

PMP-Based Power Management Strategy of Fuel Cell Hybrid Vehicles Considering Multi-Objective Optimization

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In order to develop more practical vehicle controllers for hybrid vehicles, performance limitations of hybrid powertrain components need to be considered when constructing power management strategies. This paper introduces a Pontryagin's Minimum Principle (PMP)-based power management strategy for fuel cell hybrid vehicles which considers not only the fuel consumption minimization but also the requirement on limiting battery state of charge (SOC) usage or on prolonging fuel cell system lifetime.

The battery SOC constraint problem is solved by introducing a new cost function to the PMP-based optimal control problem. The limitation requirements on the battery SOC are satisfied by this solution. In order to take into account the lifetime of a fuel cell system, another cost function is defined and added to the PMP-based optimal control problem. Simulation results show that the lifetime of the fuel cell system can be prolonged by this method. Global optimality is discussed for the two extended cases. The proposed PMP-based power management strategy saves much time compared to dynamic programming (DP) approach while it guarantees global optimality under reasonable battery assumptions.

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1. Introduction

A PMP-based power management strategy optimizes the power distribution between power sources and minimizes the performance measure by instantaneously providing necessary optimality conditions. One of the major advantages of the PMP-based strategy is that there is usually only one parameter to be tuned in this strategy in order to obtain optimal results over a specific driving cycle.¹ Moreover, the core of this strategy is implementable in a real-time controller, even if the driving cycle information is not known in advance.² Furthermore, previous research³ proved from a mathematical point of view that the PMP-based power management strategy can serve as a global optimal solution (DP) under the assumption that the open-circuit voltage (OCV) and the internal resistance of a battery are independent of the battery SOC during the battery operation. This assumption is reasonable for charge-sustaining types of hybrid vehicles.

The PMP-based power management strategy is applied to a fuel cell hybrid vehicle in this research. Some researchers have studied this power management strategy for engine/battery powered hybrid

vehicles and plug-in hybrid vehicles as well.⁴⁻⁸ In earlier research on the PMP-based optimal control problem, the performance measure to be minimized is the total fuel consumption, the state variable of the control system is the battery SOC, and the control variable is the battery power or the engine power. Some researchers have extended the basic form of the optimal control problem to achieve some specific goals.^{4,7,8} In the research,⁴ the limitation requirement on the battery SOC usage is considered by defining a cost function related to the battery SOC and adding it to the PMP-based optimal control problem. In the research,⁷ engine oil temperature is added to the PMP-based optimal control problem as an extra state variable in order to assess the effect of engine thermal management on the fuel consumption. In the research,⁸ a parameter related to the battery aging factor is defined and added to the PMP-based optimal control problem as a second cost function in order to take into account the battery lifetime together with the total fuel consumption.

In spite of the previous research on the PMP-based power management strategy, there are still some important factors which are ignored or which need to be improved when applying this strategy to

fuel cell hybrid vehicles. In this research, factors related to performance limitations of powertrain components are considered. These factors include the battery SOC constraint and the fuel cell system lifetime, and they are considered by mathematically adding a new cost function to the PMP-based optimal control problem. In the previous research,³ the global optimality of the PMP-based strategy is proved for the basic case where there is one cost function. In this research, global optimality is discussed when the new factors are considered in the PMP-based power management strategy, and time-saving effect of this strategy is also emphasized.

2. The fuel cell hybrid vehicle

Fig. 1 illustrates the configuration and energy flows of a fuel cell hybrid vehicle. The architecture of a fuel cell hybrid vehicle is similar to that of a series hybrid electric vehicle, considering that the electric motor is the only powertrain component that is directly connected to the wheels. The fuel cell system and the battery are the power sources and they are connected to the wheels through the traction motor. The motor receives power from both the fuel cell system and the battery through the DC-DC converter and the DC-AC inverter. The motor can be controlled to operate as a generator to convert the kinetic or potential energy of the vehicle into electrical energy and store it in the battery. The arrows in Fig. 1 indicate the energy flow directions. The motor uses a map to express its efficiency, and the converters are assumed to be ideal converters with a constant efficiency of 95%. The final drive gear efficiency is considered to be a constant. The vehicle parameters used in this research are shown in Table 1. Parts of these data are sourced from available literature.⁹ In this research, a 62 kW fuel cell system and a battery with the energy capacity of 1.5 kWh are selected as power sources of the fuel cell hybrid vehicle. A 75 kW motor is also selected.

