INTERNATIONAL JOURNAL OF PRECISION ENGINEERING AND MANUFACTURING Vol. 16, No. 1, pp. 91-97

DOI: 10.1007/s12541-015-0011-7

Topology Optimization for Nonlinear Structural Problems based on Artificial Bee Colony Algorithm

Jae-Yong Park[1] and **Seog-Young Han**[2,#]

1 Department of Mechanical Engineering, Graduate School, Hanyang University, 222 Wangsimni-ro, Seongdong-gu, Seoul, South Korea, 133-791
2 School of Mechanical Engineering, Hanyang University, 222 Wangsimni-ro, Seongdong-gu, Seoul, South Korea, 133-791
Corresponding Author / E-mail: syhan@hanyang.ac.kr, TEL: +82-2-2220-0456, FAX: +82-2-2220-2299

KEYWORDS: Artificial bee colony algorithm (ABCA), Fitness, Geometrically nonlinear, Materially nonlinear, Nonlinear structural problems, Waggle index

Topology algorithm is suggested for nonlinear structural problems based on artificial bee colony algorithm (ABCA). Although ABCA has been successfully applied to static and dynamic stiffness topology optimizations, it has never been applied to nonlinear structural problems. In order to examine whether the topology algorithm is suitable for nonlinear problems, it is applied to geometrically nonlinear, materially nonlinear, and both geometrically nonlinear and materially nonlinear topology optimization. Waggle index update rule and changing filter scheme were implemented with ABCA to obtain a robust and stable optimized topology. Some examples were presented to show the applicability and effectiveness of the suggested algorithm and compared with solid isotropic material with penalization (SIMP). It was concluded that the proposed algorithm can be applied to the above three kinds of nonlinear structural problems and has been verified as effective and applicable topology algorithm.

Manuscript received: March 31, 2014 / Revised: September 5, 2014 / Accepted: September 27, 2014

1. Introduction

Until now, various topology optimization techniques such as the homogenization method,[1] the solid isotropic material with penalization (SIMP) method,[2] the evolutionary structural optimization (ESO/ BESO) method,[3] which correspond to parameterizations, and genetic algorithm (GA)[4] and simulated annealing (SA)[5] as stochastic methods have been developed and applied to linear structural problems. However, when a very large load is applied or the structural deformation is very large, geometric nonlinearity may occur. Also, when a structure is made in a nonlinear material, both material and geometric nonlinearities may occur. In these cases, the nonlinearities must be considered in the analysis as well as design in order to obtain a more stable and robust structural topology.

Stiffness designs of geometrically nonlinear structures using topology optimization based on the SIMP (solid isotropic material with penalization) method have been performed.[6] It was found that minimization of complementary elastic work rather than compliance as an objective function, should be used for geometrically nonlinear topology optimization. In addition, it was verified that there is no difference between the obtained topologies of four- and nine-node rectangular elements when a mesh-independent filtering scheme is employed, while the computation speed of the four-node element is much faster than that of the nine-node element. Huang and Xie[3,7] also carried out nonlinear topology optimization with load and displacement constraints using a bidirectional evolutionary structural optimization (BESO) method.

It is known that the SIMP and BESO methods widely used in topology optimization may provide different topologies under the same constraints due to the parameters used in each method. For example, even in the case of linear topology optimization in the SIMP and BESO methods, the obtained optimized topologies may be different due to parameters of the filtering scheme or the element removal ratio.[2,3] It is expected that these phenomena may be severe in nonlinear topology optimization since large displacements may cause the tangent stiffness matrix in low-density elements to become indefinite or even negative definite,[7] and optimal topology may be affected very sensitively by the intermediate topology caused by the nonlinearity of geometry and material. Therefore, a new nonlinear topology optimization scheme is needed to obtain a stable and robust optimal topology regardless of the filtering parameter or removal ratio.

Nature-inspired computation methods such as GA[4] and the artificial bee colony algorithm (ABCA)[8] have been applied for optimum designs using natural or physical phenomena with an evolutionary optimization

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process. As opposed to local search methods, these techniques are close to global search methods. In general, stochastic and probabilistic methods are implemented. The ABCA imitates bee colony behavior of gathering nectar. Recently, it has been applied to determine a variable of concrete dam,[9] and in linear topology optimization for static[10] and dynamic[11] stiffness problems.

