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Observer-based Dynamic Parameter Identification for Wheeled Mobile Robots

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This paper presents a novel method for online dynamic parameter identification in wheeled mobile robots with nonholonomic constraints. First, the direct dynamic model of a mobile robot is obtained and reformulated in the linear form of dynamic parameters. Then, an adaptive estimation routine is proposed in order to identify the robot dynamic parameters with high accuracy and in finite time without requiring the measurement of the acceleration state vector. Based on Solidworks/SimMechanics, a virtual prototype with the structure of a real mobile robot system is established to implement in the simulation. The simulation results demonstrate the effectiveness of the proposed approach for identifying the mobile robot dynamic parameters.

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NOMENCLATURE

oxy = Global coordinate system

- OXY = Coordinate system fixed to the mobile platform
- O_p = Mass center of the WMRs without wheels and rotors

 $O_{w1,2}$ = Center of mass of the left and right wheel

- $O_{r1,2}$ = Center of mass of the left and right rotor
- I_p = Moment of inertia of the mobile platform about O₁
- I_{wm} = Moment of inertia of the wheel about its diameter
- I_w = Moment of inertia of the wheel about its rotation axle
- I_m = Moment of inertia of the rotor about its diameter
- I_r = Moment of inertia of the rotor about its rotation axle
- k_r = Gear reduction ratio
- m_p = Mass of the WMRs without wheels and rotors
- m_w = Mass of each wheel
- $m_r =$ Mass of each rotor
- a =Distance from O to $O_{w1,2}$

b =Distance from O to contact point of wheel and ground (d_x, d_y) = Position of O_p with respect to OXY coordinate (c_x, c_y) = Position of O_{r1,2} with respect to OXY coordinate *r* = Wheel radius

 $\theta = [\theta_R, \theta_L]^T =$ Angular displacement vector of wheels $\eta = [\eta_1, \eta_2]^T =$ Angular velocity vector of wheels

1. Introduction

Wheeled mobile robots (WMRs) are becoming increasingly important in advanced application in fields such as manufacturing, medicine, and health care. In literature, the control research for nonholonomic WMRs has been centered on stabilization and tracking problems. Most proposed control algorithms rely on the kinematic model with nonholonomic assumptions such as backstepping,¹ pure pursuit,² neural networks,³ neural fuzzy,⁴ and linearization.⁵ These algorithms completely neglect the robot dynamics. The control inputs, usually motor voltages, are assumed to instantaneously establish the desired robot velocities, known as perfect velocity tracking. In the case of heavy WMRs with good performance, however, the above assumption is no more valid in reality. In order to overcome this problem, some control schemes based on a full dynamic model have been proposed so far by accounting for dynamic effects caused by mass, friction, and inertia.⁶⁻¹⁰ The control performance of the dynamic based controller has been shown to be better than that of the kinematics-based controller. Despite the better performance could be achieved, these algorithms mainly rely on an accurate physical dynamic model that is difficult to obtain in practice. Moreover, WMRs sometimes have to handle unknown payloads, which can cause changes in the location of the center of gravity, the mass, and the inertia of the system. In order to apply successful high performance control



algorithms, the accurate estimates of the robot dynamic parameters are required.

The dynamic parameters also play a key role in fault detection and diagnosis in WMRs. In many applications, it is important not only to control a robot tracking a reference trajectory with high performance but also maintain robotic performance at a high level of safety and reliability. Because most of WMRs are designed to act autonomously, any faults can cause serious damage to both the robotic system and the working environment. The fault problems in WMRs have been investigated in many papers.¹¹⁻¹³ In references^{14,24} the authors proposed a novel method to detect and isolate WMRs process faults by taking into account the robot dynamic model. The sensitivity of that fault diagnosis mostly depends on the accuracy of the estimated dynamic parameters. If the dynamic parameters with high accuracy are applied, the system is able to detect and isolate faults earlier. Hence, the effect of a fault on a robot system will be reduced.

In recent years, various effective algorithms have been proposed to identify dynamic parameters in robotic systems. These identification techniques can be classified into two approaches: inverse dynamic identification model (IDIM)^{15,25,26} method and output error (OE)^{15,27,28} method. The IDIM method is based on the inverse dynamic model, which is linear with respect to the dynamic parameters. This allows the use of the linear least-squares technique to estimate the parameters. This method has been successfully applied to identify inertial and friction parameters of industrial robots. However, this method requires a well-tuned derivative band pass filtering of joint positions in order to calculate the joint velocities and accelerations. The OE method is based on the minimization of the quadratic error between an actual output and a simulated output of the system, assuming that both the actual and simulated systems have the same input. The difficulties in this method arise from the choice of initial conditions, resulting in multiple or local solutions. Both the IDIM and OE methods only show the efficiency in the offline process due to the computational complexity. The sensitivity of joint accelerations with noise is also a reason why the IDIM method is impossible in online processes. In WMRs, online identification of dynamic parameters can be critical for system performance when an unknown payload must be addressed. In Ref. 16, the authors present a mutual information based observable metric for the online dynamic parameter identification of mobile manipulators. The simulation and experimental results show the effectiveness of their algorithm. However, the results still depend on the assumption that the accelerations are measurable.

