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A Novel Method for Estimating External Force: Simulation Study with a 4-DOF Robot Manipulator

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This paper proposes an algorithm to estimate external force exerted on the end-effector of a robot manipulator using information from joint torque sensors (JTS). The algorithm is the combination of Time Delay Estimation (TDE) and input estimation technique where the external force is considered as an unknown input to the robot manipulator. Based on TDE's idea, the estimator which does not require an accurate dynamics model of the robot manipulator is developed. The simultaneous input and state estimation (SISE) is used to reject not only nonlinear uncertainties of the robot dynamics but also the noise of measurements. The performance of the proposed estimation algorithm is evaluated through simulation and experiment of a four degree-of-freedom manipulator and it demonstrates the stability and feasibility in estimating the external force. The estimation results show that this approach allows inexpensive sensors as joint torque sensors to be used instead of expensive ones as force/torque sensors in robot applications.

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1. Introduction

For years, position control has been commonly utilized in robot applications. This scheme requires a manipulator to track a desired trajectory strictly, contacting with the environment that causes damage to itself and objects in its way. As a result of the increasing use of robots in various elds of industry and services, many tasks performed by a robot manipulator require it to interact with its environment with the appropriate performance, such as pushing, scraping, grinding, pounding, polishing, twisting, cutting, and excavating. To perform these tasks, not only position control but also force control is required to deal with controlling interactions between the manipulator and its environment. Hence, information on the external force exerted on a robot manipulator during the interaction between the robot and its environment becomes an important factor for force control.

To measure the external force information, a force sensor is generally used. Force sensors are often installed in the wrist or the base of robot manipulators to extract the external force acting on the tip of the end-effector. However, sensors are often too expensive to be used in many applications. Technically, force sensors are sensitive to shock. They do not usually generate reliable performance when they are exposed to tough environments. In addition, if the external force exerts at other points on the robot-body the force sensor cannot detect such external force. To deal with these drawbacks of force sensors, inexpensive sensors, such as joint torque sensors (JTS) that are equipped at the joints of the robot manipulator, may be used. An estimation method to measure the external force by using JTS will be proposed in the next sections.

During the last two decades many external force estimation methods have been suggested to realize the external force applied to a robot manipulator without force sensor. Hacksel and Salcudean used the model-based observer with accurate knowledge of the robot's dynamic model as well as measured joint angle and actuator torques.¹ In Ref. 2, Murakami and coworkers implemented a robot compliance controller employing a model-based disturbance observer that estimates the external reaction force. In Ref. 3-5 Ohishi et al. used a robot contact force observer utilizing a motor torque observer and the robot inverse dynamic model. These model-based observers demonstrated good results on direct drive manipulators with negligible unmodeled nonlinear uncertainty dynamics. Smith⁶ used a neural network to learn the entire dynamical model of their three degree-of-freedom (DOF) haptic device ofine. They assumed that the dynamics of the system does not change after the initial neural network training. With the trained network, they used the motor torque predictions to predict the



external force applied to the end-effector.

Regarding sensor-less external force estimation, some methods use information on robot performance, such as motor current, voltage, and displacement, to measure the external force. For example, Simpson⁷ used both servo motor current and position with an accurate model to estimate applied force. Similarly, Aksman⁸ combined a robot dynamic model with current feedback to calculate the external force exerted on the 2-DOF harmonically driven manipulator. Taking advantage of a piezoelectric actuator that can enable simultaneous sensing and actuation, Ronkanen⁹ used current and actuator input voltage together with displacement measurement in a force estimator. Due to its small size, however, this motor is just suitable for applications of microrobotics.

All of the techniques mentioned rely on either the dynamic model or the motor currents of the robot manipulator. Therefore, any inaccuracies on the model or motor current will cause errors in force estimation. In fact, from a practical view, obtaining an accurate dynamic model of a manipulator is arduous and time-consuming. Meanwhile, currents are notoriously unreliable due to noise with unwanted signals, friction, and other disturbance torques in actuators. Because of these limitations, most of the above methods have been either evaluated by simulation or performed on a simple low-DOF robot manipulator such as a 2-DOF planar arm. To overcome this limitation, a method for external force estimation with no accurate dynamic model is developed in Ref. 10. The external force is estimated based on Time Delay Estimation (TDE), where it is assumed that nonlinear dynamics is continuous or piecewise continuous and that the time delay is sufciently small. Then, with a small sampling time, the dynamics of system is not changed, and the external force will be extracted using information of joint torques of the manipulator measured by JTS. However, in situations with high uncertainty mechanism of a robot manipulation, the method will perform poorly.

