a}} = V_3 + \frac{1}{2}\tilde{\zeta}^T T_\zeta^{-1}\tilde{\zeta} \tag{46}$$

The time derivative of Eq. (46) is given by

$$\begin{aligned}\dot{V}_{\mathrm{a}} &= \dot{V}_3 - \frac{1}{2}\tilde{\zeta}^T T_\zeta^{-1}\dot{\hat{\zeta}}\\ &= -e^T K_1 e + z_2^T\left(W^T e - \Gamma(\Omega) + \zeta + \bar{u} - \dot{\Omega}_{\mathrm{c}}\right) - \tilde{\zeta}^T T_\zeta^{-1}\dot{\hat{\zeta}}\\ &= -e^T K_1 e + z_2^T\left(W^T e - \Gamma(\Omega) + \left(\hat{\zeta} + \tilde{\zeta}\right) + \bar{u} - \dot{\Omega}_{\mathrm{c}}\right) - \tilde{\zeta}^T T_\zeta^{-1}\dot{\zeta}\\ &= -e^T K_1 e + z_2^T\left(W^T e - \Gamma(\Omega) + \hat{\zeta} + \bar{u} - \dot{\Omega}_{\mathrm{c}}\right) + \tilde{\zeta}^T\left(z_2 - T_\zeta^{-1}\dot{\zeta}\right)\end{aligned} \tag{47}$$

Hence, the control input is chosen as

$$\bar{u} = -W^T e + \Gamma(\Omega) - \hat{\zeta} + \dot{\Omega}_{\mathrm{c}} - K_2 z_2 \tag{48}$$

where $K0$ is a positive definite matrix.

Thus, the commanded torque is expressed as follows

$$\tau = I_f\bar{u} = -I_f W^T e + \Omega \times I_f\Omega - I_f\hat{\zeta} + I_f\dot{\Omega}_{\mathrm{c}} - I_f K_2 z_2 \tag{49}$$

Then, Eq. (37) becomes the following.

$$\dot{V}_{\mathrm{a}} = -e^T K_1 e - z_2^T K_2 z_2 + \tilde{\zeta}^T\left(z_2 - T_\zeta^{-1}\dot{\hat{\zeta}}\right) \tag{50}$$

The update law can be designed so that the inequality holds

$$\tilde{\zeta}^T\left(z_2 - T_\zeta^{-1}\dot{\hat{\zeta}}\right) \le 0 \tag{51}$$

Therefore, the adaptive law is determined as

$$\dot{\hat{\zeta}} = T_\zeta z_2 \tag{52}$$

Finally, from Eqs. (39) and (41) we obtained $\dot{V}_a \le -e^T K_1 e - z_2^T K_2 z_2$. It means that all signals e and z_2 are bound.

In order to compute Ω_c without differentiation, two tracking differentiators base on formula (26) are employed with the input signals are Θ_d and Ω_d, respectively.

4. Control Algorithm

This section presents the hierarchical control algorithm design based on adaptive schemes for tracking position and attitude controller.

First, the desired trajectory $P_d(t) = [x_d(t), y_d(t), z_d(t)]^T$ and desired yaw angle ψ_d are given.

Next, the virtual control signal for the adaptive tracking position scheme and the adaptive law for uncertainties in position subsystem are given as Eqs. (21) and (24), where desired acceleration $\ddot{P}_d$ is calculated by employing tracking differentiator Eq. (23).

Base on the virtual control signal $u = [u_1, u_2, u_3]^T$ and the desired yaw angle ψ_d, the reference pitch and roll angles are calculated as Eqs. (30) and (31).

In adaptive tracking attitude controller scheme, the desired angular velocity Ω_d is given in Eq. (37).

The command angular velocity Ω_c and its velocity $\dot{\Omega}_c$ are calculated by using tracking differentiator Eq. (23).

Then, the virtual control signal and adaptive law for uncertainties in attitude subsystem are chosen as Eqs. (48) and (52).

Finally, the control force T and the control torque τ sent to quadrotor dynamic systems are given in Eqs. (32) and (49).

