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Study of the Influence of Material and Geometry on the Anisotropy of Dimensional Change on Sintering of Powder Metallurgy Parts

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In the conventional press and sinter process, dimensional change on sintering determines the precision of the final parts, providing that a good dimensional precision of green parts is ensured. Anisotropic dimensional change on sintering may be detrimental to the precision of Powder Metallurgy (PM) parts, and it should be considered in the design step. The effect of material and geometry on the anisotropic dimensional change is studied in this work. Four different iron alloys and five different geometries were considered. Dimensions were measured both on green and on sintered parts and the anisotropy of dimensional change was evaluated and correlated to the material and geometry. The effect of neglecting anisotropy in the design step was investigated, in terms of dimensional tolerances, which can be obtained with different process capabilities. A model to describe the effect of material and geometry on the anisotropic dimensional change is also being developed.

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NOMENCLATURE

 D_{ext} = dimensional change in the external diameter normalized by the isotropic dimensional change

 D_{int} = dimensional change in the internal diameter normalized by the isotropic dimensional change

- d_N = nominal dimension [mm]
- d_g = dimension of the green part [mm]
- d_s = dimension of the sintered part [mm]
- e = process capability error

 $H =$ dimensional change in height normalized by the isotropic dimensional change

 $h =$ height of the part (disks - rings) [mm]

- h_g = height of the green part (disks rings) [mm]
- h_N = nominal height [mm]

 h_s = height of the sintered part (disks - rings) [mm]

 $K =$ anisotropy parameter

 l_g = dimension in the green state [mm]

 $R =$ ratio between the internal and the external diameter in the green parts

- $t =$ wall thickness of the rings [mm]
- V_g = volume of the green part (disks rings) [mm³]
- V_s = volume of the sintered part (disks rings) [mm³]
- α = geometrical parameter relating the dimensional changes in the internal and external diameters
	- $γ =$ geometrical parameter relating R and $α$
	- Δ V/V_g = volume change
	- ε_{d} = actual dimensional change
	- ε_h = dimensional change in height
	- $\varepsilon_{\rm iso}$ = isotropic dimensional change
- ε_{ref} = dimensional change relevant to a known reference geometry
- ε_{ϕ} = dimensional change in the external diameter for disks
- $\varepsilon_{\text{best}}$ = dimensional change in the external diameter for rings
- $\varepsilon_{\text{d}_{\text{int}}}$ = dimensional change in the internal diameter for rings
- ϕ = external diameter of the disks [mm]

ρ_s = sintered density [g/cm³]

1. Introduction

Powder Metallurgy (PM) is a cost effective net-shape technology used for the production of parts characterized by an excellent combination of mechanical properties and dimensional/geometrical precision. In the conventional PM process the powder is cold compacted in rigid dies to obtain the so-called green part, which is then sintered to form the metallic bonding between the particles. The formation and growth of the interparticle neck during sintering is accompanied by either volumetric contraction (shrinkage) or expansion (swelling), depending on the material, the green density and the sintering parameters; the effect of these variables on dimensional change on sintering is extensively reported by German.¹ Such a dimensional change has to be taken into account in designing the compaction tools and strategy, in order to obtain, at least within certain limits, the precision required by the final products.

Dimensional change on sintering of cold compacted green parts is anisotropic, and this subject has been systematically investigated in some recent works. Cristofolini et al. investigated the anisotropy of sintering shrinkage of parts produced with a Fe-Cu-P alloy.² showing that shrinkage along the compaction direction is larger than that in the compaction plane. The trend was confirmed on 3% Cr-0.5%Mo^{3,4} and on a 1.5%Cr-0,2%Mo steel parts,⁵ even if an inversion (larger shrinkage in the compaction plane) was observed on this latter steel when sintering was carried out at very high temperature.⁵ Cu steels, which swell during sintering, were also studied reporting that swelling along the compaction direction is smaller than that in the compaction plane.⁶