A method that can overcome the deterministic uncertainty of a robot manipulator is proposed in Ref. 11. This estimation algorithm is built by combining the extended Kalman filter (EKF) with an adaptation law for nonlinear stochastic systems with the external force that is considered an unknown input. With this scheme, the state variables are estimated by the modied EKF, and the unknown inputs are estimated through an adaptation law derived from the Lyapunov stability condition. Similar to this estimation algorithm, an algorithm for simultaneous input and state estimation (SISE) is developed in Ref. 12. However, instead of solving the linearization of the system as in the above approach, this algorithm is developed based on a discrete-time linear system. This characteristic makes SISE simpler and easier to implement in reality. In addition, unlike the above-discussed works,¹⁻¹¹ the SISE algorithm has stability properties that have been proven explicitly. This factor is very important when using SISE to design a robot controller further on.

Therefore, except for SISE and EKF, as much accuracy as possible of the model of the system is required in most of above-discussed methods. In this research, an estimation algorithm that does not require an accurate dynamic model is considered. Hence, the above-discussed approaches are not accepted in our study, because they either need an accurate dynamic model of a robot manipulator or can be sensitive to the uncertainties of the system and measurement noise. The new estimation algorithm proposed in this research combines the advantages of TDE and SISE. The external force is considered an unknown input to the robot manipulator, the model of which is derived from TDE. Meanwhile, the input estimation based on the linear system minimum variance unbiased estimation is applied to the model to reduce nonlinear uncertainty dynamics as well as the noise of measurement. The external force is then extracted based on this combination. Estimation performance is illustrated by a 4-DOF manipulator. This paper presents simulation studies and experimental results to show the feasibility of the proposed algorithm.

2. Proposed Method

2.1 Model of the robot manipulator

The general dynamic equation of an n-DOF robot manipulator in the joint space coordinates is given by

$$\tau = M(q)\ddot{q} + V(q,\dot{q}) + G(q) + \tau_d - J(q)^T F_{ext}$$
(1)

where $\tau \in \Re^n$ is the vector of the torques applied by motors at the joints, and $F_{ext} \in \Re^6$ is the vector of the external force exerted on the end-effector of the robot manipulator; q, \dot{q} and $\ddot{q} \in \Re^n$ denote the joint angle, the joint velocity, and the joint acceleration, respectively; $M(q) \in \Re^{m \times n}$ is inertia matrix, which is the positive definite; $V(q, \dot{q}) \in \Re^n$ includes the Coriolis and centrifugal torque, and $G(q) \in \Re^n$ is the gravitational forces; τ_d represents all other unmodeled torques, such as frictions, unknown disturbance torque caused by non-linear uncertainty, etc.; J(q) is the jacobian of the robot manipulator.

Practically, obtaining accurate model parameters such as M, V, and G is challenging. In most practical cases, only estimation of the robot model is available for controller design. The disturbance torque τ_d is also unknown. Therefore, estimation technique of nonlinearities and uncertainty system is necessary. The TDE^{10,13} is a simple but effective technique for our approach. This technique utilizes time delayed signals of system variables to estimate the current system behavior. The derivation of proposed method is explained as follows:

For simplicity, $h(q, \dot{q}, \ddot{q}) = V(q, \dot{q}) + G(q)$ is denoted so that Eq. (1) can be rewritten as

$$\tau = M(q)\ddot{q} + h(q,\dot{q},\ddot{q}) + \tau_d - J(q)^T F_{ext}$$
⁽²⁾

The robot dynamic Eq. (2) can be expressed as

$$\tau = \overline{M}\ddot{q} + \overline{h} - J(q)^T F_{ext} + \tau_d \tag{3}$$

where

$$\overline{h} = [M(q) - \overline{M}] \ddot{q} + h(q, \dot{q}, \ddot{q}) \tag{4}$$

and the constant matrix $\overline{M} \in \Re^{n \times n}$ is an estimation of M. This \overline{M} will be explained more detail in next section.

