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A Novel Fuzzy Second-Order Sliding Mode Observer-Controller for a T-S Fuzzy System With an Application for Robot Control

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This paper investigates an algorithm for the tracking performance of a Takagi-Sugeno (T-S) fuzzy system using the second-order sliding mode observer/controller technique. First, the original second-order nonlinear system is represented by a T-S fuzzy model, in which most of the parameters can be computed offline. A novel fuzzy second-order sliding mode observer (FSOSMO), which combines the T-S fuzzy model and the second-order sliding mode observer (SOSMO), is then designed to estimate the velocity. Also, a new fuzzy second-order sliding mode control (FSOSMC), which combines the T-S fuzzy model and the second-order sliding mode control (SOSMC), is proposed to stabilize and guarantee the exact motion tracking for the T-S fuzzy system. By integrating the T-S fuzzy model with SOSMO/C, the resulting observer/controller scheme preserves the advantages of both techniques, such as the low online computational burden of the T-S fuzzy model, and low chattering, fast response, and finite time convergence of the SOSMO/C. Moreover, the stability and convergence of the proposed closed loop observer-based controller strategy is theoretically proven by the Lyapunov method. Finally, the simulation results of a two-link robot manipulator are presented to demonstrate the effectiveness of the proposed approach.

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NOMENCLATURE

FSMC = Fuzzy sliding mode control
FSOSMC = Fuzzy second-order sliding mode control
FSOSMO = Fuzzy second-order sliding mode observer
LMI = Linear matrix inequalities
SM = Sliding mode
SMC = Sliding mode control
SMO = Sliding mode observer
SMC/O = Sliding mode controller/observer
SOSM = Second-order sliding mode
HOSM = High-order sliding mode
SOSMO = Second-order sliding mode observer
SOSMC = Second-order sliding mode controller
SOSMO/C = Second-order sliding mode observer/controller
PDC = Parallel distributed control

1. Introduction

Due to characteristics such as reduced mathematical complexity, good approximation, and easy implementation for describing a complex nonlinear system, the Takagi-Sugeno (T-S) fuzzy model has been widely applied to model nonlinear systems.[1-11] The basic idea of the T-S fuzzy approach is to first decompose the model of a nonlinear system into a set of linear subsystems, and then smoothly connect them by a fuzzy membership function. Although the nonlinear system can be represented by the suitable selection of the membership function, additional uncertainties are generated from the mismatch between the T-S fuzzy system and the original nonlinear models. To compensate for these additional uncertainties, the linear fuzzy observer and controller design methods such as parallel distributed compensation (PDC) and linear matrix inequality (LMI)-based design techniques have been proposed.[2-7] However, the linear method approaches do not guarantee the finite time convergence of the states observer and tracking control in the presence of the system uncertainties. In addition, the complex nonlinear system makes the LMIs more complicated and difficult to solve in real applications.

Due to the robustness and fast transient response properties, sliding mode control has been studied and applied to control nonlinear systems.[8-17] Recently, for the T-S fuzzy system, conventional sliding mode control (SMC)[8-12] and sliding mode observer (SMO)[12,13] have also been utilized to compensate for the uncertainties. In these approaches, the T-S fuzzy model with the most offline computing parameters is first used to represent the nonlinear system, and then the SMC/O is used to compensate for the uncertainty and stabilize the T-S fuzzy system. By integrating both techniques, the results were shown to be able to decrease the online computational burden and have robust for the uncertainties. However, a common drawback of the conventional SMC/O is the chattering phenomenon. To eliminate the chattering and obtain higher accuracy, high-order sliding mode techniques[18] have been studied and applied successfully for real applications, such as a controller for nonlinear system[19-25] as well as robotic system[26-28] and state observation.[29-33] Unlike the classical SMC, which works on the first time derivative of the sliding variable, HOSM works with the discontinuous control acting on the higher order time derivative. By moving the switching to the higher derivatives of the control, a chattering reduction is achieved because the control signal now is continuous. For the states observer, the SOSM observer inherits two basic advantages:[31,32] 1) providing the exact velocity estimation without filtration, and 2) allowing properties of equivalent control to be used to identify the unknown inputs. To the best of the authors' knowledge, up to now, there has not been an investigation into the T-S fuzzy system using the SOSM observer and controller.

In this paper, a second-order sliding mode controller/observer is applied for the T-S fuzzy system to obtain both the state estimation and tracking control. Our objective is to take advantage of these two techniques (T-S fuzzy model and SOSMO/C), by proposing a novel fuzzy second-order sliding mode observer/controller. First, a T-S fuzzy system is employed to represent the nominal nonlinear system. Then, a novel FSOSMO, which combined the T-S fuzzy model with a SOSMO observer, is designed to estimate the velocity. Based on the estimated velocity, a new FSOSMC controller, which combined the T-S fuzzy model with a SOSMC controller is designed to compensate for the uncertainties and guarantee exact motion tracking of the system. By combining the T-S fuzzy and SOSM observer/controller technique, the proposed method benefits from both techniques, such as the low online computational burden of the T-S fuzzy model, low chattering, fast transient response, and finite time convergence of the SOSM observer/controller. The stability and convergence of the proposed closed loop observer-based controller are proved theoretically by the Lyapunov method.

