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A Novel Adaptive Finite-Time Control Method for a Class of Uncertain Nonlinear Systems

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This paper presents a novel adaptive finite-time control (AFTC) method for a class of uncertain nonlinear systems. First, a new nonsingular terminal sliding mode surface is proposed. Then an adaptive finite-time controller with proper adaptive laws is designed to guarantee the occurrence of the sliding motion in finite time without prior knowledge of the upper bounds of the uncertainties and external disturbances. The globally finite-time stability of the closed-loop system is analytically proven. The numerical simulation results are presented to illustrate the effectiveness of the proposed method.

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NOMENCLATURE

AFTC = Adaptive Finite-time Control
SMC = Sliding Mode Control
TSM = Terminal Sliding Mode
FTSM = Fast Terminal Sliding Mode
NTSM = Nonsingular Terminal Sliding Mode

1. Introduction

In practical applications, second-order nonlinear systems are a broad class of many physical systems, such as robotic manipulators,[1] magnetic levitation system,[2] a passive maglev tray system,[3] a moving permanent magnet linear synchronous motor,[4] a variable-length pendulum,[5] a double pendulum,[6] chaotic system,[7] a micro-electro-mechanical system.[8] On the other hand, some complex high-order system control problems can be transformed into second-order nonlinear system control problems, such as permanent magnet synchronous motors, point mass planar satellites, and so on.[9,10] The control of practical nonlinear systems is a challenging problem due to the effects of uncertainties and external disturbances, which are not known in advance. Many advanced control schemes have been introduced for nonlinear systems, including proportional-integral adaptation scheme, adaptive control, robust control, intelligent control, Least Quadratic Regulator control, sliding mode control (SMC) and so on.[1-14]

SMC is an efficient control method that has been widely used for both linear and nonlinear systems due to its robustness against bounded uncertainties and matched disturbances.[12,13] However, the traditional SMC scheme only guarantees asymptotic stability; that is, the system states do not converge to the equilibrium point in finite time because a linear manifold is used. Recently, terminal sliding mode (TSM) control methods,[15,16,18] which use nonlinear sliding surfaces instead of linear surfaces, have been developed. The TSM control schemes cause the system states to reach the equilibrium point in finite time, and also offer some superior properties such as faster convergence, improved transient performance, and higher precision. X. Yu and Z. Man[17] and S. Yu[18] have adopted a fast terminal sliding mode (FTSM) that improves the convergence rate when the system states are far from equilibrium. However, both the TSM and FTSM methods both have a singularity problem. In order to overcome this, some nonsingular TSM (NTSM) methods [19-21] have been proposed.

However, most of the aforementioned works require the knowledge of the upper bounds of the uncertainty and external disturbance terms, which could be, from a practical point of view, a hard requirement to achieve. Some methods, which integrate adaptive control or intelligent control to TSM or NTSM to cope with this problem, have been proposed. Z. Man et al.[22] and M. Neila and D. Tarak[23] have proposed adaptive TSM controls for rigid robotic manipulators. L. Y. Wang et al.[24] introduced a neural-network-based TSM control for robotic manipulator including actuator dynamics. Li et al.[25] presented an adaptive fuzzy TSM controller for robotic manipulators. Some researchers have used adaptive TSM control techniques to chaos control or anti-synchronization for chaotic systems.[26-28] C. C. Yang[29] discussed an adaptive NTSM control which was designed for synchronization of identical F^6 oscillators. J. Fei and W. Yan[30] proposed an adaptive control of MEMs gyroscope using global fast terminal sliding mode control and fuzzy-neural-network. M. P. Aghababa and H. P. Aghababa[31] established a general nonlinear adaptive control scheme for finite-time synchronization of chaotic systems with uncertain parameters and nonlinear inputs. However, these mentioned methods still suffer from several problems. For example, some methods[22-26] used TSM surfaces that include the singularity problem and these methods also required huge initial values of estimated parameters to prove the finite-time convergence property. Although an indirect method was employed to avoid the singularity problem, the resulting control laws are discontinuous across the TSM surfaces, which may excite undesired high frequency dynamics. In L. Fang's work[27] or L. Yang's work[28] the finite-time convergence has been not considered completely beside the mentioned problems. Meanwhile, C. C. Yang's work[29] or J. Fei's work[30] only guarantees the asymptotical stability. Although adaptive laws have been applied, the prior knowledge of the upper bounds of the parameter uncertainties, the un-modeled uncertainties and external disturbances are required.[31]

In this paper, the above-mentioned problems are addressed. A novel adaptive finite-time control (AFTC) method for a class of uncertain nonlinear systems is proposed. Based on a proposed novel nonsingular terminal sliding mode surface, an AFTC law with appropriate update laws is designed to drive the system states to reach the sliding surface and to converge to zero in a finite amount of time. The globally finite-time stability of the closed-loop system is strictly proven.

