

# Pattern Generation and Control of a Double Pendulum using a Nonlinear Oscillator

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*Prior to developing the wearable walking assistant robot that supports walking for hemiplegia patients, a neural oscillator, a type of CPG (central pattern generation), was applied to 2-DOF double pendulum, which can replace the leg of a robot. The walking pattern generation method was proposed using the gait pattern of the non-affected side of hemiplegia patients. Because it is difficult for hemiplegia patients to distinguish the intended action signal of the patient wearing a robot on their affected side, we had to utilize a limited amount of information to get the maximum effect. We needed an effective solution for robot control in an outside environment where many unknown disturbances exist. In order to deal with these two problems, we used a nonlinear oscillator with a double pendulum as a test bed to explore the possibility of producing a robot walking pattern.*

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## 1. Introduction

The central pattern generator (CPG) refers to the biological neuronal circuit that exists within the spinal cord of the vertebrate. Through it, the vertebrate breathes, walks, flies, and swims without being particularly aware of these activities. One of the characteristics of CPG is that it can autonomously generate periodic rhythms without the presence of an external input signal.<sup>1</sup> CPG is widely applied to the biped control of a humanoid robot or a quadruped robot, but biological CPG has a very complicated structure and many parameters, since it was developed based on physiological data obtained from experiments.<sup>2</sup> Therefore, direct use biological CPG is almost impossible. The neural oscillator is a simplified version of the CPG, and it has a similar design as CPG. As it has a simple structure and maintains the key mechanisms of biological CPG, it could become a good replacement for biological CPG. There are three types of widely used oscillators: recurrent neural oscillator,<sup>3-8</sup> lamprey (phase) oscillator,<sup>9-12</sup> and the well-known Van der Pol oscillator.<sup>13,14</sup> The Lamprey oscillator has its foundation in Hopf oscillator. The typical recurrent neural oscillator is from Matsuoka. With the Lamprey and Van der Pol oscillator, the controller needs to be designed separately for synchronization of input and output signals. Aside from periodic input signals, signals do not synchronize. However, the Matsuoka neural oscillator, a type of recurrent oscillator,

automatically synchronizes input and output signals without having a separate controller. It also has constant inputs that are not periodic signals, which makes it more applicable to a wearable robot than the other two types of oscillators. Therefore, this study proposes the use of a recurrent neural oscillator to generate walking patterns. To verify the results in the experiment, the neural oscillator network was applied to a 2-DOF double pendulum, which will replace the leg of the wearable robot. The experiment used the walking pattern of the unaffected side of hemiplegia patient as an input signal to see if a usable walking pattern on the affected side is created. In Section 2, explanations about the design of a single neural oscillator unit, its local stability analysis, and the neural oscillator network design, which is composed of stable units, are presented. Furthermore, a walking pattern creation of a double pendulum and its means of use will be explained.

## 2. Neural Oscillator Design

### 2.1 Neural Oscillator unit design

Matsuoka presented a dynamics model of neural oscillators and a mutual inhibition network to generate numerous periodic patterns using neural oscillators and to control the frequency of the period.<sup>6,15</sup> Eqs. (1) ~ (5) show the dynamics model of neural oscillators proposed by

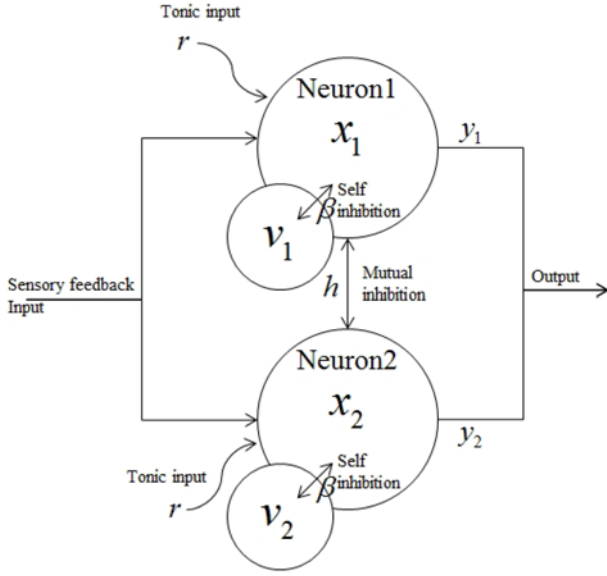


Fig. 1 Neural oscillator

Matsuoka, while Fig. 1 shows the internal structure of the neural oscillators.