2.1 The fuel cell system

A fuel cell system consists of a fuel cell stack and other auxiliary components. A fuel cell stack converts chemical energy of reactants into electrical energy and provides power to the vehicle and to other

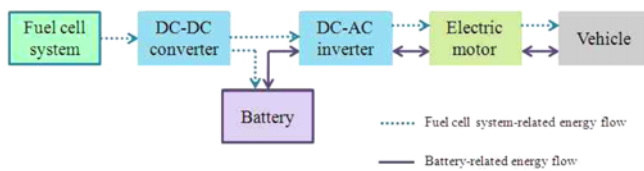


Fig. 1 Configuration and energy flows of a fuel cell hybrid vehicle

Table 1 Parameters of the vehicle

Item	Value
Vehicle total mass (kg)	1700
Final drive gear efficiency (%)	95
Tire radius (m)	0.29
Aerodynamic drag coefficient	0.37
Vehicle frontal area (m ²)	2.59
Air density (kg/m ³)	1.21
Rolling resistance coefficient	0.014

auxiliary components of the fuel cell system. A fuel cell stack is composed of many single cells connected in series. Here, these cells are assumed to be identical in performance. The voltage of a single cell is calculated as follows:^{10,11}

$$v_{fc} = E - v_{act} - v_{ohm} - v_{conc} \quad (1)$$

Here, E is the OCV. v_{act} , v_{ohm} , and v_{conc} represent activation loss, ohmic loss, and concentration loss. These losses are considered by physical and empirical equations here.

The stack-provided power P_{stack} is related to the stack current I_{stack} and the cell voltage v_{fc} , as follows:

$$P_{stack} = N_{cell} \cdot v_{fc} \cdot I_{stack} \quad (2)$$

Here, N_{cell} represents the cell number of the stack. The stack-provided power is partially used to maintain the auxiliary devices of the fuel cell system. The part of power used to propel the vehicle is called net power. The relationship between the fuel cell system net power P_{fcs} and the stack-provided power P_{stack} is as follows:

$$P_{fcs} = P_{stack} - P_{aux} \quad (3)$$

Here, P_{aux} represents the power consumption of the auxiliary components of the fuel cell system such as an air compressor. The power consumption of the air compressor is modeled by a map which is obtained from the compressor model,^{10,11} and the power consumption of the other auxiliary components is modeled by a constant. Fig. 2 illustrates the stack-provided power, auxiliary power, and net power of the fuel cell system used in this research. The parameters related to the fuel cell system model are listed in Table 2.

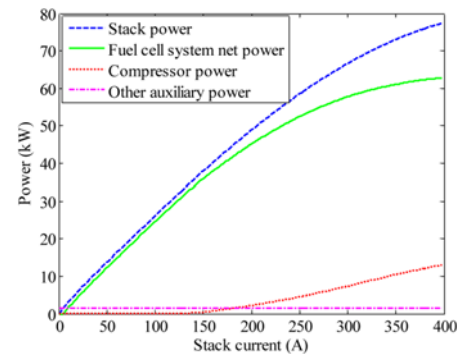


Fig. 2 Stack-provided power, auxiliary power, and net power of the fuel cell system used in this research

Table 2 Parameters used in the fuel cell system model

Item	Value
Maximum stack power (kW)	77
Maximum net power (kW)	62
Maximum stack current (A)	400
Anode pressure (hydrogen) (atm)	2
Cathode pressure (air) (atm)	1-2.5
Stack temperature (°C)	80
Cell number	350
Active area (cm ² /cell)	280
Membrane thickness (cm)	0.01275
Compressor efficiency (%)	80

For a fuel cell stack, the fuel consumption rate \dot{m}_{h_2} is related to the stack current according to the following equation:

$$\dot{m}_{h_2} = \frac{N_{cell} \cdot M_{h_2}}{n \cdot F} \cdot I_{stack} \cdot \lambda \quad (4)$$

In equation (4),^{12,13} M_{h_2} represents the molar mass of hydrogen, n represents the number of electrons acting in the reaction, F is the Faraday constant, and λ is the hydrogen excess ratio.