Rewriting Eq. (3) in time-variant gives

$$\tau(t) = \overline{M}\ddot{q}(t) + \overline{h}(t) - J(q(t))^{T} F_{ext}(t) + \tau_{d}(t)$$
(5)

Introducing t_s as time delay of system, value τ at the previous time or at time $(t-t_s)$ will be

$$\tau(t-t_s) = \overline{M}\ddot{q}(t-t_s) + \overline{h}(t-t_s)$$

$$-J(q(t-t_s))^T F_{ext}(t-t_s) + \tau_d(t-t_s)$$
(6)

From Eqs. (5) and (6), we obtain

$$\tau(t) = \tau(t-t_{s}) + M(\ddot{q}(t) - \ddot{q}(t-t_{s})) + h(t) - h(t-t_{s}) + J(q(t))^{T} F_{ext}(t) - J(q(t-t_{s}))^{T} F_{ext}(t-t_{s}) + \tau_{d}(t) - \tau_{d}(t-t_{s})$$
(7)

The above equation can be described by

$$\tau(t) = \tau(t - t_s) + \overline{M}d(t) + \overline{h}(t) - \overline{h}(t - t_s) + \omega(t)$$
(8)

where

$$\omega(t) = \tau_d(t) - \tau_d(t - t_s) \tag{9}$$

and

$$d(t) = \ddot{q}(t) - \ddot{q}(t-t_s)$$
¹[$J(q(t))^T F_{ext}(t) - J(q(t-t_s))^T F_{ext}(t-t_s)$]
(10)

Here, the external force is considered unknown input that affects the performance of robot motion. On the other hand, the term d(t) is expressed as a function of unknown input of Eq. (8).

Simplifying Eq. (8) gives

 $-\overline{M}$

$$\tau(t) = \tau(t - t_s) + M^* u(t) + \omega(t) \tag{11}$$

where

$$M^* = \begin{bmatrix} \overline{M} & I \end{bmatrix} \tag{12}$$

and

$$u(t) = [d(t) \quad \overline{h}(t) - \overline{h}(t - t_s)]$$
(13)

Since the robot is controlled by a digital processor, if t_s is sampling time of the controller, the dynamics becomes

$$\tau_k = \tau_{k-1} + M^* u_k + \omega_k \tag{14}$$

Introducing y_k as signal obtained from joint torque sensor at all joints at time k, we have

$$y_k = \tau_k + \nu_k \tag{15}$$

where u_k is a noise of measurement.

From Eqs. (14) and (15), a linear system is obtained as

$$\begin{cases} \tau_k = \tau_{k-1} + M^* d_k + \omega_k \\ y_k = \tau_k + v_k \end{cases}$$
(16)

2.2 Simultaneously input and state estimation (SISE)

Equation system Eq. (16) can be brought in the form of the linear discrete-time system

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k \\ y_k = Cx_k + v_k \end{cases}$$
(17)

where $x_k \in \Re^n$ that correlates with joint torques represents the system state variable at time instant k, $u_k \in \Re^n$ that correlates with factor containing the external force represents the unknown input, and $y_k \in \Re^n$ that correlates JTS information of system represents the system measurement; A, B, and C are system matrices with detail values as $A = I^{(n \times n)}$, $B = M^* = [\overline{M}^{(n \times n)} I^{(n \times n)}]$, and $C = I^{(n \times n)}$. To apply the observer theory into this discrete-time linear system, the process noise and measurement noise should be mutually uncorrelated, zero-mean, and white random with variances $R_w > 0$ and $R_v > 0$, respectively.

Indeed, as determined by the Eq. (9), we have

$$\omega_k = \tau_{d_k} - \tau_{d_{k-1}} \tag{18}$$

where τ_{d_k} and $\tau_{d_{k-1}}$ are the disturbance torques of the system at time k and k-1, respectively. In most literature, the disturbance torque can be assumed as Gaussian white noise¹⁴⁻¹⁶ and independent to the state variables (joint torques).¹⁷ Therefore, ω_k is obviously Gaussian white noise and independent. However, to verify the algorithm in general case, the disturbance torque will be modeled as friction, backlash or both of them in next section.

From the observer theory for deterministic linear system,¹⁸ the input and state estimator is designed, respectively, as follows:

$$\hat{u}_k = H_k(y_{k+1} - CA\hat{x}_k) \tag{19}$$

$$\hat{x}_{k+1} = A\hat{x}_k + B\hat{u}_k + L_k(y_k - C\hat{x}_k)$$
(20)

where \hat{u}_k denotes estimation of the input estimate and \hat{x}_k the state estimation; H_k and L_k are the gain matrices of estimator that will be determined later. Equation system Eq. (17) and Eqs. (19) and (20) show the fact that x_{k+1} depends on x_k and u_k motivates \hat{x}_{k+1} to be recovered by \hat{x}_k and \hat{u}_k with a correcting term.