$$\tau_1 \dot{x}_1 = -x_1 - \beta v_1 - h y_2 + L s + r, \quad (1)$$

$$\tau_2 \dot{v}_1 = -v_1 + y_1, \quad (2)$$

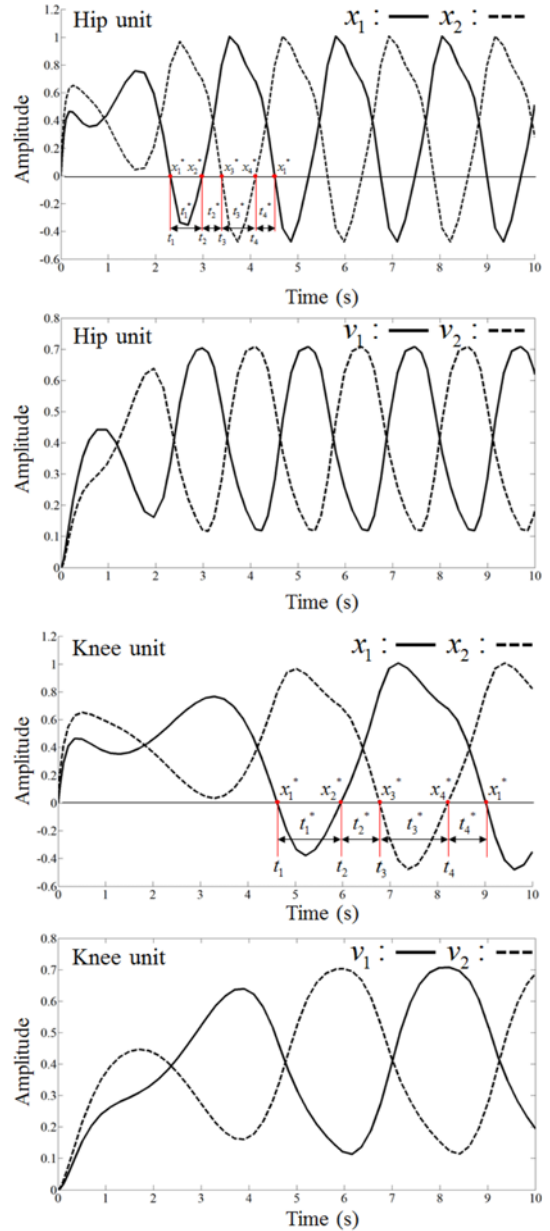
$$\tau_1 \dot{x}_2 = -x_2 - \beta v_2 - h y_1 + L s + r, \quad (3)$$

$$\tau_2 \dot{v}_2 = -v_2 + y_2, \quad (4)$$

$$y_i = f(x_i) = \max(x_i, 0), \quad i = 1, 2. \quad (5)$$

In Eqs. (1)–(5),  $\tau_1$  and  $\tau_2$  are positive time constants,  $x_i$  is the variable of the membrane voltage in a neuron,  $v_i$  is the membrane current of the slow recovery component,  $f(x_i)$  is a nonlinear function,  $y_i$  is the neuronal output,  $h$  is the gain for the interaction between the two neurons,  $\beta$  is the adaptive gain for the self-inhibition of the neuron,  $s$  is the sensory feedback,  $r$  is the tonic input, and  $L$  is the feedback gain.