5. Numerical Study

The proposed control method has been tested by the simulations of typical trajectory tracking under certain condition to verify the validity and efficiency of the control scheme employed in this work. A quadrotor model[24] is utilized in the simulation with data listed in Table 2. Additionally, the external disturbances d_t and d_r are described as normally distributed noise signal with zero mean and variance of 0.01. The unknown non-linear effects due to aerodynamic friction damping[1] are expressed as follows:

$$A_t = \begin{bmatrix} a_1 + a_2 \cdot |v_x| & 0 & 0 \\ 0 & a_3 + a_4 \cdot |v_y| & 0 \\ 0 & 0 & a_5 + a_6 \cdot |v_z| \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{53}$$

$$A_r = \begin{bmatrix} a_7 + a_8 \cdot |\Omega_x| & 0 & 0 \\ 0 & a_9 + a_{10} \cdot |\Omega_y| & 0 \\ 0 & 0 & a_{11} + a_{12} \cdot |\Omega_z| \end{bmatrix} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \tag{54}$$

where $a_i (i = 1, 2, \ldots, 12)$ are the damping coefficients with their values chosen as the following.

$$[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}]^T =$$

$$[0.06, 0.1, 0.06, 0.1, 0.06, 0.1, 0.1, 0.15, 0.1, 0.15, 0.1, 0.15]$$

The initial positions and Euler angles are $p_0 = [0; 0; 0]$ and $\Theta_0 = [0; 0; 0]$, respectively. In addition, the initial linear and angular velocities are $v_0 = [0; 0; 0]$ and $\Omega_0 = [0; 0; 0]$, respectively. And the initial estimated trajectory and attitude parameters are fixed at zero. Control parameters of the developed adaptive controller are selected as follows. $K = \mathrm{diag}\{1, 1, 1\}$; $\lambda = \mathrm{diag}\{2, 2, 2\}$; $K_1 = \mathrm{diag}\{0.1, 0.1, 0.1\}$; $K_2 = \mathrm{diag}\{0.1, 0.1, 0.1\}$. The filter gain matrices for linear tracking differentiator for calculation of $\ddot{p}_d$ and Ω_c are selected as $r = \mathrm{diag}\{30, 30, 30\}$, and the adaptation matrices are chosen as $T_\eta = \mathrm{diag}\{0.01, 0.01, 0.01\}$; $T_\zeta = \mathrm{diag}\{0.01, 0.01, 0.01\}$.

In order to evaluate the proposed controller, performances of conventional PID and non-adaptive controller have been carried out as comparison. Parameters of PID controller are chosen via trial and error, while the structure of non-adaptive controller is designed by removing the adaptive schemes in Figs. 3 and 4 and the external disturbance and dynamic uncertainties are assumed to be zero in control signal calculation as in Eqs. (21) and (48). In order to guarantee the system stability, the control gains for sliding mode controller and backstepping controller is chosen with larger values, for example, $K = \mathrm{diag}\{3, 3, 3\}$; $K_1 = \mathrm{diag}\{1.5, 1.5, 1.5\}$; $K_2 = \mathrm{diag}\{1.5, 1.5, 1.5\}$.

Case study 1: Consider a desired horizontal rectangular trajectory p_d and a desired yaw angle ψ_d given by:

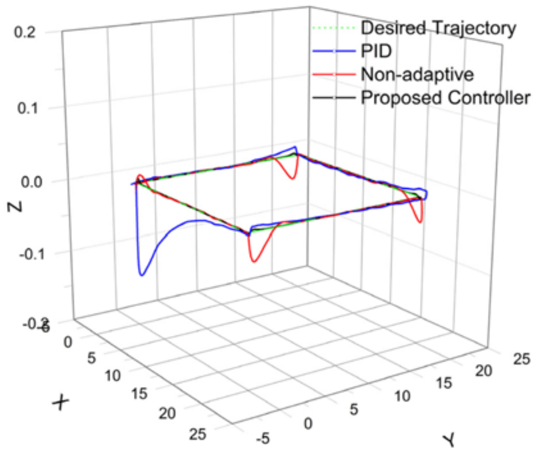

Fig. 5 3D horizontal trajectory

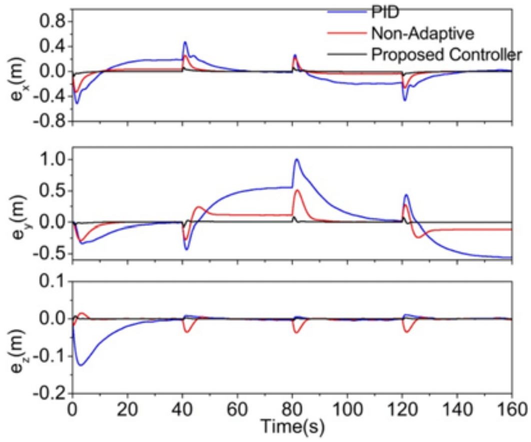

Fig. 6 Trajectory tracking Error

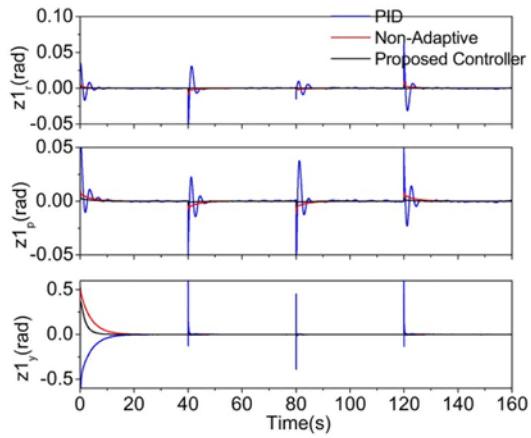

Fig. 7 Attitude tracking error

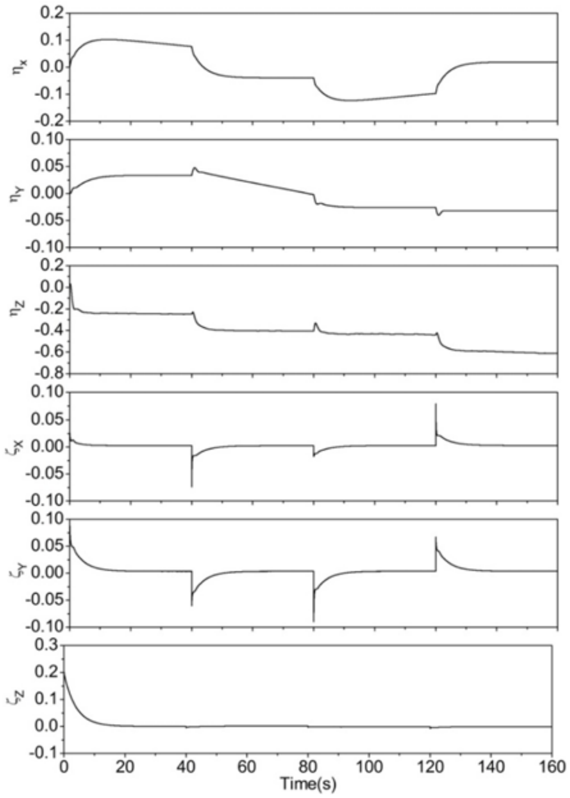

Fig. 8 Estimated values of trajectory uncertainties

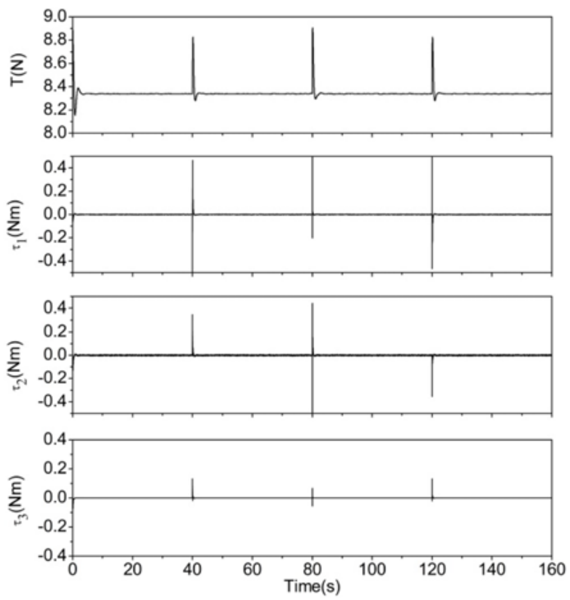

Fig. 9 Total thrust force and torque

$$
\begin{cases}
x_d = t/2F(t,0,40) + 20F(t,40,80) + (60 - t/2)F(t,80,120) \\
y_d = (t/2 - 20)F(t,40,80) + 20F(t,80,120) + (80 - t/2)F(t,120,160) \\
z_d = 0 \\
\psi_d = 0
\end{cases}
\tag{55}
$$

where F is an interval function given by the following formula:

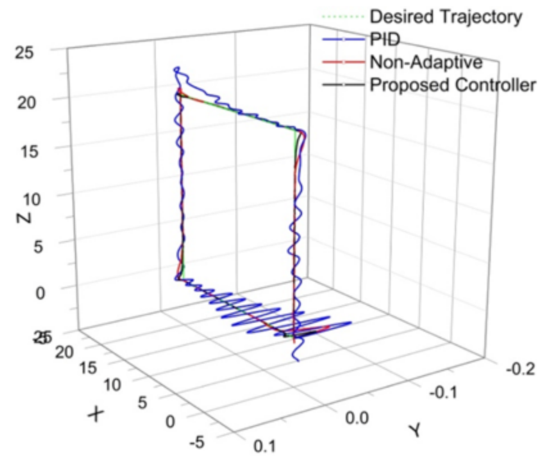

Fig. 10 3D vertical trajectory

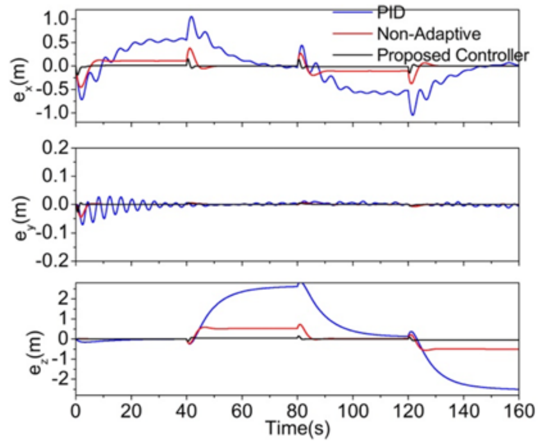

Fig. 11 Trajectory tracking error

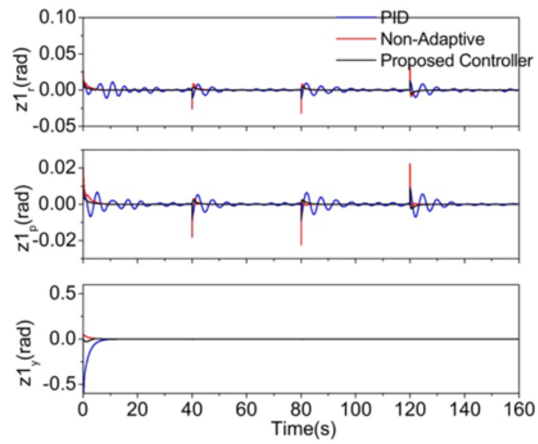

Fig. 12 Attitude tracking error

$$F(\mathrm{x,a,b}) = \frac{\mathrm{sign}(\mathrm{x-a}) - \mathrm{sign}(\mathrm{x-b})}{2}$$