The fuel cell system net power and the fuel consumption rate have a specific relationship, as both of them are related to the fuel cell stack current according to Fig. 2 and equation (4). Fig. 3 illustrates the relationship between the net power and the fuel consumption rate of the fuel cell system used in this research.

In a fuel cell system, its efficiency is defined as

$$\eta_{fcs} = \frac{P_{fcs}}{\dot{m}_{h_2} \cdot LHV} \quad (5)$$

In equation (5),¹⁴ LHV is the lower heating value of hydrogen. Fig. 4 illustrates the fuel cell system efficiency versus the net power for the fuel cell system used in this research.

2.2 The battery

An internal resistance battery model^{15,16} is used in this research. This battery model consists of a voltage source (OCV) and an internal resistance component.¹⁵ In this battery model, the battery parameters are related according to the following equation:

$$\dot{SOC} = -\frac{I}{Q_{bat}}, \quad I = \frac{V(SOC) - \sqrt{V(SOC)^2 - 4R(SOC) \cdot P_{bat}}}{2R(SOC)} \quad (6)$$

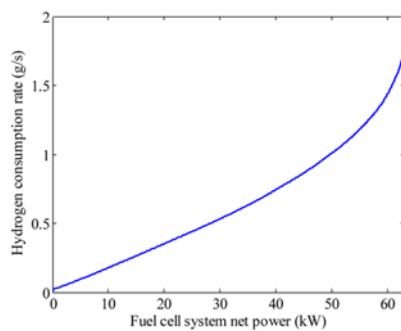


Fig. 3 Relationship between net power and fuel consumption rate of the fuel cell system used in this research

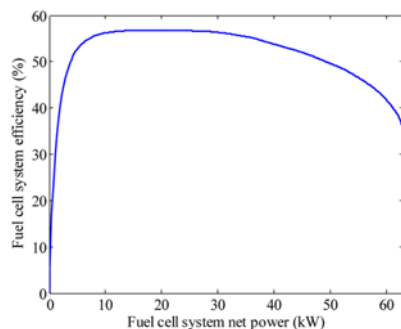


Fig. 4 Fuel cell system efficiency versus net power for the fuel cell system used in this research

Here, Q_{bat} is the battery charge capacity, I is the battery current, and P_{bat} is the battery power at the battery terminals. The OCV V and the internal resistance R are both functions of the battery SOC. Fig. 5(a) illustrates the electrical schematic of the internal resistance battery model, and (b) and (c) show the OCV and internal resistance of the battery used in this research.

3. Review of PMP-based power management strategy

PMP stems from the optimal control theory; it is a general case of the fundamental theorem of the Calculus of Variations.¹⁷ PMP instantaneously provides necessary conditions to optimal control problems to let them find optimal control laws. In our previous research,^{18,19} the basic formulation of the PMP-based optimal control problem for fuel cell hybrid vehicles is presented. The only objective of the optimal control problem in the research is to find an optimal power split trajectory which minimizes the fuel consumption when the vehicle is being driven. We solve this problem by finding out the optimal trajectory of the fuel cell system net power, which is the control variable of the optimal control problem. The battery SOC is the state variable of the optimal control problem.

