

Topology Optimization of Piezoelectric Materials and Application to the Cantilever Beams for Vibration Energy Harvesting

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A new design analysis method based on FEM and a topology optimization for piezoelectric materials was developed for the unimorph cantilevered energy harvesters which can produce the maximum electric power outputs. The optimum topology of a piezoelectric material layer on the harvesting beam has been calculated by considering natural frequencies of beams, electromechanical couplings of piezoelectric materials, tip masses and MMA (method of moving asymptotes). The piezoelectric coefficients such as elasticity, capacitance and piezoelectric coupling were interpolated by element density variables in the topology optimization. The optimum design method was verified by vibration tests and measuring voltage outputs of the harvester and a good correlation between two results has been obtained. The effects of beam geometric parameters and several piezoelectric materials (PZT, PVDF, PMN-PT and piezoelectric fiber composites) on power generation were also investigated. The beam with PMN-PT generated the largest voltage and the next largest is PZT, MFC and PVDF, respectively.

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1. Introduction

One of the most common renewable electric energy can be recovered from ambient vibrations in the surroundings such as rotating machinery, heavy traffic bridges, running automobiles, airplanes, and so on. Various fluctuation resources together with their ranges of frequencies as well as the maximum acceleration magnitudes were reported in Roundy *et al.*¹ In recent years, the piezoelectric transduction from mechanical vibration to electricity has attracted many researchers' attention greatly. The highly efficient materials for electric energy harvesting from environmental vibrations are known to be piezoelectric materials such as PZT (lead zirconate titanate), PMN-PT (lead magnesium niobate-lead titanate), MFC (macro-fiber composite) and PVDF (polyvinylidene difluoride).^{2,3} Even if piezoelectric cantilevered electric energy harvesters (EH) may generate small electric power, it is large enough to operate low power electronic devices or wearable computing systems. When the cantilevered beams with piezoelectric material patches vibrate owing to winds or some other disturbances such as bridge vibration induced from automobile traffic, the attached piezoelectric material layers strain and then straining induces an

electric power. In order to maximize the power, the external exciting frequencies of cantilevered harvesters are tuned to the first natural frequency of the harvesting beam.

Even though PZT is widely used in low power harvesting devices, it is brittle and therefore the fatigue life is limited by a repeated alternating strain level. Compared to PZT, the PVDF is more flexible and easier to be formed in a required shape, and it can withstand larger amounts of strain. Roundy *et al.*⁴ investigated low level vibrations as a power source for wireless sensor nodes and showed the different output powers although the volume, 1 cm³, of tested energy harvesters is the same. Beeby *et al.*⁵ investigated the performance of cantilevered electric energy harvesters, according to a cantilever size, a tip mass, input frequencies, amplitude and acceleration, and reported the size of energy harvester beam is very important. Hu *et al.*⁶ studied the properties of PVDF and electric outputs by adjusting the resonant frequency of a PVDF bimorph power harvester. Inman and Erturk^{7,8} derived correction factors for improving the simple SDOF harmonic base excitation model of cantilevered PZT energy harvesters. Karami *et al.*⁹ and Bilgen *et al.*¹⁰ performed an experimental and theoretical electromechanical characterization of cantilevered unimorph beams with several

piezoelectric materials such as PZT-5A and PMN-PZT. Wardle *et al.*¹¹ investigated the proof mass effects on vibration energy harvester performance analytically and experimentally. Yang *et al.*¹² suggested the vibration energy harvester using macro-fiber composites (MFC) instead of piezoceramic materials. The above-mentioned reviewed papers did not consider a subject on the design optimization of vibration energy harvesting for improving the performance of the devices further.