The remainder of this paper is organized as follows: in section 2, the problem formulation is presented. In section 3, a FSOSM observer is designed to estimate the velocity. In section 4, the design of the T-S fuzzy model-based SOSM control is described. The proposed observer/controller scheme is applied for a two-link robot manipulator in section 5. Section 6 outlines the conclusions.

2. Problem Formulation

Consider the following nonlinear second-order mechanical systems:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x)+g(x)u \\ y &= x_1 \end{aligned} \tag{1}$$

where $x_1 \in \Re^n$, $x_2 \in \Re^n$, $x = (x_1, x_2)^T \in \Re^n$ denotes the system state and $u \in \Re^n$ is the control input; $f(x)$ and $g(x)$ are smooth functions.

The nonlinear system in Eq. (1) can be expressed by the T-S fuzzy system as follows:

Plant rule i:

If z_1 is M_1^i and … and z_g is M_g^i, then

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= A_i x + B_i u \quad i = 1, 2, \ldots, r \\ y &= x_1 \end{aligned} \tag{2}$$

where z_1, ..., z_g are the premise variables, M_1^i, ..., M_g^i are the fuzzy sets, r is the number of fuzzy rules, and A_i and B_i are the system matrices with appropriate dimensions.

The fuzzy systems are inferred as follows:[1-4]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{\sum_{i=1}^{r} \mu_i(z)\{A_i x + B_i u\}}{\sum_{i=1}^{r} \mu_i(z)} \\ &= \sum_{i=1}^{r} h_i(z)\{A_i x + B_i u\} \\ y &= x_1 \end{aligned} \tag{3}$$

where

$$\begin{aligned} \mu_i(z) &= \prod_{j=1}^{g} M_j^i(z_j) \\ h_i(z) &= \frac{\mu_i(z)}{\sum_{i=1}^{r} \mu_i(z)} \\ z &= [z_1, z_2, \ldots, z_r] \end{aligned} \tag{4}$$

and $M_j^i(z_j)$ is the membership grade of z_j in M_j^i. According to Refs.,[1-4] we have

$$h_i(z) > 0 \quad \text{and} \quad \sum_{i=1}^{r} h_i(z) = 1. \tag{5}$$

Based on the T-S fuzzy model representing the nonlinear model as described in Eq. (3), the plant model of the nonlinear system in Eq. (1) with modeling uncertainties can be described using a T-S fuzzy model such that

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \sum_{i=1}^{r} h_i(z)\{A_i x + B_i u\} + \Delta(t) \\ y &= x_1 \end{aligned} \tag{6}$$

where $\Delta(t) \in \Re^n$ represents the system uncertainties that include the approximation error of the fuzzy model, the disturbance, and the interaction dynamics resulting from the other subsystems.

The objective of this paper is to design a FSOSMC controller for the uncertainty of the nonlinear second-order nonlinear system, which can be described by Eq. (6) based on a novel FSOSMO observer such that the system state will reach the desired trajectory. The following assumptions are assumed to be fulfilled henceforth.

Assumption 1: There exists a known constant such that

$$\|\Delta(t)\| < \bar{\Delta} \tag{7}$$

where $\bar{\Delta}$ is a known constant.

3. Fuzzy Second-Order Sliding Mode Observer (FSOSMO) Design and Stability Analysis

In this section, a novel fuzzy second-order sliding mode observer is proposed for the state estimation of the nonlinear system described by Eq. (6). The T-S fuzzy model of the observer scheme has the same rule as the plant rules. The i^{th} rule of the proposed observer is defined as: Observer rule i:

If z_1 is M_1^i and … and z_g is M_g^i, then

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + \alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1)\\ \dot{\hat{x}}_2 &= A_i\hat{x} + B_iu + \alpha_2 sign(\tilde{x}_1)\end{aligned}, \tag{8}$$

where $x=(x_1,x_2)^T$, $\hat{x}_1$ and $\hat{x}_2$ are the state estimation of x_1 and x_2, respectively, $\tilde{x}=x-\hat{x}$ is the state estimation error, and α_1, α_2 are sliding gains to be designed. The FSOSMO observer is then given as

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + \alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1)\\ \dot{\hat{x}}_2 &= \frac{\sum_{i=1}^{r}\mu_i(z)\{A_i\hat{x}+B_iu\}}{\sum_{i=1}^{r}\mu_i(z)} + \alpha_2 sign(\tilde{x}_1)\\ &= \sum_{i=1}^{r}h_i(z)\{A_i\hat{x}+B_iu\} + \alpha_2 sign(\tilde{x}_1).\end{aligned} \tag{9}$$

Substituting Eq. (6) into Eq. (9), we obtain the estimation error of the states:

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{x}_2 + \alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1)\\ \dot{\tilde{x}}_2 &= \sum_{i=1}^{r}h_i(z)A_i\tilde{x} + \Delta(t) - a_2 sign(\tilde{x}_1)\end{aligned}. \tag{10}$$

Let $F(x,\hat{x},t) = \sum_{i=1}^{r}h_i(z)A_i\tilde{x}+\Delta(t)$. Based on the assumption 1, there exists a constant such that

$$\|F(x,\hat{x},t)\| < f^+, \tag{11}$$

which holds for possible $x, \hat{x}, t$. Subsequently, based on the Lyapunov approach introduced by Moreno and Osorio in Ref.,[33] the stability of the FSOSMO observer is given below.

