

Performance Evaluation of Five-DOF Motion in Ultra-precision Linear Stage

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In this study, the performance of five-DOF motion in ultra-precision linear stage is evaluated accurately by extending the application of the ISO 230-2 International Standard, which is focused only on the linear displacement motion of a linear stage. The bidirectional accuracy and bidirectional repeatability of positioning in five-DOF motion are calculated by measured geometric errors. Five geometric errors except for the linear displacement error of a linear stage are measured simultaneously using the optimal measurement system, which is designed to enhance the standard uncertainty of the estimated geometric errors. The geometric errors are in good agreement with those from the laser interferometer. In addition, the confidence intervals of the performances are determined by the uncertainties of the equipment used in the experiment.

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1. Introduction

An ultra-precision linear stage is used widely in various systems, such as semiconductor equipment and the diamond turning machine.¹⁻³ Therefore, the measurement and evaluation of errors that affect the performance (accuracy, repeatability) of the stage are important for ensuring confidence interval showing stochastic characteristics. Geometric errors, which are one of the primary error sources in the stage, have a strong influence on the form accuracy of the machined part so the measurement and evaluation for the errors are required.⁴⁻⁶ For example, the machined surface is affected mainly by the geometric errors of two axes in a diamond turning machine, which generally consists of *X*-axis and *Z*-axis. The horizontal straightness error of *X*-axis and linear displacement error of *Z*-axis are only measured and compensated.^{7,8}

The linear stage is designed to follow linear motion but it also has unexpected additional motion. This motion is defined as the geometric error under the quasi-static command. The geometric error is generally categorized as the PDGEs (Position Dependent Geometric Errors) and PIGEs (Position Independent Geometric Errors).⁹⁻¹² The PDGEs are caused by an imperfection of the linear stage components and are defined in terms of the three position errors (linear displacement error, horizontal straightness error and vertical straightness error) and three rotation errors (roll, pitch and yaw). The PIGEs are caused primarily by

the assembly between the stages. These errors are defined as the squareness error and offset error.

A range of methods have been used to measure the geometric error in a linear stage. A six-DOF measurement system was developed using three laser interferometers and two quadrant photo detectors for the geometric errors.¹³ The MDFM (Multi-Degree-of-Freedom Measurement) system, which consists of four position sensitive detectors for five geometric errors was proposed.¹⁴ The multi-DOF measurement system was developed to measure the geometric error of mMT (Miniaturized Machine Tool) using five capacitive sensors.¹⁵ In addition, the optimal measurement system was proposed to minimize the standard uncertainty of the estimated geometric errors by determining the optimal position of the reference coordinate system.¹⁶

The ISO 230-2 International Standard, as the evaluation method under no-load condition, focuses only on linear displacement motion of a linear stage and the angular positioning motion of a rotary table. This standard specifies the method for evaluating the accuracy and repeatability of the positioning, which are stochastic characteristics with a coverage factor of $k=2$, in a numerically controlled machine tool.¹⁷

In this study, the performance of five-DOF motion in the ultra-precision linear stage is evaluated by extending the application of the ISO 230-2 International Standard. The optimal measurement system is explained briefly and the standard uncertainty of estimated geometric error according to the number of sensor used for the measurement is

analyzed in Section 2. The performance with the measurement uncertainty is evaluated using the measured geometric errors by the optimal measurement system in Section 3. Finally, the conclusions and applications are summarized in Section 4.

2. The Five-DOF Measurement System

A linear stage is designed to follow linear motion but it has unexpected motion caused by geometric errors. The linear stage coordinates system $\{X\}$ can be modeled using six geometric errors, as shown in Fig. 1. The six geometric errors consist of three position errors (linear displacement δ_{xx} , horizontal straightness δ_{yx} and vertical straightness δ_{zx}) and three angular errors (roll ε_{xx} , pitch ε_{yx} and yaw ε_{zx}).

The optimal measurement system consists of a reference mirror and five capacitive sensors for measuring the five geometric errors, as shown in Fig. 2(a). The standard uncertainty of the measured data u_{DEVICE} is analyzed to enhance the standard uncertainty of the estimated geometric errors from the optimal measurement system. This standard uncertainty is determined mainly by the uncertainty of the sensor, reference mirror and environmental factors (temperature, humidity and pressure etc.) in the experiments. The environmental factors that are dependent on time are not considered and the effects of the equipment only are analyzed. The standard uncertainty of the sensor is determined primarily by the resolution, linearity and accuracy, and the standard uncertainty of reference mirror is governed by the flatness. These factors are assumed to have a uniform distribution,¹⁸ and the standard uncertainty of the measured data is expressed as Eq. (1).