In the present paper, a novel approach is presented for the online dynamic parameter identification of WMRs. First, the kinematic and dynamic model of WMRs is obtained based on the assumption of pure rolling motion and Lagrangian formulation. The effects of the mass and inertia of the electric motors are also taken into account. It is worth noting that the result of the inertia matrix is a constant positive definite symmetric matrix. With this notable result, the direct dynamic model is arranged according to linear relations resulting in a nonlinear system with linear parameterized of dynamic parameters. There have been many contributions in parameter estimation of linear parameterized nonlinear systems.¹⁷⁻¹⁹ In this paper, the novel parameter estimation routine¹⁸ is applied with some modifications. The estimation results are guaranteed under the presence of modeling uncertainties and external



Fig. 1 Differential-driven WMRs

disturbance. It is helpful to note that there is no need for acceleration measurements in this approach. In order to verify the proposed approach, a virtual platform is built up based on Solidworks and Matlab SimMechanics software.^{20,21} First, the WMRs is designed and assembled using Solidworks 3D CAD software. The robot dynamic parameters can be obtained with high accuracy by analyzing the assembly robot model. Then, the virtual prototype is generated by importing the assembly model from Solidworks into Matlab Simulink/SimMechanics. The simulation results based on the generated virtual prototype show that the developed approach is very effective for identifying the robot dynamic parameters.

This paper is organized as follows: In Section 2, the robot kinematics and dynamics are described. In Section 3, the proposed algorithm for online dynamic parameter identification is presented in detail. The virtual prototype based on Solidworks/SimMechanics is given in Section 4. The simulation results are shown in Section 5. Section 6 contains the research conclusions.

2. Kinematic and Dynamic Models of The Wheeled Mobile Robot

In this section, we briefly describe the modeling of the WMRs used in our paper. First, the kinematic model is derived based on the assumption of the pure rolling motion of each wheel. Then, the dynamic model is formulated by using the Lagrangian formulation.

The geometric model of a differential-drive WMRs is shown in Fig. 1. This WMRs has two drive wheels and two ball caster wheels. Each drive wheel is driven by an actuator (electric motor) and is sensed by an incremental encoder. The effect of the ball castor cannot be modeled exactly and will be treated as un-modeled dynamic disturbances. In many papers, in order to simplify the dynamic model, many assumptions are made as follows. 1) The effects of motor gearbox mass and inertia are neglected. 2) The mass center of the platform is in the geometric symmetric line of the platform. 3) The mass center of the wheel coincides with the contact point between the wheel and the ground from the top view. These assumptions do not hold in reality, especially when WMRs have to handle the unknown payloads. In this paper, we consider the general case of WMRs when these assumptions are removed.

The configuration of the WMRs can be described by five generalized coordinates: $q = [x, y, \phi, \theta_R, \theta_L]^T$ where $X = [x, y, \phi]^T$ are the Cartesian coordinates of O and the robot's orientation with reference to the o-xy frame and $\theta = [\theta_R, \theta_L]^T$ is the angular displacement vector of the wheels.

2.1 Kinematic model

We assume that the WMRs has pure rolling motion. The velocity of O must be in the direction of the axis of symmetry OX_c and the wheels do not slip. Thus, the three following constraints are assumed

$$-\dot{x}\sin\phi + \dot{y}\cos\phi = 0 \tag{1}$$

$$\dot{x}\cos\phi + \dot{y}\sin\phi + b\dot{\phi} = r\dot{\theta}_R \tag{2}$$

$$\dot{x}\cos\phi + \dot{y}\sin\phi - b\dot{\phi} = r\dot{\theta}_L \tag{3}$$

The constraints Eqs. (1)~(3) can be rewritten as

$$A(q)\dot{q} = 0 \tag{4}$$

Where

$$A(q) = \begin{bmatrix} -\sin\varphi\cos\varphi & 0 & 0 & 0\\ \cos\varphi & \sin\varphi & b & -r & 0\\ \cos\varphi & \sin\varphi & -b & 0 & -r \end{bmatrix}$$
(5)

