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# Polymers in Wave Fnergy Converter Application Polymers in Wave Energy Converter Application

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This paper presents a design, modeling and control of a novel wave energy converter (WEC) using Dielectric electro active polymer (DEAP). Application of DEAP in WEC has attracted a lot of works due to development of renewable energy and increasing of human energy demand. However, various challenges of the WEC using DEAP must be overcome before going to realistic application. Firstly, stretch ratio has significant influence on energy conversion efficiency of DEAP. It cannot exceed the limitation value due to mechanical or electrical breakdown, whereas small stretch ratio reduces the energy conversion efficiency significant. Secondly, WEC has to be controlled to maximize the absorbed energy. Therefore, this study employs an innovative device which can adjust the inertia of the floating buoy. A variable inertia hydraulic flywheel is attached on the main thrust shaft to control stretch ratio based on change in the hydrodynamic behavior of the system. A proportional–integral–derivative (PID) controller is designed to optimize stretch ratio under different regular waves. Consequently, the overall energy conversion efficiency of the proposed WEC can reach up to 25%.

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- $\lambda$  = Stretch ratio
- $\lambda_{w}$  = Wave length
- $M_a$  = Added mass
- $M_h$  = Buoy mass
- $M_{eq}$  = Equivalent mass
- $M_s$  = Supplementary mass
- $N =$ Number of ADG
- 
- $n =$ Pump speed
- $n_n$  = Pump speed
- $p_s$  = Supplied fluid pressure
- $p_{\text{pre}}$  = Gas pre-charge pressure
- $p_{set}$  = Setting pressure of relief valve
- $P_w$  = Mean wave power
- $Q$  = Electric charges
- $Q_d$  = Discharge flowrate
- $Q_p$  = The pump flowrate
- $Q_r$  = Flowrate through the relief valve
- $R_i$  = Radiation damping coefficient
- $R_f$  = Viscous friction coefficient
- $R_u$  = User's coefficient
- $r_h$  = Flywheel radius
- $r_{ni}$  = Pinion radius
- $\rho$  = Density of sea water
- $\rho_h$  = Density of hydraulic fluid
- $\sigma_{me}$  = Maxwell stress
- $\sigma_{el}$  = Elastic stress
- $l_0$  = Initial length
- $t =$ Thickness
- $S$  = The active electrode surface
- $S_b$  = The hydrostatic stiffness
- $V_c$  = Charge voltage
- $V_{vol}$  = Volume of DEAP material
- $V_f$  = Volume of hydraulic chamber
- $V_0$  = Initial volume of fluid
- $V_{pre}$  = Pre-charge volume of gas chamber
- $w$  = the wave elevation
- $w_0$  = DEAP width
- $\omega$  = Wave angular frequency
- $\omega_n$  = PTO natural frequency
- $\omega_n$  = Pump rotational speed
- $X_0$  = Stretch ratio at equilibrium position
- $\ddot{z}(t)$  = Buoy acceleration

## 1. Introduction

Dielectric elastomers (DE) have been known as suitable candidates for energy harvesting due to some distinct advantages, such as low cost, lightweight, high energy density and good impedance matching to many energy sources.<sup>1</sup> These vibration energy sources include human motion, vehicles, transportation and civil structures which are discussed in Ref. 2. Soft generators using DE is presented by Thomas G. McKay  $y_n = W$ ave length<br>  $y_n = W$ ave length<br>  $d_a =$  Added mass<br>  $d_{eq} =$  Equivalent<br>  $d_s =$  Equivalent<br>  $d_s =$  Supplements<br>  $d_s =$  Supplements<br>  $d_s =$  Suppled flui<br>  $r =$  Number of A<br>  $r =$  Pump speed<br>  $r =$  Gas pre-charge  $r =$  Gas pre-cha  $I_a =$  Added mass<br>  $I_a =$  Added mass<br>  $I_b =$  Buoy mass<br>  $I_s =$  Buoy mass<br>  $I_s =$  Equivalent<br>  $I_s =$  Supplementa<br>  $I_s =$  Supplementa<br>  $I_s =$  Pump speed<br>  $I_s =$  Pump speed<br>  $I_s =$  Supplied fluid<br>  $I_w =$  Gas pre-cha<br>  $I_t =$  Setting pres  $\frac{1}{b}$  = Buoy mass<br>  $\frac{1}{b}$  = Buoy mass<br>  $\frac{1}{s}$  = Equivalen<br>  $\frac{1}{s}$  = Supplemen<br>
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ielectric elastomers (DE)<br>
inergy harvesting due to<br>
lightweight, high energy c



Fig. 1 The basic working principle of DE

et al. in Ref. 3. Moreover, applications of DE in energy harvesting fields are performed on human passive motion, automobile tires and wave power are reported in Refs. 4-7.