At first, we should define the parameters of a neural oscillator that satisfy Eqs. (6)–(8). If the above conditions are satisfied, a limit cycle can be generated and bounded.<sup>16</sup> That is, the stability conditions set forth in Eqs. (6)–(8) are conditions for which a limit cycle can be generated and bounded. Hence, we do not have to examine the global stability of the neural oscillator, but we should examine the local stability based on a certain initial value in a certain range. In addition, in Eqs. (9)–(10), we can determine the frequency of output signals.<sup>16</sup> The fact that CPGs can independently generate rhythmic patterns even without sensory feedback has already been established,<sup>1</sup> and sensory feedback does not affect the stability of neural oscillators. Therefore, in this section, we examine the local stability of the neural oscillator when sensory feedback signals are 0 (i.e.,  $L=0$ ). We designed two types of neural oscillator units: one unit controlling the hip joint and the other unit controlling the knee joint. Internal parameters of the hip and knee units are shown in (6) and (7), respectively. In the internal parameters, the tau and external signal frequency of the unit are in an inverse relationship. As the value of tau increases, the output frequency

Fig. 2 Plot of the states  $x_1$  and  $x_2$  in the time domain

decreases.

$$\tau_1 = 0.25, \quad \tau_2 = 0.5, \quad \beta = 2, \quad h = 2, \quad r = 2. \quad (6)$$

$$\tau_1 = 0.5, \quad \tau_2 = 1, \quad \beta = 2, \quad h = 2, \quad r = 2. \quad (7)$$

$$\beta > h - 1, \quad (8)$$

$$h > 1 + \frac{\tau_1}{\tau_2}, \quad (9)$$

$$r \neq 0, \quad (10)$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1 + \left(\frac{\beta}{h} - 1\right) \left(\frac{\tau_1}{\tau_2} + 1\right)}{\tau_1 \tau_2}} \quad (11)$$

$$\text{If } \beta = h, \quad f = \frac{1}{2\pi} \sqrt{\frac{1}{\tau_1 \tau_2}} \quad (12)$$

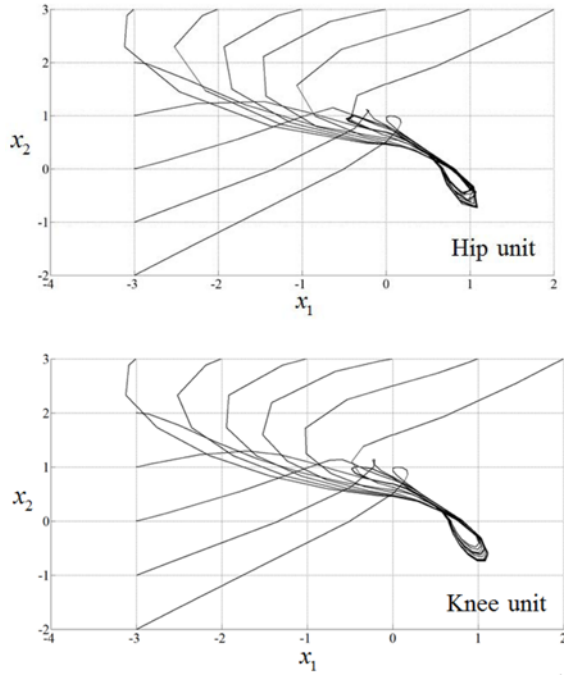


Fig. 3 Convergence of the hip and knee units in the  $x_1$  and  $x_2$  planes with different initial values

In Eqs. (1)~(5), the neural oscillator can be regarded as a piecewise linear system, and a piecewise linear method can be adopted to analyze the local stability.<sup>18</sup> The switching surface is given by  $S_i = C_i x + d_i$ , where  $C_i$  is a row vector and  $i = 1, \dots, 4$ . If it is assumed that at  $t_1$ , the initial switch occurs on switching surface  $S_1$ , and the state of the neural oscillator at the moment will be  $x_1^*$ . When the time between  $t_1$  and  $t_2$  is assumed to be  $t_1^*$ , and the second switch occurs on switching surface  $S_2$ , then the state of the neural oscillator will be  $x_2^*$ . Since four switches occur in each limit cycle of the neural oscillator's outputs,  $x_5^* = x_1^*$  and  $t_5^* = t_1^*$ . In Fig. 2, output signals of hip and knee units are shown.