Let S(q) be a full rank matrix formed by a set of smooth and linearly independent vector fields spanning the null space of A(q) so that A(q)S(q)=0

S(q) can be selected as

$$S(q) = \begin{bmatrix} \frac{r}{2}\cos\phi \frac{r}{2}\cos\phi \\ \frac{r}{2}\sin\phi \frac{r}{2}\sin\phi \\ \frac{r}{2b} - \frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$
(6)

The kinematic model of the WMRs is given by

$$\dot{q} = S(q)\eta \tag{7}$$

where $\eta = [\eta_1 \ \eta_2]^T = [\dot{\theta}_R \ \dot{\theta}_L]^T$ is the vector of the angular velocities of two wheels.

2.2 Dynamic model

With the Lagrangian formulation, the equation of the motion of the WMRs is given by:

$$M_{0}(q)\ddot{q} + V_{0}(q,\dot{q}) + F_{0}(\dot{q}) + \tau_{0}^{d} = B\tau - A^{T}(q)\lambda$$
(8)

where $M_0(q)$ is the symmetric positive definite inertia matrix, $V_0(q, \dot{q})$ is the centripetal and Coriolis matrix, $F_0(\dot{q})$ is the vector of the friction force, τ_0^d represents the unknown disturbances, *B* is the input transformation matrix, τ is the torque input vector, and λ is the vector of the constraint forces. The details of the matrices $M_0(q)$ and $V_0(q, \dot{q})$ are shown in Appendix 1.

In order to eliminate the constraint matrix $A^{T}(q)\lambda$, we differentiate Eq. (7), substitute that result into Eq. (8), and then multiply by S^{T} . We obtain the resulting dynamic model described by

$$M\dot{\eta} + V(\eta) + F(\eta) + \tau_d = \tau \tag{9}$$

Where

$$M_{11} = I_w + k_r^2 I_r + (mb^2 + I - 2m_p bd_y) \frac{r^2}{4b^2}$$
(10)

$$M_{12} = M_{21} = (mb^2 - I)\frac{r^2}{4b^2}$$
(11)

$$M_{22} = I_w + k_r^2 I_r + (mb^2 + I + 2m_p bd_y) \frac{r^2}{4b^2}$$
(12)

$$V_{11} = V_c(\eta_1 \eta_2 - \eta_2^2)$$
(13)

$$V_{21} = V_c(\eta_1 \eta_2 - \eta_1^2)$$
(14)

with

$$I = I_p + m_p (d_x^2 + d_y^2) + 2(I_{mw} + m_w a^2) + 2(I_{mr} + m_r (c_x^2 + c_y^2))$$
$$m = m_a + 2m_w + 2m_r$$

and $V_c = (m_p d_x + 2m_r c_x)r^3/4b^2$

where I_p is the moment of inertia of the mobile platform about O_1 , I_{wm} is the moment of inertia of the wheel about its diameter, I_w is the moment of inertia of the wheel about its rotation axle, I_{rm} is the moment of inertia of the rotor about its diameter, I_r is the moment of inertia of the rotor about its diameter, I_r is the moment of inertia of the rotor about its diameter, I_r is the moment of inertia of the rotor about its rotation axle, k_r is the gear reduction ratio, m_p is the mass of the WMRs without wheels and rotors, m_w is the mass of each wheel, m_r is the mass of each rotor, a is the distance from O to $O_{w1,2}$, b is the distance from O to contact point of wheel and ground, (d_x, d_y) are the position of O_p with respect to OXY coordinate, (c_x, c_y) are the position of $O_{r1,2}$ with respect to OXY coordinate, r is the wheel radius.

The friction term in Eq. (9) is of the form²²

$$F(\eta) = F_v \eta + F_d \tag{15}$$

where F_v and F_d are the coefficient matrices of the viscous friction and the dynamic friction terms, respectively.

Assumption 1: The friction is uncoupled among the joints. Since friction is a local effect, so that the viscous friction and dynamic friction are assumed to have the following forms, respectively

$$F_{v}\eta = diag(F_{vi})\eta \tag{16}$$

$$F_d = diag(F_{di})sgn(\eta) \tag{17}$$

where F_{vi} , F_{di} (*i* = 1, 2) are constant friction coefficients.

Assumption 2: Each element of the unmodeled dynamic disturbance is bounded by

$$\left|\tau_{di}\right| \le \tau_{Bi} \quad (i=1,2) \tag{18}$$

where $\tau_{Bi} > 0$ (*i* = 1, 2) are bounded known constants of the unmodeled dynamic disturbances.