Applying the proposed control method, simulation results of the

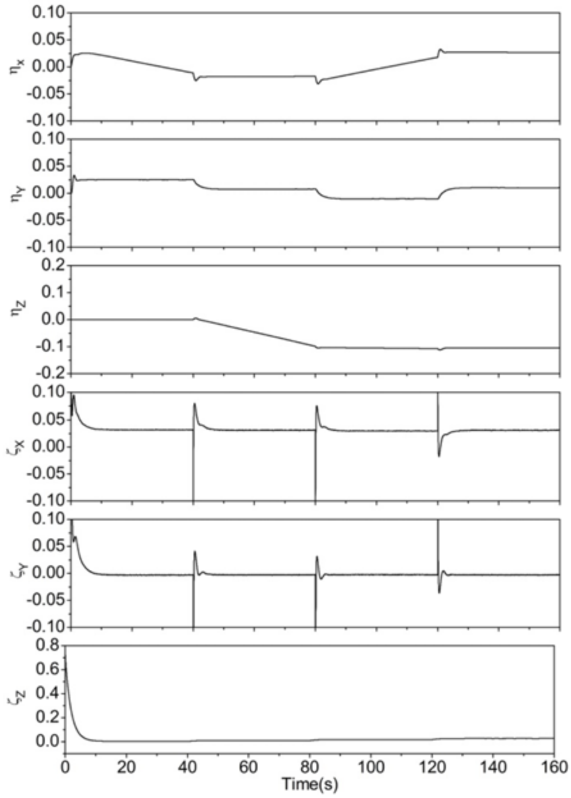

Fig. 13 Estimated values of trajectory uncertainties

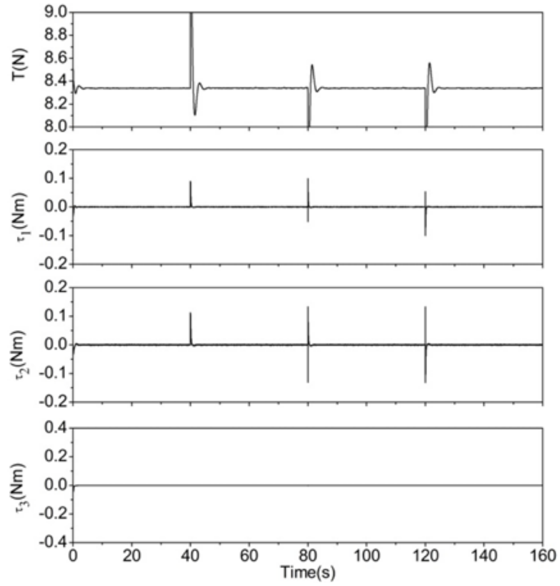

Fig. 14 Total thrust force and torque

quadrotor system for horizontal trajectory tracking task are shown in Figs. 5 to 9. The three-dimension trajectory was performed under the condition of set-point position and angle control as shown in Fig. 5. As expected, the design methodology provided for excellent responses with much higher tracking accuracy and faster response than did the non-adaptive controllers and PID controllers as depicted in Figs. 6 and 7. In spite of the reference position and angle were changed in every moment and the quadrotor suffering from uncertainties in the aerodynamic parameter and exogenous disturbance in working condition, the proposed controller managed to effectively hold the quadrotor's position and attitude in finite-time. Those demonstrate the robustness and effectiveness of the designed controller. In addition, the convergence of estimated trajectory and attitude uncertainties were confirmed in Fig. 8 and the time history of thrust force and torque input are displayed in Fig. 9.

Case study 2: In order to investigate more the effective of the proposed controller, we consider a vertical rectangular trajectory as

$$\begin{cases} x_d = t/2F(t,0,40) + 20F(t,40,80) + (60-t/2)F(t,80,120) \\ y_d = 0 \\ z_d = (t/2-20)F(t,40,80) + 20F(t,80,120) \\ \quad + (80-t/2)F(t,120,160) \\ \psi_d = 0 \end{cases} \tag{56}$$

Applying the same controller, the 3D trajectory, position tracking errors, attitude tracking error, estimated values of trajectory uncertainties, and thrust force and torque input signals are presented from Figs. 10 to 14, respectively. With this case study, the effectiveness of the proposed controller over the conventional PID and non-adaptive controller is strongly confirmed Figs. 10, 11 and 12 presented the performances of the quadcopter system with three controllers utilized, which shows that the position tracking errors and attitude tracking errors resided in the prescribed set uniformly while the two remain controllers failed to do the same. It can be seen from Figs. 11 and 12, the conventional PID controller even gave the oscillated responses. Moreover, the desired controller has the faster response in comparison with non-adaptive controller and conventional PID controller. This, once again, illustrate the feasibility of the proposed control method. Finally, the approximated values of uncertainties and disturbances are given in Fig. 13 and the control signals are shown in Fig. 14.

It is noted that the total thrust force and torque control input, as shown in Figs. 9 and 14, are continuous as desired and become feasible to apply in an experimental test. The performance shows not only the stability robustness against the parameter variation and the external disturbance but also the fast response and excellent tracking capacity.