Yoon *et al.*¹³ suggested a design of curved rectangular PZT unimorph beams in shoes for energy harvesting, based on a thin shell theory and parametric optimization. Dietl and Garcia¹⁴ developed an analytical model of bending vibration based on long, slender beams with tip masses and investigated power harvesting by changing beam shapes obtained from shape optimization. Zheng *et al.*¹⁵ applied a topology optimization technique to improve mechanical to electrical energy conversion in a static sense for piezoelectric plates. Rupp *et al.*¹⁶ developed a general methodology to analyze and design piezoelectric energy harvesting systems based on multilayer plate and shell structures with piezoelectric layers coupled to an external circuit. Kim *et al.*¹⁷ derived two conditions that the penalty exponents for topology optimization with PZT had to satisfy for stable convergence for one-dimensional problems, and their effectiveness for two-dimensional problems was studied. Kang and Wang¹⁸ investigated the topology optimization of bending actuators with multilayer piezoelectric material and the distribution of control voltage and the actuator structures were optimized. Park *et al.*¹⁹ developed a size design optimization of piezoelectric energy harvester subject to tip rotary motion. They used the rectangular piezoelectric (PZT, PVDF) sheets.

The attempt to maximize the energy harvesting performance by the topology optimization for piezoelectric materials emerged only a few years ago. Therefore, only a limited number of research articles about the topology optimization of piezoelectric materials are available. In the articles were proposed some basic ideas and algorithms on piezoelectric topology optimization; however, most of them were not proved by experiments. This paper presents an efficient topology optimization method for vibrating cantilevered beam-like piezoelectric energy harvesting devices and optimum design results were verified by vibration tests and measuring voltage outputs. Topology optimization was applied for four piezoelectric materials, PZT, PMN-PT, MFC and PVDF to compare their voltage variations using a developed MATLAB code. Voltage generation was calculated with the finite element method coupled with piezoelectric behaviors. In addition, parametric (size) optimization was carried out and voltage results were compared to the topology optimization results in order to emphasis on the need for the electro-mechanical topology optimization of piezoelectric materials.

2. Topology Optimization Theory of Piezoelectric Materials

The constitutive equations of piezoelectric materials are described as follows,²⁰

$$S_1 = s_{11}^E T_1 + d_{31} E_3 \quad (1)$$

$$D_3 = d_{31} T_1 + \varepsilon_{33}^T E_3 \quad (2)$$

where S_1 is strain, S_{11}^E is compliance in a constant electric field, T_1 is stress, d_{31} is the piezoelectric coefficient, E_3 is the electric field, D_3 is the electric displacement field and ε_{33}^T is the dielectric permittivity at a constant stress. The subscript 1 denotes the direction of the harvester beam, while the subscript 3 denotes the thickness direction. The poling direction d_{31} of piezoelectric materials is normal to the electrode.

The density-based topology optimization method in the present work is based on the PEMAP-P (piezoelectric material with penalization and polarization) model²¹ which employs the material density ($0 \leq \rho \leq 1$) as a design variable to describe the distribution of piezoelectric material in each finite element. The material density is interpolated between solid material ($\rho = 1$) and no material ($\rho = 0$). The value $\rho = 0$ is usually not applied to avoid the stiffness matrix becoming singular. Since the presence of intermediate values of ρ causes some problems when generating the final topology, a penalization factor p is employed that penalizes intermediate densities and pushes it to limiting values of zero or unity.

The PEMAP-P model is expressed as,²¹

$$s^E = \rho^{ps} \bar{s}^E \quad (3)$$

$$d = \rho^{pd} \bar{d} \quad (4)$$

$$\varepsilon^T = \rho^{pe} \bar{\varepsilon}^T \quad (5)$$

where ρ^{ps} , ρ^{pd} and ρ^{pe} are the density interpolation functions, the superscripts ps , pe , and pe are penalization factors related to mechanical and electrical properties, and \bar{s}^E , \bar{d} and $\bar{\varepsilon}^T$ are the elasticity modulus, piezoelectric coefficient, and dielectric permittivity, respectively.

For an energy harvester structure consisting of piezoelectric and other substrate materials, the following semi-empirical equation for an electromechanical coupling coefficient, k_{ec} , can be applicable,¹⁷

$$k_{ec}^2 = \frac{\omega_o^2 - \omega_s^2}{\omega_o^2} \quad (6)$$

where ω_o is the natural frequency of an open circuit and ω_s is that of a short circuit with no external electric resistance.