The estimation errors is evaluated by the covariance matrices defined as

$$P_k^u = \mathbb{E}[\tilde{u}_k \quad \tilde{u}_k^T], \quad P_k^{ux} = \mathbb{E}[\tilde{u}_k \quad \tilde{x}_k^T], \quad P_k^x = \mathbb{E}[\tilde{x}_k \quad \tilde{x}_k^T]$$
(21)

where $\tilde{u}_k = u_k - \hat{u}_k$ and $\tilde{x}_k = x_k - \hat{x}_k$, and "E" indicates expectation. These covariance matrices that affect to the stability of estimator will be define later.

Our estimator is developed based on an algorithm about simultaneous input and state estimation. The algorithm that we use in this paper is established on the ground of minimum variance unbiased estimation (MVUE), which has been commonly used to address input and state estimation.¹⁹⁻²¹ From Ref. 12, the procedures of estimation are given as Algorithm 1.

Algorithm 1: Estimate the external force **Ininitialize:** $\hat{x}_0 = E(x_0)$, $P_0^x = p_0 I$, $F_{ext_0} = 0$ (where is a large positive value) **FOR** k = 0 \rightarrow N-1 **DO**

$$\hat{O}_{k} = C\hat{P}_{k}^{x}C^{I} + CR_{w}C^{I} + R_{v};$$

$$\hat{H}_{k}^{*} = (B^{T}C^{T}\hat{O}_{k}^{-1}CB)^{-1}B^{T}C^{T}\hat{O}_{k}^{-1};$$

$$\hat{P}_{k}^{u} = \hat{H}_{k}^{*}\hat{O}_{k}\hat{H}_{k}^{*T};$$

IF K < N-1

$$\begin{split} \hat{P}_{k}^{ux} &= -\hat{H}_{k}^{*}CA\hat{P}_{k}^{x}; \\ \hat{Q}_{k} &= \begin{bmatrix} \hat{p}_{k}^{x} (\hat{p}_{k}^{ux})^{T} \\ \hat{p}_{k}^{ux} & \hat{p}_{k}^{u} \end{bmatrix}; \\ \hat{S}_{k} &= \begin{bmatrix} A & B \end{bmatrix} \hat{Q}_{k} \begin{bmatrix} A & B \end{bmatrix}^{T}; \\ \hat{T}_{k} &= \begin{bmatrix} A & B \end{bmatrix} \hat{Q}_{k} \begin{bmatrix} C & 0 \end{bmatrix}^{T}; \\ \hat{U}_{k} &= \begin{bmatrix} C & 0 \end{bmatrix} \hat{Q}_{k} \begin{bmatrix} C & 0 \end{bmatrix}^{T} + R_{v}; \\ \hat{L}_{k}^{*} &= \hat{T}_{k}U_{k}^{-1}; \\ \hat{x}_{k+1} &= A\hat{x}_{k} + B\hat{u}_{k} + \hat{L}_{k}^{*}(y_{k} - C\hat{x}_{k}); \\ \hat{P}_{k+1}^{x} &= \hat{S}_{k} - \hat{L}_{k}^{*}U_{k}^{T} + R_{w}; \end{split}$$

END IF

 $\hat{u}_k = \hat{H}_k^*(y_{k+1} - CA\hat{x}_k) \Rightarrow F_{ext}$ by Eqs. (10) and (13) END FOR

By inspecting SISE algorithm,¹⁶ it is found that its stability depends on the stability of covariance P_k^x and P_k^u , the covariance matrices of state estimation and input estimation error, respectively described in Eq. (21). Thus, the stability analysis of algorithm becomes the analysis of the upper bound of P_k^x and P_k^u as $k \to \infty$. This analysis is properly analyzed in Ref. 12.