The fact that many equilibrium points or one limit cycle exists in each stable nonlinear system is well known. The limit cycle of the neural oscillator designed by us is shown in Fig. 3. The oscillator's local stability can be proved using Poincaré maps.<sup>19</sup> If all eigenvalues of the Jacobian of  $P$  exist in the unit disk when a Poincaré map in a small area around point  $x_1^*$  and  $S_1$  is considered, the limit cycle should be locally stable.<sup>18,20</sup>

If the Jacobian of the Poincaré map  $P$  of the hip unit is as follows:

$$W_{hip} = \begin{bmatrix} -0.2427 & 0.2858 & 0.2406 & -0.2942 \\ 0.0364 & -0.0433 & -0.0361 & 0.0446 \\ 0 & 0 & 0 & 0 \\ -0.0030 & -0.0023 & 0.0034 & 0.0040 \end{bmatrix}$$

then the eigenvalues of  $W$  are as follows:

$$\begin{bmatrix} -0.2883 \\ 0.0064 \\ 0 \\ 0 \end{bmatrix}$$

If the Jacobian of the Poincaré map  $P$  of the knee unit is as follows:

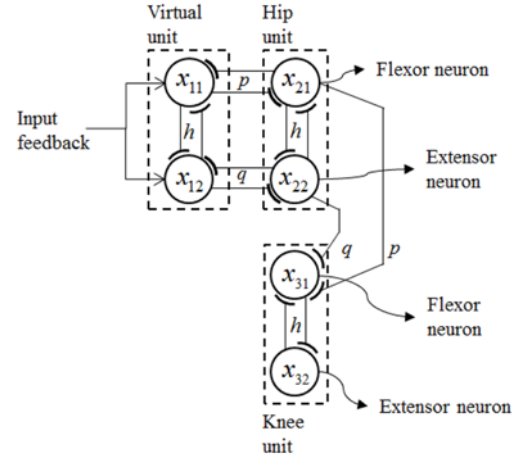


Fig. 4 The neural oscillator network of a neural oscillator

$$W_{knee} = \begin{bmatrix} -0.2332 & 0.3773 & 0.2286 & -0.3961 \\ 0.0152 & -0.0248 & -0.0149 & 0.0261 \\ 0 & 0 & 0 & 0 \\ -0.0070 & 0.0097 & 0.0073 & -0.0086 \end{bmatrix}$$

then the eigenvalues of  $W$  are as follows:

$$\begin{bmatrix} -0.2694 \\ 0.0029 \\ 0 \\ 0 \end{bmatrix}$$

Consequently, it can be seen that all eigenvalues exist in the unit disk. Therefore, the limit cycle is locally stable.

## 2.2 Neural Oscillator network design

Earlier, we designed a single neural oscillator and analyzed its stability. In this section, the many neural oscillators that are fully interconnected will be described. The  $n_{th}$  order multiple neural oscillators are expressed in Eqs. (11)~(16).  $n$  refers to the total number of oscillators and  $x_{i1}$  and  $x_{i2}$  refer to the state variables of the two neurons existing in the  $i_{th}$  oscillator.  $\tau_{i1}$  and  $\tau_{i2}$  are time constants, while and refer to weights for the connections among neural oscillators.

$S_i$  is the sensory feedback,  $L$  the feedback gains.  $r_i$  is the tonic input. If Eqs. (17)~(18) is satisfied and sensory feedback  $S_i$  is bounded, then the neural oscillators' states  $x_{i1}$ ,  $v_{i1}$ ,  $x_{i2}$  and  $v_{i2}$  are bounded.<sup>16</sup>

$$\tau_{i1} \dot{x}_{i1} = -x_{i1} - \beta_i v_{i1} - h_i v_{i2} + r_i - \sum_{j=1, j \neq i}^n (p_{ij}^1 y_{j1} + q_{ij}^1 y_{j2}) + L S_i, \quad (13)$$