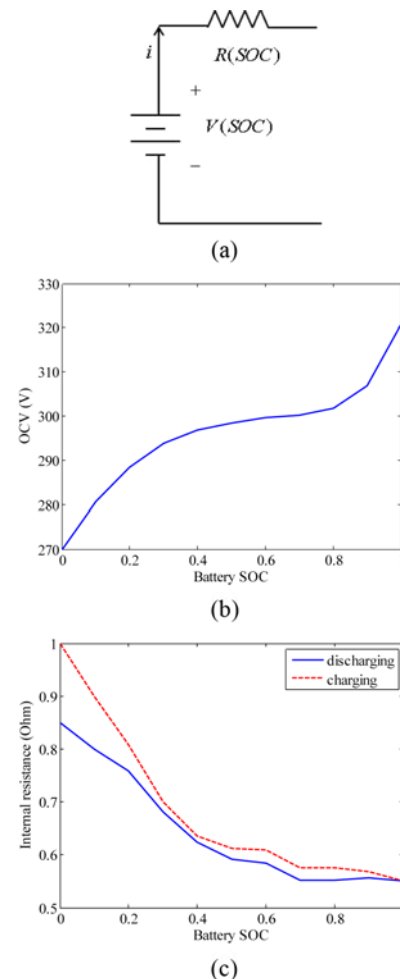


Fig. 5 (a) Electrical schematic of the internal resistance battery model, (b) OCV of the battery used in this research, (c) Internal resistance of the battery used in this research

The state equation of the system, which describes the dynamics of the state variable, is given in (6). Considering that the internal resistance and OCV of the battery are functions of the battery SOC, equation (6) can be simplified using a function f , as follows:

$$\dot{SOC}(t) = f(SOC(t), P_{bat}(t)) \quad (7)$$

The power required to propel the vehicle P_{req} , the fuel cell system net power P_{fcs} , and the battery power P_{bat} have the following relationship:

$$P_{bat}(t) = P_{req}(t) - P_{fcs}(t) \quad (8)$$

As the power required to propel the vehicle can be derived when selecting a driving cycle, we can transform the state equation (7) into

$$\dot{SOC}(t) = F(SOC(t), P_{fcs}(t)) \quad (9)$$

using a different function F .

Performance measure to be minimized here is the total fuel consumption when the fuel cell hybrid vehicle drives over a specified driving cycle from time t_0 to time t_f . Given that the fuel cell system net power and the fuel consumption rate are related to each other as shown in Fig. 3 and that the state equation (9) is a constraint of the optimal control problem, the performance measure J is expressed as follows,

$$J(P_{fcs}(t)) = \int_{t_0}^{t_f} \{ \dot{m}_{h_2}(P_{fcs}(t)) + p(t) \cdot (F(SOC(t), P_{fcs}(t)) - \dot{SOC}(t)) \} dt \quad (10)$$

where p is the Lagrange multiplier, which is also called the costate in the PMP-based control. The only cost function in this section is the first term inside of the integration sign, which is related to the fuel consumption.

The objective of the optimal control problem here is to minimize the total fuel consumption while the dynamic state equation (9) is satisfied. Thus, necessary conditions of the optimal control problem are given when the variation of the performance measure δJ from equation (10) is zero.^{3,17} If we introduce a Hamiltonian H ,^{3,17} which is defined as

$$H(SOC(t), P_{fcs}(t), p(t)) = \dot{m}_{h_2}(P_{fcs}(t)) + p(t) \cdot F(SOC(t), P_{fcs}(t)) \quad (11)$$

then the necessary conditions that derive optimal trajectories are as follows:

$$\begin{aligned} \frac{\partial H}{\partial p} &= \dot{SOC} \\ \frac{\partial H}{\partial SOC} &= -\dot{p} \\ \frac{\partial H}{\partial P_{fcs}} &= 0 \end{aligned} \quad (12)$$

The necessary conditions in (12) should be satisfied all the time in order to obtain the optimal results. The first necessary condition is the state equation (9), which is a constraint of the optimal control problem.

The second necessary condition is called the costate equation that determines the optimal trajectory of the costate p when the initial value of the costate is given. The third necessary condition determines the optimal trajectory of the control variable P_{fcs} .