The organization of this paper is as follows. Preliminaries and problem formulation are given in section 2. Section 3 deals with the design of the proposed AFTC method. In section 4, numerical simulation results for a chaotic horizontal platform system are provided to illustrate the effectiveness of the proposed control method. Finally, some concluding remarks are given in section 5.

2. Preliminaries and Problem Formulation

2.1 Preliminaries

Several preliminary results that will be utilized for the control design are presented in this section.

Definition 1:[32] The nonlinear system

$$\dot{x} = \varphi(x)\,,\ \ \varphi(x) = 0\,,\ \ x \in D \subset R^n\,,\ \ x(0) = x_0 \tag{1}$$

the origin is said to be a finite-time stable equilibrium of Eq. (1) if there exists an open neighborhood $N \subseteq D$ of the origin and a function T: $N\backslash\{0\} \to (0, \infty)$ which is called the settling time function, such that the following statements hold:

- Finite-time convergence: For every $x_0 \in N/\{0\}$, every solution $x(t, x_0)$ is defined for $t \in [0, T(x_0))$, $x(t, x_0) \in N\backslash\{0\}$, for $t \in [0, T(x_0))$ and $\lim_{t \to T(x_0)} x(t, x_0) = 0$.
- Lyapunov stability: for every open set U_s such that $0 \in U_s \subseteq N$ there exists an open set U_δ such that $0 \in U_s \subseteq N$ and such that for every $x_0 \in U_\delta/\{0\}$, $x(t, x_0) \in U_s$ for all $t \in [0, T(x_0))$.

The origin is said to be a globally finite-time stable equilibrium if it is a finite-time stable equilibrium and $D = N = R^n$.

Definition 2:[33] (Homogeneous) A family of dilations Δ_s^r is a mapping that assigns to every real $\varepsilon > 0$ a diffeomorphism

$$\Delta_\varepsilon^r(x_1, \ldots, x_n) = (\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) \tag{2}$$

where $x_1, \ldots, x_n$ are suitable coordinates on R^n and $r = (r_1, \ldots, r_n)$ with the dilation coefficients $r_1, \ldots, r_n$ positive real numbers.

A vector field $\varphi(x) = (\varphi_1(x), \ldots, \varphi_n(x))^T$ is homogeneous of degree $q \in R$ with respect to the family of dilations if

$$\varphi_l(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^{q + r_l} \varphi_l(x), \quad l = 1, 2, \ldots, n \tag{3}$$

System Eq. (1) is called homogeneous if its vector field φ is homogeneous.

Lemma 1:[33] Consider the following system

$$\dot{x} = \varphi(x) + \hat{\varphi}(x)\,,\ \ \varphi(0) = 0\,,\ \ x \in R_n \tag{4}$$

where $\varphi(x)$ is a continuous homogeneous vector field of degree $q<0$ with respect to $(r_1, \ldots, r_n)$, and $\hat{\varphi}$ satisfies $\hat{\varphi}(0) = 0$. Assume $x=0$ is an asymptotically stable equilibrium of the system $\dot{x} = \varphi(x)$. Then $x=0$ is a locally finite-time stable equilibrium of the system Eq. (4) if

$$\lim_{\varepsilon \to 0} \frac{\hat{\varphi}_l(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n)}{\varepsilon^{q + r_l}} = 0\,,\ \ l = 1, 2, \ldots, n\,,\ \ \forall x \neq 0 \tag{5}$$

The following notation is introduced for simplicity of expression:

$$x^{[\alpha]} = sign(x)|x|^\alpha\,,\ \ \alpha > 0 \tag{6}$$

It can be verified that as $\alpha \geq 1$

$$\frac{d}{dt} x^{[\alpha]} = \alpha |x|^{\alpha - 1} \dot{x} \tag{7}$$

The following result is a special case of Proposition 8.1 in Ref. 34

Lemma 2:[34] Consider the system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \tag{8}$$

The origin of the system Eq. (8) is a globally finite-time stable equilibrium under the feedback control law

$$u = -k_1 x_1^{[\alpha_1]} - k_2 x_2^{[\alpha_2]}\,,\ \ 0 < \alpha_1 < 1\,,\ \ \alpha_2 = \frac{2\alpha_1}{1 + \alpha_1} \tag{9}$$

where k_1 and k_2 are positive constants.