A typical DE film includes a soft dielectric elastomer sandwiched between two compliant electrode layers as shown in Fig. 1, and the electroactive nanostructured polymers is reported by Ravi Shankar et al. in Ref. 8. Here, a dielectric elastomer layer is made of a relatively soft polymer such as silicone rubber, and compliant electrode layers are produced by coating carbon grease or the noble metals, such as platinum or gold. DEs have shown potential application in the actuator mode. Recently, the generator mode has gained more attention. In the actuator mode, electrical energy is converted into mechanical work by bringing opposite charges closer together. In the reversely, in the generator mode, mechanical work is converted to electrical energy by amplifying the pre-charged energy.

Some previous works that involve in application and development of DE have been taken into account. For example, the energy gain and other important material parameters are optimized to fit the requirement for energy converter in Ref. 9. A DE generator based on circular energy conversion unit is studied by Huaming W. et al. in Ref. 10 to investigate effect of bias voltage and stretch displacement on the generated energy and efficiency. Jean-Mistral C. et al. in Ref. 11 developed a reliable method for modelling DE generator based on biaxial plane. And, two elementary characteristic dielectric elastomer energy generating synergetic structures (DIESYS) design concepts for in-plane and outplane oscillations are examined by Anroniadis I. A. et al. in Ref. 12.

Being the group of Electroactive polymers (EAPs) and DE, Dielectric electro-active polymers (DEAP) is made by Danfoss Polypower. DEAP have shown great promise due to their capabilities for high strain and significant change in capacitance. There are some typical studies on this material. Firstly, elastic behavior of DEAP film in the development of actuators is reported in Ref. 13. Next, an analytical model for the electromechanical response of cone DEAP actuators is developed by Y. Zhu in Ref. 14, and hysteresis modeling and identification of DEAP actuator using an APSO based nonlinear Preisach NARX fuzzy model is presented in Ref. 15. In order to investigate the performance of DEAP generator mode in a wave energy converter (WEC), modeling and experimental investigation on DEAP generator is studied in Ref. 16. Moreover, a design and modeling of the antagonistic structure generator is developed in Ref. 17 to increase the conversion energy efficiency in wave energy harvesting application. Finally, based on the reliable analytical model, a design and modeling of an innovative WEC using DEAP is proposed in Ref. 18.

Although previous studies have taken into account valuable results, there are some challenges to face in application of DEAP in generator mode. Among, stretch ratios play important role to optimize the harvested energy and prevent mechanical breakdown of DEAP generator. Obviously, stretch ratios can be set up in laboratory environment precisely; however, these must be obtained by solving the dynamic performance in the realistic vibration source. For example, when the DEAP generator is applied in the WEC, stretch ratios must be obtained of the hydrodynamic behavior and interaction between the buoy and an incident wave.<sup>18</sup>

This paper investigates the performance of a novel WEC using DEAP generator with optimization stretch ratios. Firstly, modeling of antagonistic DEAP generator (ADG) is introduced. Then, the hydrodynamic model using the linear potential wave theory is presented and validated by experimental results under different wave profiles. Next, the complete analytical model including the hydrodynamic force, due to incident wave, coupled with the electromechanical force, due to DEAP stiffness, is expressed in time domain. Moreover, a variable supplementary inertia (SI) device is attached on the float to control the hydrodynamic performance of the float. The natural frequency of device is controlled by adjusting the SI. Consequently, reacting to the incident wave, the elevations of the float is obtained by solving the hydrodynamic behavior. Based on the float elevations, the response stretch ratios are determined and controlled to reach the desired values under different regular waves (different amplitudes and frequencies). Here, the PID controller is employed to generate the SI of device. Finally, a numerical simulation is built in MATLAB®/Simulink® to investigate performances of WEC under hydrodynamic behavior. By setting some parameters, simulation results indicate that stretch ratios can be controlled to optimize the harvested energy and to satisfy the realistic requirement.