PMP is a general case of the Euler-Lagrange equation of the Calculus of Variation,³ in which the third necessary condition in (12) is expressed as follows:

$$H(SOC^*(t), P_{fcs}^*(t), p^*(t)) \leq H(SOC^*(t), P_{fcs}(t), p^*(t)) \quad (13)$$

The advantage of form (13) is that it can be applied to a non-linear or a non-differentiable or a non-convex function.³ In the computer calculation of the PMP-based optimal control, the optimal P_{fcs} is obtained at every calculation time step by finding out the P_{fcs} among all admissible fuel cell system net power values, which minimizes the Hamiltonian H . Now, the necessary conditions of the PMP-based optimal control can be written in a specific form, as follows:

$$\begin{aligned} \dot{SOC}^*(t) &= \frac{\partial H}{\partial p}(SOC^*(t), P_{fcs}^*(t), p^*(t)) = F(SOC^*(t), P_{fcs}^*(t)) \\ \dot{p}^*(t) &= -\frac{\partial H}{\partial SOC}(SOC^*(t), P_{fcs}^*(t), p^*(t)) \\ &= -p^*(t) \cdot \frac{\partial F}{\partial SOC}(SOC^*(t), P_{fcs}^*(t)) \\ H(SOC^*(t), P_{fcs}^*(t), p^*(t)) &\leq H(SOC^*(t), P_{fcs}(t), p^*(t)) \end{aligned} \quad (14)$$

Boundary condition also needs to be satisfied other than the necessary conditions in order to satisfy the condition that the value of δJ is zero, which is as follows:¹⁷

$$[-p^*(t_f)]^T \cdot \delta SOC_f + [H(SOC^*(t_f), P_{fcs}^*(t_f), p^*(t_f))] \cdot \delta t_f = 0 \quad (15)$$

Previous research^{17,20} theoretically investigated the relationship between the Hamilton-Jacobi-Bellman (HJB) equation, which is the recurrence relation of DP, and the PMP, and concluded that the PMP can be derived from the HJB equation under certain conditions. Previous research³ also proved from a mathematical point of view that the PMP-based power management strategy can work as a global optimal solution (DP) under the assumption that the internal resistance and OCV of a battery do not depend on the battery SOC when the battery operates. This assumption is reasonable for fuel cell hybrid vehicles, considering that batteries in fuel cell hybrid vehicles are operated in a certain SOC range because fuel cell hybrid vehicles are charge-sustaining types of hybrid vehicles. For a fuel cell hybrid vehicle, the second necessary condition in (14) can be transformed to the following equation, as it can be assumed that the function F does not depend on the battery SOC.

$$\dot{p}^*(t) = -p^*(t) \cdot \frac{\partial F}{\partial SOC}(P_{fcs}^*(t)) = 0 \quad (16)$$

From equation (16), previous research³ proved that the PMP-based power management strategy can work as a global optimal solution (DP). This conclusion is derived for the case where there is one cost function, and it will be extended for two-cost function cases in the next section.

4. PMP-based power management strategy of fuel cell hybrid vehicles considering multi-objective optimization

The PMP-based power management strategy reviewed in section 3 is the basic one, in which there is only one cost function and the state variable is not constrained. In a fuel cell hybrid vehicle, however, the battery SOC usage should be limited because of battery chemical characteristics. Prolonging the fuel cell system lifetime is also important considering that the price of the fuel cell system is very high currently. In this section, the basic formulation reviewed in section 3 is mathematically extended considering two important factors which are limitations on the battery SOC usage and requirements on prolonging the fuel cell system lifetime. Global optimality of the extended cases is also discussed in this section.

4.1 PMP-based power management strategy considering battery SOC constraint

Most batteries should be operated in a certain SOC range in the charge-sustaining types of hybrid vehicles. Thus the battery SOC should be constrained in the optimal control problem. In this subsection, the constraints on the battery SOC usage are considered by defining a new cost function and adding this cost function to the PMP-based optimal control problem. The objective of the optimal control problem in this subsection is to minimize the fuel consumption while the battery SOC usage boundary is satisfied. The formulation of the optimal control problem here is introduced below.