Theorem 1: Suppose that the condition in Eq. (11) holds for a system expressed as Eq. (10). If the sliding mode gains of the proposed observer scheme in Eq. (8) are chosen as

$$\begin{aligned}&\alpha_1 > 0\\ &\alpha_2 > 3f^+ + 2\frac{f^{+2}}{\alpha_1^2}\end{aligned}, \tag{12}$$

then the observer scheme is stable and the observer states in Eq. (8) $(\hat{x}_1, \hat{x}_2)$ converge to the true states (x_1, x_2) of the system in Eq. (6) in a finite time.

Proof:

The state estimation errors described by Eq. (10) can be rewritten as

$$\begin{aligned}\dot{\tilde{x}}_1 &= \tilde{x}_2 + \alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1)\\ \dot{\tilde{x}}_2 &= F(x,\hat{x},t) - a_2 sign(\tilde{x}_1)\end{aligned}. \tag{13}$$

Let us consider the Lyapunov candidates as

$$L(\tilde{x}) = 2\alpha_2\|\tilde{x}_1\| + \frac{1}{2}\tilde{x}_2^2 + \frac{1}{2}(\alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1) - \tilde{x}_2)^2. \tag{14}$$

The Lyapunov form expressed in Eq. (14) can be written in quadratic form as $L=\zeta^T P\zeta$, where $\zeta^T = [\alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1) - \tilde{x}_2]$ and $P = \frac{1}{2}\begin{bmatrix}4\alpha_2+\alpha_1^2 & -\alpha_1\\ -\alpha_1 & 2\end{bmatrix}$.

As shown in Ref.,[33] L is continuous, positive, radially unbound, and continuously differentiable everywhere except at $\tilde{x}_1 = 0$, thus

$$\lambda_{\min}(P)\|\zeta\|^2 \le L(\tilde{x}) \le \lambda_{\max}(P)\|\zeta\|^2, \tag{15}$$

where $\|\zeta\|^2 = \|\tilde{x}_1\| + \tilde{x}_2^2$ is the Euclidian norm of ζ, and the time derivative of $L(\tilde{x})$ is obtained as

$$\dot{L} = -\frac{1}{\|\tilde{x}_1\|^{1/2}}\zeta^T T_1\zeta + F(x,\hat{x},t)T_2\zeta, \tag{16}$$

where $T_1 = \frac{\alpha_1}{2}\begin{bmatrix}2\alpha_2+\alpha_1^2 & -\alpha_1\\ -\alpha_1 & 1\end{bmatrix}$ and $T_2^T = [-\alpha_1\ 2]$.

Recalling that $\|F(x,\hat{x},t)\| < f^+$, after algebraic manipulation, it can be shown that

$$\dot{L}(\tilde{x}) \le -\frac{1}{\|\tilde{x}_1\|^{1/2}}\zeta^T T_3\zeta, \tag{17}$$

where $T_3 = \frac{\alpha_1}{2}\begin{bmatrix}2\alpha_2+\alpha_1^2-2f^+ & -\alpha_1-\frac{2f^+}{\alpha_1}\\ -\alpha_1-\frac{2f^+}{\alpha_1} & 1\end{bmatrix}$.

Under the conditions expressed in Eq. (12), T_3 becomes bigger than zero ($T_3 > 0$), and $\dot{L}(\tilde{x})$ is negative, so that the stability property is proved.

4. Fuzzy Second-Order Sliding Mode Controller (FSOSMC) Design and Stability Analysis

In this section, the proposed fuzzy second-order sliding mode control is described. First, a conventional sliding mode control is designed to stabilize the T-S fuzzy system. Then, in order to eliminate the chattering and obtain higher accuracies, a second-order sliding mode control is employed.

4.1 Design of Fuzzy Sliding Mode Control (FSMC)

The design procedure of the sliding mode control includes two main steps.[14] The first step involves the construction of the desired sliding surface, which is chosen such that when it converges to zero, the desired control is achieved. The next step is to select a control law that forces the system state to reach the sliding surface in a finite time.

The first step is to choose a proper switching surface that is defined as follows:

$$s = \dot{e} + \lambda e , \tag{18}$$

where $e = x_1 - x_d$, x_d, is the desired trajectory; and λ is a strictly positive constant. It is obvious that the desired control task $e \to 0$ can be achieved if the system state remains on the sliding surface.