Since topology optimization methods are based on an iterative optimization process, the computing time is very important. It is known that ABCA is a very fast and accurate algorithm among evolutionary algorithm (EA), particle swarm optimization (PSO) and differential evolution (DE) approaches for multi-dimensional numeric problems,[12] and the genetic algorithm (GA), particle swarm algorithm (PSO) and particle swarm-inspired evolutionary algorithm (PS-EA) in finding a global minimum for multivariable functions.[12,13] It was also verified that a topology optimization method based on ABCA is faster than SIMP method for static stiffness problems[10] and BESO for dynamic stiffness problems.[11] Furthermore, a topology optimization method based on ABCA has not been applied to topology optimization for nonlinear structures. For these reasons, ABCA was adopted as a topology optimization algorithm in this study.

In this study, a new topology optimization algorithm based on ABCA is proposed for nonlinear structural problems. The algorithm can be applied to both linear and nonlinear structures. The distribution of material is expressed by the density of each element in the application of ABCA. In order to prevent a checkerboard pattern and to improve the robust and stability of the optimized layouts, waggle index update rule and a changing filter scheme were implemented. The optimized topologies were compared with those of the SIMP[2] method to evaluate the effectiveness and applicability of the proposed algorithm.

2. Topology Optimization

2.1 Waggle index update rule

Swarm-intelligence algorithms applied for topology optimization must have an intermediate variable such as the pheromone of the ACO algorithm.[14] The intermediate variable plays a role in accumulating information of the colony. In other words, swarm-intelligence algorithms induce a candidate solution by gradually closing in on an optimal solution by accumulating information of efficient elements obtained from the every iteration. It is the reason that the ABCA, as a swarm-intelligence algorithm, also needs an intermediate variable. In this paper, the ABCA adopts the amount of information shared by bee colony as the intermediate variable. It is defined as waggle index I_i.

A waggle index update rule is developed as a simple algebraic equation based on the amount of information shared on each element from the previous iteration and the presence/absence of the employed bees in the present iteration, which is updated by Eq. (1):

$$I_i^{(k)} = \delta \times I_i^{(k-1)} + (1-\delta) \times e_i^{(k)} \tag{1}$$

where,

k : the number of the k-th iteration

I_i : waggle index (amount of information shared on the i-th element)

e_i : employed bee presence / absence ($1/\chi_{\min}$)

δ : waggle index update coefficient

where, the waggle index update coefficient δ is empirically appropriate to be 0.8 for nonlinear topology optimization. It represents the amount of information that should be transferred to bee colony in the next iteration.

Applying the waggle index to the SIMP model to calculate elemental sensitivity,[2] likewise the pheromone of the ACO algorithm for topology optimization, the material density, ρ and Young's modulus E are functions of the intermediate design variable. That is, waggle index I_i as follows:

$$\begin{aligned} \rho(I_i) &= I_i \rho^1 \\ E(I_1) &= I_i^p E^1 \quad (0 < I_{min} < I \le 1) \end{aligned} \tag{2}$$

where, ρ^1 and E^1 are the material density and Young's modulus of solid elements, respectively. p is the penalty factor. $I_{\min}$ is the minimum value of I_i (e.g., 10^{-3}) to denote a void element.

2.2 Nonlinear topology optimization

2.2.1 Formulation for geometrically nonlinear topology optimization

In geometrically nonlinear topology optimization under a load constraint, the complementary work W^C shown in Fig. 1 as an objective function provides a better result than the compliance.[6] Nonlinear topology optimization used complementary work as an objective function, can be formulated as follows:

$$\begin{aligned} &\text{Minimize} &&: f(x) = W^C \\ &\text{Subjected to} &&: \text{equilibrium and } V_s \le \overline{V}_s \end{aligned} \tag{3}$$

where, $\overline{V}_s$ is the prescribed volume constraint of the final structure. V_s is a volume of the present topology. The complementary work can be expressed as follows:

$$W^C = \lim_{n\to\infty} \frac{1}{2}\sum_{i=1}^{n} (\mathbf{F}_i^T - \mathbf{F}_{i-1}^T)(\mathbf{U}_i + \mathbf{U}_{i-1}) \tag{4}$$

where, $\mathbf{F}_i$ and $\mathbf{F}_{i-1}$ are incremental loads between i and i-1, respectively. $\mathbf{U}$ is the displacement vector, i is the incremental number of the load vector and n is the total number of load increments.