All these studies confirm the results obtained by other authors. Swelling phenomena due to liquid phase sintering of Fe-Cu parts were studied by Wanibe et al.,⁷ and Griffo et al.⁸ analysed the influence of several variables on the dimensional behaviour of Fe-Cu-C parts, highlighting the effect of Cu content. Raman et al.⁹ observed an axial swelling smaller than that in the compaction plane in Fe-Cu-C parts. Dimensional change on sintering of Mo alloyed steels was investigated by Danninger,¹⁰ highlighting that anisotropy is mainly due to the different dimensional variations occurring in the solid state, with no significant effect due to the formation of the liquid phase. Anisotropy depends on several parameters, related to the material, the process and the geometry of the parts, so that the complexity of the phenomenon is definitely high. The design of sintered parts is approached with an empirical methodology, based on the designer experience, which very

Fig. 1 Schematic representation of the parts studied (h is height, f the external diameter and t the wall thickness of rings)

frequently leads to the necessity to re-design compaction tools before obtaining the dimensional precision required by the final products.

Zavaliangos and Bouvard¹¹ approached the study of anisotropy of dimensional change by numerical simulation, and concluded that modelling results can be helpful provided that the inhomogeneity of the modifications introduced by cold compaction in the powder particles is included in the theoretical analysis. The complexity of the relevant phenomena is determined by both the morphological and the structural modifications of the metallic powder, which, in turn, influence the sintering stress and, according to a recent work by Molinari et al.¹² the mass transport mechanisms responsible for sintering.

To investigate the effect of material and geometry on the anisotropic dimensional changes, an experimental study was planned, based on the systematic analysis of the dimensional behaviour of different parts made of different materials. Five axisymmetric parts and four different materials were considered. The quite simple geometry of the five parts, both disks and rings, comes close to that of many typical products of the PM industry. The four materials were selected with the aim of investigating both shrinking and swelling systems, as well as both solid state and liquid phase sintering processes. To this purpose the following materials were selected: iron (solid state sintering - shrinkage), ironphosphorous (liquid phase sintering - shrinkage) and two iron-copper alloys (liquid phase sintering - swelling). The parts were cold compacted to the same green density and sintered at different temperatures in order to investigate a wide range of dimensional changes.

The results confirm that anisotropy is strongly dependent on geometry and material. A preliminary analysis of the results is reported in a previous work by Cristofolini et al. 13 In the present paper the influence of the anisotropic dimensional changes on the precision of sintered parts is investigated and a model, correlating the anisotropy of dimensional changes to the material and the geometry of the parts, is proposed.

2. Experimental Procedure

The axial-symmetric parts shown in Fig. 1 were considered in this study to investigate the effect of the geometry.

The effect of increasing height is investigated by this sampling, as well as the effect of the presence of the hole. Comparing disks and rings, parts with the same height, as well as parts with the same h/ϕ ratio (h/t respectively) are considered, ϕ being the external diameter of disks and t being the thickness of rings. The thick ring shown in Fig. 1 may be regarded as an intermediate part, having the same height of the low disk and the high ring, and the same h/t ratio of the high disk and the

Table 1 Density and densification for the different materials sintered at 1120°C

Material	Green parts	Sintered parts	Densification %		
	$\rho_{\rm g}$ (g/cm ³)	$\rho_{\rm s}$ (g/cm ³)	$[(\rho_{\rm s} - \rho_{\rm g})/\rho_{\rm g}] \cdot 100$		
Fe	7.14	7.16	0.28		
$Fe-0.45%P$	7.11	7.14	0.42		
$Fe-2\%Cu$	7.17	7.06	-1.53		
$Fe-4\%$ Cu	719	6.86	-4.59		

low ring.

The use of different materials allowed comparing both shrinking systems (Fe and Fe-0.45%P) and swelling systems (Fe-2%Cu and Fe-4%Cu). All the materials are based on a water atomized iron powder, to which either an elemental copper powder (72% particles $\leq 45 \mu m$) or Fe3P micrometric particles were added to obtain the different compositions. Amidewax was used as compaction lubricant. Parts were uniaxially cold compacted up to the densities shown in Table 1 and sintered at different temperatures: 1000°C and 1120°C in a belt furnace under a $95\%N_2/5\%H_2$ atmosphere, 1280°C and 1350°C in a vacuum furnace with nitrogen backfilling at 1000°C. Isothermal holding time is 30 minutes for all the sintering cycles. In the first two sections of the next chapter, reference is made to the 1120°C sintering cycle; in the third section the results of all the sintering experiments will be considered.