For the second step, to ensure the trajectories of the system approach the sliding surface, the derivative of the sliding surface $\dot{s} = 0$ should be satisfied such that

$$\begin{aligned}\dot{s} &= \ddot{e} + \lambda \dot{e} \\ &= \sum_{i=1}^{r} h_i(z)\{A_iB + B_iu\} + \Delta(t) - \ddot{x}_d + \lambda(x_2 - \dot{x}_d) \\ &= \sum_{i=1}^{r} h_i(z)A_ix + \sum_{i=1}^{r} h_i(z)B_iu + \Delta(t) - \ddot{x}_d + \lambda(x_2 - \dot{x}_d)\end{aligned} \tag{19}$$

According to the sliding mode design procedure, we choose[10,11]

$$u = u_{eq} + u_{SMC} \tag{20}$$

The equivalent control signal u_{eq} is obtained by equation $\dot{s} = 0$ without considering the presence of the system uncertainties. It yields:

$$u_{eq} = \left(\sum_{i=1}^{r} h_i(z)B_i\right)^{-1}\left[\ddot{x}_d - \lambda(x_2 - \dot{x}_d) - \sum_{i=1}^{r} h_i(z)A_ix\right], \tag{21}$$

and u_{SMC} is the term that compensates for the effect of the uncertainties. It is designed such that

$$u_{SMC} = -\left(\sum_{i=1}^{r} h_i(z)B_i\right)^{-1} \rho sign(s) , \tag{22}$$

where ρ is a constant chosen based on the upper bound of the modeling uncertainties in the system, that is, $\rho > \overline{\Delta}$. The stability of the T-S fuzzy system under the controller scheme in Eq. (20) is demonstrated in Theorem 2.

Theorem 2: Considering that the uncertain robot manipulators with the dynamic model are described by Eq. (6), the proposed controller law given by Eq. (20) ensures that the sliding surface s asymptotically converges to zero in a finite time.

Proof: Let the Lyapunov function be $V = \frac{1}{2}s^Ts$. Differentiating V with respect to time gives

$$\begin{aligned}\dot{V} = s^T\dot{s} &= s^T\left(\begin{aligned}&\sum_{i=1}^{r} h_i(z)A_ix + \sum_{i=1}^{r} h_i(z)B_i(u_{eq} + u_{SMC}) \\ &\quad + \Delta(t) - \ddot{x}_d + \lambda(x_2 - \dot{x}_d)\end{aligned}\right) \\ &= s^T(-\rho sign(s) + \Delta(t)) \\ &\leq s^T(-\rho sign(s) + \overline{\Delta})\end{aligned} \tag{23}$$

If $\rho \geq \overline{\Delta}$ is satisfied, then $\dot{V} < 0$ is sufficiently ensured. This means that the sliding surface s asymptotically converges to zero.

4.2 Design of the Fuzzy Second-Order Sliding Mode Control (FSOSMC)

The main drawback of the conventional sliding mode control is the chattering and high control efforts. To reduce it, a SOSMC is employed. Unlike another SOSM algorithm, such as a sub-optimal or twisting algorithm that depends on the derivative of the sliding variable, the super-twisting algorithm (STW) only depends on the sliding variable. Because the STW algorithm contains a discontinuous function under the integral, chattering is not eliminated, but much attenuated.

The FSOSMC is designed as:

$$u = u_{eq} + u_{SOSMC}. \tag{24}$$

where u_{eq} is designed as Eq. (21). The differentiation of the sliding surface is now obtained as:

$$\begin{aligned}\dot{s} &= \sum_{i=1}^{r} h_i(z)A_ix + \sum_{i=1}^{r} h_i(z)B_i(u_{eq} + u_{SOSMC}) + \Delta(t) \\ &\qquad - \ddot{x}_d + \lambda(x_2 - \dot{x}_d) \\ &= \sum_{i=1}^{r} h_i(z)B_iu_{SOSMC} + \Delta(t)\end{aligned} \tag{25}$$

From assumption 1, the uncertainties $\Delta(t)$ are bounded. In addition, the following assumption is imposed for the existence of $\left(\sum_{i=1}^{r} h_i(z)B_i\right)^{-1}$.

Assumption 2: The matrices $\sum_{i=1}^{r} h_i(z)B_i$ are nonsingular for all possible states x.

Hence, based on Ref.,[33] the super-twisting SOSMC is designed as:

$$\begin{aligned}u_{SOSMC} &= -\left(\sum_{i=1}^{r} h_i(z)B_i\right)^{-1}(k_1\|s\|^{1/2}sign(s) - z) \\ \dot{z} &= -k_2 sign(s)\end{aligned}. \tag{26}$$

Substituting Eq. (26) into Eq. (25), it yields:

$$\begin{aligned}\dot{s} &= -k_1\|s\|^{1/2}sign(s) + z + \Delta(t) \\ \dot{z} &= -k_2 sign(s)\end{aligned}. \tag{27}$$

The stability and convergence of the closed loop system in Eq. (27) under the proposed FSOSMC is given in Theorem 3.

Theorem 3: Suppose that assumption 1 is guaranteed. If the sliding gains of the SOSMC given in Eq. (26) for the system in Eq. (27) satisfy the following condition

$$k_1 > 2\overline{\Delta} \text{ and } k_2 > k_1\frac{5k_1 + 4\overline{\Delta}}{(2k_1 - 4\overline{\Delta})}\overline{\Delta} , \tag{28}$$

then the sliding surface s is stable and converges to zero in a finite time.

Proof: As a similar way to represent the proof for Theorem 1, and based on Ref.,[33] let us consider the Lyapunov candidate

$$V = 2k_2\|s\| + \frac{1}{2}(k_1\|s\|^{1/2}sign(s) - z)^2 + \frac{z^2}{2}. \tag{29}$$

Eq. (29) can be written in quadratic form:

$$V = \xi^T P \xi \tag{30}$$

where $\xi = [\|s\|^{1/2}sign(s), z]^T$ and $P = \frac{1}{2}\begin{bmatrix} k_1^2 + 4k_2 & -k_1 \\ -k_1 & 2 \end{bmatrix}$.