Remark 1: M is symmetric, positive definite and independent of the state vector and its derivatives.

Remark 2: After eliminating the kinematic constraints, V still depends on $\dot{\phi}$, but this dependence can eliminated by using the following relationship $\dot{\phi} = (r\dot{\theta}_R - r\dot{\theta}_I)/2b$.

3. Dynamic Parameter Identification

In this section, an online algorithm for identifying the robot dynamic parameters is presented in detail. Based on the inverse dynamic model in Section 2, the direct dynamic model is obtained and arranged in the linear form of the dynamic parameters. Then, an adaptive estimation routine is proposed to estimate the dynamic parameters.

The WMRs dynamic model can be represented in the following form

$$\dot{\eta} = M^{-1} (\tau - V - F_v \eta - F_d) + \mathcal{G}$$
⁽¹⁹⁾

where $\mathcal{G} = -M^{-1}\tau_d$ is assumed to satisfy a known upper bound $|\mathcal{G}| \leq \mathcal{G}_B < \infty$.

By inserting Eqs. (15)~(17) into Eq. (19), we obtain

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{pmatrix} m_{11} m_{12} \\ m_{21} m_{22} \end{pmatrix} \begin{pmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} - V_c \begin{bmatrix} \eta_1 \eta_2 - \eta_2^2 \\ \eta_1 \eta_2 - \eta_1^2 \end{bmatrix} - \begin{bmatrix} F_{v_1} & 0 \\ 0 & F_{v_2} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

$$- \begin{bmatrix} F_{d1} sgn(\eta_1) \\ F_{d2} sgn(\eta_2) \end{bmatrix} + \mathcal{G}$$

$$(20)$$

where $m_{2\times 2} = M^1$ is the inverse of the inertia matrix with the corresponding elements as follows

$$m_{11} = \frac{M_{22}}{M_{11}M_{22} - M_{12}M_{21}}; \quad m_{12} = \frac{-M_{21}}{M_{11}M_{22} - M_{12}M_{21}}$$
$$m_{21} = \frac{-M_{21}}{M_{11}M_{22} - M_{12}M_{21}}; \quad m_{22} = \frac{M_{11}}{M_{11}M_{22} - M_{12}M_{21}}$$

We consider the online identification process of the following base dynamic parameters

$$\theta_0 = [m_{11} \ m_{12,21} \ m_{22} \ V_c \ F_{v1} \ F_{v2} \ F_{d1} \ F_{d2}]^T$$
(21)

These parameters play an important key in both robot control⁸ and robot fault diagnosis.¹⁴ If the estimated high accuracy parameters are

applied, the tracking control performance will be much improved. The method of using neural networks,^{7,9} fuzzy,¹⁰ and sliding mode²³ methods to track the robot dynamics is no longer needed. In the fault diagnosis field,^{14,24} the sensitivities of the fault detection and isolation thresholds are also improved. The fault will be detected and isolated earlier and the damage caused by the fault can be reduced. However, the parameters in Eq. (21) cannot be estimated directly. In the following process, the algorithm is proposed in order to estimate the intermediate parameters. The estimate of the base parameters in Eq. (21) can then be obtained from the estimated results of the intermediate parameters.

The dynamic model in Eq. (20) can be rearranged in the linear parameterized form as follows

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} g(\eta, \tau)\theta_1 \\ g(\eta, \tau)\theta_2 \end{bmatrix} + \mathcal{G}$$
(22)

where θ_1 , θ_2 are the vectors of the intermediate parameters with the corresponding elements as follows

$$\theta_1 = \left[m_{11} \ m_{12} \ m_{11} V_c \ m_{12} V_c \ m_{11} F_{v1} \ m_{12} F_{v2} \ m_{11} F_1 + m_{12} F_{d2} \right]^T$$
(23)

$$\theta_2 = [m_{21} \ m_{22} \ m_{21}V_c \ m_{22}V_c \ m_{21}F_{v1} \ m_{22}F_{v2} \ m_{21}F_1 + m_{22}F_{d2}]^T$$
(24)

and the $g(\eta, \tau)_{1\times 7}$ is a known smooth vector as

$$g(\eta, \tau) = [\tau_1 \ \tau_2 \ \eta_2^2 - \eta_1 \eta_2 \ \eta_1^2 - \eta_1 \eta_2 \ -\eta_1 \ -\eta_2 \ -1]$$
(25)

Here, we assume that the robot wheels move only in the positive direction so that $sgn(\eta_1)=1$ $sgn(\eta_2)=1$. Thus the number of intermediate parameters is decreased and the computation time is also reduced. The parameters θ_1 , θ_2 are assumed to lie inside a max-min region defined by $[\theta_{1,2}^{min}, \theta_{1,2}^{max}]$.