The sensitivity number α indicates the effectiveness of the i-th element on the objective function. The sensitivity number for a load constraint α_e in geometrically nonlinear topology optimization can be expressed as the following equation.[3]

$$\alpha_e = \frac{\partial f(x)}{\partial x_e} = \lim_{n\to\infty} \frac{1}{2}\sum_{i=1}^{n} \left[(\mathbf{F}_i^T - \mathbf{F}_{i-1}^T)\left(\frac{\partial \mathbf{U}_i}{\partial x_e} + \frac{\partial \mathbf{U}_{i-1}}{\partial x_e} \right) \right] \tag{5}$$

where, x_e is the design variable for density at the i-th element.

An adjoint equation is introduced by adding a series of vectors of Lagrangian multiplier λ_i into the objective function as follows:

$$f(x) = \lim_{n\to\infty} \frac{1}{2}\sum_{i=1}^{n} [(\mathbf{F}_i^T - \mathbf{F}_{i-1}^T)(\mathbf{U}_i + \mathbf{U}_{i-1}) + \lambda_i^T(\mathbf{R}_i + \mathbf{R}_{i-1})] \tag{6}$$

where, $\mathbf{R}_i$ and $\mathbf{R}_{i-1}$ are residual forces, defined as the discrepancy between the internal force vector and the external force vector at the increment at the increment between i and i-1, respectively. $\mathbf{R}_i + \mathbf{R}_{i-1}$ is as follows:

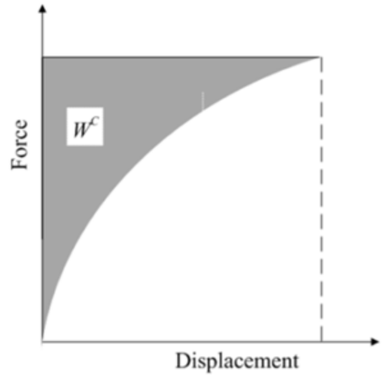


Fig. 1 Load-deflection curves in nonlinear finite element analysis[3]

$$\mathbf{R}_i+\mathbf{R}_{i-1} = \mathbf{F}_i-\mathbf{F}_i^{int}+\mathbf{F}_{i-1}-\mathbf{F}_{i-1}^{int} = 0 \tag{7}$$

where, $\mathbf{F}_i^{\mathrm{int}}$ is the internal load.

The sensitivity number for the modified objective function of Eq. (6) can be expressed as follows:

$$\begin{aligned}\alpha_e = \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\Big[&(\mathbf{F}_i^T-\mathbf{F}_{i-1}^T)\Big(\frac{\partial \mathbf{U}_i}{\partial x_e}+\frac{\partial \mathbf{U}_{i-1}}{\partial x_e}\Big)\\ &+\lambda_i^T\Big(\frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i}\frac{\partial \mathbf{U}_i}{\partial x_e}+\frac{\partial \mathbf{R}_{i-1}}{\partial \mathbf{U}_{i-1}}\frac{\partial \mathbf{U}_{i-1}}{\partial x_e}+\frac{\partial(\mathbf{R}_i+\mathbf{R}_{i-1})}{\partial x_e}\Big)\Big]\end{aligned} \tag{8}$$

If the increment of the load is small enough, the relationship between the load and displacement can be assumed to be linear, where $\mathbf{R}_i$ / $\mathbf{U}_i$ and $\mathbf{R}_{i-1}$ / $\mathbf{U}_{i-1}$ can be expressed using the tangential stiffness matrix $\mathbf{K}_i^t$ at the i-th increment as follows:

$$\frac{\partial \mathbf{R}_i}{\partial \mathbf{U}_i} = \frac{\partial \mathbf{R}_{i-1}}{\partial \mathbf{U}_{i-1}} = -\mathbf{K}_i^t \tag{9}$$

The sensitivity number for the objective function can be written using the following equation.