On the other hand, in order to guarantee the stability of our proposed algorithm, the convergence analysis of estimator should be carried out. Besides, the estimation error will be examined relied on it covariance matrix properties. We have propagation of governed by

$$\hat{p}_{k+1}^{x} = D\hat{Q}_{k}D^{T} - D\hat{Q}_{k}N^{T}(N\hat{Q}_{k}N^{T} + R_{v})^{-1}N\hat{Q}_{k}D^{T} + R_{w}$$
(22)

where $D = \begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} I & B \end{bmatrix}$, $N = \begin{bmatrix} I & 0 \end{bmatrix}$ and

$$\hat{Q}_{k} = \begin{bmatrix} \hat{p}_{k}^{x} (\hat{p}_{k}^{ux})^{T} \\ \hat{p}_{k}^{ux} & \hat{p}_{k}^{u} \end{bmatrix}$$
(23)

Substituting D, N, and \hat{Q}_k into Eq. (22) gives

$$\hat{p}_{k+1}^{x} = R_{v} + 2R_{w} \tag{24}$$

Therefore, it is found that the variance P_k^x always converges to a fixed point that depends on R_v and R_w , variances of the measurement noise v_k and the process noise w_k , respectively.

3. Simulation Study

To verify the analytical issues of section 2 and demonstrate the efficiency of the proposed method of estimation for external force, simulations were carried out on a 4-DOF robot manipulator where its end-effector is exerted by arbitrary external force. The results are reported in this section. Fig. 1 depicts the 4-DOF manipulator. Table 1 shows its



Fig. 1 The 4-DOF robot manipulator

Table 1 Parameters of the robot manipulator

Parameters	Values	
l_1, l_2, l_3, l_4	319, 172.7, 300.5, 399.7 (mm)	
m_1, m_2, m_3, m_4	6.9, 4.7, 4.7, 3.3(kg)	

parameters.

The dynamical equation of the manipulator in Fig. 1 can be written as

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) + \tau_d = \tau + J^T F_{ext}$$
⁽²⁵⁾

where M(q), $V(q, \dot{q})$, G(q) and $J^{T}(q)$ will be derived from Lie Group formulation of robot dynamics.^{23,24} Analyzing kinematics of the 4-DOF manipulator with structure shown in Fig. 1, the joint twists are calculated as below

Due to the property of proposed method, only M(q) will be considered in terms of the robot dynamical equation. This inertia matrix is derived from the product of the exponentials formula based on the Lie group formulation of robot dynamics. Because of space limitations, only the final result is presented as

$$M(q) = \begin{bmatrix} M_{11} M_{12} M_{13} M_{14} \\ M_{21} M_{22} M_{23} M_{24} \\ M_{31} M_{32} M_{33} M_{34} \\ M_{41} M_{42} M_{43} M_{44} \end{bmatrix}$$
(27)

where

$$M_{11} = I_{z1} + (m_2 l_1 l_2 + m_3 l_1^2 + m_4 l_1^2 + I_{y3} + I_{y4}) \sin^2(q_2)$$
$$+ (I_{z2} + I_{z3} + I_{z4}) \cos^2(q_2)$$
$$M_{12} = M_{21} = (I_{z3} + I_{z4}) \sin(q_3) \cos(q_2)$$

$$M_{13} = M_{31} = (m_4 l_1 l_3 \cos(q_4) - m_4 l_1^2 \cos(q_4)) \sin(q_2)$$

$$-(m_3 l_1^2 + m_4 l_1 l_3 + I_{y3} + I_{y4}) \sin(q_2)$$

$$M_{14} = M_{41} = (m_4 l_1 l_3 + I_{y4}) \sin(q_2)$$

$$M_{22} = m_2 l_1^2 + I_{x2} + (m_3 l_2^2 + m_4 l_2^2 + I_{z3} + I_{z4}) \sin^2(q_3)$$

$$+ (m_3 l_1^2 + m_4 l_1^2 + I_{x3} + I_{x4}) \cos^2(q_3)$$

$$+ 2(m_3 l_1 l_2 + m_4 l_1 l_2) \sin(q_3) \cos(q_3)$$

$$M_{23} = M_{32} = 0$$

$$M_{24} = M_{42} = 0$$

$$M_{33} = M_{43} = m_3 l_1^2 + (m_3 + m_4) l_2^2 + m_4 l_3^2 + I_{y3} + I_{y4}$$

$$+ 2m_4 l_1 (l_1 - l_3) \sin(q_4) + 2m_4 l_1 (l_1 - l_3) \cos(q_4)$$

(28)

$$M_{34} = m_4(l_2^2 + l_3^2) + I_{y4} + m_4 l_2(l_1 - l_3)\cos(q_4) + m_4 l_2(l_1 - l_3)\cos(q_4)$$
$$M_{44} = m_4(l_2^2 + l_3^2) + I_{y4}$$