Density was determined as the ratio between the weight (measured by a precision balance) and the volume calculated from dimensions. As shown in Table 1, the expected increase in density after sintering at 1120°C is confirmed for Fe and Fe-P parts, as well as the decrease for Fe-Cu parts, especially pronounced for Fe-4%Cu.

Five specimens for each geometry were measured by a coordinate measuring machine (CMM), the same part was measured in the green and sintered condition. All the surfaces were measured by continuous scan (accuracy $3.4/120 \ \mu m/sec$ according to ISO 10360-4, 2000,¹⁴ which gives the scanning probe error, referred to a well-defined scan path, performed over 120 sec). The cylindrical surface was measured at five different distances from the upper plane (that in contact with the upper punch during cold compaction). Planes and cylindrical surfaces were calculated processing the measured data according to a procedure described by Cristofolini et al.¹⁵ Reliable dimensions were derived from the surfaces, in terms of distances between planes and diameters of the cylindrical surfaces.

3. Results and Discussion

3.1 Evaluation of anisotropy

Considering axial-symmetric PM parts, anisotropy is observed when the dimensional change in the axial direction (z) is different from that in the compaction plane $(x \text{ and } y \text{ directions})$, as by Eq. (1). Axial symmetry provides that the dimensional change in the compaction plane is the same in all the directions considered, as by Eq. (2), where l_g is the dimension in the green state.

$$
\left(\frac{\Delta l}{l_g}\right)_{z} \neq \left(\frac{\Delta l}{l_g}\right)_{y}, \left(\frac{\Delta l}{l_g}\right)_{x}
$$
\n(1)

Fig. 2 Dimensions used to evaluate anisotropy for disks and rings

$$
\left(\frac{\Delta l}{l_g}\right)_x = \left(\frac{\Delta l}{l_g}\right)_y\tag{2}
$$

Referring to Fig. 2, anisotropy is observed for disks when the dimensional change of height is different from that of diameter, as by Eq. (3), and for rings when the dimensional change of height is different from that of thickness, as by Eq. (4).

$$
\frac{\Delta h}{h_g} \neq \frac{\Delta \phi}{\phi_g} \tag{3}
$$

$$
\frac{\Delta h}{h_g} \neq \frac{\Delta t}{t_g} \tag{4}
$$

Thickness being not a design parameter, dimensional change of external and internal diameters, compared to the dimensional change of height, will be used to evaluate the anisotropy of rings, as by Eq. (5).

$$
\frac{\Delta h}{h_g} \neq \frac{\Delta \phi_{ext}}{\phi_{ext_g}}, \frac{\Delta \phi_{int}}{\phi_{int_g}}
$$
(5)

It must be observed that, in principle, the dimensional change of the external diameter may differ from the dimensional change of the internal diameter, as by Eq. (6).

$$
\frac{\Delta \phi_{ext}}{\phi_{ext_g}} \neq \frac{\Delta \phi_{int}}{\phi_{int_g}}
$$
(6)

On the basis of the considerations above, the following parameters have been used to investigate anisotropy:

- for disks, the dimensional change in height - e_h - and in diameter - e_f - as by Eqs. (7) and (8), where h_s represents the height of the sintered part and h_g the height of the green part, f_s the diameter of the sintered part and f_g the diameter of the green part.

εh

$$
E_h = \frac{h_s - h_g}{h_g} \tag{7}
$$

$$
\varepsilon_{\phi} = \frac{\phi_s - \phi_g}{\phi_g} \tag{8}
$$

- for rings, the dimensional change in height as defined above - e_h - in external diameter - $e_{f ext}$ - and in internal diameter - $e_{f int}$ - as by Eqs. (7), (9) and (10), where f_{ext} s represents the external diameter of the sintered part and f_{ext} g the external diameter of the green part, $f_{int s}$ the internal diameter of the sintered part and $f_{int g}$ the internal diameter of the green part.

g

g

$$
\varepsilon_{\phi_{ext}} = \frac{(\phi_{ext_s} - \phi_{ext_g})}{\phi_{ext_s}}
$$
(9)

$$
\frac{\Delta I}{I_g} \neq \left(\frac{\Delta I}{I_g}\right)_x \left(\frac{\Delta I}{I_g} \right)_x \tag{1}
$$
\n
$$
\varepsilon_{\phi_{int}} = \frac{\left(\phi_{int_s} - \phi_{int_g}\right)}{\phi_{int_o}} \tag{10}
$$