Combining Eqs. (27) and (30), the time derivative of V becomes

$$\dot{V} = -\frac{1}{\|s\|^{1/2}}(\xi^T B_1 \xi - \Delta B_2 \xi), \tag{31}$$

where $B_1 = \frac{k_1}{2}\begin{bmatrix} 2k_2 + k_1^2 & -k_1 \\ -k_1 & 1 \end{bmatrix}$ and $B_2 = \begin{bmatrix} 2k_2 + \frac{k_1^2}{2}, & -\frac{k_1}{2} \end{bmatrix}^T$.

Based on assumption 1, Eq. (31) can be further manipulated algebraically to

$$\dot{V} \le -\frac{1}{\|s\|^{1/2}}\xi^T B_3 \xi, \tag{32}$$

where $B_3 = \frac{k_1}{2}\begin{bmatrix} 2k_2 + k_1^2 - \left(\frac{4k_2}{k_1} + k_1\right)\bar{\Delta} & -(k_1 + 2\bar{\Delta}) \\ -(k_1 + 2\bar{\Delta}) & 1 \end{bmatrix}$.

In this case, if the condition in Eq. (28) is satisfied, $B_3 > 0$, and then, from Eq. (32), $\dot{V}$ is a negative-definite matrix, providing its stability.

On the other hand, from Eq. (30) with k_1 and k_2 as positives, the following condition is satisfied:

$$\lambda_{\min}(P)\|\xi\|^2 \le V \le \lambda_{\max}(P)\|\xi\|^2, \tag{33}$$

where $\|\xi\|^2 = \|s\| + \|z\|^2$ is the Euclidian norm of ξ.
Using Eqs. (32) and (33), and the fact that

$$\|s\|^{1/2} \le \|\xi\| \le \frac{V^{1/2}}{\lambda_{\max}^{1/2}(P)}, \tag{34}$$

It follows that

$$\dot{V} \le -\frac{1}{\|s\|^{1/2}}\xi^T B_3 \xi \le -\delta V^{1/2}, \tag{35}$$

and hence

$$\dot{V} \le -\delta V^{1/2}, \tag{36}$$

where

$$\delta = \frac{\lambda_{\min}^{1/2}(P)\lambda_{\min}(B_3)}{\lambda_{\max}(P)} > 0.$$

Since the solution of the differential equations

$$\dot{y} = -\delta y^{1/2}, \quad y(0) = y_0 \ge 0$$

is given as

$$y(t) = \left(y_0^{1/2} - \frac{\delta}{2}t\right)^2.$$

Here, $y(t)$ converges to zero in a finite time, and reaches zero after $t = \frac{2}{\delta}y_0^{1/2}$. Combining this result with the comparison principle from Ref.[17] leads to the relation $V(t) \le y(t)$, when $V_0 \le y_0$. Therefore, the sliding surface s converges to zero after $T = \frac{2}{\delta}V_0^{1/2}$.

Remark 1: To simplify the analysis, the FSMC and FSOSMC are designed based on the assumption that the velocity is measurable, however in this paper it is not available. In this case, the theoretically exact velocity estimation $\hat{x}_2$, which is obtained from the FSOSMO observer, is used to replace the velocity measurement x_2 in the controller scheme in Eqs. (20) and (24). For example, the equivalent controller is now designed as:

$$u_{eq} = \left(\sum_{i=1}^{r} h_i(z)B_i\right)^{-1}\left[\ddot{x}_d - \lambda(\hat{x}_2 - \dot{x}_d) - \sum_{i=1}^{r} h_i(z)A_i\hat{x}\right]. \tag{37}$$

Remark 2: No matter what the control input is, the FSOSMO observer can obtain theoretically exact velocity estimations. However, due to noise and sampling rate restrictions, the exact velocity estimation cannot be achieved for a real application. Thus, the velocity estimation converges in a finite time toward the true value plus a small error value. To handle the velocity estimation error, the sliding mode gains ρ, k_1, k_2, in Eqs. (20) and (24) should be chosen to be large enough to stabilize the closed loop observer-controller system.

Remark 3: Based on the Eq. (1), the conventional SOSM observer is designed as:[31,32]

$$\begin{aligned} \dot{\hat{x}}_1 &= x_2 + \alpha_1\|\tilde{x}_1\|^{1/2}sign(\tilde{x}_1) \\ \dot{\hat{x}}_2 &= f(x) + g(x)u + \alpha_2 sign(\tilde{x}_1) \end{aligned} \tag{38}$$

And the conventional sliding mode control is designed:[14]

$$u = u_{eq} + u_{SMC} \tag{39}$$

where

$$u_{eq} = (g(x))^{-1}[\ddot{x}_d - \lambda(x_2 - \dot{x}_d) - f(x)] \tag{40}$$

and,

$$u_{SMC} = (-g(x))^{-1}\rho sign(s) \tag{41}$$

Similarly, the conventional SOSM controller is designed:[27]

$$\begin{aligned} u_{SOSMC} &= (-g(x))^{-1}(k_1\|s\|^{1/2}sign(s) - z) \\ \dot{z} &= -k_2 sign(s). \end{aligned} \tag{42}$$