$$\tau_{i2} \dot{v}_{i1} = -v_{i1} + y_{i1}, \quad (14)$$

$$\tau_{i1} \dot{x}_{i2} = -x_{i2} - \beta_i v_{i2} - h_i v_{i1} + r_i - \sum_{j=1, j \neq i}^n (p_{ij}^2 y_{j1} + q_{ij}^2 y_{j2}) + L S_i, \quad (15)$$

$$\tau_{i2} \dot{v}_{i2} = -v_{i2} + y_{i2}, \quad (16)$$

$$y_{i1} = f(x_{i1}) = \max(x_{i1}, 0), \quad (17)$$

$$y_{i2} = f(x_{i2}) = \max(x_{i2}, 0), \quad i = 1, 2, \dots, n. \quad (18)$$

$$-h_i v_{i2} + L S_i \leq \sum_{j=1, j \neq i}^n (p_{ij}^1 y_{j1} + q_{ij}^1 y_{j2}) + c_{i1}, \quad (19)$$

$$-h_{ij}y_{i1} + Ls_{i2} \leq \sum_{j=1, j \neq i}^n (p_{ij}^2 y_{ji}^2 + q_{ij}^2 y_{j2}^2) + c_{i2}, \quad (20)$$

where  $C_{i1}$  are  $C_{i2}$  constants.

When the single units of neural oscillators are stable, and those stable units are fully interconnected, each output signal is proved to be bounded. Then this neural oscillator network can be applied to the robot. Fig. 4 shows the design of the neural oscillator network. Three units are used in the network. The hip unit designed in chapter 2.1 is used for the virtual hip unit and robot hip unit, and the previously designed knee unit is used in the robot knee unit. Each unit creates rhythm through stimulation and inhibition activities. As shown in Fig. 4, the output signal of the hip unit is used as actuation signal to the hip joint motor of the double pendulum, and the output signal of the knee unit is used as an actuation signal to the knee joint motor of the double pendulum.<sup>17</sup> The virtual unit synchronizes the hip joint angle position information of the unaffected side of the patient, which acts as the input signal here. The output signal of virtual unit is not actually used to drive the robot, but it can be first synchronized with the input signal to create the walking pattern of a double pendulum adjusted to the changes of input signal. Then adjusted walking pattern can be created by a hip unit and a knee unit connected to a virtual unit, based on the changes in the input signal.

### 3. Experiment

The experiment was conducted in a way that the walking pattern information obtained on the hemiplegia patient was used to create a walking pattern of a double pendulum to which a neural oscillator network is applied. Walking pattern information from a hemiplegia patient was measured by attaching a goniometer to the hip joint and knee joint of both legs of the patient. He was made to walk five meters on flat ground. The measurement was made three times. The patient was a male in his 50s, with stage 4 Brunnstrom and hemiplegia on the right side. The joint angles in all graphs in this section and next section have are positive in the flexion direction.

Fig. 5 shows how the walking patterns of the hemiplegia patient were measured. The patient was standing at the starting point with both hands down, walking at the start signal, and stopping at the stopping point. This was repeated three times, and the distance between starting and stopping point was five meters. Due to spatial limitations and the experimental range of the wired equipment, the experiment could not be done at a distance longer than five meters. Therefore, we extracted on cycle of the most stable walking pattern observed during those five meters, excluding starting and stopping motions. We generated repeated patterns of the cycle and created a continuous walking state. Fig. 6 indicated the most stable walking pattern among the three walking patterns. In this walking pattern, we used one cycle from 2.5s~4s from the data of the unaffected side's left hip joint angle position. In other words, the most stable cycle of the unaffected hip joint angle position of hemiplegia patient was continually used as an input signal.