Lemma 3:[35] Assume that a continuous positive definite function satisfies the differential inequality

$$\dot{V}(t) \leq -\kappa V^\delta\,,\ \ \forall t \geq t_0\,,\ \ V(t_0) \geq 0 \tag{10}$$

where $\kappa > 0$ and $0 < \delta < 1$ are constants. Then for any given t_0, $V(t)$

satisfies the inequality

$$V^{1-\delta}(t) \le V^{1-\delta}(t_0) - \kappa(1-\delta)(t-t_0), \quad t_0 < t < t_1 \tag{11}$$

and $V(t) = 0$, $\forall t \ge t_1$, with t_1 given by

$$t_1 = t_0 + \frac{1}{\kappa(1-\delta)} V^{1-\delta}(t_0) \tag{12}$$

Lemma 4:[36] Jensen's inequality

$$(\Sigma_{i=1}^{m} \theta_i^{\lambda_2})^{1/\lambda_2} \le (\Sigma_{i=1}^{m} \theta_i^{\lambda_1})^{1/\lambda_1}, \quad 0 < \lambda_1 < \lambda_2 \tag{13}$$

with $\theta_i \ge 0$, $1 \le i \le m$.

2.2 Problem formulation

Consider a class of uncertain nonlinear systems in the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x,t) + d(x,t) + g(x,t)u(t) \end{cases} \tag{14}$$

where $x = [x_1, x_2]^T$ is the state vector, $f(x, t)$ and $g(x, t) \neq 0$ are given nonlinear functions, $d(x, t)$ presents the uncertainties and external disturbances, and $u(t)$ is the control input.

Assumption 1: The uncertainty and disturbance term $d(x, t)$ is a bounded function satisfying

$$|d(x,t)| \le b_0 + b_1 \|x\| \tag{15}$$

where b_0 and b_1 are unknown positive constants.

The control objective of this paper is to design a novel AFTC strategy in the presence of unknown uncertainties and disturbances, such that the system states converge to zero within finite time.

3. Main Results

In this section, the design of the proposed adaptive finite-time controller for the system in Eq. (14), which involves two major phases, is presented. First, a novel nonsingular terminal sliding surface is introduced such that the system states are finite-time stable. Then the suitable adaptive control law is designed to make the prescribed manifold reachable in spite of unknown uncertainties and disturbances.

3.1 Nonsingular terminal sliding surface design

In this paper, a novel nonsingular terminal sliding surface is proposed as follows:

$$s = x_2 + \int_0^t (k_1 x_1^{[\alpha_1]} + k_2 x_2^{[\alpha_2]} + k_3 x_1 + k_4 x_1^3) d\tau \tag{16}$$

where k_1, k_2, k_3 and k_4 are positive constants, $0 < \alpha_1 < 1$ and $\alpha_2 = \dfrac{2\alpha_1}{1+\alpha_1}$.

When the system operates in sliding mode, the SMC theory give us the following property:[37]

$$s = 0 \tag{17}$$

Combing Eq. (17) with Eq. (16), we have

$$x_2 + \int_0^t (k_1 x_1^{[\alpha_1]} + k_2 x_2^{[\alpha_2]} + k_3 x_1 + k_4 x_1^3) d\tau = 0 \tag{18}$$

Then, the following sliding mode dynamics can be obtained

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_1 x_1^{[\alpha_1]} - k_2 x_2^{[\alpha_2]} - k_3 x_1 - k_4 x_1^3 \end{cases} \tag{19}$$

Theorem 1: The origin of the sliding mode dynamics Eq. (19) is a globally finite-time stable equilibrium.

Proof. The following proof will proceed in two steps.

First, prove the origin of the system Eq. (19) is globally asymptotically stable.

For the sliding mode dynamics Eq. (19), consider a Lyapunov function candidate as follows:

$$V_1 = \frac{k_1}{\alpha_1 + 1} |x_1|^{\alpha_1 + 1} + \frac{1}{2} x_2^2 + \frac{k_3}{2} x_1^2 + \frac{k_4}{4} x_1^4 \tag{20}$$

Taking the derivative of V_1 in Eq. (20) and applying Eq. (19) yields

$$\begin{aligned} \dot{V}_1 &= k_1 x_1^{[\alpha_1]} \dot{x}_1 + x_2 \dot{x}_2 + k_3 x_1 \dot{x}_1 + k_4 x_1^3 \dot{x}_1 \\ &= k_1 x_1^{[\alpha_1]} x_2 + x_2(-k_1 x_1^{[\alpha_1]} - k_2 x_2^{[\alpha_2]} - k_3 x_1 - k_4 x_1^3) + k_3 x_1 x_2 + k_4 x_1^3 x_2 \\ &= -k_2 |x_2|^{1+\alpha_2} \end{aligned} \tag{21}$$

Therefore, according to LaSalle's invariant principle,[38] the origin of the system in Eq. (19) is globally asymptotically stable equilibrium.

Second, we will show that the equilibrium at the origin of the system in Eq. (19) is locally finite-time stable.