The objective function for topology optimization of piezoelectric material can be expressed as follows,¹⁷

$$\text{Minimize } f_{cost} = \frac{1}{k_{ec}} \alpha + (\omega_{eh}^2 - \omega_{ee}^2)(1 - \alpha) \quad (7)$$

where ω_{eh} is a natural frequency of the energy harvester and ω_{ee} is that of external excitation. The weighting factor, α , which is used to put the emphasis on either the electromechanical coupling coefficient or the gap between natural frequencies, takes the value of 0 between 1 and in this study, 0.8 was utilized. The energy harvester vibrates usually in resonance so that the second term of RHS in Eq. (7) is very small.

For elastic materials, the optimum value for the penalization factor ps was found to be 3 by Bendsoe and Sigmund.²² To obtain the best topology of piezoelectric materials, proper values for penalization factors must be chosen. By the fact that the electromechanical coupling coefficient (ECC) k_{ec} increases as the density of piezoelectric material increases, an additional constraint for choosing penalization factors can be imposed on as in¹⁷

$$k_{ec}^2 = \frac{(\rho^{pd} \bar{d})^2}{(\rho^{ps} \bar{s})(\rho^{pe} \bar{\epsilon}^T)} = \rho^{2pd-(ps+pe)} \frac{\bar{d}}{\bar{s} \cdot \bar{\epsilon}^T} \quad (8)$$

$$2pd - (ps + pe) \geq 0 \quad (9)$$

If the penalization factors are chosen satisfying Eq. (9), then the best topology can be obtained. In this paper the different combination of penalization factors were used for different piezoelectric materials.

3. Parametric Optimization of Harvesters

The initial design of a cantilevered energy harvester is shown in Fig. 1. The dimensions of an aluminum substrate and a piezoelectric material layer are 40 (length) × 20 (width) × 1 mm (substrate thickness) and the piezoelectric material layer is 0.1 mm thick. The brass tip mass is a cube of 20×20×20 mm.

The size optimization was performed by ANSYS. The piezoelectric material and the substrate were modeled with SOLID5 and SOLID45 elements, respectively. One node on each piezoelectric material surface is selected as an electrode and all the other voltage nodes of piezoelectric finite elements are connected to each electrode. The voltage on the top surface of the piezoelectric layer is calculated, while the bottom surface of the piezoelectric layer was set to be 0 V. Piezoelectric coefficients and material properties for the materials used for optimization are summarized in Table 1. The effective properties of MFC were calculated,

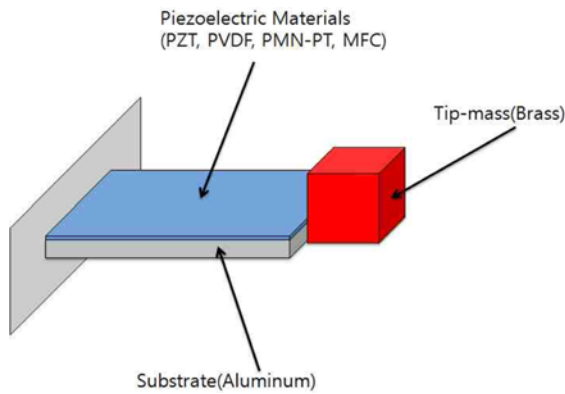


Fig. 1 The initial design domain of a cantilevered energy harvester with a tip mass

Table 1 Material properties of an energy harvester²³

Materials	Density (kg/cm ³)	Dielectric constants	Piezoelectric coefficients
		$\epsilon_{11}, \epsilon_{33}$	d_{31}, d_{33}, d_{15} (C/m ²)
PVDF	1780	6.482, 6.761	-0.01036 ^o , -0.02788, -0.01224
PZT5H	7500	1530, 1500	-5.4, 15.8, 12.3
PMN-PT	8000	1335, 4200	-4.8, 18.6, 6.9
MFC	7500	1030, 627	-0.02, 0.04,
		Young's modulus (GPa)	Poisson's ratio
Aluminum	2700	72	0.33
Brass	8400	105	0.32

based on the references.^{24,25}

The size optimization was performed to maximize the output voltage of the EV together with constraints as follows,