The geometric errors are estimated from the combination of the measured data m_i ($i=1, \dots, 5$). Hence, the standard uncertainty, u_{DEVICE} affects the standard uncertainty of the estimated geometric errors. In addition, the standard uncertainty of the estimated geometric error is dependent on the relative position between the reference coordinate system $\{F\}$ and sensor s_i ($i=1, \dots, 5$) due to Abbe's error. The standard uncertainty of the estimated geometric error should be minimized for precise measurements and can be reduced by determining the optimal position of reference coordinate system $\{F\}$.

The position of the reference coordinate system $\{F\}$ can be described using the offset o_x , o_y and o_z . The position of the sensor s_3 is modeled to investigate the effect by relative position of sensors (s_3 , s_4 and s_5), as shown in Fig. 2(b). The parameter w , h represents the width and height of the measurement area at the reference mirror, respectively. The position of the sensors s_i ($i=1, \dots, 5$) from the reference coordinate system $\{F\}$ is defined and the measured data m_i are calculated using the HTM (Homogeneous Transform Matrix). The geometric errors of a linear stage are estimated by solving the simultaneous equations from the measured data m_i . The standard uncertainty of the measured data are assumed to be the same, $u_{DEVICE} = u_{DEVICE,1} = u_{DEVICE,5}$.¹⁹ The conditions for minimizing the standard uncertainty are defined and the offset o_x , o_y , o_z and parameter l are calculated using the partial differential equation. The offset o_x , o_y , o_z and parameter l are determined to be $h/4$, $w/2$, $h/2$ and $h/4$ respectively. The optimal position of the sensor and reference coordinate system $\{F\}$ is determined from these results, as shown in Fig. 2. The relationship between the geometric errors and measured data are determined as shown in Eq. (2). The position shape of the three

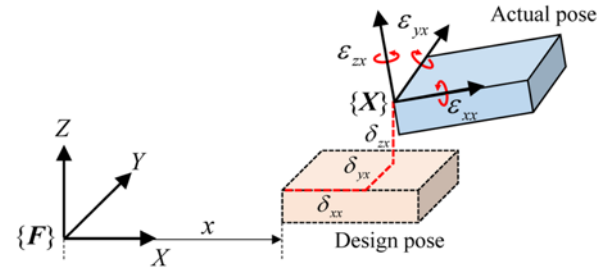


Fig. 1 Geometric errors of a linear stage

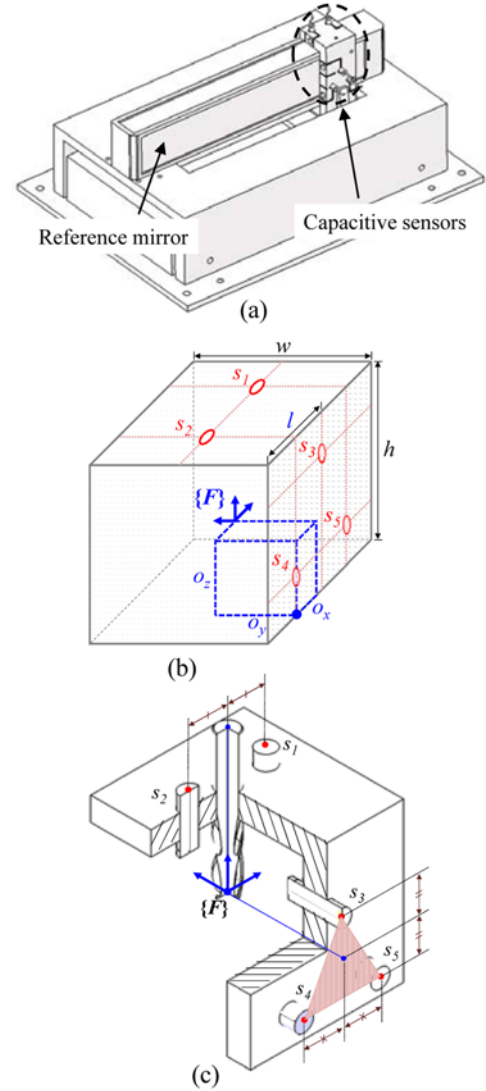


Fig. 2 Five-DOF measurement system¹⁶ (a) System configuration, (b) Position modeling of the reference coordinate system, (c) Optimal position of the reference coordinate system

sensors located on the side is an isosceles triangle. The position of the reference coordinate system $\{F\}$ is located at the middle of the height in the isosceles triangle.