Fig. 3 Dimensional changes for the different materials and geometries sintered at 1120°C

In case of isotropic dimensional change, $\varepsilon_h = \varepsilon_f = e_{iso}$ for disks, and $\varepsilon_h = \varepsilon_{fext} = \varepsilon_{fint} = \varepsilon_{iso}$ for rings. The isotropic dimensional change ε_{iso} can be derived from the change in volume, as by Eq. (11), where V_s is the volume of the sintered part and V_g the volume of the green part. ε_{iso} will be used as reference to evaluate the anisotropy for the different materials and geometries.

$$
1 - \frac{V_s - V_g}{V_g} = (1 + \varepsilon_{iso})^3 \Rightarrow \varepsilon_{iso} = \sqrt{\frac{V_s - V_g}{V_g} + 1} - 1
$$
 (11)

Fig. 3 shows the dimensional changes measured for the different geometries in shrinking systems (Fe and Fe-0.45%P) and in swelling systems (Fe-2%Cu and Fe-4%Cu). The values are compared to that of the isotropic dimensional change. In the following, $\varepsilon_{\phi \text{ ext}}$ corresponds to ε_{ϕ} for disks.

Concerning shrinking systems, Fe parts (solid state sintering) show very small and significantly anisotropic dimensional changes, while in Fe-P parts (liquid phase sintering) dimensional changes are somehow larger and less anisotropic. For both materials anisotropy appears to be more pronounced for rings. Concerning swelling systems (liquid phase sintering), in Fe-2%Cu parts dimensional changes are quite small and anisotropic, while in Fe-4%Cu parts dimensional changes are significantly large and almost isotropic. From these results, the presence of liquid phase has a strong effect in decreasing anisotropy, which has been investigated in depth by dilatometry.

Nevertheless, dimensional changes are noticeably different for the different geometries and materials, so that it is difficult to derive the influence of the geometry and the material on the anisotropic behaviour from a direct comparison. Dimensional changes have been normalized by the isotropic dimensional variation, defining parameters H, D_{ext}, D_{int} as by Eqs. (12), (13), and (14).

$$
H = \frac{\varepsilon_h}{\varepsilon_{iso}}\tag{12}
$$

$$
D_{ext} = \frac{\varepsilon_{\phi_{ext}}}{\varepsilon_{iso}} \tag{13}
$$

$$
D_{int} = \frac{\varepsilon_{\phi_{int}}}{\varepsilon_{iso}} \tag{14}
$$

Fig. 4 Anisotropy parameters for Fe disks (left) and rings (right)

Fig. 5 Anisotropy parameters for Fe disks and rings with the same height

Representation in Fig. 4 clearly depicts the trend of the parameters defined above for Fe disks and thin rings, corresponding to the material with the most significantly anisotropic dimensional changes. The same scheme for representation has been maintained for both disks and rings to allow a comparison, although in the case of disks only H and D_{ext} are reported, being $D_{in}=0$ by definition. The isotropic condition is represented for disks by the black broken line, the limit-points of which correspond to the unit value for all the anisotropic parameters, and for rings by the triangle defined by the black broken lines, the vertices of which correspond to the unit value for all the anisotropic parameters.

Increasing the height, anisotropy tends to decrease for disks and to increase for rings. Moreover, for rings a slight difference between D_{ext} and D_{int} is observed.

Parts with the same height are compared in Fig. 5.

Although the thick ring appears slightly less anisotropic than the other parts, no definite trend relevant to the presence/dimension of the hole may be derived from this comparison.

The dimensional variations in the compaction plane are generally significantly lower than those in the axial direction, and the difference between D_{ext} and D_{int} highlighted in Fig. 4 is even larger for the thick ring.

Same evaluation is made for Fe-4%Cu parts, those showing the less anisotropic dimensional changes. Fig. 6 shows the trend of the parameters defined above for Fe-4%Cu disks and thin rings.

In this case the differences among all the parameters are definitely small: the large amount of Copper, which enhances liquid phase sintering and particle rearrangement, significantly decreases anisotropy. Concerning the effect of height, despite the small differences, the same trend highlighted for Fe parts is observed (increasing the height, anisotropy tends to decrease for disks and to increase for rings). On the contrary, no differences between D_{ext} and D_{int} are observed.