With the kinematics parameters based on Lie group theory and dynamics parameters of the links shown in Table 1, rewriting Eq. (28) obtains M(q) as

$$M(q) = \begin{bmatrix} \alpha + 0.0086 & 0 & \beta & \gamma \\ 0 & \delta + 0.4845 & 0 & 0 \\ \beta & 0 & \xi + 1.0249 & \sigma + 0.4005 \\ \gamma & 0 & \sigma & 0.4005 \end{bmatrix}$$
(29)

with

$$\alpha = 1.0803 \sin^{2}(q_{2})$$

$$\beta = -(0.0195 \cos(q_{4}) + 0.8046) \sin(q_{4})$$

$$\gamma = 0.3205 \sin(q_{2})$$

$$\delta = (0.4886 \sin(q_{3}) + 0.9023 \cos(q_{3}))^{2}$$

$$\zeta = 0.0211(\sin(q_{2}) + \cos(q_{2}))$$

$$\sigma = 0.0105(\sin(q_{2}) + \cos(q_{2}))$$
(30)

In simulation, the constant matrix \overline{M} mentioned in Eq. (4) will be obtained by directly using the diagonal elements of M(q).¹³ Let λ_1 , λ_2 , λ_3 and λ_4 be the four eigenvalues of M(q). After some computations, the lower bound of λ_1 , λ_2 , λ_3 and λ_4 for all q_1 , q_2 , q_3 and q_4 was found to be $\lambda = (5.11, 4.06, 3.17, 1.25)$. Thus, \overline{M} is determined as

$$\overline{M} = \begin{bmatrix} 5.11 & 0 & 0 & 0\\ 0 & 4.06 & 0 & 0\\ 0 & 0 & 3.17 & 0\\ 0 & 0 & 0 & 1.25 \end{bmatrix}$$
(31)

In practice, \overline{M} can be tuned regardless to system parameters. It is possible to begin with a small positive initial value of \overline{M} , and then increase the diagonal elements to tune the system performance.

Remembering the discrete-time linear system of the robot manipulator, the system matrices are determined as

$$A = C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$
(32)



Fig. 2 The procedures of simulation study

Table 2 Estimated errors in cases

Case	Percentage error (%)	RMSE
А	1.72	0.1541
В	2.48	0.2538
С	2.63	0.2913
D	3.27	0.3712

$$B = \begin{bmatrix} 5.11 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4.06 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3.17 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.25 & 0 & 0 & 1 \end{bmatrix};$$
(33)
$$R_{w} = R_{v} = \begin{bmatrix} 0.15 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \\ 0 & 0 & 0 & 0.15 \end{bmatrix}$$
(34)

where R_w and R_v are variances of $\{w_k\}$ and $\{v_k\}$. In fact, these factors can be modified as a threshold for reducing the noise of result.

Fig. 2 shows the whole procedures of the simulation study. In this section, suppose that robot move along an arbitrary trajectory as Fig. 3 while an external force is acting on the end-effector of robot manipulator during its motion as seen in Fig. 4. When this external force is applied to inaccurate dynamic models, including various assumed nonlinear uncertainties of the robot manipulator developed on MATLAB and Simulink, the supposed signal of JTS will be obtained respectively. Based on these JTS values, the external force will be estimated based on the proposed method and shown in detail in following cases.

3.1 Nonlinear uncertainty dynamics as Gaussian noise

By applying the algorithm derived in section 2 to the robot dynamics system in which nonlinear uncertainty dynamics is assumed as Gaussian white noise, the external force estimation results are shown in Fig. 5. The estimated external forces are close to the actual values. Table 2 shows the error of estimation result in this case.

3.2 Nonlinear uncertainty dynamics as Gaussian noise and nonlinear friction

In this assumption, a nonlinear uncertainty dynamics model including





Fig. 3 The trajectory of joints



Fig. 4 The desired external force applied to the end-effector of robot manipulator



Fig. 5 The desired external force and estimated force with nonlinear uncertainty dynamics as Gaussian noise

Coulomb and Viscous friction and Gaussian white noise is added to the robot dynamics model. Among many friction models, the LuGre²⁵ is

one of the most well-known dynamics friction model used in most robot applications. In this concept, the model has the form







Fig. 6 The desired external force and estimated force with nonlinear uncertainty dynamics as Gaussian noise and friction



Fig. 7 The desired external force and estimated force with nonlinear uncertainty dynamics as Gaussian noise and backlash

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z \tag{35}$$

$$F = \sigma_0 z + \sigma_1(v) \frac{dz}{dt} + f(v)$$
(36)

where *z* denotes the average bristle deflection. The function g(v) models the Stribeck effect, and f(v) is the viscous friction. The model behaves like a spring for small displacements. For simplicity, we used the modeling block built in MATLAB & Simulink to estimate the friction of robot performance with the model parameters are assumed to be constant: stiffness coefficient $\sigma_0 = 0.5$ N/m, and damping coefficient $\sigma_1 = 0.5$ N/m.