$$\alpha_e = \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\Big[(\mathbf{F}_i^T-\mathbf{F}_{i-1}^T)\Big(\frac{\partial \mathbf{U}_i}{\partial x_e}+\frac{\partial \mathbf{U}_{i-1}}{\partial x_e}\Big)+\lambda_i^T\frac{\partial(\mathbf{R}_i+\mathbf{R}_{i-1})}{\partial x_e}\Big] \tag{10}$$

In order to remove the ($\mathbf{U}_i / x_e + \mathbf{U}_{i-1} / x_e$) term, the Lagrangian multiplier is calculated as follows:

$$\lambda_i\mathbf{K}_i^t = \mathbf{F}_i^T-\mathbf{F}_{i-1}^T = \mathbf{K}_i^t(\mathbf{U}_i-\mathbf{U}_{i-1}) \tag{11}$$

$$\lambda_i = \mathbf{U}_i-\mathbf{U}_{i-1} \tag{12}$$

Using Eqs. (11) and (12), the sensitivity number can be expressed as follows:

$$\begin{aligned}\alpha_e &= \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\Big[\lambda_i^T\frac{\partial(\mathbf{R}_i+\mathbf{R}_{i-1})}{\partial x_e}\Big]\\ &= -\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\Big[(\mathbf{U}_i^T-\mathbf{U}_{i-1}^T)\Big(\frac{\partial \mathbf{F}_i^{int}}{\partial x_e}+\frac{\partial \mathbf{F}_{i-1}^{int}}{\partial x_e}\Big)\Big]\end{aligned} \tag{13}$$

2.2.2 Formulation for materially nonlinear topology optimization

In the case of materially nonlinear problems, a relationship between the effective stress ($\bar{\sigma}$) and the effective strain ($\bar{\varepsilon}$) can be expressed as follows:

$$\bar{\sigma} = K\Phi(\bar{\varepsilon}) \tag{14}$$

where, $\Phi(\bar{\varepsilon})$ is a function representing the material property, K is a constant related with elastic modulus. In the power-law material model, if $\Phi(\bar{\varepsilon})$ is expressed using work-hardening exponent n, and material interpolation scheme is introduced in order to consider solid and void elements, the effective stress $\bar{\sigma}(x_e)$ of each element can be written as follows:

$$\bar{\sigma}(x_e) = x_e^pK\Phi(\bar{\varepsilon}^0) \tag{15}$$

where, $\bar{\varepsilon}^0$ is the effective strain of solid element. Internal load for each element becomes as follows:

$$\mathbf{F}^{int} = \sum_{e=1}^{M}x_e^p\mathbf{C}^{eT}\mathbf{F}_{e0}^{int} \tag{16}$$

where, M is the number of total elements. $\mathbf{C}^{eT}$ is the transformation matrix to transform the i-th elemental nodal load vector to the global nodal load vector. $\mathbf{F}_{e0}^{int}$ is the internal load vector of solid element.

Substituting this equation into Eq. (13), the following equation can be obtained.

$$\frac{\partial f(x)}{\partial x_e} = -\lim_{n\to\infty}\frac{1}{2}px_e^{p-1}\sum_{i=1}^{n}(\mathbf{U}_i^T-\mathbf{U}_{i-1}^T)(\mathbf{C}^{eT}\mathbf{F}_{e,i}^{int}+\mathbf{C}^{eT}\mathbf{F}_{e,i-1}^{int}) \tag{17}$$

where, $\mathbf{F}_{e,i}^{int}$ is the internal load vector of solid element in the i-th increment. (–) indicates that the complementary work is reduced as the design variable x_e is increased. In order to minimize the complementary work, the sensitivity number of each element can be defined as follows:[3]

$$\begin{aligned}\alpha_e &= -\frac{1}{p}\frac{\partial f(x)}{\partial x_e}\\ &= x_e^{p-1}\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{U}_i^T-\mathbf{U}_{i-1}^T)(\mathbf{C}^{eT}\mathbf{F}_{e,i}^{int}+\mathbf{C}^{eT}\mathbf{F}_{e,i-1}^{int})\\ &= x_e^{p-1}\lim_{n\to\infty}\sum_{i=1}^{n}(E_i^e-E_{i-1}^e)\\ &= x_e^{p-1}E_n^e\end{aligned} \tag{18}$$

where, E_n^e is the final elastic-plastic strain energy for each element.