Parts with the same height are compared in Fig. 7.

Fig. 6 Anisotropy parameters for Fe-4%Cu disks (left) and rings (right)

Fig. 7 Anisotropy parameters for Fe-4%Cu disks and rings with the same height

No definite trend relevant to the presence/dimension of the hole may be derived from this comparison, also due to the slightly significant differences among the parameters. The strong effect of the liquid phase in decreasing anisotropy remains the most significant result.

Anisotropic dimensional changes have been observed, affected by both the material and the geometry. The effect on the precision of parts will be evaluated in the next paragraph.

3.2 Anisotropy and precision

The precision of PM parts is firstly determined by the process capability, coming from both the compaction and the sintering steps, as well as from post treatments, if any. Process capability, which is commonly expressed in terms of an interval of attainable dimensions around a nominal value, or in terms of ISO IT classes, gives the best precision which can be obtained. Such a precision, or minimum attainable tolerance, can only be obtained if parts are designed taking into account both the entity and the anisotropy of dimensional changes on sintering. As highlighted above, both material and geometry affect the anisotropic dimensional change, so that a precise knowledge on the dimensional change of a specific part is not often available. Neglecting the anisotropic dimensional changes can imply a worsening in the attainable tolerances. To investigate the entity of such a worsening for the different materials and geometries studied, two different approaches are proposed.

The first approach neglects the anisotropy of dimensional changes, and green parts are designed in the hypothesis of isotropic dimensional changes, than the dimension of the green part d_g is calculated by Eq. (15), where d_N is the nominal dimension required for the sintered part

$$
d_g = \frac{d_N}{1 + \varepsilon_{iso}}\tag{15}
$$

The dimension of the sintered part d_s will however be determined

$d_{\rm w}$	Nominal dimension	d_N			
	Green dimension in the hypothesis of isotropic dimensional change	$d_g = \frac{d_N}{1 + \varepsilon_{ice}}$			
d,	Sintered dimension (actual dimensional change)	$d_s = d_o(1 + \varepsilon_d)$			
\rightarrow	Process capability error addition	$d_{x} \pm e$			
$ d_s + e - d_N $	Estimated highest deviation from the nominal dimension	$D = \max\{d_s + e - d_N ; d_s - e - d_N \}$			
ISO IT	Estimated ISO IT class	ISO IT class = $f(d_N, D)$			

Fig. 8 Procedure for tolerances quantification

by its actual dimensional change, ε_d , according to Eq. (16)

$$
d_s = d_g(1 + \varepsilon_d) = d_N \frac{(1 + \varepsilon_d)}{(1 + \varepsilon_{iso})}
$$
(16)

From Eq. (16), the larger the difference between e_d and e_{iso} - that means the larger the anisotropy of dimensional change - the larger the difference between d_s and d_N . To quantify the attainable tolerances for the sintered height, process capability has to be considered, according to the procedure shown in Fig. 8.

Table 2 summarizes the attainable tolerances for the external diameter when parts are designed in the hypothesis of isotropic dimensional changes, in case of process capability ISO IT 9 and ISO IT 8.

As expected, an "isotropic design approach leads to tolerances which are always higher than the process capability - bold values in Table 2 are the closest to the lowest attainable value. Although the tolerance classes derive from the contribution of both the anisotropic behaviour and the entity of the dimensional changes, anisotropy appears as the most significant parameter. In fact, for any geometry, the best tolerance classes are found in correspondence to the less anisotropic material (Fe-4%Cu). A strong effect of the geometry is also revealed: for any material, tolerances are quite good for disks and the thick ring, while a general worsening is observed for thin rings (ISO IT 12-13 is an unacceptable value for dimensions in the compaction plane). Moreover, the effect of anisotropy on the precision is mostly significant when the tolerance relevant to the process capability is low, that means when the production process is well controlled. In fact, it is observed that the tolerance class deriving from an isotropic approach often remains the same for both the process capabilities considered, but obtaining IT 10 when the process capability is IT 8 is definitely worse than obtaining IT 10 when the process capability is IT 9.