- Minimize $-V_{out}$ (voltage output)
- Subject to σ_{max} (max. stress) ≤ 50 MPa (Al strength)
- ω_1 (1st natural frequency) ≤ 120 Hz
- 30 mm $\leq L$ (beam length) ≤ 70 mm
- 10 mm $\leq W$ (beam width) ≤ 40 mm
- 0.5 mm $\leq t$ (substrate thickness) ≤ 2 mm
- 5 mm $\leq L_p$ (piezoelectric material length) ≤ 20 mm
- 5 mm $\leq W_p$ (piezoelectric material width) ≤ 20 mm

Fig. 2 shows the objective function converges after 11 iterative calculations. The results of optimum designs for four different piezoelectric materials are summarized in Table 2. There is almost no change in the dimensions of the substrate beam because it has no influence on voltage generation; however, the length of piezoelectric material has been shortened significantly. The higher strain occurs near the wall of the cantilever beam, so the piezoelectric materials remains close to the wall region after optimization, as in Fig. 3. Fig. 4 shows clearly that the resulting voltage output can be increased after optimization with PMN-PT.

4. Topology Optimization of Harvesters

To maximize the voltage output, topology optimization was performed for the cantilevered unimorph harvesters consisting of a piezoelectric

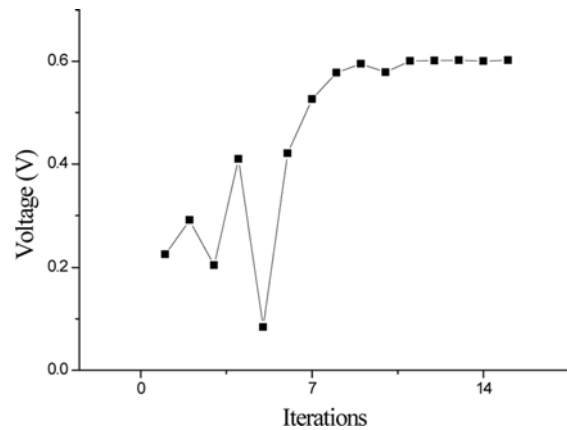


Fig. 2 Convergence history of size optimization

Table 2 Size optimization results of the cantilevered piezoelectric energy harvester

	l (mm)	w (mm)	t (mm)	P_l (mm)	P_w (mm)
Initial Design	40.0	20.0	0.1	40.0	20.0
Optimum PVDF	41.0	20.0	0.1	8.7	14.7
Optimum MFC	40.0	19.9	0.1	8.4	16.0
Optimum PZT-5H	40.1	20.0	0.1	7.9	17.1
Optimum PMN-PT	39.1	20.1	0.1	8.6	17.4

l, w and t : length, width and thickness of substrate, respectively. P_l and P_w : piezoelectric material length and width.

material (i.e., PZT, PVDF, PMN-PT, or MFC) and the aluminum substrate with the volume fraction of 0.5 on the basis of the topology optimization algorithm (Eqs. (7)–(9)). As an instance, the procedure for ECC (Eq. 8) gives the value of $k_{ec} = 0.0156$ during optimization as in Fig. 5(a). The same initial design domain used for the size optimization above was applied for comparing with two results of voltage outputs.

As results of topology optimization of the cantilevered electric harvester, the optimum topology solutions for four different piezoelectric materials were obtained as shown in Fig. 5 and the penalization factors applied for computation are described in Fig. 5. The convergence histories of the electromechanical coupling coefficients and the first natural frequencies for each piezoelectric material are also plotted together with optimum topologies in Fig. 5. Especially, the electromechanical coupling coefficients are related to the objective function, as in Eq. (7). Similar optimum topologies were obtained as results of optimization regardless of kinds of piezoelectric materials like that the piezoelectric materials in the corner and center region were removed. The optimum topologies of PZT, MFC and PMN-PT materials are much closer each other than the topology of PVDF which has far lower density and dielectric constants than other three materials, as in Table 1. Some materials also exist near the tip mass in the cases of PZT, MFC and PMN-PT materials. It was reported previously that the trapezoidal cantilever beam with PMN-PT generated more power than a rectangular one.²⁶