Finally, if two discrete design variables are used, Eq. (18) can be expressed as follows:

$$\alpha_e = \begin{cases}E_n^e & x_e = 1\\ x_e^{p-1}E_n^e & x_e = x_{min}\end{cases} \tag{19}$$

Therefore, the sensitivity number for nonlinear topology optimization can be defined by using Eqs. (2) and (19) as follows:

$$\alpha_e = \begin{cases}E_n^e & I_e = 1\\ I_e^{p-1}E_n^e & I_e = I_{min}\end{cases} \tag{20}$$

3. ABCA for Nonlinear Topology Optimization

As the fitness, fit_i, increases, the candidate solution improves in

ABCA. The fitness value of the i-th element is defined using the sensitivity number of each element as follows:[10,11]

$$fit_i = \begin{cases} \dfrac{1}{1+\alpha_i} & \alpha_i \geq 0 \\ 1-\alpha_i & \alpha_i < 0 \end{cases} \tag{21}$$

α_i : sensitivity number of the i-th element

The topology optimization procedure using the suggested ABCA proceeds as follows.[10]

1. Establish the design domain and parameters of topology optimization using ABCA.
2. Calculate the fitness values for the initial design domain using finite element analysis.
3. Perform the employed bee phase. Determine the positions of the x_i and x_k elements and the randomly chosen temporary candidate solution v_i. The positions are determined by applying the following equations.

$$\begin{aligned} x_i &= INT[rand\,[0,1] \times (number\ of\ all\ food\ source)] \\ x_k &= INT[rand\,[0,1] \times (number\ of\ all\ food\ source)] \\ v_i &= x_i + INT[rand\,[0,1](x_i - x_k)] \end{aligned} \tag{22}$$

INT = interger value
$rand\,[0, 1]$ = random number between 0 and 1

4. Perform the onlooker bee phase. Search for the new employed bee position determined by an onlooker bee. Calculate the probability of each element, p_i and select the temporary candidate solution, v_i.

$$p_i = \frac{fit_i}{\sum_{i=1}^{n} fit_i} \tag{23a}$$

$$v_i = x_i + INT[rand\,[0,1](x_i - x_k)] \quad \text{for} \quad rand\,[0,1] < p_i \tag{23b}$$

5. Perform the scout bee phase. Search occupied elements (employed bee colony) and abandoned elements (scout bee colony). Compare the fitness of employed bee colony with that of the scout bee colony.
6. Obtain a candidate solution satisfying all constraints.
7. Calculate the complementary work for the updated design domain using finite element analysis. In this step, a suggested changing filter scheme[11] is implemented to prevent a checkerboard pattern and unreasonable structures such as cleft or biased structural parts in topology optimization. To overcome this problem, filtering size, $r_{\min}$ needs to be controlled to become gradually larger as a candidate solution becomes close to an optimum solution by using mesh-independency scheme[15] as follows:

$$\alpha_i = \frac{\sum_{j=1}^{M} w(r_{ij})\alpha_j^n}{\sum_{j=1}^{M} w(r_{ij})}, \quad w(r_{ij}) = r_{min} - r_{ij} \quad (j = 1, 2, \ldots, M) \tag{24}$$

$r_{\min}$ = element size × $\sqrt{2}$ (iteration < limit value)
$r_{\min}$ = element size × 2 (limit value ≤ iteration < limit value × 2)

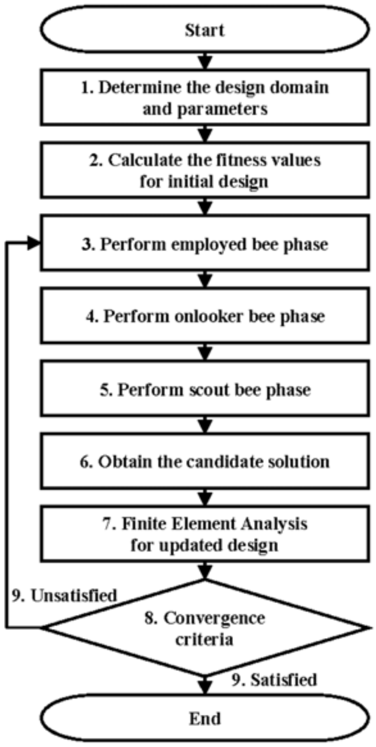


Fig. 2 Flowchart of the artificial bee colony algorithm[10]