$$u_{DEVICE} = \sqrt{(u_{SENSOR})^2 + (u_{MIRROR})^2} \quad (1)$$

where,

$$\begin{aligned}
u_{SENSOR} &= \sqrt{(u_{LINEARITY})^2 + (u_{RESOLUTION})^2} \\
&= \frac{1}{2\sqrt{3}} \sqrt{(s_{LINEARITY}^+ - s_{LINEARITY}^-)^2 + (r)^2} \\
u_{MIRROR} &= \frac{m_{FLATNESS}^+ - m_{FLATNESS}^-}{2\sqrt{3}}
\end{aligned}$$

This maximizes the average effect due to its symmetric location from the reference coordinate system $\{F\}$. Therefore, the geometric errors ε_{yx} , ε_{zx} and δ_{zx} are estimated from a combination of the two sensors data and the geometric errors ε_{xx} and δ_{yx} are estimated from a combination of the three sensors data. The standard uncertainty is defined as root sum square error (RSSE) according to the ISO 230-9 International Standard. The estimated geometric error using the three measured data has a relatively small standard uncertainty. As shown in Eq. (3), the standard uncertainty of the estimated geometric errors ε_{xx} and δ_{yx} is approximately 13.4% smaller than the standard uncertainty of the estimated geometric errors ε_{yx} , ε_{zx} and δ_{zx} . Similarly, opposite results can be obtained when three and two sensors are installed on the top and side, respectively. In addition, when six sensors are used to measure the geometric errors using an additional sensor on the top, the standard uncertainties of the straightness errors (vertical, horizontal) are the same.

In previous studies, the optimal position of the reference coordinate system $\{F\}$ is not considered so the position is normally located on a sensor. When two sensors are located on the top, the vertical straightness error is measured using a sensor data. The measured error is affected directly by the standard uncertainty of the measured data u_{DEVICE} . On the other hand, the error is measured using two sensors data by determining the optimal position of the reference coordinate system $\{F\}$. Therefore, the coefficient of standard uncertainty is decreased by the average effect and the value is given in Eq. (3). Thus, the result shows an approximately 30% decrease in the coefficient by the optimal position of the reference coordinate system. The coefficient of standard uncertainty is determined by the number of used sensors for the measurement and the position of the reference coordinate system $\{F\}$, as shown in Fig. 3.

$$\varepsilon_{xx} = -\frac{2}{h}m_3 + \frac{1}{h}m_4 + \frac{1}{h}m_5 \quad (2)$$

$$\varepsilon_{yx} = -\frac{2}{h}m_1 + \frac{2}{h}m_2$$

$$\varepsilon_{zx} = -\frac{2}{h}m_4 + \frac{2}{h}m_5$$

$$\delta_{yx} = \frac{1}{2}m_3 + \frac{1}{4}m_4 + \frac{1}{4}m_5$$

$$\delta_{zx} = \frac{1}{2}m_1 + \frac{1}{2}m_2$$

$$u(\varepsilon_{xx}) = \frac{\sqrt{6}}{h}u_{DEVICE} \quad (3)$$

$$u(\varepsilon_{yx}) = \frac{2\sqrt{2}}{h}u_{DEVICE}$$

$$u(\varepsilon_{zx}) = \frac{2\sqrt{2}}{h}u_{DEVICE}$$

$$u(\delta_{yx}) = \frac{\sqrt{6}}{4}u_{DEVICE}$$

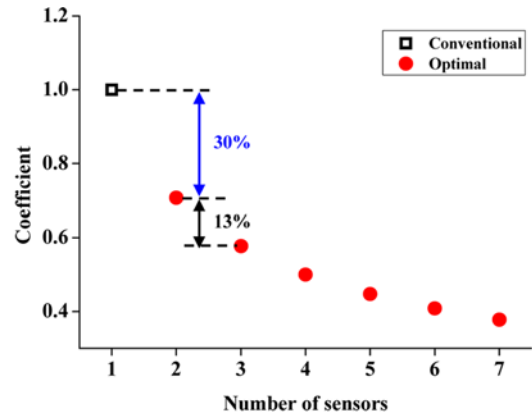


Fig. 3 Standard uncertainty coefficient by the number of sensors used for the vertical straightness error