The system in Eq. (19) can be given by

$$\dot{x} = \varphi(x) + \hat{\varphi}(x) \tag{22}$$

where

$$\varphi(x) = \begin{pmatrix} x_2 \\ -k_1 x_1^{[\alpha_1]} - k_2 x_2^{[\alpha_2]} \end{pmatrix}, \quad \hat{\varphi}(x) = \begin{pmatrix} \hat{\varphi}_1(x) \\ \hat{\varphi}_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ -k_3 x_1 - k_4 x_1^3 \end{pmatrix} \tag{23}$$

when $\hat{\varphi}(x) \equiv 0$ the system in Eq. (22) is exactly in the form of the closed-loop system Eqs. (8) and (9), which is globally finite-time stable, from Lemma 2.

Therefore, based on Lemma 1, the origin of the system Eqs. (22) or (19) is locally finite-time stable equilibrium if the perturbation vector $\hat{\varphi}(x)$ satisfies Eq. (5).

According to Definition 2, it is not difficult to show that the system $\dot{x} = \varphi(x)$ is homogeneous of negative degree $q = \alpha_1 - 1 < 0$ with respect to the dilation $(r_1, r_2) = (2, 1+\alpha_1)$. It is obvious that $r_1 - r_2 - q > 0$ Thus, we have

$$\lim_{\varepsilon \to 0} \frac{\hat{\varphi}_2(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2)}{\varepsilon^{r_2+q}} = \lim_{\varepsilon \to 0} \frac{-k_3 \varepsilon^{r_1} x_1 - k_4 \varepsilon^{3r_1} x_1^3}{\varepsilon^{r_2+q}} = 0, \quad \forall x \neq 0 \tag{24}$$

Consequently, $\hat{\varphi}(x)$ satisfies condition in Eq. (5), that is, the origin of the system Eq. (19) is locally finite-time stable equilibrium. The proof of Theorem 1 is completed.

Remark 1: The reported terminal sliding surfaces and fast terminal sliding surfaces[16,17] are expressed as

$$s = x_2 + \beta x_1^{q/p}, \quad s = x_2 + \alpha x_1 + \beta x_1^{q/p} \tag{25}$$

respectively, where $\alpha > 0$, $\beta > 0$, $p > q > 0$, and p and q are odd integers.

The control signals contain $x_1^{p/q-1}x_2$, which may cause a singularity to occur if $x_2 \neq 0$ when $x_1 = 0$.

Yu et al.[18] proposed terminal sliding surfaces as follows:

$$s = x_2 + \beta sign(x_1)|x_1|^{\upsilon}, \quad s = x_2 + \alpha x_1 + \beta x_1^{q/p} \tag{26}$$

where $\alpha > 0$, $\beta > 0$ and $0 < \upsilon < 1$.

And Zhankui and Sun[8] recently proposed the sliding surface as

$$s = x_2 + k_1 x_1 + k_2 sign(x_1)|x_1|^{\upsilon_1} + k_3 sign(x_1)|x_1|^{\upsilon_2} \tag{27}$$

where $k_i > 0$, $(i = 1, 2, 3)$, $0 < \upsilon_1 < 1$, and $\upsilon_2 > 0$. The control inputs still involve $|x_1|^{\upsilon-1}x_2$ or $|x_1|^{\upsilon_1-1}x_2$, which can cause a singularity problem as well if $x_2 \neq 0$ when $x_1 = 0$. However, our proposed nonsingular terminal sliding surface in Eq. (16) does not contain any of the mentioned singularities.

3.2 Control law design

Once a suitable sliding surface is established, the next phase is to construct the robust adaptive finite-time controller to drive the system Eq. (14) to the expected surface Eq. (16) in a finite time. The proposed control strategy is summarized in Theorem 2.

Theorem 2. Consider the uncertain nonlinear system in Eq. (14). If the proposed AFTC law is designed as in Eq. (29) with the update laws designed as in Eqs. (30)~(31) and the nonsingular terminal sliding surface is chosen as in Eq. (16), then $\hat{b}_0$ and $\hat{b}_1$ have upper bounds; that is there exist positive constants $\hat{b}_0$ and $\hat{b}_1$ such that

$$\hat{b}_i \le b_i^*, \quad i = 0, 1; \quad \forall t \ge 0 \tag{28}$$

Moreover, the system trajectories will converge to the sliding surface $s = 0$ in a finite time; that is the globally finite-time stability of the closed-loop system is achieved.

$$\begin{aligned} u(t) = -g(x,t)^{-1}[&f(x,t) + k_1 x_1^{[\alpha_1]} + k_2 x_2^{[\alpha_2]} + k_3 x_1 + k_4 x_1^3 + \\ &+ \eta s + (\hat{b}_0 + \hat{b}_1\|x\|)sign(s)] \end{aligned} \tag{29}$$

$$\dot{\hat{b}}_0 = \kappa_0 |s| \tag{30}$$

$$\dot{\hat{b}}_1 = \kappa_1 |s| \|x\| \tag{31}$$

where $\hat{b}_0$ and $\hat{b}_1$ are the estimations of the unknown upper bound b_0 and b_1, respectively, and η, κ_0 and κ_1 are positive constants.