The "isotropic design approach" quantitatively highlighted the effect of the anisotropic dimensional change on the precision of PM parts. Aiming at improving the attainable tolerances, an "anisotropic design approach" is proposed in the following.

In this case, green parts are designed in the hypothesis of the anisotropic dimensional changes relevant to a known "reference geometry" - ε_{ref} - then the dimension of the green part d_g is calculated by Eq. (17), where d_N is the nominal height required for the sintered part

$$
d_g = \frac{d_N}{1 + \varepsilon_{ref}}\tag{17}
$$

$\rm Fe$	Fe-2%Cu	11/11	11/10	10 11 _l	10/10	11/11	11 / 11	12 / 11	10/10	12/12	13/12
$Fe-0.45\%P$	Fe-4%Cu	11/11	11/10	11/11	10/10	11 ′ 11	/10	11/11	10/9	12/12	

Table 2 ISO IT classes for external diameter designed in the hypothesis of isotropic dimensional changes - values obtained considering different process capability - IT 9 (left) / IT 8 (right)

Table 3 ISO IT classes for external diameter designed in the hypothesis of isotropic dimensional changes - values obtained considering different process capability - IT 9 (left) / IT 8 (right)

Fe	$Fe-2\%Cu$										
$Fe-0.45\%P$	Fe-4%Cu										
				10/9	10/10	10/9	10/9	10/9	11/10	11/11	14/14
				10/9	11/11	10/9	10/9	10/10	11/11	11/10	10/9
I		10/9	10/10			10/9	11/10	10/10	10/9	11/11	14/14
		10/9	11/11			10/9	11/11	10/10	11/10	11/10	12/12
31 X		10/9	10/9	10/9	10/10			10/10	11/11	11/11	14/14
		10/9	10/9	10/9	11/11			11/10	11/11	10/9	10/9
\Rightarrow		10/9	10/10	10/10	10/9	10/10	11/10			11/10	14/14
		10/9	11/10	10/9	10/10	11/10	11/10			11/11	11/11
		11/10	13/13	11/10	13/13	11/11	13/13	11/10	14/14		
	\sim	10/10	10/9	10/10	11/11	10/9	10/9	11/11	11/11		

The dimension of the sintered part d_s will be determined by its actual dimensional change, ε_d , according to Eq. (18)

$$
d_s = d_g(1 + \varepsilon_d) = d_{N} \frac{(1 + \varepsilon_d)}{(1 + \varepsilon_{ref})}
$$
(18)

To take into account the anisotropy in the design step, the possibility of individuating a unique "anisotropic reference geometry" for the different materials and geometries is investigated. This "reference geometry" would be helpful to the designer, it would allow to avoid the necessity of considering for each part its own anisotropy, without neglecting the anisotropy, as by the isotropic approach. Table 3 collects ISO IT classes relevant to the external diameter attainable with an "anisotropic design approach". Each column reports the IT classes obtained when the green part is designed on the hypothesis of the anisotropic behaviour of the reference geometry shown in the relevant row. Again, the two process capabilities IT 9 and IT8 are considered.

The "anisotropic design approach" allows obtaining lower tolerances than the "isotropic design approach", as by the comparison between Tables 2 and 3, which confirms the significant influence of the anisotropic dimensional changes on the precision of PM parts, particularly when the tolerance relevant to the process capability is low. Aiming at identifying a unique "reference geometry", the thin high ring has to be avoided, while the thick ring is the most promising for each other geometry.

Same calculation has been made for the internal diameter and for the height, the thick ring being confirmed as the most promising "reference geometry".

Concerning the height, controlling tolerances in the axial direction might appear not so important, from the industrial point of view, given that axial dimensions are easier to adjust with a proper compaction strategy. Nevertheless, such a control gains in importance in the case of axial dimensions deriving from stepped dies.

The analysis above highlighted the importance of considering the anisotropy of dimensional changes in the design step. In the next paragraph a first study on the development of a model to describe the anisotropic dimensional change is proposed.

3.3 Developing a model for the anisotropic dimensional change

To develop a model for the anisotropic dimensional change, which accounts for the influence of material and geometry, the case of disks was considered first.