#### 2. Working Principle of DEAP Generator

Several energy cycles were studied in Ref. 19. Based on a constant charge Q, Fig. 2 illustrates basic operating principle of DEAP generator in an energy harvesting. Firstly, DEAP material is pre-stretched in order to decrease the charged voltage and to increase the performance. Then, the mechanical energy is applied to stretch DEAP generator, which causes reduction in thickness and increasing the surface area. Its capacitance is given by the equation:

$$
C = \varepsilon_r \varepsilon_0 \frac{S}{t} = \varepsilon_r \varepsilon_0 \frac{V_{vol}}{t^2}
$$
 (1)

After stretch material, a pre – charge  $Q = CV_c$  is placed upon the two electrode layers where positive charges are on one side and negative ones are one the other side. Due to the Coulomb's law, an electric field is induced between two electrode layers. Then, the stress due to electrostatic force, as calculated by Krakovesky I. et al. in Ref. 20 can be expressed in Eq. (2). It is assumed that only electrostatic phenomenon is represented.

$$
\sigma_{me} = \varepsilon_0 \varepsilon_r E^2 \tag{2}
$$

At this moment, the input charged energy into DEAP can be expressed in the following equation:

$$
e_1 = 0.5 C_c V_c^2 \tag{3}
$$



Fig. 2 Basic working principle of DEAP energy cycle

On path 3-4, the DEAP material is allowed to retract point 4 based on the equilibrium between the elastic recovering force and electrostatic force. Then, the stored energy in DEAP at this point is expressed in Eq. (4):

$$
e_2 = 0.5 C_d V_d^2 \tag{4}
$$

Finally, the output energy of DEAP generator per cycle defined as the difference of Eqs.  $(4)$  and  $(3)$  is obtained in Eq.  $(5)$ 

$$
e_{cyc} = e_2 - e_1 = 0.5 C_c V_c^2 (C_c / C_d - 1)
$$
\n(5)

Here, the second equality can be expanded due to constant charge  $Q = CV_c$ .

#### 3. Modeling Mechanical Behavior of ADG

In order to describe dynamic behavior of ADG, a complete model including mechanical coupling electrical system is depicted in Fig. 3. Two DEAP generators are antagonistic connection by a cable and pulley system. The reactive force imposed in each DEAP generator consists of the elastic recovering force and Maxwell force. Using the Newton second's law, the dynamic equation of the attached mass motion in term of stretch ratio is expressed in the set of Eq. (6).

$$
M_{eq}l_0 \frac{d^2 \lambda_z}{dt^2} = F_h - F_1 + F_2
$$
  

$$
\lambda_z = \frac{l_0 + X_0 + z}{l_0}
$$
 (6)

To determine the reactive force, the Mooney-Rivlin model is employed to describe stretch ratio and force relationship. Stress in term of stretch ratio was given in Ref. 21. Defining  $\lambda$  is stretch ratio of DEAP 1, the elastic recovering force is obtained by multiplying the stress and current section area, and presented in Ref. 16. It is re-written in Eq. (7).

$$
F_{el1} = w_0 t_0 \left[ 2 \left( C_1 + \frac{C_2}{\lambda} \right) \left( \lambda - \frac{1}{\lambda^2} \right) \right] \tag{7}
$$

And, the Maxwell force is presented in Eq. (8)

$$
F_{me1} = \varepsilon_0 \varepsilon_r \frac{w_0 V_c^2 \lambda}{t_0} \tag{8}
$$



Fig. 3 Schematic of the ECU

Then, the reactive force of DEAP 1 generator is expressed in Eq. (9).

$$
F_1 = w_0 t_0 \left[ 2 \left( C_1 - \frac{C_2}{\lambda} \right) \left( \lambda - \frac{1}{\lambda^2} \right) \right] - \varepsilon_r \varepsilon_0 \frac{w_0 V_c^2 \lambda}{t_0} \tag{9}
$$