$r_{\min}$ = element size × $2\sqrt{2}$ (limit value × 2 ≤ iteration)

Here, M is the total number of nodes in the circular sub-domain, $w(r_{ij})$ is the linear weight factor, α_j^n is the nodal sensitivity number of the j-th node, $r_{\min}$ is the length scale parameter, and r_{ij} is the distance between the center of the element i and the j-th node.

Limit value is defined as a predefined value to change employed bee with scout bee when there is no improvement in the amount of nectar from a certain food source for a while in original ABCA. If a scout accidentally discovers a rich, previously unknown food source without any guidance as to the location, it becomes an employed bee.[8] The limit value is used to escape from a local minimum and develop the changing filter scheme in the suggested ABCA.

8. Check whether the updated topology converged to the optimized topology using the following convergence criterion.[15]

$$error = \frac{\left|\sum_{i=1}^{N'} (W_{k-i+1}^C - W_{k-N'-i+1}^C)\right|}{\sum_{i=1}^{N'} W_{k-i+1}^C} \tag{25}$$

Here, W^C is the complementary work, τ is an allowable convergence error, k is the present iteration number, and N is the integral number which results in a stable complementary work in at least ten successive iterations.

9. If the updated topology is not converged, calculate fitness values for the updated design domain with the density of entire elements updated by the density updating scheme using finite element analysis. Return to step 3 and repeat the above steps until an optimized topology is obtained. A schematic diagram of the suggested ABCA is described in Fig. 2.

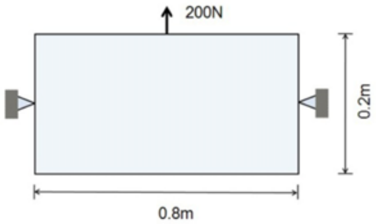
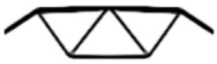


Fig. 3 Design domain of a simply supported beam

Table 1 Comparison of optimal topologies between example 1 and SIMP method

Cases	ABCA (No. of iteration)	SIMP (No. of iteration)
Linear	 (41)	 (63)
Materially Nonlinear	 (43)	 (above 150)
Geometrically Nonlinear	 (48)	 (above 150)
Geometrically and Materially Nonlinear	 (46)	 (52)

4. Numerical Examples

4.1 A simply supported beam

A simply supported beam having dimensions of 0.8 m × 0.2 m × 0.001 m is subjected to 200 N at the center of the top surface as shown in Fig. 3. Design domain is divided into 160×40 by four-node rectangular element. The material is assumed to have Young's modulus of 1 GPa, Poisson's ratio of 0.3 and yield strength of 2 MPa. The ABCA is applied to linear and three kinds of nonlinear topology optimization. The materially nonlinear model is assumed to be bilinear material and tangent elastic modulus for topology optimization is 0.2 GPa. The filtering radius $r_{\min}$ is restricted by the value less than 3 in the SIMP method. The penalty factor p is set to 1 for nonlinear analysis. The limit value is set to 10. The allowable convergence error τ is set to 0.001. The objective is to obtain the stiffest structure under a volume constraint of 20% of the original volume.

Optimal topologies for linear and geometrically nonlinear cases of the ABCA and SIMP method are shown in Table 1, respectively. From the comparison of the optimized topologies, it is found that the optimal topologies of the ABCA are very similar to those of the SIMP method. The difference between the topologies is caused only by the grey elements in SIMP method, and the discrete elements in ABCA.

Next, 10 trials of topology optimization for the simply supported beam were performed in order to show the stability and robustness of the suggested ABCA. The evolutionary histories of iteration for only one typical case out of 10 trials for simplicity are shown in Fig. 4 to verify the stability, and the optimized layouts obtained for all 10 trials are listed in Fig. 5 to verify the robustness. It can be found that the objective function converges stably, and the proposed ABCA is a robust algorithm.