6. Conclusion

This paper has presented an adaptive trajectory tracking control algorithm for a small quadrotor unmanned aircraft. The adaptive sliding mode control and adaptive backstepping control are carried out to drive the quadrotor follow the desired trajectory and keep the attitude stable. In addition, online tuning estimators and robust control laws are added to compensate the parameter uncertainties and external disturbances in quadrotor dynamic. The asymptotic stability property has been proven using special Lyapunov method. In order to eliminate the analytic computation of differentiation, tracking differentiators are integrated in sliding mode technique and command filtered backstepping approach. The effectiveness of the proposed control scheme has been illustrated via numerical simulations. Future work includes the extensions to obstacle avoidance as well as the real-time experiment validation.

ACKNOWLEDGEMENT

This material is based upon work supported by the Ministry of Trade, Industry & Energy (MOTIE, Korea) under Industrial Technology Innovation Program. No.10067184

REFERENCES

1. Dierks, T. and Jagannathan, S., “Output Feedback Control of a Quadrotor UAV using Neural Networks,” IEEE Transactions on Neural Networks, Vol. 21, No. 1, pp. 50-66, 2010.
2. Pounds, P., Mahony, R., and Corke, P., “Modelling and Control of a Quad-Rotor Robot,” Proc. of Australasian Conference on Robotics and Automation, 2006.
3. Hoffmann, G. M., Waslander, S. L., and Tomlin, C. J., “Quadrotor Helicopter Trajectory Tracking Control,” Proc. of AIAA Guidance, Navigation and Control Conference and Exhibit, Paper No. AIAA 2008-7410, 2008.
4. Bouabdallah, S. and Siegwart, R., “Towards Intelligent Miniature Flying Robots,” in: Field and Service Robotics, Corke, P., Sukkariah, S., (Eds.), Springer, pp. 429-440, 2006.
5. Slotine, J. E. and Li, W., “Applied Nonlinear Control,” Prentice Hall, 1991.
6. Dinh, T. X., Nam, D. N. C., and Ahn, K. K., “Robust Attitude Control and Virtual Reality Model for Quadrotor,” International Journal of Automation Technology, Vol. 9, No. 3, pp. 283–290, 2015.
7. Ahn, K. K., Truong, D. Q., and Islam, M. A., “Modeling of a Magneto-Rheological (MR) Fluid Damper using a Self Tuning Fuzzy Mechanism,” Journal of Mechanical Science and Technology, Vol. 23, No. 5, pp. 1485-1499, 2009.
8. Anh, H. P. H. and Ahn, K. K., “Hybrid Control of a Pneumatic Artificial Muscle (PAM) Robot Arm using an Inverse NARX Fuzzy Model,” Engineering Application of Artificial Intelligence, Vol. 24, No. 4, pp. 697–716, 2011.
9. Danh, L. T. and Ahn, K. K., “Experimental Investigation of a Vibration Isolation System using Negative Stiffness Structure,” International Journal of Mechanical Sciences, Vol. 70, pp. 99-112, 2013.
10. Madani, T. and Benallegue, A., “Sliding Mode Observer and Backstepping Control for a Quadrotor Unmanned Aerial Vehicles,” Proc. of American Control Conference, pp. 5887-5892, 2007.
11. Guenard, N., Hamel, T., and Moreau, V., “Dynamic Modeling and Intuitive Control Strategy for an “X4-Flyer”,” Proc. of International Conference on Control and Automation, pp. 141-146, 2005.
12. Kendoul, F., Lara, D., Fantoni, I., and Lozano, R., “Real-Time Nonlinear Embedded Control for an Autonomous Quadrotor Helicopter,” Journal of Guidance, Control, and Dynamics, Vol. 30, No. 4, pp. 1049-1061, 2007.
13. Zuo, Z., “Trajectory Tracking Control Design with Command-Filtered Compensation for a Quadrotor,” IET Control Theory & Applications, Vol. 4, No. 11, pp. 2343-2355, 2010.
14. Benallegue, A., Mokhtari, A., and Fridman, L., “High-Order Sliding-Mode Observer for a Quadrotor UAV,” International Journal of Robust and Nonlinear Control, Vol. 18, Nos. 4-5, pp. 427-440, 2008.
15. Xu, R. and Özgüner, Ü., “Sliding Mode Control of a Class of Underactuated Systems,” Automatica, Vol. 44, No. 1, pp. 233-241, 2008.
16. Zheng, E.-H., Xiong, J.-J., and Luo, J.-L., “Second Order Sliding Mode Control for a Quadrotor UAV,” ISA Transactions, Vol. 53, No. 4, pp. 1350-1356, 2014.
17. Raffo, G. V., Ortega, M. G., and Rubio, F. R., “Nonlinear H∞ Controller for the Quad-Rotor Helicopter with Input Coupling,” IFAC Proceedings Volumes, Vol. 44, No. 1, pp. 13834-13839, 2011.
18. Leitner, J., Calise, A., and R. Prasad, J., “Analysis of Adaptive Neural Networks for Helicopter Flight Control,” Journal of Guidance, Control, and Dynamics, Vol. 20, No. 5, pp. 972-979, 1997.
19. Roberts, A. and Tayebi, A., “Adaptive Position Tracking of VTOL UAVs,” IEEE Transactions on Robotics, Vol. 27, No. 1, pp. 129-142, 2011.
20. Zuo, Z., “Adaptive Trajectory Tracking Control Design with Command Filtered Compensation for a Quadrotor,” Journal of Vibration and Control, Vol. 19, No. 1, pp. 94-108, 2013.
21. Castillo, P., Dzul, A., and Lozano, R., “Real-Time Stabilization and Tracking of a Four-Rotor Mini Rotorcraft,” IEEE Transactions on Control Systems Technology, Vol. 12, No. 4, pp. 510-516, 2004.
22. Choi, I.-H. and Bang, H.-C., “Adaptive Command Filtered Backstepping Tracking Controller Design for Quadrotor Unmanned Aerial Vehicle,” Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, Vol. 226, No. 5, pp. 483-497, 2012.
23. Han, J., “From PID to Active Disturbance Rejection Control,” IEEE Transactions on Industrial Electronics, Vol. 56, No. 3, pp. 900-906, 2009.
24. Bresciani, T., “Modelling, Identification and Control of a Quadrotor Helicopter,” M.Sc. Thesis, Department of Automatic Control, LUND University, 2008.
25. Gautam, D. and Ha, C., “Control of a Quadrotor using a Smart Self-Tuning Fuzzy PID Controller,” International Journal of Advanced Robotic Systems, Vol. 10, 2013. (DOI: 10.5772/56911)
26. Tran, X.-T. and Kang, H.-J., “Adaptive Hybrid High-Order Terminal Sliding Mode Control of MIMO Uncertain Nonlinear Systems and Its Application to Robot Manipulators,” Int. J. Precis. Eng. Manuf., Vol. 16, No. 2, pp. 255-266, 2015.