A relationship between the dimensional changes in height and in diameter has been established through the change in volume, given by Eq. (19) in the hypothesis of isotropic dimensional change and by Eq. (20) in the hypothesis of anisotropic dimensional change

$$
\left(1 + \frac{\Delta V}{V_g}\right) = \left(1 + \varepsilon_{iso}\right)^3\tag{19}
$$

$$
\left(1 + \frac{\Delta V}{V_g}\right) = \left(1 + \varepsilon_\phi\right)^2 \left(1 + \varepsilon_h\right) \tag{20}
$$

Consequently, the relationship given by Eq. (21) is derived

$$
(1 + \varepsilon_{\phi}) = \sqrt{\frac{(1 + \varepsilon_{iso})^3}{(1 + \varepsilon_{h})}}
$$
(21)

Plotting these entities on the graph shown in Fig. 9, the points relevant the different dimensional changes will lie on the bisector line, and the far they will be from the point relevant to the isotropic dimensional changes, the larger the anisotropy of dimensional changes.

On the basis of the consideration above, the parameter K is proposed as anisotropy parameter, given by Eq. (22)

$$
K = \frac{\left|(1 + \varepsilon_{\phi}) - (1 + \varepsilon_{iso})\right|}{\left|\varepsilon_{iso}\right|} = \frac{\left|\varepsilon_{\phi} - \varepsilon_{iso}\right|}{\left|\varepsilon_{iso}\right|} \tag{22}
$$

In Fig. 10 K is plotted versus $|\varepsilon_{iso}|$ - data coming from different

Fig. 9 Representation of anisotropic dimensional change

Fig. 10 Anisotropy parameter K vs. $|\varepsilon_{iso}|$ for disks

sintering temperatures were used (1000°C, 1120°C, 1280°C, 1350°C). All the materials were considered, data were distinguished only by the geometry (height of the disk).

A good correlation between K and $|\varepsilon_{iso}|$ is given by Eq. (23), for these experimental data.

$$
K = \exp\left(\frac{A}{\varepsilon_{iso}} + B\right) - 1\tag{23}
$$

Considering that data are plotted irrespective to the material, it may reasonably deduced that the influence of material is given by $|\varepsilon_{iso}|$, i.e., by its volume change, as affected by the sintering temperature, while the influence of the geometry is given by constants A , B , and n , assuming different values for the high and low disks, as shown in Fig. 10. The constant n has always been considered as $2/3$, being the best fitting value which allows the comparison between the curves relevant to the different geometries.

To carry out this procedure on rings, under the same assumptions regarded for disks, ε_{f} and ε_{f} in have to be considered, they being different in principle. The change in volume is given for rings by Eq. (24)

$$
\left(1+\frac{\Delta V}{V_g}\right) = \left(1+\varepsilon_h\right)\frac{\left(1+\varepsilon_{\phi_{ext}}\right)^2 - R^2 \left(1+\varepsilon_{\phi_{int}}\right)^2}{1-R^2}
$$
\n
$$
R = \frac{\phi_{g_{int}}}{\phi_{g_{ext}}}
$$
\n(24)

An additional relationship between ε_{f} and ε_{f} int has to be found, aiming at identifying an anisotropy parameter K for rings too.

Fig. 11 ε_{fext} vs. ε_{fint} for rings - different materials grouped by geometry

The relationship defined by Eq. (25) has been found experimentally, as shown in Fig. 11, which collects data coming from different sintering temperatures (1000°C, 1120°C, 1280°C), distinguished only by the geometry.

$$
\frac{1 + \varepsilon_{\phi_{int}}}{1 + \varepsilon_{\phi_{ext}}} = \alpha = 1 + 0.25 \cdot 10^{-3}
$$
 (25)

The dimensional variation of the internal diameter can now be expressed in terms of the dimensional variation of the external diameter, so that the change in volume is expressed by Eq. (26), introducing the geometry parameter γ

$$
\left(1 + \frac{\Delta V}{V_g}\right) = (1 + \varepsilon_h)(1 + \varepsilon_{\phi_{ext}})^2 \frac{1 - \alpha^2 R^2}{1 - R^2}
$$

$$
y^2 = \frac{1 - \alpha^2 R^2}{1 - R^2}
$$
 (26)

The change in volume can be expressed in terms of the isotropic dimensional change and the anisotropy parameter K is defined for rings by Eq. (27). It may be observed that Eq. (27) becomes Eq. (22) in the case of disks, R being equal to zero and γ equal to one consequently.

$$
K = \frac{\left(1 + \varepsilon_{\phi_{ext}}\right)\gamma - \left(1 + \varepsilon_{iso}\right)}{\left|\varepsilon_{iso}\right|} \tag{27}
$$

In Fig. 12 K is plotted versus $|\varepsilon_{iso}|$ - data coming from different sintering temperatures were used (1000°C, 1120°C, 1280°C). All the materials were considered, data were distinguished only by the geometry of the rings.