By analogy, the reactive force of DEAP 2 generator is obtained by determining the stretch ratio  $\lambda_2$  in term of  $\lambda$ . Then, the reactive force of DEAP 2 in term of  $\lambda$  is obtained in Ref. 17 and given in the set of Eq. (10)

$$
F_2 = 2w_0 t_0 \left( C_1 - \frac{C_2}{2S_0 - \lambda} \right) \left( 2S_0 - \lambda - \frac{1}{(2S_0 - \lambda)^2} \right) - \varepsilon_r \varepsilon_0 w_0 \frac{V_c^2 (2S_0 - \lambda)}{t_0}
$$
  

$$
S_0 = (\lambda_2 + \lambda)/2
$$
 (10)

Substituting Eqs. (9) and (10) into Eq. (6), the coupled equation of motion can be obtained as follow:

$$
M_a I_0 \ddot{\lambda} + 2 w_0 t_0 \left[ \left( C_1 + \frac{C_2}{2S_0 - \lambda} \right) \left( 2S_0 - \lambda - \frac{1}{(2S_0 - \lambda)^2} \right) - \left( C_1 + \frac{C_2}{\lambda} \right) \left( \lambda - \frac{1}{\lambda^2} \right) \right] + 2 \varepsilon_0 \varepsilon_r \frac{w_0 V_c^2}{t_0} (\lambda - S_0) = F_h
$$
\n(11)

Material parameters  $C_1$  and  $C_2$  can be determined by curve fitting experiment results with Eq. (11). Identification DEAP material parameters is presented in Ref. 16, and these results are shown in Table 1. Employing the test rig in Fig. 4, the analytical model is validated by

#### Table 1 the identified DEAP parameters





Fig. 4 Test rig of ADG



Fig. 5 Relative force error between modeling and measurement with random input stretch ratio

acceptable agreement between simulation results and that of measurement. Fig. 5 illustrates the response forces comparison. The same input stretch ratio is applied to the DEAP generators, the induced forces are measured and compared with the simulation results. It is indicated that the relative error is bounded in 5%.



Fig. 6 Test rig for validation of wave buoy interaction in water tank



In order to investigate hydrodynamic performance of a buoy and wave interaction, a test rig is designed, fabricated and assembled in the Research Institute of Small & Medium Shipbuilding (RIMS). The test layout is shown in Fig. 6. It includes the buoy, the power take-off (PTO) system and the peripheral interface devices. Using the Newton second's law, the acceleration of the buoy in the heave mode is obtained in Eq. (12)

$$
\ddot{z} = \frac{F_h + F_u}{M_b} \tag{12}
$$

where the hydrodynamic force  $F_h$  is calculated in Ref. 22 and presented in the following equation:

$$
F_h = F_e + F_r + F_v + F_f + F_s \tag{13}
$$

The excitation force is usually decomposed in two components: Froude-Krylove force and the diffraction force. Here, the Froude-Krylove force is caused by the undisturbed wave field and can be represented for the excitation force. The diffraction force can be negligible due to small floating buoy. The excitation force  $F_e$  is a function of the amplitude, frequency and phase of waves, and it is given in the following equation:

$$
F_e = f_3 A \sin(\omega t + \alpha) \tag{14}
$$

where  $f_3$  is the force amplitude, and  $\alpha$  is the phase difference between the wave and the excitation force.

The radiation force induced by buoy oscillations is required to move the buoy, and it is obtained in Eq. (15)

$$
F_r = -M_a \ddot{z} - R_r \dot{z} \tag{15}
$$

where  $M_a$  is the added mass at an infinite frequency, and  $R_r$  is the radiation damping coefficient.

The viscous damping force induced by relative turbulent flow is calculated by the following equation:



Fig. 7 The buoy elevation comparisons for validation hydrodynamic model in water tank

$$
F_v = -\frac{1}{2}\rho C_v A_w (z - \dot{\psi}) |z - \dot{\psi}| \tag{16}
$$

where  $C_{dr}$  is drag coefcient, and  $A_w$  is the water plane area of the buoy at rest.