Proof. First, the upper bound property Eq. (28) is considered. We define the following Lyapunov function candidate

$$V_2 = \frac{1}{2}s^2 + \frac{1}{2\kappa_0}(\hat{b}_0 - b_0)^2 + \frac{1}{\kappa_1}(\hat{b}_1 - b_1)^2 \tag{32}$$

Differentiating Eq. (32) with respect to time yields

$$V_2 = s\dot{s} + \frac{1}{\kappa_0}(\hat{b}_0 - b_0)\dot{\hat{b}}_0 + \frac{1}{\kappa_1}(\hat{b}_1 - b_1)\dot{\hat{b}}_1 \tag{33}$$

Based on Eq. (14), the time derivative of the sliding surface in Eq. (16) can be represented as

$$s = f(x,t) + d(x,t) + g(x,t)u(t) + k_1 x_1^{[\alpha_1]} + k_2 x_2^{[\alpha_2]} + k_3 x_1 + k_4 x_1^3 \tag{34}$$

From Eqs. (33) and (34), we have

$$\begin{aligned} \dot{V}_2 = s[&f(x,t) + d(x,t) + g(x,t)u(t) + k_1 x_1^{[\alpha_1]} + k_2 x_2^{[\alpha_2]} + k_3 x_1 + k_4 x_1^3] \\ &+ \frac{1}{\kappa_0}(\hat{b}_0 - b_0)\dot{\hat{b}}_0 + \frac{1}{\kappa_1}(\hat{b}_1 - b_1)\dot{\hat{b}}_1 \end{aligned} \tag{35}$$

By substituting Eqs. (29) to (31) into (35) and using Eq. (15), then

$$\begin{aligned} \dot{V}_2 &= s[d(x,t) - \eta s - (\hat{b}_0 + \hat{b}_1\|s\|)sign(s)] \\ &\quad + (\hat{b}_0 - b_0)|s| + (\hat{b}_1 - b_1)|s|\|x\| \\ &\le -\eta s^2 + |d(x,t)||s| - (\hat{b}_0 + \hat{b}_1\|x\|)|s| \\ &\quad + (\hat{b}_0 - b_0)|s| + (\hat{b}_1 - b_1\|x\|)|s| \\ &\le -\eta s^2 + |d(x,t)||s| - (b_0 + b_1\|x\|)|s| \\ &\le -\eta s^2 \le 0 \end{aligned} \tag{36}$$

Using the Lyapunov stability theorem,[38] the estimation values $\hat{b}_0$ and $\hat{b}_1$ are bounded. That is, there exist positive constants b_i^* such that $\hat{b}_1 \le b_i^*$, $i = 0, 1$, for $\forall t \ge 0$ This completes the proof the upper bound property Eq. (28).

Now it will be proven that the system states in Eq. (6) reach the nonsingular terminal sliding surface $s = 0$ within a finite time. The proof procedure is inspired by F. Plestan's work.[14]

The following Lyapunov function candidate is considered

$$V_3 = \frac{1}{2}s^2 + \frac{1}{2\gamma_0}(\hat{b}_0 - b_0^*)^2 + \frac{1}{2\gamma_1}(\hat{b}_1 - b_1^*)^2 \tag{37}$$

where γ_0 and γ_1 are positive constants. Taking the time derivative of Eq. (37) and using the same procedure that was used to get Eqs. (35) and (36), we can obtain

$$\begin{aligned} \dot{V}_3 &\le -\eta s^2 + |d(x,t)||s| - (\hat{b}_0 + \hat{b}_1\|x\|)|s| \\ &\quad + \frac{\kappa_0}{\gamma_0}(\hat{b}_0 - b_0^*)|s| + \frac{\kappa_1}{\gamma_1}(\hat{b}_1 - b_1^*)|s|\|x\| \end{aligned} \tag{38}$$

Combining Eqs. (38) and (15) yields

$$\begin{aligned} \dot{V}_3 &\le (b_0 + b_1\|x\|)|s| - (\hat{b}_0 + \hat{b}_1\|x\|)|s| + \frac{\kappa_0}{\gamma_0}(\hat{b}_0 - b_0^*)|s| + \frac{\kappa_1}{\gamma_1}(\hat{b}_1 - b_1^*)|s|\|x\| \\ &\le (b_0 + b_1\|x\|)|s| - (\hat{b}_0 + \hat{b}_1\|x\|)|s| + (b_0^* + b_1^*\|x\|)|s| \\ &\quad - (b_0^* + b_1^*\|x\|)|s| + \frac{\kappa_0}{\gamma_0}(\hat{b}_0 - b_0^*)|s| + \frac{\kappa_1}{\gamma_1}(\hat{b}_1 - b_1^*)|s|\|x\| \\ &\le [(b_0^* - b_0) + (b_1^* - b_1)\|x\|]|s| \\ &\quad + (\hat{b}_0 - b_0^*)\left(-|s| + \frac{\kappa_0}{\gamma_0}|s|\right) + (\hat{b}_1 - b_1^*)\left(-|s|\|x\| + \frac{\kappa_1}{\gamma_1}|s|\|x\|\right) \end{aligned} \tag{39}$$