Fig. 7 shows the schematic diagram of the test bed where Fig. 8 represents the actual one. Here,  $l_1$ ,  $l_{c1}$ ,  $m_1$  represents the length of link

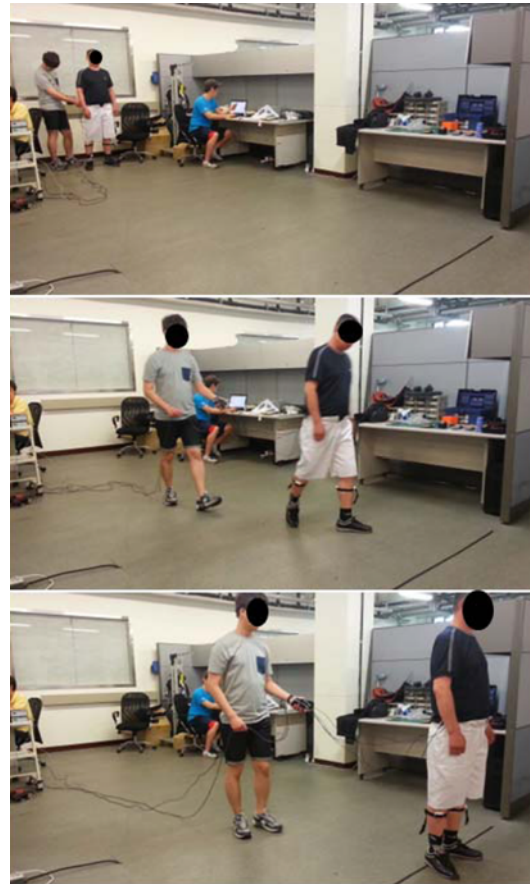


Fig. 5 Measuring walking pattern of the hemiplegia patient

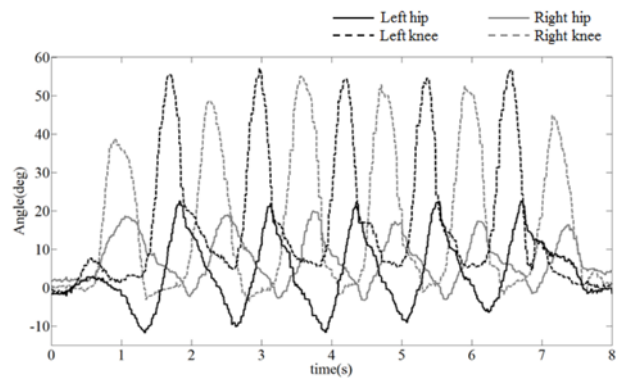


Fig. 6 Result of walking pattern measurement of the hemiplegia patient

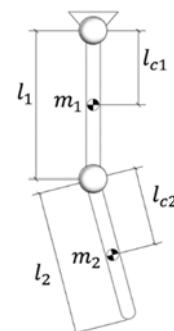


Fig. 7 Schematic diagram of double pendulum test bed



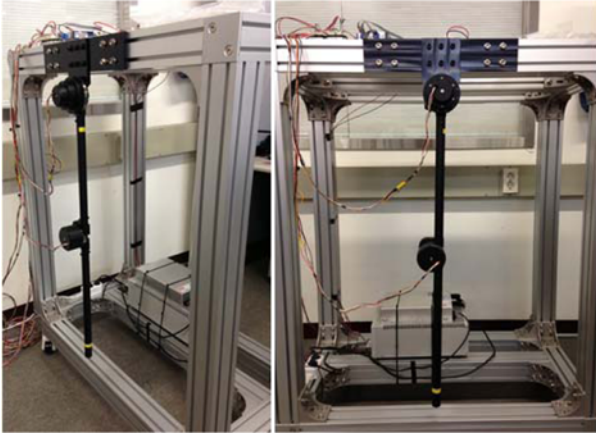


Fig. 8 Double pendulum test bed

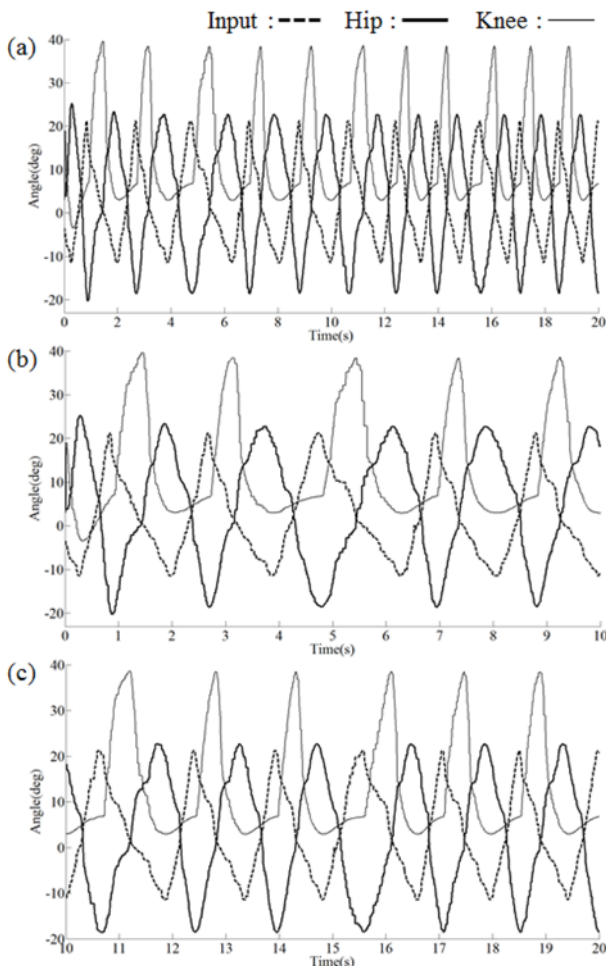


Fig. 9 Experiment result of double pendulum. (a) Result of the entire experiment, (b) Zoomed result of the experiment from 0s to 10s, (c) Zoomed result of the experiment from 10s to 20s

1, center of link 1 and mass of link 1. Similarly,  $l_2$ ,  $l_{c2}$ ,  $m_2$  represents the length of link 2, center of link 2 and mass of link 2. All these values are chosen suitable for the human wearer. Links are made of Aluminum. Moreover, 200 w and 100 w motors were used for hip and knee joints, respectively. The hip joint motor was anchored to a box made of Aluminum profiles to act as a pivot point. The robot independently

controlled the joint, and each sub-controller used a motor controller capable of automatically running. The previously obtained angle position data was iterative sent to the neural oscillator network as the input signal and the input and input signal frequency varied from time to time.