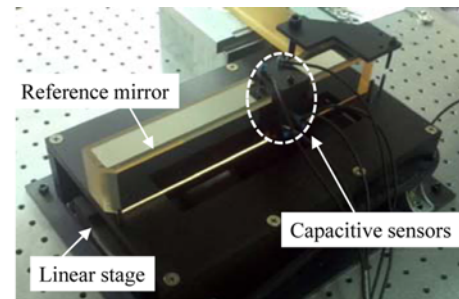


Fig. 4 Setup of the measurement system on a linear stage

$$u(\delta_{zx}) = \frac{\sqrt{2}}{4}u_{DEVICE}$$

In addition, it is necessary to check the reduction of the standard uncertainty coefficient according to the number of the sensors used in measurement. It shows a reduction of 13% using three sensors. Hence, it is unnecessary to use the extra sensor for the reduction of the standard uncertainty considering the measurement cost. Therefore, the measurement system consists of three and two sensors on the side and top, respectively.

3. Experimental Study

3.1 Measurement and verification of geometric errors

An experiment is conducted to measure the geometric errors of a linear stage (ABL1000, Aerotech Inc., U.S.A.) using the optimal measurement system. The capacitive sensors (4810 module, 2812 probe, ADE Technology Inc., U.S.A.) fixed on the table are moved to eliminate of Abbe's error and the reference mirror (Nitto Optical Co., Ltd., Japan) is stationary during the measurement, as shown Fig. 4. Specifications of the reference mirror, capacitive sensor and linear stage used in the experiment are summarized in Table 1. The uncertainties of the equipment are used to calculate the standard uncertainties of the estimated geometric errors and the results are listed in Table 2. The measurement uncertainties are calculated with a coverage factor of $k=2$. These results are used in the performance evaluation of five-DOF motion in Section 3.2.

Table 1 Specifications of the equipment used in the experiment

	Parameter	Specification
Reference mirror	Range	300 mm
	Flatness	± 30 nm, F.S.
	Width, w	35 mm
	Height, h	50 mm
Capacitive sensor	Range	± 100 μ m
	Linearity	± 60 nm, F.S.
	Resolution	0.8 nm
Linear stage	Stroke	100 mm
	Resolution	10 nm
	Drives	Linear brushless servomotor

Table 2 Standard uncertainties of the estimated errors

Parameter	Unit	Standard uncertainty	Measurement uncertainty
$u(\varepsilon_{xx})$	arcsec	0.14	0.28
$u(\varepsilon_{yy})$	arcsec	0.16	0.32
$u(\varepsilon_{zx})$	arcsec	0.16	0.32
$u(\delta_{yx})$	nm	23.72	47.44
$u(\delta_{zx})$	nm	27.39	54.78

The measured range is 100 mm, which is the stroke of the stage and the measurement interval is 10 mm. The experiments are carried out five times to measure the geometric errors in the bi-direction. The measured data of the sensors indicates large value due to a setup error of the reference mirror with respect to the reference coordinate system $\{F\}$. To minimize the effect of the setup error, the slope values of the estimated straightness errors δ_{yx} and δ_{zx} are calculated using the least square method and these values are subtracted from the estimated errors mathematically. In the experimental results, the pitch error indicates a larger value than the other angular errors, as shown in Fig. 5. This error might be affected by the flatness of base fixture between the linear stage and vibration isolation table.

To verify the measured geometric errors, a laser interferometer (XL80, Renishaw Plc., U.K.) is used and four geometric errors (two straightness errors, pitch error and yaw error) of a linear stage are compared. The experiments are conducted five times to improve the reliability of the measured results and the average of the measured data is used. In the experimental results, the maximum differences between the measured data by the two measurement systems are 0.11 arcsec for angular errors and 0.2 μ m for the position errors. This difference is within the measurement uncertainty of a laser interferometer, therefore measured results from the five-DOF measurement system shows the validity.