Using Eqs. (28), (39) can be rewritten as

$$\begin{aligned} \dot{V}_3 &\le -[(b_0^* - b_0) + (b_1^* - b_1)\|x\|]|s| \\ &\quad - \left(-|s| + \frac{\kappa_0}{\gamma_0}|s|\right)\left|\hat{b}_0 - b_0^*\right| - \left(-|s|\|x\| + \frac{\kappa_1}{\gamma_1}|s|\|x\|\right)\left|\hat{b}_1 - b_1^*\right| \end{aligned} \tag{40}$$

We denote

$$\beta_s = (b_0^* - b_0) + (b_1^* - b_1)\|x\|$$

$$\beta_0 = -|s| + \frac{\kappa_0}{\gamma_0}|s|$$

$$\beta_1 = -|s|\|x\| + \frac{\kappa_1}{\gamma_1}|s|\|x\|$$

We always chose b_i^* and γ_i, $(i = 0, 1)$ such that $b_i^* > b_i$ and $\kappa_i > \gamma_i$, which yields $\beta_s > 0$ and $\beta_i > 0$ The result is as follows:

$$\begin{aligned}\dot{V}_3 &\le -\beta_s|s| - \beta_0|\hat{b}_0 - b_0^*| - \beta_1|\hat{b}_1 - b_1^*| \\ &\le -\sqrt{2}\beta_s\frac{|s|}{\sqrt{2}} - \beta_0\sqrt{2\gamma_0}\frac{|\hat{b}_0 - b_0^*|}{\sqrt{2\gamma_0}} - \beta_1\sqrt{2\gamma_1}\frac{|\hat{b}_1 - b_1^*|}{\sqrt{2\gamma_1}} \\ &\le -\min\{\sqrt{2}\beta_s, \beta_0\sqrt{2\gamma_0}, \beta_1\sqrt{2\gamma_1}\}\left(\frac{|s|}{\sqrt{2}} + \frac{|\hat{b}_0 - b_0^*|}{\sqrt{2\gamma_0}} + \frac{|\hat{b}_1 - b_1^*|}{\sqrt{2\gamma_1}}\right)\end{aligned} \quad (41)$$

Applying Lemma 4 and denoting $\beta = \min\{\sqrt{2}\beta_s,\ \beta_0\sqrt{2\gamma_0},\ \beta_1\sqrt{2\gamma_1}\}$ results in the following:

$$\dot{V}_3 \le -\beta\left[\left(\frac{|s|}{\sqrt{2}}\right)^2 + \left(\frac{|\hat{b}_0 - b_0^*|}{\sqrt{2\gamma_0}}\right)^2 + \left(\frac{|\hat{b}_1 - b_1^*|}{\sqrt{2\gamma_1}}\right)^2\right]^{1/2} = -\beta V_3^{1/2} \quad (42)$$

By using Lemma 3, it is concluded that the system trajectories in Eq. (14) converge to the terminal sliding surface $s = 0$ in a finite time, $T \le \frac{2V_3^{1/2}(0)}{\beta}$. This completes the proof of Theorem 2.

Remark 2.[12] In order to eliminate the possible chattering phenomenon, we can use the function $s / (|s| + \rho)$, (here ρ is a small positive constant) to approximate $sign(s)$ in the control law Eq. (29).

Remark 3. According to Theorems 1 and 2, the proposed AFTC law Eq. (29) with the adaptive laws in Eqs. (30)~(31) and the nonsingular terminal sliding surface in Eq. (16) can make the state trajectories of the system Eq. (14) converge to zero in a finite time.