Comparison of Eq. (9) with Eq. (38), Eq. (21) with Eq. (40), Eq. (22) with Eq. (41) and Eq. (26) with Eq. (42) shows that the T-S fuzzy based SOSM observer/controller does not need to compute the nonlinear term $f(x)$, $g(x)$ and $(-g(x))^{-1}$ online. In addition, the system parameter A_i, B_i and B_i^{-1} of T-S fuzzy based observer/controller can be computed offline. Therefore, the T-S fuzzy model approach may significantly alleviate the online computational burden and hence increase the opportunity for real observer/controller implementation.

5. Application to a two-link robot manipulator

In this section, the proposed observer-controller scheme is applied for a two-link robot manipulator, as shown in Fig. 1. We assumed that the robot works in normal operation without actuator and sensor faults, the dynamic equation of the two-link robot is described as follows:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+F(\dot{q})+G(q)+\tau_d=\tau, \tag{43}$$

where

$$M(q)=\begin{bmatrix}(m_1+m_2)l_1^2 & m_2l_1l_2(s_1s_2+c_1c_2)\\ m_2l_1l_2(s_1s_2+c_1c_2) & m_2l_2^2\end{bmatrix}$$

$$C(q,\dot{q})=m_2l_1l_2(c_1s_2-s_1c_2)\begin{bmatrix}0 & -\dot{q}_2\\ -\dot{q}_1 & 0\end{bmatrix}$$

$$G(q)=\begin{bmatrix}-(m_1+m_2)l_1gs_1\\ -m_2l_2gs_2\end{bmatrix},$$

and $F(\dot{q})$ and τ_d are the friction matrix and load disturbance matrix, respectively, which are known as the modeling uncertainties. It is assumed as:

$$F(\dot{q})=\begin{bmatrix}5\sin(q_1)\\ 5\sin(q_2)\end{bmatrix} \tag{44}$$

and

$$\tau_d=\begin{bmatrix}0.2sign(q_1)\\ 0.2sign(q_2)\end{bmatrix} \tag{45}$$

and $q=[q_1, q_2]^T$, q_1, q_2 are the generalized coordinates, τ is the control input, m_1 and m_2 are the link masses, l_1 and l_2 are the link lengths, gravity $g=9.8(m/s^2)$, and $s_1=\sin(q_1)$, $s_2=\sin(q_2)$, $c_1=\cos(q_1)$, and $c_2=\cos(q_2)$. Let $x_1=q_1$, $x_2=\dot{q}_1$, $x_3=q_2$, $x_4=\dot{q}_2$, $x=[x_1,x_2,x_3,x_4]^T$, and $u=\tau$ as the control input. The dynamic equation in Eq. (38) can be represented in state space form as follows:[6]

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=f_1(x)+g_{11}(x)u_1+g_{12}u_2+\omega_1\\ \dot{x}_3&=x_4\\ \dot{x}_4&=f_2(x)+g_{21}(x)u_1+g_{22}u_2+\omega_2,\end{aligned} \tag{46}$$

where

$$\begin{aligned}f_1(x)=&\frac{(s_1c_2-c_1s_2)}{l_1l_2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)]}\\ &\times[m_2l_1l_2(s_1s_2+c_1c_2)x_2^2-m_2l_2^2x_4^2]\\ &+\frac{1}{l_1l_2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)]}\\ &\times[(m_1+m_2)l_2gs_1-m_2l_2gs_2(s_1s_2+c_1c_2)]\end{aligned}$$

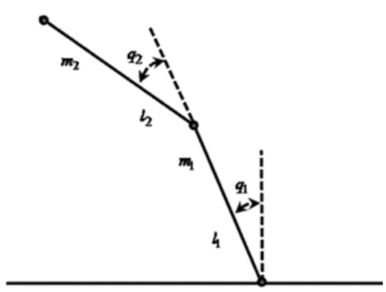

Fig. 1 Configuration of the two-link robotic system

$$\begin{aligned}f_2(x)=&\frac{(s_1c_2-c_1s_2)}{l_1l_2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)]}\\ &\times[-(m_1+m_2)l_1^2x_2^2+m_2l_1l_2(s_1s_2+c_1c_2)x_4^2]\\ &+\frac{1}{l_1l_2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)]}\\ &\times[-(m_1+m_2)l_1gs_1(s_1s_2+c_1c_2)+(m_1+m_2)l_1gs_1]\end{aligned}$$

$$g_{11}(x)=\frac{m_2l_2^2}{m_2l_1^2l_2^2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)^2]}$$

$$g_{12}(x)=g_{21}(x)=\frac{-m_2l_1l_2(s_1s_2+c_1c_2)}{m_2l_1^2l_2^2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)^2]}$$

$$g_{22}(x)=\frac{(m_1+m_2)l_1^2}{m_2l_1^2l_2^2[(m_1+m_2)-m_2(s_1s_2+c_1c_2)^2]},$$