The friction force of the PTO system can be modeled by proposing Armstrong's method in Ref. 23, and approximated with the following equation:

$$
F_f = (F_c + (F_{br} - F_c)e^{(-c_v|\dot{z}|)})sign(\dot{z}) + R_f \dot{z}
$$
 (17)

where  $F_c$  is the Coulomb friction which opposes motion with a constant force at any velocity; the breakaway friction force  $F_{bc}$  is the sum of the Coulomb and static frictions at zero velocity; the transition approximation coefficient  $c<sub>v</sub>$  is used for the approximation of the transition between the static and the Coulomb frictions.

The hydrostatic restoring force, due to excursion of the buoy from its equilibrium position, can be calculated by using the Archimedes principle, and it is simply given in Eq. (18):

$$
F_s = -S_b z \tag{18}
$$

The user's force reacting to the PTO is the required force to drive the PTO system. It is usually calculated as follow:

$$
F_u = -R_u \dot{z} \tag{19}
$$

The equation of motion for the buoy is given in Eq. (12) and rewritten in Eq. (20) by substituting Eqs. (13)~(19) into Eq. (12):

$$
(M_b + M_a)\ddot{z} + (R_r + R_f + R_u)\dot{z} + S_b z + \frac{1}{2}\rho C_v A_w(\dot{z} - \dot{\psi})|\dot{z} - \dot{\psi}|
$$
  
+ 
$$
(F_c + (F_{br} - F_c)e^{(-c_v|\dot{z}|)})sign(\dot{z}) = f_3 A sin(\omega t + \alpha)
$$
(20)

#### 5. A Novel Design of a WEC System

## 5.1 Design concept & working principle

The general conceptual design of the proposed WEC is illustrated in Fig. 8. There are five main components, such as a stationary platform, a variable inertia hydraulic flywheel (VIHF), DEAP transducers, and a



Fig. 8 The proposed WEC including multiple ADG and VIHF

floating buoy and main control unit (MCU). The platform is held stationary relative to free water surface on the sea. The VIHF is mounted on the platform and rotates around its axis. A plurality of DEAP transducers are fixed on the platform at one end and the other ends are fixed on the floating buoy. The main shaft of the buoy is coupled with the VIHF thanks to rack and pinion mechanism. Moreover, the MCU is employed to collect data and send control signal to operate the VIHF.

Reacting to the incident waves, the floating buoy oscillates up and down. Then, DEAP transducers are also stretched and relaxed. Since two DEAP generators are antagonistic connection in each ADG, they are stretched and relaxed inversely. One DEAP generator is stretched, the other DEAP generator is allowed to retract. On the other hand, the elastic force of one DEAP generator in relaxing phase is used to stretch the opposite one. Thus, the required forces using to drive a transducer are reduced in the same stretch ratio. Moreover, for increasing the absorbed energy, the equivalent inertia of floating buoy structure must be tuned to the values which bring the natural frequencies of device to close to wave spectrum frequencies (resonances behavior). The VIHF plays a role as the SI. By controlling fluid inside chamber, mass of the VIHF will be changed to adjust its inertia. Once the SI is changed, the equivalent inertia will be adjusted. Consequently, the buoy elevation (BE) and stretch ratio can be controlled to improve the performance and efficiency of the device.

Compared with previous ideas about controlling SI, this paper shows convenient mechanism to satisfy the realistic requirement of WEC using DEAP. The proposed design keeps the floating buoy closed to the water surface to have good radiation capabilities while it maintains the same draft compared to the adjusted draft in Ref. 18. It is also controllable device compared to two splited bodies in Refs. 24-25, or rigid supplementary mass in Ref. 26.