Remark 4. The proposed RAFTC approach in Theorems 1 and 2 can be extended to a broader class of n- order $(n > 2)$ nonlinear dynamic systems, which are expressed as follows:

$$\begin{cases}\dot{X}_1 = X_2 \\ \dot{X}_2 = F(X_1, X_2, t) + D(X_1, X_2, t) + G(X_1, X_2, t)U(t)\end{cases} \quad (43)$$

where $X_1 = [x_1, x_2, \ldots, x_n]^T$, $F(X_1, X_2, t)$ are vector functions, $D(X_1, X_2, t)$ represents the uncertainties and external disturbances satisfying $\|D\| \le B_0 + B_1\|X_1\| + B_2\|X_2\|$, $G(X_1, X_2, t)$ is a nonsingular matrix, and $U(t) = [u_1, u_2, \ldots, u_n]^T$ is a control vector. The proposed AFTC law for the system in Eq. (43) can be designed as follows

Choose the nonsingular terminal sliding surfaces as

$$S = X_2 + \int_0^t (K_1X_1^{[A_1]} + K_2X_2^{[A_2]} + K_3X_1 + K_4X_1^3)d\tau \quad (44)$$

where $K_j = diag(k_{1j}, k_{2j}, \ldots, k_{nj})$, $k_{lj} > 0$ for $l = 1, 2, \ldots, n$ and $j = 1, 2, 3, 4$, $A_i = diag(\alpha_{1i}, \alpha_{2i}, \ldots, \alpha_{ni})$, $\alpha_{li} > 0$ for $l = 1, 2, \ldots, n$ and $i = 1, 2$, and the following notations are employed

$$X_1^{[A_i]} = [x_1^{[\alpha_{1i}]}, x_2^{[\alpha_{2i}]}, \ldots, x_n^{[\alpha_{ni}]}]^T \quad (45)$$

Then, if the AFTC law is designed as in Eq. (46) and the adaptive laws are designed as in Eqs. (47) to (49), the state trajectories of the high-order system in Eq. (43) will converge to the sliding surface Eq. (44) and zero along $S = 0$ in a finite time, respectively:

$$\begin{aligned}U(t) = &-G(X_1, X_2, t)^{-1}[F(X_1, X_2, t) + K_1X_1^{[A_1]} + K_2X_2^{[A_2]} \\ &+ K_3X_1 + K_4X_1^3 + \mathrm{H}S + (\hat{B}_0 + \hat{B}_1\|X_1\| + \hat{B}_2\|X_2\|)sign(S)]\end{aligned} \quad (46)$$

$$\dot{\hat{B}}_0 = \kappa_0\|S\| \quad (47)$$

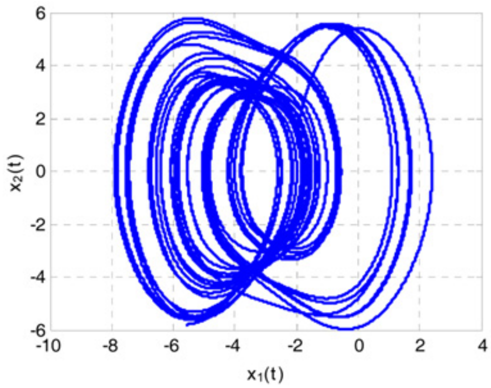

Fig. 1 Chaotic state space

$$\dot{\hat{B}}_1 = \kappa_1\|S\|\|X_1\| \quad (48)$$

$$\dot{\hat{B}}_2 = \kappa_2\|S\|\|X_2\| \quad (49)$$

where $\mathrm{H} = diag(\eta_1, \eta_2, \ldots, \eta_n)$, $\eta_i > 0$ for $i = 1, 2, \ldots, n$; $\hat{B}_m$ are the estimations of B_m, $m = 0, 1, 2$ and $\kappa_m > 0$, $m = 0, 1, 2$.

Therefore, the proposed control strategy in this paper can be applied to any uncertain nonlinear system that is of the form in Eq. (43) or can be transformed to the form in Eq. (43). For example, it can be used for the stabilization or trajectory tracking of mechanical systems as magnetic levitation systems, robotic systems, etc.; or for the chaos control, synchronization and anti-synchronization of uncertain chaotic systems.

4. Numerical Simulation

In this section, we present an illustrative example to verify the validity of the proposed control scheme by considering suppression of chaotic behavior in horizontal platform system (HPS). The motion equations of this HPS are given by[7]

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -ax_2 - b\sin x_1 + q\cos x_1 \cdot \sin x_1 + h\cos\omega t + d(x, t) + u(t)\end{cases} \quad (50)$$

where the parameter values that are employed to simulate the HPSs system are a=4/3, b=3.776, q=4.6×10^{-6}, h=34/3, ω=1.8, and the unknown uncertainty and external disturbance value is assumed to be $d(x, t) = 0.2\sin(3t)x_2 + 0.1\cos(2t)$, and the initial conditions are set to $x_1(0) = -2$ and $x_2(0) = 1$.

The chaotic behavior of the HPS (50) without control is shown in Figs. 1 and 2. The control objective is to suppress the oscillations of this system by using the proposed AFTC approach.

The parameters of the sliding surface Eq. (16) are then selected as k_1=5, k_2=10, k_3=10, k_4=5, α_1=1/2, and $\alpha_2 = 2/3$ and the control parameters of the proposed AFTC in Theorem 2 are chosen as κ_0=1, α_1=0.05, η=15, and $\hat{b}_0(0) = \hat{b}_1(0) = 0$.