The correlation between K and $|\varepsilon_{iso}|$ by Eq. (23) is very good for the thick ring, and quite good for the thin high ring. The difference between theoretical and experimental values of K for the thin high ring has been calculated, and the relevant difference in the dimensional changes has been derived, which corresponds to 0.2% maximum.

The data relevant to the thin low ring are quite dispersed, and no well represented by the correlation in Eq. (23). The reason might be found in the shape distortions, which are not negligible in the low thin rings in certain process conditions, so that the relationship given by Eq. (21) is not strictly applicable. Further experiments are in progress to analyse these results in depth.

On the basis of these results, a design procedure accounting for anisotropic dimensional change is proposed, assuming that change in

Fig. 12 anisotropy parameter K vs. $|\varepsilon_{iso}|$ for the thick ring (upper) and the thin rings (lower)

Design procedure

Fig. 13 design procedure accounting for the anisotropic dimensional changes for disks (left) and rings (right)

volume is only function of the material and process - so that ε_{iso} is only function of the material and process - and shape distortions are negligible - so that a relationship between ε_f and ε_h can be derived. A scheme for the design procedure is summarised in Fig. 13.

 ε_{iso} is given by the change in volume, while the nominal dimensions of the part allow to identify the anisotropy constants A , B , n and the parameter γ . In the design procedure, the parameter R given by the ratio between the internal and external diameter in the green state, which allows obtaining the parameter γ , has been substituted by the ratio between the relevant nominal values $(R=\phi_{int N}/\phi_{ext N})$. It has been demonstrated that the difference between R^2 calculated by the diameters in the green state and R^2 calculated by the nominal values is definitely negligible. The anisotropy parameter K is obtained by Eq. (23) for disks and by Eq. (27) for rings and it allows deriving the dimensional change of the external diameter ε_{ϕ} ext. In the case of rings, the dimensional change of the internal diameter $\varepsilon_{\phi int}$ is given by Eq. (25). The dimensional change of height is finally given by Eq. (21) for disks and Eq. (26) for rings. The dimensional change being known, the green dimensions, which allow obtaining the required nominal dimensions taking into account the anisotropy, are derived.

4. Conclusions

This work studied the effect of material and geometry on the dimensional change on sintering of PM parts. The dimensions of disks and ring-shaped parts made by both shrinking and swelling materials have been measured in the green and sintered state and the dimensional changes have been derived and analysed. The main conclusions are following summarized.

• The anisotropy of dimensional change on sintering depends both on material and on geometry.

• Effect of the material:

° shrinking systems: in Fe parts (solid state sintering) dimensional changes are very small and significantly anisotropic, while in Fe-P parts (liquid phase sintering) dimensional changes are somehow larger and less anisotropic.

° swelling systems (liquid phase sintering): in Fe-2%Cu parts dimensional changes are quite small and anisotropic, while in Fe-4%Cu parts dimensional changes are significantly large and negligibly anisotropic. A strong effect of the liquid phase in decreasing anisotropy has been revealed.

• Effect of the geometry: increasing height, anisotropy tends to decrease for disks, while the opposite is generally observed for rings. No univocal trend concerning the influence of the diameter, and thickness in case of rings, was observed.

• Influence on precision: attainable dimensional tolerances significantly worsen if anisotropic dimensional change is neglected in the design step, even if the dimensional variations are small, particularly if the dimensional tolerance relevant to the process capability is low.

• Development of a model: a model for designing disks and rings accounting for the anisotropic dimensional change is being developed, assuming that the change in volume is only function of material and process.

Work is in progress, aiming both at evaluating more complex geometries and at investigating the mechanisms, which determine the influence of material and geometry on anisotropy.

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