Fig. 6 represents the finite element model of a cantilevered EH with the optimum topology of a piezoelectric material adhered to an aluminum substrate with epoxy. Piezoelectric material was modeled with SOLID227 and others were modeled with SOLID45. Vibration

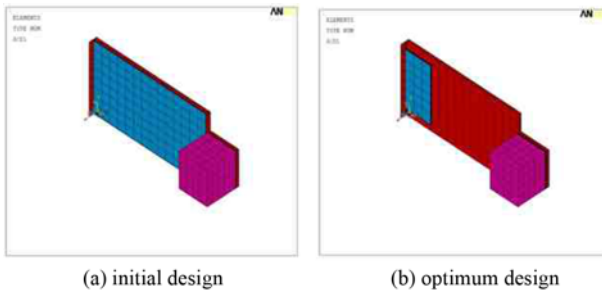


Fig. 3 Changes in the PZT area after size optimization

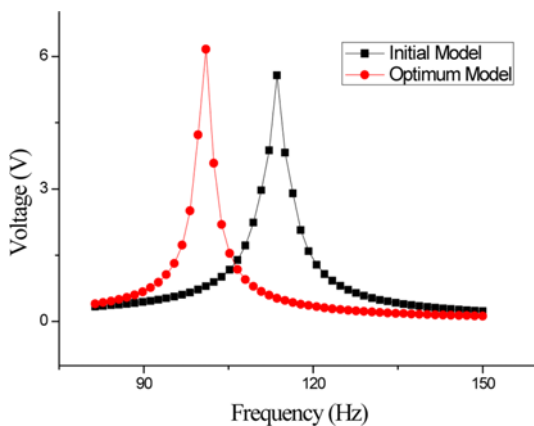
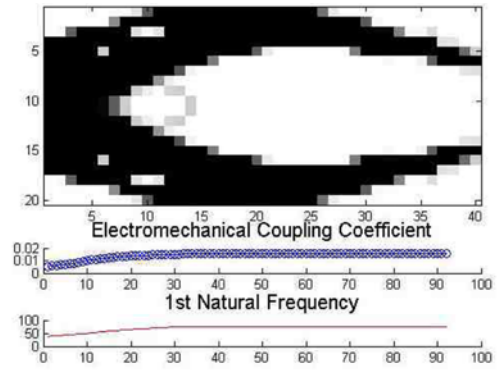
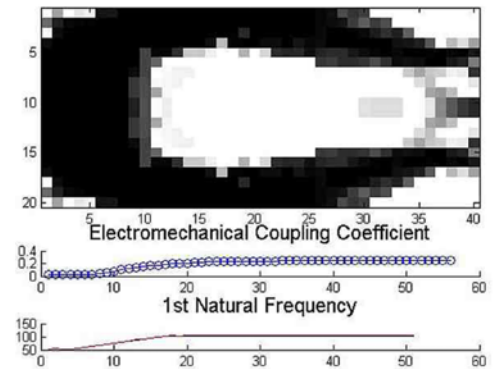


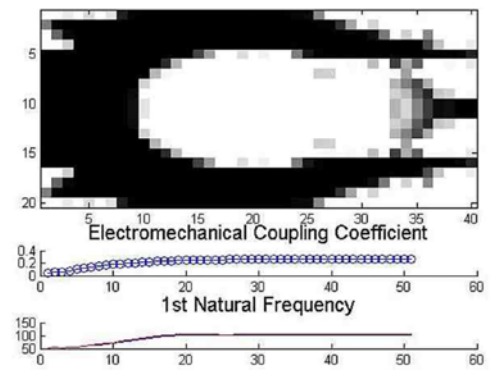
Fig. 4 Voltage peaks at resonance frequencies after and before the size optimization of PMN-PT