The following parameter adaptation is proposed¹⁸

$$\begin{bmatrix} \dot{\hat{\eta}}_1\\ \dot{\hat{\eta}}_2 \end{bmatrix} = \begin{bmatrix} g(\eta, \tau)\hat{\theta}_1\\ g(\eta, \tau)\hat{\theta}_2 \end{bmatrix} + ke_\eta + \begin{bmatrix} \dot{w}\hat{\theta}_1\\ w\hat{\theta}_2 \end{bmatrix}$$
(26)

where $\hat{\theta}_1$, $\hat{\theta}_2$ is a parameter estimate generated by the update law $\hat{\theta}_1$, $\dot{\hat{\theta}}_2$ in the later part of Eqs. (35)~(36), k > 0 is the design constant, $e_\eta = \eta - \hat{\eta}$ is the prediction error and $w_{1\times7}$ is the output of the auxiliary filter

$$\dot{w} = g(\eta, \tau) - kw \quad w(t_0) = O_{1 \times 7}$$
 (27)

From Eq. (22) and Eq. (26), we can derive the state prediction error

$$\dot{e}_{\eta} = -ke_{\eta} + \begin{bmatrix} g(\eta,\tau)\tilde{\theta}_{1} \\ g(\eta,\tau)\tilde{\theta}_{2} \end{bmatrix} - \begin{bmatrix} w\dot{\hat{\theta}}_{1} \\ w\dot{\hat{\theta}}_{2} \end{bmatrix} + \mathcal{G}$$
(28)

with the initial condition $e_{\eta}(t_0) = \eta(t_0) - \hat{\eta}(t_0)$. By defining the auxiliary variable $\xi_{2\times 1}$ as $\xi = e_{\eta} - [w\tilde{\theta}_1 \ w\tilde{\theta}_2]^T$, it follows from Eqs. (27) and (28) that ξ can be generated from

$$\begin{split} \dot{\xi} &= \dot{e}_{\eta} - \begin{bmatrix} \dot{w}\tilde{\theta}_{1} \\ \dot{w}\tilde{\theta}_{2} \end{bmatrix} + \begin{bmatrix} w\dot{\hat{\theta}}_{1} \\ w\dot{\hat{\theta}}_{2} \end{bmatrix} \\ &= ke_{\eta} + \begin{bmatrix} g(\eta,\tau)\tilde{\theta}_{1} \\ g(\eta,\tau)\tilde{\theta}_{2} \end{bmatrix} + g - \begin{bmatrix} (g(\eta,\tau) - kw)\tilde{\theta}_{1} \\ (g(\eta,\tau) - kw)\tilde{\theta}_{2} \end{bmatrix} = -k\xi + g \end{split}$$

$$\end{split}$$

$$(29)$$

Since \mathcal{G} is the unmodeled disturbances, the estimate of ξ is generated by

$$\dot{\xi} = -k\hat{\xi}, \ \hat{\xi}(t_0) = e_{\eta}(t_0)$$
 (30)

In order to prove the auxiliary error $\tilde{\xi} = \xi - \tilde{\xi}$ are bounded, let's consider the following Lyapunov function

$$V_{\xi} = \frac{1}{2} \tilde{\xi}^T \tilde{\xi} = \frac{1}{2} (\tilde{\xi}_1^2 + \tilde{\xi}_2^2)$$
(31)

The time-derivative of $V_{\tilde{\xi}}$ is given by

$$\dot{V}_{\xi} = \tilde{\xi}^{T} \dot{\tilde{\xi}} = \tilde{\xi}^{T} (-k\tilde{\xi} + \vartheta) = -k\tilde{\xi}^{T}\tilde{\xi} + \tilde{\xi}^{T}\vartheta$$

$$\leq -k \|\tilde{\xi}\|^{2} + \|\tilde{\xi}\| \vartheta_{B} = -k (\|\tilde{\xi}\| - \frac{\vartheta_{B}}{2k})^{2} + \frac{\vartheta_{B}^{2}}{4k}$$
(32)

This implies that $\dot{V}_{\xi} < 0$ if $|\xi| > k^{-1} \mathcal{G}_B$. Since $|\xi(t_0)| = 0$ by definition, then the auxiliary error is bounded by $|\xi| < k^{-1} \mathcal{G}_B$. This can be set arbitrarily small by selecting a suitable *k*.