By applying the estimation algorithm to this assumed model of the robot manipulator based on information from JTS values, the estimated external force are shown in Fig. 6. It describes that the estimated external force are almost close to the actual values. However, error between desired and estimated value of external force is getting greater but still small with percentage error of 2.48%, as shown in Table 2.

3.3 Nonlinear uncertainty dynamics as Gaussian noise and backlash

Disturbance torque caused by Gaussian noise and backlash will be added to the robot dynamics model. A simplification of the exact physical model, also found in almost robot application regarding backlash, called dead-zone model is used in this simulation. The detail information of this model is described in Ref. 26-29, where the shaft torque, being proportional to the shaft twist $\theta_s(rad)$, is given by

$$T_s = k_s \theta_s = k_s D_\alpha(\theta_d) \tag{37}$$

where the dead-zone function is described by

$$D_{\alpha}(x) = \begin{cases} x - \alpha & x > \alpha \\ 0 & |x| = \alpha \\ x + \alpha & x < -\alpha \end{cases}$$
(38)

gives directly as a static function of the displacement θ_d of joint. The dead-zone model is a static and the shaft in this model is modeled as a pure spring. Similarly to friction model, the modeling block about backlash built in MATLAB & Simulink is used. The dead-band width



Fig. 8 The desired external force and estimated force with nonlinear uncertainty dynamics as Gaussian noise, friction, and backlash



Fig. 9 The error between desired external force and estimated force in cases

is set to be $\alpha = 0.05$ (*rad*) and $k_s = 10$ *Nm/rad*. With this assumption, Fig. 7 shows the results of the estimated external force. Not only is the estimated external force close to the actual values but also the characteristics of backlash affect the accuracy of estimated results. Indeed, changing the direction of the external force exerted on the robot manipulator will generate backlash torque. This causes error of estimated values of the external force of 2.63% shown in Table 2 confirms the feasibility of estimation algorithm.

3.4 Nonlinear uncertainty dynamics as Gaussian noise, friction, and backlash

In this assumption, all the main nonlinear uncertainties of the system, such as the Coulomb and Viscous friction, the Gaussian white noise, and the backlash, are added to the dynamic model of robot manipulator. Fig. 8 shows the result of the simulation, similar to the processes realized above. Compared with the above assumptions, this case makes greater error due to the higher nonlinear uncertainty of system. However, with a percentage error of 3.27%, this is such a good



Fig. 10 The distribution of estimation errors in cases

result and conrms the accuracy of the estimator algorithm with the presence of high nonlinear uncertainty dynamics in the robot manipulator. There are jumps of estimated external force at points that the actual external force changes direction in Figs. 7 and 8. In these cases, since the noncontinuous of backlash, and friction, little big errors occur where the applied force changes direction.

3.5 The accuracy and stability of estimation algorithm

Fig. 9 shows the error between the actual forces and the estimated forces. These errors were quickly damped out in about 0.1 (*s*) and lightly moved around zero with small RMSE (in Table 2). The error distribution graph in Fig. 10 describes the difference of estimation errors in cases of assumption. Due to the different effects caused by nonlinear dynamics uncertainty in above cases, the magnitude of errors is different. However, they all have a form of normal distribution. That means the estimation errors are almost zero-mean as analyzed in section 2.

In addition, in the simulation, we also find that \hat{P}_k^x quickly converges to \overline{P}_k :

$$\overline{\tilde{P}} = \begin{vmatrix} 0.7040 & 0 & 0 & 0 \\ 0 & 0.8441 & 0 & 0 \\ 0 & 0 & 0.8854 & 0 \\ 0 & 0 & 0 & 1.3967 \end{vmatrix}$$
(39)

This result confirms the stability and accuracy of estimation algorithm as mentioned in section 2. The trace of shown in Fig. 11 demonstrates the monotonically converging trend. This suggests that, in addition to being upper bounded, is likely to have certain convergence property and show the robust of the estimation.