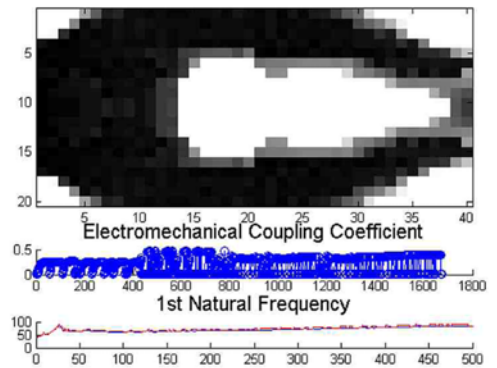
(a) PVDF (penalization factors used: $p_s=3, p_d=7, p_e=4$)



(b) PZT-5H ($p_s=3, p_d=8, p_e=4$)



(c) MFC ($p_s=3, p_d=7, p_e=4$)



(d) PMN-PT ($p_s=3, p_d=8, p_e=4$)

Fig. 5 The optimum topology solutions of four piezoelectric materials layered on the upper surface of the cantilevered EH were obtained numerically, and the histories of corresponding electromechanical coupling coefficients (ECC) and the first natural frequencies were also plotted together

responses to harmonic excitation are computed for the energy harvesters which have different natural frequencies depending on piezoelectric materials such as in Table 3. Voltage outputs computed by finite element analyses for three different cases of energy harvesters (i.e., an initial, a size-optimized, and a topology-optimized beam) of PMN-PT piezoelectric material are shown in Fig. 7. The voltage by the topology optimization case increases 9.42% compared to the size optimization case. The generated voltage from the arbitrary initial design is smallest among three cases. The voltage output of the cantilevered EH with topology-optimized piezoelectric material is

larger than that of the simply size-optimized case. This means that the topology optimization of piezoelectric materials is efficient in improving the performance of an electric EH structure. In addition, piezoelectric materials can be saved by topology optimization.

5. Validation Experiment

A vibration test was performed to measure the voltage output of the energy harvester with the optimum topology PVDF material, as in Fig. 6 and the experimental result was compared to the analysis. The PVDF

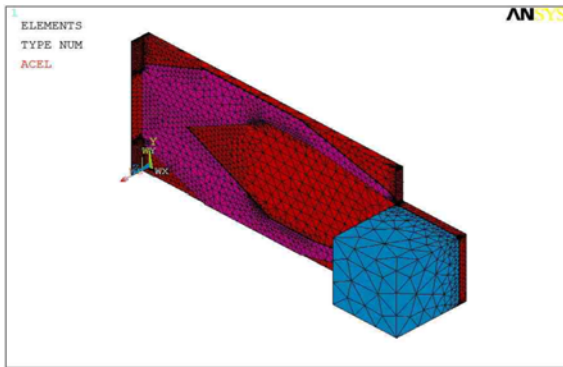


Fig. 6 The Cantilevered EH with the optimum topology of a piezoelectric material on an Al substrate

Table 3 Voltage outputs calculated depending on kinds of piezoelectric materials

Piezoelectric Material	1 st Natural Frequency (Hz)	Output Voltage (V)
PVDF	95.09	3.55
MFC	111.07	4.36
PZT-5H	120.35	4.70
PMN-PT	109.06	6.74