#### 5.2 Mathematical model

As shown in Fig. 8, a variable SI is taken into account to analyze the combined model of the proposed WEC. The forces acting on the floating buoy consist of the reactive force, the supplementary force and the hydrodynamic force. The resistive force from the PTO involves the force required to drive all ADG. The supplementary force involves change in hydrodynamic performance of the proposed WEC by adjusting the natural frequency. By using the wave linear theory, behavior of the buoy is analyzed as a single body system with one degree of freedom along the vertical  $z$ -axis.<sup>22</sup> According to the Newton second's law, the equation of motion of the buoy is obtained in Eq. (21):

$$
\ddot{z} = \frac{F_h + F_D + F_{sup}}{M_b} \tag{21}
$$

$$
F_D = N K_{eq} z \tag{22}
$$

$$
F_{sup} = -\frac{I_{fl}\ddot{\theta}}{r_p} = -\frac{I_{fl}\ddot{z}}{r_p^2}
$$
 (23)

Substituting Eqs. (13), (22) and (23) into Eq. (21), the complete motion equation of floating buoy is expressed in Eq. (24):

$$
(M_b + M_a + I_f/r_p^2)\ddot{z} + (R_r + R_f + R_u)\dot{z} + (S_b + NK_{eq})z
$$
  
+ 
$$
\frac{1}{2}\rho C_v A_w(\dot{z} - \dot{\psi})|\dot{z} - \dot{\psi}| + (F_c + (F_{br} - F_c)e^{(-c_v|\dot{z}|)})sign(\dot{z}) \qquad (24)
$$
  
= 
$$
f_3 A \sin(\omega t + \alpha)
$$

#### 5.3 Optimum SI

For the given buoy and the given incident wave, the absorbed energy can be maximized by bringing the natural frequency of the PTO devices  $\omega_n$  close to wave excitation frequency  $\omega$  (resonance behavior).<sup>22</sup> This can be achieved by attaching an adaptive SI:

$$
\omega_n = \sqrt{\frac{S_b + NK_{eq}}{M_b + M_a + I_f / r_p^2}} = \omega
$$
\n(25)

Then, the optimum SI of the flywheel is obtained

$$
I_{fl} = r_p^2 \left[ \frac{(S_b + NK_{eq})}{\omega^2} - M_b - M_a \right]
$$
 (26)

 $K_{eq}$  is the equivalent stiffness of the ADG, and it is defined as follow:

$$
K_{eq} = \frac{abs(F_1 - F_2)}{z} \tag{27}
$$

However, due to limitation of stretch ratio for mechanical breakdown

strength of DEAP. The optimum SI must be derived to satisfy the desired values.

#### 5.4 Modeling of variable inertia hydraulic flywheel

The current inertia of the hydraulic flywheel includes the initial inertia without fluid and the inertia of fluid pumped into hydraulic chamber, and it is expressed in Eq. (28):

$$
I_{fl} = I_i + I_f = I_i + \frac{\rho V_f r_h^2}{2}
$$
 (28)

The fluid inside the accumulator is a function of the supplied hydraulic pressure, pre-charged pressure, accumulator capacity and specific heat ratio. It is calculated by the following:

$$
V_f = \begin{cases} 0, & \text{if } P_S \le P_{pre} \\ V_{pre} & \end{cases}
$$
  

$$
V_{pre} \left[ 1 - \left( \frac{P_{pre}}{P_S} \right)^{\frac{1}{n}} \right], \quad \text{if } P_S > P_{pre}
$$
 (29)

Here, the pressure inside the flywheel-accumulator is calculated by the continuity equation:

$$
\frac{dP_s}{dt} = \frac{\beta}{(V_f + V_0)} (Q_p - Q_d - Q_r)
$$
\n(30)

The pump flowrate  $Q_p$  is calculated as:

$$
Q_p = \eta_v D \omega_p \tag{31}
$$

The discharge flowrate is depended on the value of current fluid pressure  $P<sub>S</sub>$  and can be obtained as:

$$
Q_d = C_d A_p \sqrt{\frac{2P_S}{\rho}}
$$
 (32)

The flowrate going through the relief valve is calculated as:

$$
Q_r = \begin{cases} 0, & \text{if } P_s \le P_{set} \\ C_{dr} A_r \sqrt{\frac{2(P_s - P_{set})}{\rho}} & \text{if } P_s > P_{set} \end{cases}
$$
 (33)  
where *e*(*t*) is the error between the des

## 6. Control Strategy

For controlling of the stretch ratio, the flowchart control is proposed in Fig. 10. For the given wave profiles and specifications of the PTO system, the hydrodynamic parameters are pre-computed by WAMIT software.<sup>27</sup> An analytical model is built to investigate the BE. The responded stretch ratio is obtained in term of the BE. Stretch ratio closed-loop control is applied to generate the required SI. Here, the relative error between desired stretch ratio and actual stretch ratio is sent to the controller. The controller sends the output signal to generate the required inertia of the VIHF. The supplementary mass is calculated and sent to the analytical model.