Let Γ be generated from

$$\dot{\Gamma} = w^T w, \ \Gamma(t_0) = \alpha I_{7 \times 7}$$
(33)

The inverse of Γ can be obtained by

$$\dot{\Gamma}^{-1} = -\Gamma^{-1}(w^{T}w)\Gamma^{-1}, \ \Gamma^{-1}(t_{0}) = \frac{1}{\alpha}I_{7\times7}$$
(34)

The following update laws are proposed

$$\dot{\hat{\theta}}_{1} = \begin{cases} \Gamma^{-1} w^{T}(e_{\eta 1} - \hat{\xi}_{1}) & \text{if } \hat{\theta}_{1k} \in [\theta_{1k}^{\min}, \theta_{1k}^{\max}] \\ 0 & Otherwise \end{cases}$$
(35)

$$\dot{\hat{\theta}}_2 = \begin{cases} \Gamma^{-1} w^T (e_{\eta 2} - \hat{\xi}_2) & \text{if } \hat{\theta}_{2k} \in [\theta_{2k}^{\min}, \theta_{2k}^{\max}] \\ 0 & Otherwise \end{cases}$$
(36)

The initial condition for the parameter estimate $\hat{\theta}_1(t_0)$, $\hat{\theta}_2(t_0)$ can be set arbitrarily in the max-min region [$\theta_{1,2}^{min}$, $\theta_{1,2}^{max}$]. In this paper, we select the initial condition by using the nominal values of the parameters. The update laws in Eqs. (35)~(36) guarantee that the estimate parameters are always in the max-min region [$\theta_{1,2}^{min}$, $\theta_{1,2}^{max}$].

Theorem 3.1: Consider the dynamic model Eq. (22) and the parameter adaptation Eq. (26). Let w be the output of the auxiliary filter Eq. (28). The update laws are given in Eqs. (35)~(36) together with the strong condition that

$$\lim_{L \to +\infty} \lambda_{\min}(\Gamma) = +\infty \tag{37}$$

Then, the estimate parameters $\hat{\theta}_1$, $\hat{\theta}_2$ converge to a neighborhood of the true values.

Proof: Consider the following Lyapunov function

$$V_{\tilde{\theta}} = \tilde{\theta}_1^T \Gamma \tilde{\theta}_1 + \tilde{\theta}_2^T \Gamma \tilde{\theta}_2$$
(38)

The time-derivative of $V_{\tilde{\theta}}$ is given by

$$\dot{V}_{\hat{\theta}} = \tilde{\theta}_{1}^{T} \dot{\Gamma} \tilde{\theta}_{1} + 2 \tilde{\theta}_{1}^{T} \dot{\tilde{\theta}}_{1} + \tilde{\theta}_{2}^{T} \dot{\Gamma} \tilde{\theta}_{2} + 2 \tilde{\theta}_{2}^{T} \dot{\tilde{\theta}}_{2}$$

$$= \tilde{\theta}_{1}^{T} \dot{\Gamma} \tilde{\theta}_{1} - 2 \tilde{\theta}_{1}^{T} \dot{\Gamma} \dot{\tilde{\theta}}_{1} + \tilde{\theta}_{2}^{T} \dot{\Gamma} \tilde{\theta}_{2} - 2 \tilde{\theta}_{2}^{T} \dot{\tilde{\theta}}_{2}$$

$$(39)$$



Fig. 2 3D mobile robot model established in Solidworks

Table 1 Mobile robot parameters

Parameter	Nominal	True	Donomoton	Nominal	True
	Value	Value	Parameter	Value	Value
I_p (kg.m ²)	4	4.98	m_r (kg)	0.05	0.0442
m_p (kg)	55	60.90	I_{rm} (kg.m ²)	1e-05	3.74e-05
$d_{x}(\mathbf{m})$	0.15	0.1781	I_r (kg.m ²)	1e-6	1.24e-06
$d_{y}(\mathbf{m})$	0	0.0119	$c_{x}(\mathbf{m})$	0.05	0.075
I_{wm} (kg.m ²)	0.025	0.03755	$c_{y}(\mathbf{m})$	0.08	0.107
I_w (kg.m ²)	0.05	0.0738	k_r	100	100
m_w (kg)	4	4.98	<i>r</i> (m)	0.16	0.16
<i>a</i> (m)	0.23	0.2286	<i>b</i> (m)	0.23	0.23