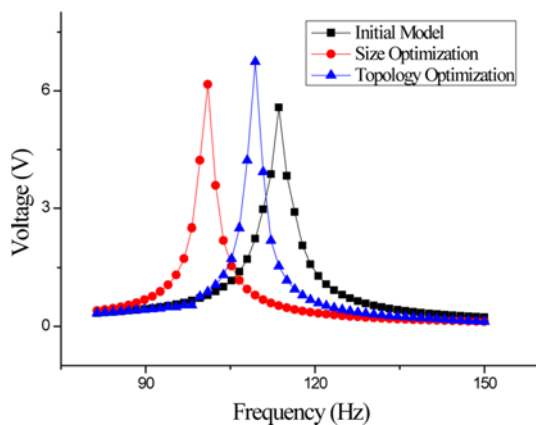


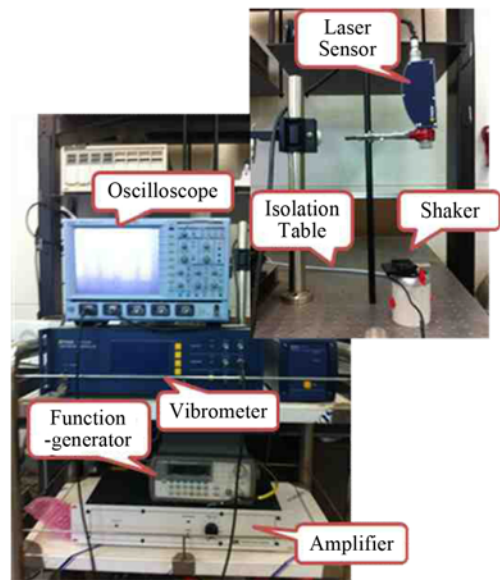
Fig. 7 Comparison of voltage outputs computed for 3 different energy harvesters (an initial, a size-optimized and a topology-optimized beam) of PMN-PT

Table 4 Comparison of two voltage results

EH	Experimental Voltage (V)	Computed Voltage (V)	Difference
PVDF	3.36	3.55	5.4%



(a)



(b)

Fig. 8 (a) The energy harvester with optimum topology; (b) the energy harvester installed on a shaker and experimental setup

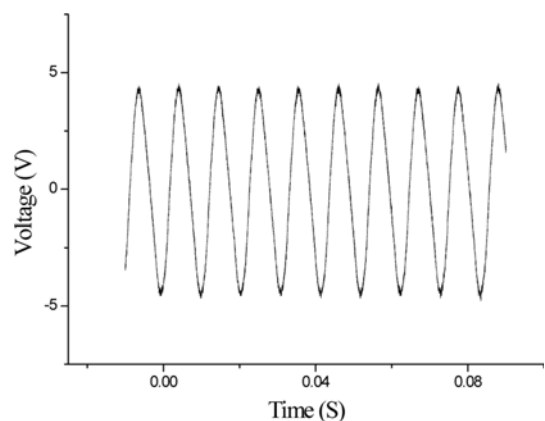


Fig. 9 Experimental voltage output at the 1st resonant frequency 98.6 Hz of the topologically optimized energy harvester

piezo-polymer was selected for the validation experiment due to that it is flexible enough to tailor the material in an optimum topology. Fig. 8 shows the optimum cantilevered energy harvester with a tip mass, installed on a shaker, used for the experiment.

The experimental setup consists of a function generator (Agilent 33220A) for generation of a wave form, an amplifier (LSD PA-25E) and a shaker (LSD V201), and a vibrometer (Polytec OFV-5000), a laser sensor (Polytech OFV-534), Oscilloscope (Lecroy LT354) and DAQ (National Instrument USB-6356) for measurement of signal and voltage. The resulting output voltages were summarized in Table 4 and Fig. 9 presents the experimental voltage output at the 1st resonant frequency 98.6 Hz of the topologically optimized energy harvester. The measured voltage was a little lower than the computed one by 5.4%. The two results have a good agreement on voltage generation. The result proves that topology optimization of piezoelectric materials used for energy harvesters can increase the voltage output all under the same condition further.

6. Conclusions

An efficient and powerful design method based on topology optimization was developed to determine the material layout of piezoelectric materials and then maximize the energy harvesting performance of a piezoelectric harvesting system. The voltage outputs calculated by the proposed method were compared to those from vibration tests with the same cantilevered energy harvester. The good correlation between two proves the efficiency of the proposed design method. The voltage outputs generated by the energy harvesting beams with a topology-optimized piezoelectric material layer were higher than simply size-optimized or un-optimized cases. The harvesting beam with PMN-PT generated the largest voltage and the next largest is PZT, MFC and PVDF in the order. Therefore, topology optimization of piezoelectric materials is proven to be very effective for improving the energy harvesting performance.

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