WEC using DEAP is very complex to build the transfer function or state-space observer. The nonlinearities are mainly caused by flowpressure characteristics of hydraulic system, viscous friction of hydrodynamic and mechanical behaviour of DEAP. In this study, a proportional flow control valve is employed to adjust flow rate of the VIHF. Once the supplying pressure is adjusted, it will change the



Fig. 9 Specification of the VIHF with hydraulic circuit



Fig. 10 Schematic control diagram for stretch ratio

performance of the proposed WEC. PID controller is often used due to its simple structure and easy design. The control signal  $u(t)$  is expressed in the time domain as follow:

$$
u(t) = K_p e(t) + K_I \int_0^t e(t) dt + K_D \frac{de(t)}{dt}
$$
\n(34)

where  $e(t)$  is the error between the desired stretch ratios and the measured values,  $K_P$  is the proportional gain,  $K_I$  is the integral gain and  $K_D$  is the derivative gain.

## 7. Energy Conversion Efficiencies

For a given regular wave, the mean wave power is given in Ref. 22. It is a function of the mean wave energy density per unit horizontal area and the group velocity. And, the mean wave power are expressed in the set of Eq. (35):

$$
P_w = ec_g, \quad (kW/m)
$$
  
\n
$$
e = e_k + e_p = \frac{1}{8}\rho g^2 H^2, \quad (J/m^2)
$$
\n(35)

For constant water depth  $h$ , the group velocity is given in Ref. 22 and expressed in the set of Eq. (36):

$$
c_g = \frac{D(kh)}{2\tanh(kh)}c_p = \frac{g}{2\omega}D(kh)
$$
  

$$
D(kh) = \left[1 + \frac{2kh}{\sinh(2kh)}\right] \tanh(kh)
$$
 (36)  

$$
k = 2\pi/\lambda
$$



#### Table 2 Specification of DEAPs generator



Parameters	Value
Operating environment	Dry environment
Operating frequency	$0.5$ Hz
Sampling time	$0.01$ s
Maximum stretch ratio	122.5%
$m_{eq}$	$0.7 \text{ kg}$
V,	1300 $V$
Gravitational acceleration	9.81 m/s <sup>2</sup>

Table 4 Wave profile and PTO's parameters for simulation



where  $D(kh)$  is the depth function.

Time dependency of mean wave power of an incident wave is calculated by integrating Eq. (35) over time domain:

$$
e_w = \int_0^t P_w, \quad (J) \tag{37}
$$

In analogy with Eq. (37), the harvested energy is obtained by integrating Eq. (5) over time domain:

$$
e_{har} = \int_0^t e_{cyc}, \quad (J)
$$
 (38)

Finally, the total efficiencies of the proposed WEC defined by ratio results of Eqs. (38) and (37) is calculated by Eq. (39):

$$
\eta_t = \frac{e_{har}}{e_w} \tag{39}
$$

# 8. Simulation Results and Discussions

Before investigating the performance of the proposed WEC, some parameters must be prescribed in advance. Here, Specifications of DEAP generator is given by Danfoss PolyPower and shown in Table 2. Table 3 shows working conditions. The wave profiles and specification of the floating buoy are shown in Table 4. And, hydrodynamic parameters are obtained by using WAMIT software and plotted in Fig. 11.

Numerical model is built in MATLAB®/Simulink®. Then, two types of simulations are carried out to investigate the performance. The first type involves in response of the proposed WEC with different SI



Fig. 11 Hydrodynamic parameters for the heave mode of the floating truncated vertical cylinder of radius a=0.5 m and of draft b=0.5 m in water of depth h=1.2 m



Fig. 12 Change in BE, stretch ratio and total theoretical harvested energy for various SI:  $0.5 \text{ kgm}^2$ ,  $0.625 \text{ kgm}^2$ ,  $0.75 \text{ kgm}^2$ ,  $0.875 \text{ kgm}^2$ and 1 kgm<sup>2</sup> under wave profile  $\psi_1$ 

separately. The second type involves in response of the WEC device with stretch ratio closed loop control.