4. Experiments

In this section, to enhance the feasibility of the proposed method in practical use, an application to impedance control of manipulator is implemented. The aim of impedance control is to maintain a desired dynamic behavior between the robot end-effector and the environment.



Fig. 11 Trace of P_k^x vs k



Fig. 12 The position-based impedance controller



Fig. 13 The robot is wiping the table

The proposed algorithm provides the estimation of the external forces exerted on robot during the task to support for this performance.

4.1 Wiping task and impedance controller design

The robot shall wipe in straight lines across the table and apply a constant force in vertical direction for cleaning. Meanwhile, an impedance behavior is applied for maintaining the contact force as well as the safety strategy of the robot manipulator when interacting with the stiffness of the table surface.

Assuming that the environment surface is a linear spring with known stiffness, the contact force can be expressed by

$$F_{ext} = k_e (x - x_e) \tag{40}$$



Fig. 14 Measured torques at joints and estimated contact forces during the wiping table

where x, x_e denote the position of the robot and the environment, respectively, in task space.

Considering the structure of the position-based impedance control,³⁰ there are two loops in this algorithm as shown in Fig. 12. The inner position control loop is to control the robot following the desired trajectory. Meanwhile, using information of the contact force, the outer impedance control loop modifies the desired trajectory to achieve the desired contact force. To regulate F_{ext} to track a desired contact force F_d , the desired trajectory x_d must be appropriately generated as follows:

$$x_d = x_e + \frac{F_d - F_{ext}}{k_e} \tag{41}$$

In the demonstration, the table will get wiped with a desired contact force of 2N.

4.2 Results

The demonstration is carried out on a 4-DOF robot manipulator called Safe and Speedy arm III (SS-arm III). SS-arm III is a serial chain of rigid links with the structure and specifications are shown in Fig. 1. For the purpose of the experiments estimating external forces acting on the robot manipulator, each joint is integrated with a JTS developed in Ref. 30. The F/T sensor is located between the cleaning tool and the robot end-effector to measure the contact forces. These values are used as references in comparison with the estimation results. The overall experiment operation is shown in Fig. 13.

Three different experiments are carried out to enhance the effectiveness of our proposed method. The time history of the measured joint torque by JTS, the measured external force by force/torque sensor, and the estimated external force are described in Fig. 14. The joint torques shown in Fig. 14 (a) are generated when the robot end-effector applies a contact force to the table surface in a vertical direction during movement. Although they are not influenced by a vertical contact force generated while wiping the table, JTS 1 and 2 are still nonzero. This error is due to the oblique direction between the robot end-effector and the plane. It might be caused by friction between the cleaner and the table surface which affected the result of estimation of external force.

However, it seems that errors have insignificantly affected the wiping performance of the robot. The sensor noise in the JTS has minimal effect on the experimental results by reliable external force estimation. The experiments validate that the robot manipulator controlled by impedance control maintains the contact force limited around 2N as desired within a reasonable accuracy. The estimated external force using the proposed method in wiping the table is shown in Fig. 14(b).

In comparison to cases without using impedance controller and using impedance control based on force sensor, our proposed algorithm shows better results with a small error of about 4.15%. It is found that force sensors sometimes do not usually generate a reliable performance when they are exposed to tough environments. This approach allows inexpensive sensors, such as JTS, to be used instead.

5. Conclusions

This paper proposes an algorithm to extract an external force exerted on the end-effector of a robot manipulator from the information of joint torque sensors. The algorithm is designed by combining TDE and input estimation technique to estimate the external force that is considered an unknown input of the robot system. Taking advantages of the TDE concept, the estimator is developed. This estimator does not require an accurate dynamic model of a robot manipulator. Also with the assumption that the disturbance torques generated by nonlinear uncertainties and system noise are the main reasons for the estimation error, the estimator based on minimum variance unbiased estimation technique with the guaranteed stability is used to reduce the error.

The proposed algorithm is applied in a 4-DOF robot manipulator and computer simulations on MATLAB & Simulink are performed. The results of the simulations reported in section 3 and Ref. 22 show good properties of the external force estimation method. It is also good to design a robot force controller such as impedance control in the task of wiping a table. The experimental results show the accuracy and the stability of the external force estimation method in which the compliant motion based on the proposed method maintains the contact forces around the desired forces with a small error during the operation.

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