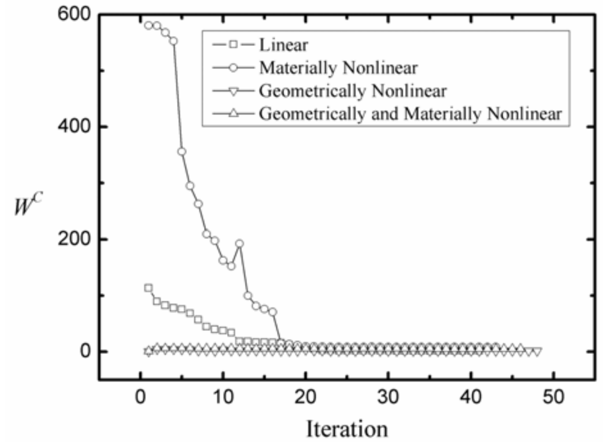


Fig. 4 Iteration histories of the complementary work of a simply supported beam

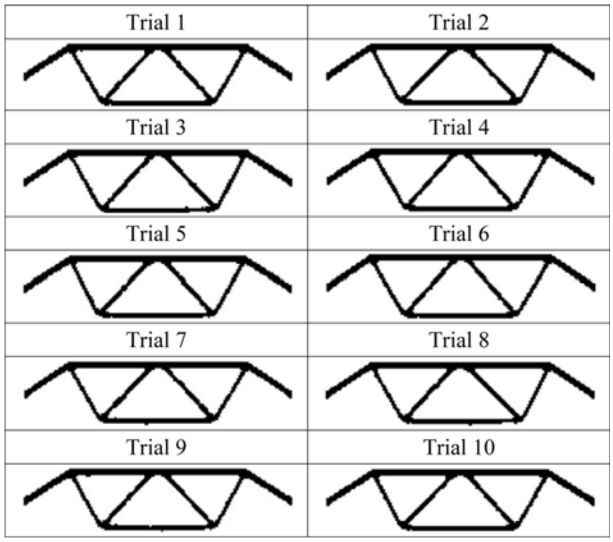


Fig. 5 Robustness of optimal topologies for 10 trials

The complementary work of the optimized topology based on the ABCA is calculated as 1.743 J for the linear case, 8.408 J for the materially nonlinear case, 1.597 J for the geometrically nonlinear case and 5.610 J for both the materially and geometrically nonlinear cases. The complementary work of the optimized topology based on the SIMP method is calculated as 1.850 J for the linear case, 7.983 J for the materially nonlinear case, 1.424 J for the geometrically nonlinear case and 5.017 J for both the materially and geometrically nonlinear cases.

As we can see, the convergence rate of ABCA is faster than that of SIMP method, and the values of the complementary work of SIMP method are smaller than those of ABCA except the linear case. Because of the grey portion of the optimized topology from SIMP method, the complementary work may be calculated a little bit smaller than ABCA.

In addition, the convergence rate of materially and geometrically nonlinear cases is a little bit slow. It seems that optimal topology may be affected very sensitively by the intermediate topology caused by the

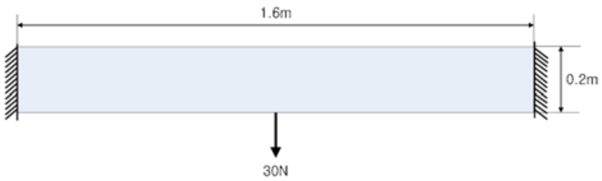
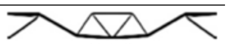


Fig. 6 Design domain of a clamped long beam under a given load

Table 2 Comparison of optimal topologies between example 2 and SIMP method

Cases	ABCA (No. of iteration)	SIMP (No. of iteration)
Linear	 (38)	 (99)
Materially Nonlinear	 (59)	 (above 150)
Geometrically Nonlinear	 (34)	 (114)
Geometrically and Materially Nonlinear	 (52)	 (122)

change of material and geometry. It is verified that the developed filtering scheme implemented with ABCA is very effective and stable in nonlinear topology optimization.