Substituting Eqs. (34)~(36) into Eq. (39), we obtain

$$\begin{aligned} \dot{V}_{\tilde{\theta}} &= \tilde{\theta}_{1}^{T} w^{T} w \tilde{\theta}_{1} - \tilde{\theta}_{1}^{T} w^{T} (e_{\eta 1} - \hat{\xi}_{1}) + \tilde{\theta}_{2}^{T} w^{T} w \tilde{\theta}_{2} - \tilde{\theta}_{2}^{T} w^{T} (e_{\eta 2} - \hat{\xi}_{2}) \\ &= -(e_{\eta 1} - \hat{\xi}_{1} - \tilde{\xi}_{1})(e_{\eta 1} - \hat{\xi}_{1} + \tilde{\xi}_{1}) - (e_{\eta 2} - \hat{\xi}_{2} - \tilde{\xi}_{2})(e_{\eta 2} - \hat{\xi}_{2} + \tilde{\xi}_{2}) \\ &= -(e_{\eta 1} - \hat{\xi}_{1})^{2} - (e_{\eta 2} - \hat{\xi}_{2})^{2} + \tilde{\xi}_{1}^{2} + \tilde{\xi}_{2}^{2} \\ &\leq -(e_{\eta 1} - \hat{\xi}_{1})^{2} - (e_{\eta 2} - \hat{\xi}_{2})^{2} + (k^{-1} \theta_{R})^{2} \end{aligned}$$
(40)

The last term in Eq. (40) is a bounded quantity. Therefore, $V_{\hat{\partial}}$ decreases until the solutions reach a compact set determined by the right hand side of Eq. (40). According to the strong condition Eq. (37), we can conclude that the estimate parameters $\hat{\theta}_1$, $\hat{\theta}_2$ converge to a neighborhood of the true values.

4. Solidworks/SimMechanics-based Platform Design

In order to show the reality of applying the proposed algorithm proposed in Section 3, a virtual prototype with the same structure as real mobile robot system is established for implementation in the simulation. That virtual prototype is based on Solidworks/ SimMechanics.²¹ The modeling process is summarized in the following steps:

Step 1: Establish the 3D CAD model of the WMRs system in Solidworks; each part of the WMR is designed separately and assembled by using suitable joints (mates). The details of the WMRs structure (including: motors, drivers, battery, communication system, computer,



Fig. 3 Virtual prototype of the mobile robot in Matlab/Simulink



Fig. 4 Mobile robot model in Simulink

sensors, and payloads) are shown in Fig. 2. Solidworks also supports a special tool to verify the dynamic parameters of each component and the whole robot platform. The results of process verification are shown in Table 1.

Step 2: Download the SimMechanics link plug-in from Mathworks official website, and install it in Solidworks;

Step 3: Save the Solidworks drawing models established in Step 1 as an XML file.

Step 4: Import the XML file into the Simulink environment.



Fig. 5 Mobile robot trajectory



Fig. 6 Torque input and velocity output

Geometry from the CAD assembly is saved as geometry files and associated with the proper body in SimMechanics as shown in Fig. 3. The mass and inertia of each part in the assembly are also imported as a rigid body in SimMechanics as shown in the right-bottom of Fig. 4.

Step 5: Add joint actuators, sensors, and viscous and dynamic frictions to the model. Some scopes are also added to measure the actuation and simulation of the model.

The details of Steps 2, 3, and 4 can be found in the official Mathworks website.

5. Simulation Results

This section presents the simulation results in order to show the effectiveness of the proposed algorithm. The mobile robot is controlled



Fig. 7 Estimated error of the intermediate parameters

in order to track a sinusoid by using backstepping method.⁸ The nominal dynamic parameters shown in Table 1 are used in the computed torque controller. The sampling time is set as T = 1(ms). The frictions are included by setting the corresponding coefficient in the Simulink model as shown in Fig. 4. The friction coefficients are included as $F_v = diag(0.01, 0.015)$ and $F_d = diag(0.045, 0.04)$. The initial values of the intermediate parameter estimates are chosen based on the nominal values as shown in Table 2. The constants parameters in the parameter adaptation are set as k = 1 and $\alpha = 5000$.

The trajectory used in the identification must be carefully selected in order to improve the convergence rate. Such a trajectory is known as a persistently exciting trajectory. In this paper, the WMRs is controlled to track a sinusoidal trajectory as shown in Fig. 5. The simulation results demonstrate that the sinusoidal trajectory is suitable for identifying robot dynamic parameters. The torque inputs and velocity outputs are shown in Fig. 6. It can be seen that the velocities of both wheel is always greater than zero, so that $sgn(\eta_1)=1$, $sgn(\eta_2)=1$. As mentioned in Section 3, if the above conditions are satisfied, the computation time is reduced.