This five-DOF measurement system can measure the geometric errors simultaneously in a short time, so it is relatively less sensitive to environmental influence. In addition, this system has a constant standard uncertainty regardless of the measurement range. Therefore, the measured geometric errors are more stable and accurate than those from the other measurement system. However, laser interferometer takes a long time to measure the geometric errors and the standard uncertainty of the straightness error increase in proportion to the measurement range.

3.2 Performance evaluation

The performance of five-DOF motion in a linear stage is evaluated

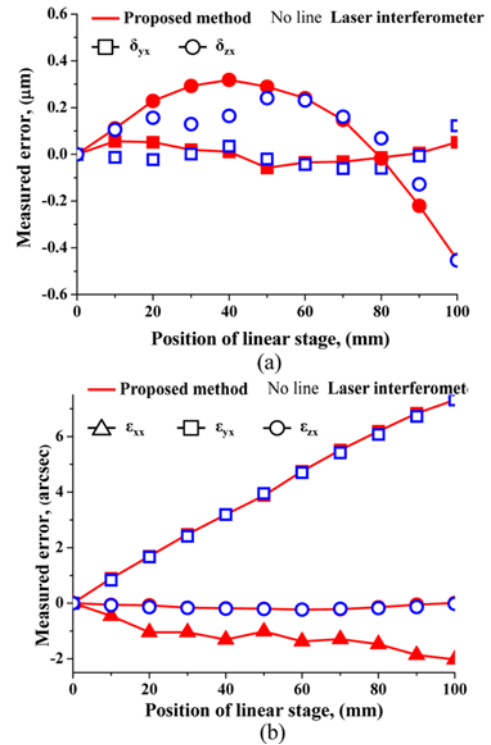


Fig. 5 Estimated geometric errors of a linear stage (a) Position errors, (b) Angular errors

by extending the application of the ISO 230-2 International Standard. The mean bidirectional positional deviation and estimator of the standard uncertainty are calculated. The evaluation results of the five-DOF motion are summarized in Table 3. As the performance, bidirectional accuracy A , and bidirectional repeatability R are evaluated using the calculated statistic parameters. The bidirectional accuracy of the position motions and angular motions are within 0.82 μ m and 7.41 arcsec, respectively. The bidirectional repeatability of the position motions and angular motions is within 0.09 μ m and 0.23 arcsec, respectively as shown in Fig. 6. The bidirectional accuracy of pitch motion has a large value, which might be affected by the base fixture as mentioned above. Nevertheless, the bidirectional repeatability of angular motion is similar (0.2 arcsec). In addition, the bidirectional repeatability of the position motions is similar as 0.1 μ m. Therefore, the performance evaluation is physically reasonable.

4. Conclusions

In this study, the performance of five-DOF motion are evaluated using an extended application of ISO 230-2 International Standard. The measurement uncertainties are derived to determine the confidence interval by calculating the uncertainty of the equipment used in the experiment. Therefore, the accuracy and repeatability with measurement uncertainty as performance are determined quantitatively. The stochastic characteristics can be used to confirm the problems that affect the performance of the stage. In addition, this diagnosis can contribute as a feedback regarding the machine tool design and assembly, thereby enhancing the precision for positioning.

Table 3 Evaluation results for five-DOF motion (unit: μm , arcsec)

Parameter		Unidirectional B	Unidirectional F	Bidirectional
Horizontal straightness motion	Mean reversal value	-	-	0.01
	Range mean bidirectional positional deviation M	-	-	0.11 \pm 0.05
	Repeatability of positioning R	0.04	0.03	0.07
	Accuracy A	0.15 \pm 0.05	0.15 \pm 0.05	0.16 \pm 0.05
Vertical straightness motion	Mean reversal value	-	-	-0.01
	Range mean bidirectional positional deviation M	-	-	0.77 \pm 0.05
	Repeatability of positioning R	0.04	0.09	0.09
	Accuracy A	0.82 \pm 0.05	0.82 \pm 0.05	0.82 \pm 0.05
Roll motion	Mean reversal value	-	-	-0.02
	Range mean bidirectional positional deviation M	-	-	2.03 \pm 0.28
	Repeatability of positioning R	0.17	0.14	0.22
	Accuracy A	2.07 \pm 0.28	2.13 \pm 0.28	2.13 \pm 0.28
Pitch motion	Mean reversal value	-	-	0.01
	Range mean bidirectional positional deviation M	-	-	7.32 \pm 0.32
	Repeatability of positioning R	0.11	0.10	0.21
	Accuracy A	8.22 \pm 0.32	8.23 \pm 0.32	8.23 \pm 0.32
Yaw motion	Mean reversal value	-	-	-0.07
	Range mean bidirectional positional deviation M	-	-	0.49 \pm 0.32
	Repeatability of positioning R	0.14	0.13	0.23
	Accuracy A	0.54 \pm 0.32	0.85 \pm 0.32	0.85 \pm 0.32