ω_1 and ω_2 represent the modeling uncertainties generated by $F(\dot{q})$ and τ_d. The uncertainties are assumed to be bounded by $\bar{\Delta}$. The uncertainty bound value $\bar{\Delta}$ of robot manipulator can be obtained by using an uncertainty observer[30] or by experiments.[34]

In this simulation, the parameters of the robot are given as $m_1=m_2=1(kg)$ and $l_1=l_2=1(m)$, and we assumed that the angular position is constrained within $[-\pi/2, \pi/2]$. The robot dynamics in Eq. (38) have the same form of the dynamic model described by Eq. (1) with modeling uncertainties. Let $X=[X_1,X_2]^T$, where $X_1=[x_1,x_3]^T$ and $X_2=[x_2,x_4]^T$, assuming that only x_1, x_3 are measurable. Therefore, the T-S fuzzy model with nine rules is used to represent the original nonlinear system whose membership functions are of triangular form:[6,7]

Rule 1: If x_1 is about $-\pi/2$ and x_3 is about $-\pi/2$, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_1X+B_1u\end{aligned}$$

Rule 2: If x_1 is about $-\pi/2$ and x_3 is about 0, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_2X+B_2u\end{aligned}$$

Rule 3: If x_1 is about $-\pi/2$ and x_3 is about $-\pi/2$, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_3X+B_3u\end{aligned}$$

Rule 4: If x_1 is about 0 and x_3 is about $-\pi/2$, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_4X+B_4u\end{aligned}$$

Rule 5: If x_1 is about 0 and x_3 is about 0, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_5X+B_5u\end{aligned}$$

Rule 6: If x_1 is about 0 and x_3 is about $\pi/2$, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_6X+B_6u\end{aligned}$$

Rule 7: If x_1 is about $\pi/2$ and x_3 is about $-\pi/2$, then

$$\begin{aligned}\dot{X}_1&=X_2\\ \dot{X}_2&=A_7X+B_7u\end{aligned}$$

Rule 8: If x_1 is about $\pi/2$ and x_3 is about 0, then

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = A_8X + B_8u$$

Rule 9: If x_1 is about $\pi/2$ and x_3 is about $\pi/2$, then

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = A_9X + B_9u \, ,$$

where triangle type membership functions are used for all of the rules model. We obtained the parameters of A_i, B_i by offline computing,[6] given in the Appendix.

To verify the effectiveness of the proposed FSOSMO and FSOSMC algorithm, we corresponding compared it with the state observer using the LMI[6] approach, and the conventional SMC[8-12] approach for observation and control of the T-S fuzzy system. The desired trajectory to be tracked is:[6]

$$\dot{x}_r(t) = A_r x_r(t) + r(t) \tag{47}$$

where

$x_r(t)$ is the reference state (desired state);

$$A_r = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -6 & -5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -6 & -5 \end{bmatrix}$$

and

$$r(t) = [0, 8\sin(t), 0, 8\cos(t)] \, .$$

Matlab/Simulink was used to perform all simulations, and the sampling time was set to $10^{-4}s$. First, we considered the state estimation performance. The sliding gains of the FSOSMO were selected as: $\alpha_1 = 10$ and $\alpha_2 = 11$. The states observer scheme using the LMI technique is shown in Eq. (11), page 383, Ref.[6] The comparison between the LMI-based observer method and the proposed FSOSMO observer is shown in Fig. 2. The figure clearly shows that both methods

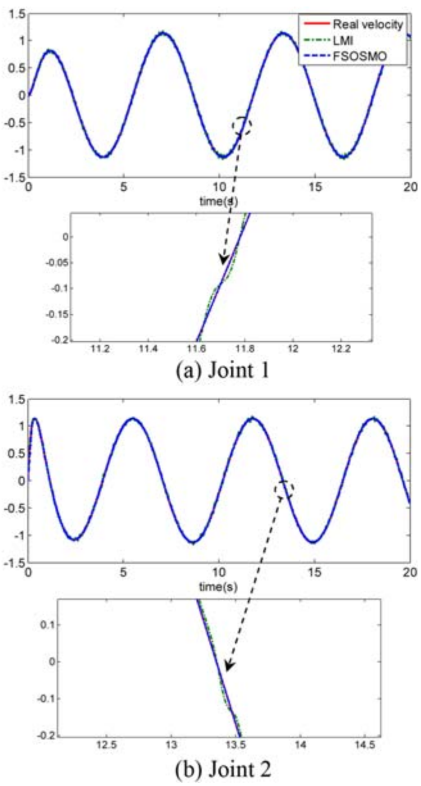

Fig. 2 Comparison between the FSOSMO observers with LMI-based observers in terms of the velocity estimation