The estimated errors of the intermediate parameters are shown in Fig. 7. It is observed that the estimated errors converge to zero in finite time. Hence, we can conclude that the estimated values of intermediate parameters converge to true values. Additionally, it is concluded that the estimated values can be used to obtain the dynamic parameters with high accuracy. The final estimated results of the intermediate parameters are shown in Table 2, where the nominal values are obtained based on our knowledge of the robot system, the true values are obtained with

Table 2 Estimated results of the intermediate parameters

Nominal	True	Estimated	Error
Value	value	Value	(%)
0.923	0.798475	0.798614	0.017
0.229	0.246692	0.246115	0.234
0.147	0.167752	0.167936	0.110
0.037	0.051828	0.051752	0.147
0	0.007985	0.007985	0.000
0	0.002467	0.003696	0.135
0	0.041807	0.045775	0.055
0.229	0.246692	0.246116	0.233
0.923	0.754828	0.754931	0.014
0.037	0.051828	0.051754	0.143
0.147	0.158583	0.158748	0.104
0	0.002467	0.002461	0.243
0	0.007549	0.011326	0.026
0	0.040061	0.041268	0.065
	Nominal Value 0.923 0.229 0.147 0.037 0 0 0 0.229 0.147 0.037 0 0.229 0.147 0 0.923 0.037 0.147 0 0 0 0 0 0 0 0 0 0	Nominal Value True value 0.923 0.798475 0.229 0.246692 0.147 0.167752 0.037 0.051828 0 0.007985 0 0.002467 0 0.041807 0.229 0.246692 0.923 0.754828 0.037 0.051828 0.037 0.051828 0.147 0.158583 0.147 0.158583 0 0.002467 0 0.007549 0 0.040061	Nominal True Estimated Value value Value 0.923 0.798475 0.798614 0.229 0.246692 0.246115 0.147 0.167752 0.167936 0.037 0.051828 0.051752 0 0.007985 0.007985 0 0.002467 0.003696 0 0.041807 0.045775 0.229 0.246692 0.246116 0.923 0.754828 0.754931 0.037 0.051828 0.051754 0.147 0.158583 0.158748 0 0.002467 0.002461 0 0.002467 0.002461 0 0.002467 0.002461 0 0.002467 0.002461 0 0.007549 0.011326 0 0.040061 0.041268

Table 3 Estimated results of the dynamic parameters

Doromot	~~	True	Estimated	Error
Faramet	ei	value	Value	(%)
Inertia Matrix	m_{11}	0.798475	0.798614	0.017
	m_{12}, m_{21}	0.246692	0.246116	0.234
	m_{22}	0.754828	0.754931	0.014
V _c		0.210090	0.210279	0.091
Viscous Friction	F_{vl}	0.010	0.009999	0.014
	$F_{\nu 2}$	0.015	0.01501	0.050
Dynamic Friction	F_{d1}	0.045	0.044992	0.020
	F_{d2}	0.040	0.039997	0.010

the Solidworks software. The estimated results of the dynamic parameters can be deduced from the intermediate parameters using Eqs. (23)~(24). The final results are shown in Table 3 with the estimation errors close to 0.1%.

6. Conclusions

In this paper, a novel approach for identifying the dynamic parameters in WMRs has been developed. Based on the linear parameterization of the dynamic parameters in the direct dynamic model of the WMRs, an adaptive estimation routine has been proposed to estimate the dynamic parameters without requiring the measurement of the acceleration vector. A virtual prototype based on Solidworks SimMechanics provides an excellent tool to verify the proposed algorithm. The simulation results are given in detail to confirm the effectiveness of our proposed method.

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APPENDIX 1

The details of the inertia matrix $M_0(q) \in R^{5\times 5}$ and the centripetal and Coriolis matrix $V_0(q, \dot{q}) \in R^{5\times 1}$

$$M_{0}(q) = \begin{bmatrix} m & 0 & -m_{p}(d_{x} \cdot \sin\phi & 0 & 0 & 0 \\ & & +d_{y} \cdot \cos\phi) & 0 & 0 & 0 \\ 0 & m & -m_{p}(d_{x} \cdot \sin\phi & 0 & 0 & 0 & 0 \\ -m_{p}(d_{x} \cdot \sin\phi - m_{p}(d_{x} \cdot \sin\phi & I & 0 & 0 & 0 \\ +d_{y} \cdot \cos\phi) & -d_{y} \cdot \cos\phi) & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & 0 & I_{mw} & 0 \\ 0 & 0 & I_{mw} & I_{mw}$$

with

$$I = I_p + m_p (d_x^2 + d_y^2) + 2(I_{mw} + m_w a^2) + 2(I_{mr} + m_r (c_x^2 + c_y^2))$$
$$m = m_p + 2m_w + 2m_r$$