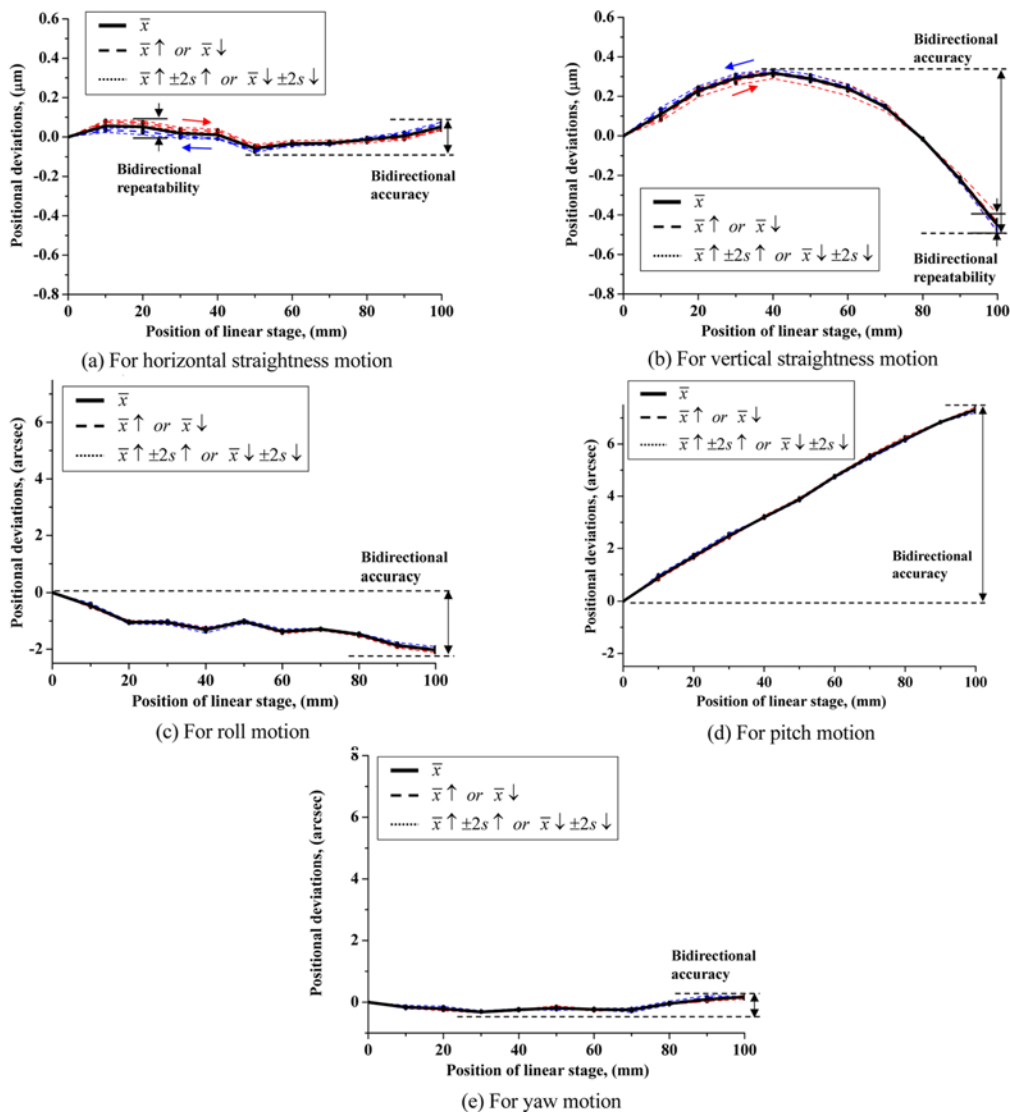


Fig. 6 Bidirectional accuracy and repeatability of positioning

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