In the first type of simulation, the performance of WEC is investigated



Fig. 13 Change in BE, stretch ratio and total theoretical harvested energy for various equivalent mass at  $0.25 \text{ kgm}^2$ ,  $0.375 \text{ kgm}^2$ ,  $0.455$ kg, 0.5 kgm<sup>2</sup> and 0.625 kgm<sup>2</sup> under wave profile  $\psi_2$ 

in two regular waves and different SI. Simulation results are plotted in Figs. 12 and 13 with case 1 and case 2 of waves, respectively. In the Fig. 12, the BE under changing in SI are shown on the top part. Response stretch ratios and harvested energy are shown in the middle and bottom part, respectively. Similar to results in Fig. 12, Fig. 13 plots simulation results for changing SI from  $0.25$  to  $0.625$  kgm<sup>2</sup> with step 0.125 kgm<sup>2</sup> under case 2 of wave. It is cleared that resonance behavior can be found at  $SI_1 = 0.875$  kgm<sup>2</sup> and  $SI_2 = 0.455$  kgm<sup>2</sup> with case 1 and case 2 of wave, respectively. The BE and stretch ratios are increasing at these points significantly. In theoretical vibration, this can be reasonable. However, due to limitation of stretch ratio for mechanical breakdown, the SI must be selected to satisfy maximum stretch ratio in the steady state. After two optimum SI are selected to generate the desired stretch ratio, the responses of WEC are plotted in Figs. 14 and 15. As shown, although stretch ratios are reaching the desired stretch ratio at the steady state, large overshoot happen at the transient state. These point can cause damage to DEAP material because of mechanical breakdown strength.

In order to eliminate the large overshoot at the transient state, stretch ratio closed loop control is applied improve the performance of the WEC. Then, the simulation results are shown in Figs. 16 and 17 with case 1 and case 2 of wave, respectively. In this work, the desired stretch ratio of DEAP material is about 1.22. The stretch ratios are reaching to desired stretch ratio without overshoot. The response of WEC system is also smooth.

Moreover, it takes a little time from transient states to the steadystate because these are strongly dependent on difference between wave elevations and desired stretch ratios. In addition, the hydraulic system and the controller have significant influence on SI which have been



Fig. 14 Ideal performance of the proposed WEC under case 1 of wave  $ψ$ <sub>1</sub> at SI=0.875 kgm<sup>2</sup>



Fig. 15 Ideal performance of the proposed WEC harvested energy in 25% efficiency under case 2 of wave  $\psi_2$  at SI=0.455 kgm<sup>2</sup>

taken into account to change the hydrodynamic performance of the WEC device. Fortunately, the transient times are so small compared with the working time of WEC device, and wave spectrum is only changed several times a day. Therefore, the proposed WEC can shows its advantage for realistic application.



Fig. 16 Response to optimum control for stretch ratio under case 1 of wave



Fig. 17 Response to optimum control for stretch ratio under case 2 of wave

#### 9. Conclusion and Future Works

This paper improves the performance of a novel WEC using DEAP generator. The complete analytical model including the hydrodynamic force, due to incident wave, coupled with the electromechanical force,

due to DEAP stiffness, is simulated by using MATLAB®/Simulink® in time domain. Here, modeling of the ADG is introduced, and the hydrodynamic model using the linear potential wave theory is presented and verified by experimental results under different wave profiles. The VIHF is proposed to control SI which adjust the natural frequency of WEC device. Stretch ratios closed loop control is applied by using the PID controller to reach the desired values under different regular waves (different amplitudes and frequencies). Consequently, simulation results indicate that the overall energy conversion efficiency of the proposed WEC can reach up to 25% and the response of WEC device is improved significantly.

This model can be proposed to adapt to any DE materials which have larger strain capacity and higher energy density. A complete test rig will be deployed at RIMS in future project to validate the proposed WEC with experimental results. Moreover, investigation of the performance in the irregular waves can be studied to satisfy the realistic requirement of the proposed WEC.

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