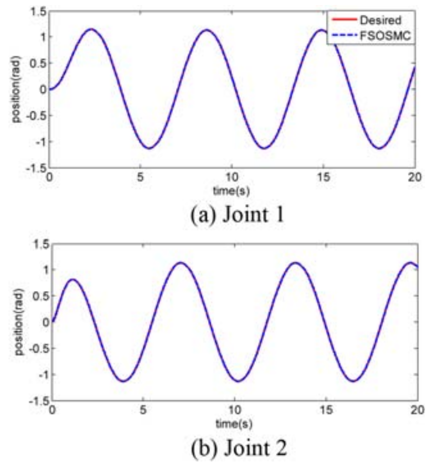

Fig. 3 Tracking performance of the proposed FSOSMC controller

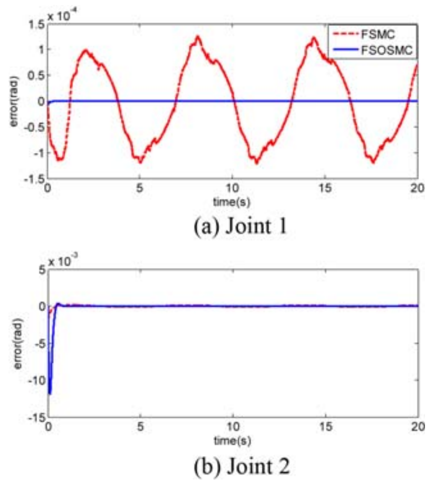

Fig. 4 Comparison between the FSOSMC with FSMC in terms of the tracking error

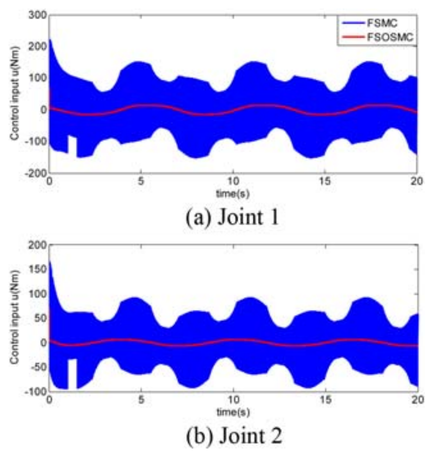

Fig. 5 Comparison between the FSOSMC with the FSMC in terms of the control efforts

obtained good velocity estimation, accuracy, and stability, but the estimated velocity of the FSOSMO was much more accurate than that of the LMI based-observer.

In turning to the tracking performance of the proposed FSOSMC algorithm, the sliding gains of the SOSM for the controller are selected as $k_1 = 6$ and $k_2 = 8$, the constant $\lambda = 10$. The good tracking performance is shown in Fig. 3. To further verify the performance of the SOSM control for T-S fuzzy system, we compared it with the conventional SMC. The sliding gain of the conventional SMC is selected as $\rho = 6$. The comparison between the FSMC and FSOSMC in terms of the tracking error and control efforts is shown in Figs. 4 and 5, respectively. They show that the FSOSMC has less chattering and control efforts than that of the FSMC, such that the tracking error is smaller.

These results demonstrate that the proposed closed loop observer-controller not only can guarantee the stability, but also obtain good tracking performance compared with existing approaches for the T-S fuzzy system.

6. Conclusions

A novel fuzzy second-order sliding mode observer-controller for a T-S fuzzy system is proposed in this paper. By integrating the T-S fuzzy model with a second-order sliding mode observer/controller, the results inherit the advantages of both methods, such as the low online computational burden, low chattering, fast response time, and finite time convergence. The stability and convergence of the closed loop observer-controller system is theoretically proven by the Lyapunov method. The simulation results for the control robot manipulator, and a comparison to the results of the existing approaches verify the effectiveness of the proposed algorithm.

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Appendix

The parameters of the T-S fuzzy model are computed offline. It is given as:[6]

$$A_1 = \begin{bmatrix} 5.927 & -0.001 & -0.315 & -8.4\times10^{-6} \\ -6.859 & 0.002 & 3.155 & 6.2\times10^{-6} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 3.0428 & -0.0011 & 0.1791 & -0.0002 \\ 3.5436 & 0.0313 & 2.5611 & 1.14\times10^{-5} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 6.2728 & 0.0030 & 0.4339 & -0.0001 \\ 9.1041 & 0.0158 & -1.0574 & -3.2\times10^{-5} \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 6.4535 & 0.0017 & 1.2427 & 0.0002 \\ -3.1873 & -0.0306 & 5.1911 & -1.8\times10^{-5} \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 11.1336 & 0.00 & -1.8145 & 0.00 \\ -9.0918 & 0.0158 & 9.1638 & 0.00 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 6.1702 & -0.0010 & 1.6870 & -0.0002 \\ -2.3559 & 0.0314 & 4.5298 & 1.1\times10^{-5} \end{bmatrix}$$

$$A_7 = \begin{bmatrix} 6.1206 & -0.0041 & 0.6205 & 0.0001 \\ 8.8794 & -0.0193 & -1.0119 & 4.4\times10^{-5} \end{bmatrix}$$

$$A_8 = \begin{bmatrix} 3.6421 & 0.0018 & 0.0721 & 0.0002 \\ 2.4290 & -0.0305 & 2.9832 & -1.9\times10^{-5} \end{bmatrix}$$

$$A_9 = \begin{bmatrix} 6.2933 & -0.0009 & -0.2188 & -1.2\times10^{-5} \\ -7.4649 & 0.0024 & 3.2693 & 9.2\times10^{-5} \end{bmatrix}$$

$$B_1 = B_5 = B_9 = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$B_2 = B_4 = B_6 = B_8 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_3 = B_7 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$