The time responses of the closed-loop system are presented in Figs. 3 to 5. The state trajectories of the closed-loop system and the control input are illustrated in Fig. 3. It is seen that the chaotic behavior of the system has been suppressed and the state trajectories reach the origin

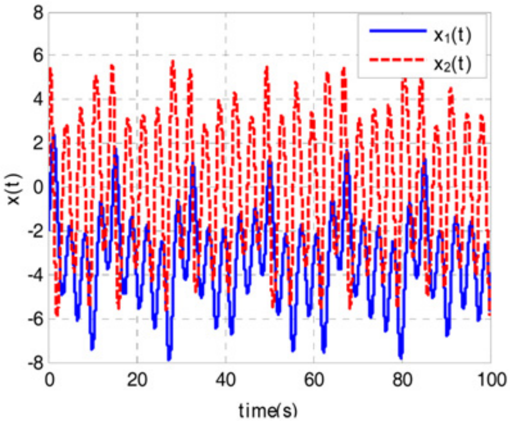

Fig. 2 System state trajectories without control

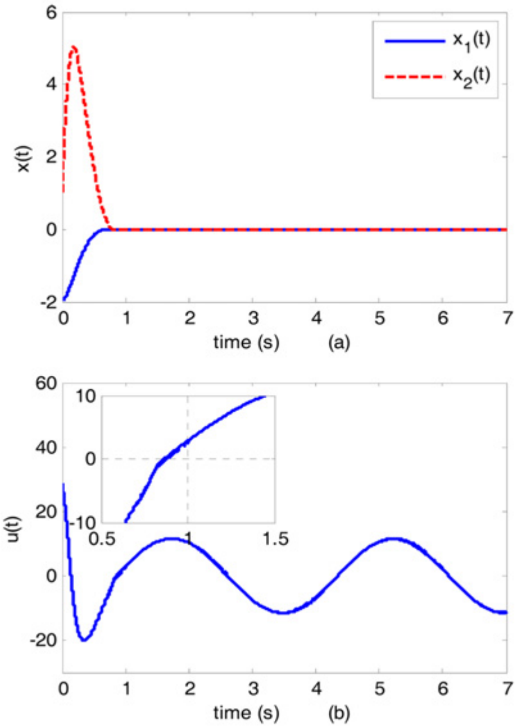

Fig. 3 Time response of the closed-loop system: (a) state trajectories, (b) control input

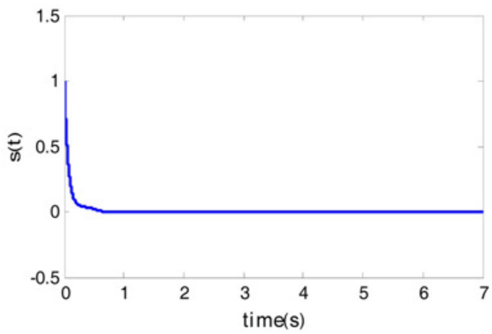

Fig. 4 Time responses of the nonsingular terminal sliding surface

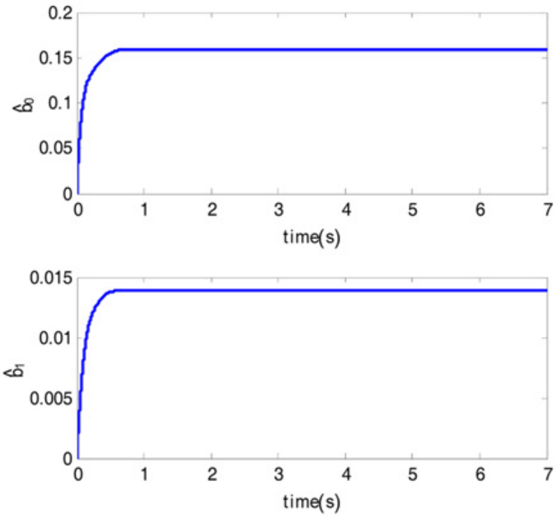

Fig. 5 Time responses of the estimated parameters

in a finite amount of time. The response of the sliding mode surface is shown in Fig. 4. It can be observed that the sliding surface converges to zero in a finite time. In Fig. 5, the estimations of the parameters b_0 and b_1 are plotted.

5. Conclusions

In this paper, a novel robust adaptive finite-time control method is proposed for a class of nonlinear systems without relying on a priori knowledge of uncertainties and unknown disturbances. The proposed control strategy makes the system states vanish at the origin in finite time without any singularity problems. The strict proof of the globally finite-time stability of the closed-loop system has been accomplished. The numerical simulation results of the chaos control of a non-autonomous horizontal platform system have demonstrated the effectiveness and robustness of the proposed method.

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