The result of this study is shown in Fig. 9. It is based on 20 seconds experimental duration. The dotted line represents the one that repeatedly used information from the unaffected hip joint angle position of the hemiplegia patient's walking motion in cycle one as an input signal. The thick solid line represents the output of the hip unit in the neural oscillator network. The thin solid line represents the output of the knee unit of the neural oscillator network. Fig. 9(a) is the result of the entire experiment; (b) is the amplification of 0~10 s section in the result graph; and (c) is the amplification of 10~20 s section. The general frequency of input signal was about 0.5 Hz, varying from 0.4 Hz, 0.6 Hz, and 0.7 Hz in the 2.5~5 s section, the 11~15 s section, and the 16~19 s section, respectively. The generated patterns show that the output signals of the hip unit and knee unit are well adjusted to the changed frequency of the input signal.

#### 4. Conclusion

In this paper, Neural Oscillator Network (NON) is designed for the walking assistance for hemiplegia patients. Using Poincaré maps, Neural Oscillator Network (NON) local stability is confirmed. Experimental study shows that NON adapts walking pattern generated by the human subject. The NON-output signal is able to generate walking patterns that are adaptable to the input signal.

Based on this work, further studies on robots that generate adjustable walking patterns by receiving on-time walking patterns from the unaffected side as well as the clinical effects and verification of these effects by wearing the robot should be conducted. In addition, more factors that increase the balance of both legs of hemiplegia patients during walking should be explored and applied to wearable robots to enhance the safety of patients.

#### ACKNOWLEDGEMENT

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