27. Tran, X.-T. and Kang, H.-J., "Adaptive Hybrid High-Order Terminal Sliding Mode Control of MIMO Uncertain Nonlinear Systems and Its Application to Robot Manipulators," Int. J. Precis. Eng. Manuf., Vol. 16, No. 2, pp. 255-266, 2015.

28. Vista IV, F. P., Singh, A. M., Lee, D.-J., and Chong, K. T., "Design Convergent Dynamic Window Approach for Quadrotor Navigation," Int. J. Precis. Eng. Manuf., Vol. 15, No. 10, pp. 2177-2184, 2014.

29. Bouabdallah, S., "Design and Control of Quadrotors with Application to Autonomous Flying," Ph.D. Thesis, Ecole Polytechnique Federale de Lausanne, 2007.

30. Bouabdallah, S. and Siegwart, R., "Backstepping and Sliding-Mode Techniques Applied TO An Indoor Micro Quadrotor," Proc. of the IEEE International Conference on Robotics and Automation, pp. 2247-2252, 2005.

31. Bouabdallah, S. and Siegwart, R., "Full Control of a Quadrotor," Proc. of IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 153-158, 2007.

32. Lee, T., Leok, M., and McClamroch, N. H., "Nonlinear robust Tracking Control of a Quadrotor UAV on SE (3)," Asian Journal of Control, Vol. 15, No. 2, pp. 391-408, 2013.