4.2 A clamped beam

A clamped beam having dimensions of 1.6 m × 0.2 m × 0.01 m is subjected to 30 N at the center of the bottom surface as shown in Fig. 6. Design domain is divided into 240×30 using symmetry by four-node rectangular element. The material is assumed to have Young's modulus of 30 MPa, Poisson's ratio of 0.3 and yield strength of 0.06 MPa. The ABCA is applied to linear and three kinds of nonlinear topology optimization. The materially nonlinear model is a power-law material model having work-hardening exponent 0.5. That is, the relation between σ and ε is $\sigma = 1.34\varepsilon^{0.5}$. The filtering radius $r_{\min}$ is restricted by the value less than 3 in the SIMP method. The penalty factor p is set to 1 for nonlinear analysis. The allowable convergence error τ is set to 0.001. The objective is to obtain the stiffest structure under a volume constraint of 20% of the original volume.

Optimal topologies for linear and geometrically nonlinear cases of the ABCA and SIMP method are shown in Table 2, respectively. From the comparison of the optimized topologies, it is found that the optimal topologies of the ABCA except for both geometrically and materially nonlinear case, are very similar to those of the SIMP method. The difference among the three topologies is caused by the grey elements in SIMP method, and the discrete elements in ABCA. In both the geometrically and materially nonlinear case, it is found that the optimized topology of ABCA shows a little bit different from that of SIMP method. The grey part of the optimized topology from SIMP method seems to be replaced by the supported parts at the both ends of that from ABCA because of solid-void materials.

Next, 10 trials of topology optimization for the clamped beam were performed in order to show the stability and robustness of the suggested ABCA. The evolutionary histories of iteration for only one typical case

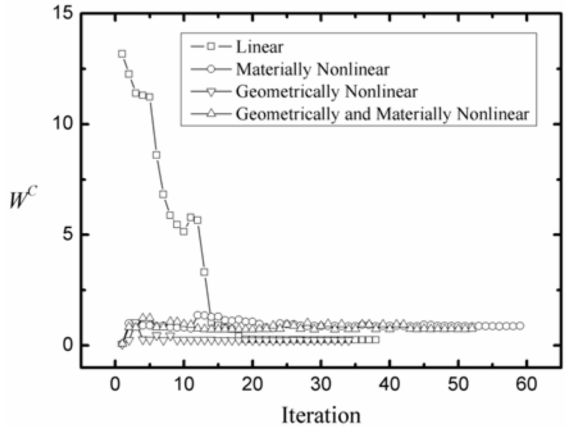


Fig. 7 Iteration histories of the complementary work of a clamped beam

out of 10 trials for simplicity are shown in Fig. 7 to verify the stability, and the optimized layouts obtained for all 10 trials were examined to verify the robustness. It can be found that the objective function converges stably, and the proposed ABCA is a robust algorithm.

The complementary work of the optimized topology based on the ABCA is calculated as 0.250 J for the linear case, 0.888 J for the materially nonlinear case, 0.241 J for the geometrically nonlinear case and 0.758 J for both the materially and geometrically nonlinear cases.

The complementary work of the optimized topology based on the SIMP method is calculated as 0.272 J for the linear case, 0.780 J for materially nonlinear case, 0.214 J for the geometrically nonlinear case and 0.674 J for both the materially and geometrically nonlinear cases.

The convergence rate of materially nonlinear case is a little bit slow. It seems that optimal topology may be affected very sensitively by the intermediate topology caused by the change of material. For the convergence rates and the values of the complementary work of the SIMP method and ABCA show very similar tendency to the example 1. It is also verified that the developed filtering scheme implemented with ABCA is very effective and stable in nonlinear topology optimization.

5. Conclusions

Topology algorithm based on ABCA is suggested for nonlinear structural problems. It has been applied to three kinds of nonlinear topology optimizations, namely, geometrically nonlinear, materially nonlinear and both geometrically and materially nonlinear cases. Waggle index and a changing filter scheme are implemented to obtain a robust and stable optimized topology. From the results, the following conclusions are obtained.

(1) The proposed topology algorithm has successfully applied to three cases of nonlinear structural problems.

(2) Waggle index update rule and changing filter scheme provide a robust and stable optimized topology in nonlinear topology optimization.

(3) Since there is no grey portion of the optimized topology from ABCA, the algorithm provides a clear and practical topology unlike that from SIMP method.

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