The state equation is the same with equation (9), as the state variable is also the battery SOC here. In the literature,^{4,21} a new cost function is defined in order to consider the battery SOC limitation factor in the fuel consumption minimization problem. However, the new cost functions are related to the state variable, and this makes the costate fluctuating according to the second necessary condition in (14). The fluctuation of the costate will consequently cause the battery SOC fluctuation, and this will shorten the battery lifetime. In order to remedy this drawback, a cost function S related to the control variable is newly defined and added to the PMP-based optimal control problem in this subsection.

The definition of the new cost function S is as follows:

$$S(P_{fcs}(t)) = \begin{cases} \alpha \cdot P_{fcs}(t) & SOC(t) \leq SOC_{\min} \\ \beta \cdot P_{fcs}(t) & SOC(t) \geq SOC_{\max} \\ 0 & Otherwise \end{cases} \quad (17)$$

Here, α and β are tuning parameters. The performance measure considering S is then

$$J(P_{fcs}(t)) = \int_{t_0}^{t_f} \{ \dot{m}_{h_2}(P_{fcs}(t)) + S(P_{fcs}(t)) + p(t) \cdot (F(SOC(t), P_{fcs}(t)) - \dot{SOC}(t)) \} dt \quad (18)$$

The necessary conditions of the PMP-based optimal control here are as follows,

$$\begin{aligned} \dot{SOC}^*(t) &= \frac{\partial H}{\partial p}(SOC^*(t), P_{fcs}^*(t), p^*(t)) = F(SOC^*(t), P_{fcs}^*(t)) \\ \dot{p}^*(t) &= -\frac{\partial H}{\partial SOC}(SOC^*(t), P_{fcs}^*(t), p^*(t)) \\ &= -P^*(t) \cdot \frac{\partial F}{\partial SOC}(SOC^*(t), P_{fcs}^*(t)) \\ H(SOC^*(t), P_{fcs}^*(t), p^*(t)) &\leq H(SOC^*(t), P_{fcs}^*(t), p^*(t)) \end{aligned} \quad (19)$$

when Hamiltonian is defined as

$$H(SOC(t), P_{fcs}(t), p(t)) = \dot{m}_{h_2}(P_{fcs}(t)) + S(P_{fcs}(t)) + p(t) \cdot F(SOC(t), P_{fcs}(t)) \quad (20)$$

Necessary conditions in (19) look the same with those in (14), but the third necessary condition S is added to the Hamiltonian here. In the previous research,^{4,21} the new cost function affects the optimal trajectory of the costate when the battery SOC is about to reach its limits, and this consequently affects other optimal trajectories including battery SOC trajectory. On the other hand, the new cost function S here has the function of shifting the optimal value of the fuel cell system net power when the battery SOC reaches its boundaries. This accordingly influences the optimal battery SOC trajectory. A lower value of the fuel cell system net power will be selected as the optimal solution when the battery SOC reaches its upper limit, and a greater value of the fuel cell system net power will be chosen when the battery SOC reaches its lower limit. Fig. 6(a) and (b) show comparison results of the PMP-based power management strategies reviewed in section 3 and here. The upper limit and lower limit of the battery SOC are set to 0.694 and 0.507, and the FTP72 urban driving cycle is used here. It can be seen that the tendency of the battery SOC trajectories is similar while the battery SOC is constrained in the case where the cost function S is used. There are no big fluctuations in the battery SOC trajectory. This is the main advantage of using the cost function S . Fig. 7(a) and (b) illustrate comparison results of the PMP-based power management strategies reviewed in section 3 and introduced in the previous research.^{4,21} The upper and lower limits are the same with those in Fig. 6. Big fluctuations in the trajectories are observed when the strategy

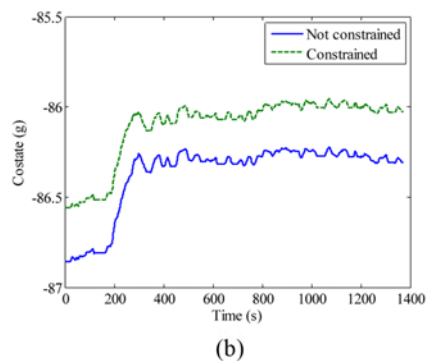
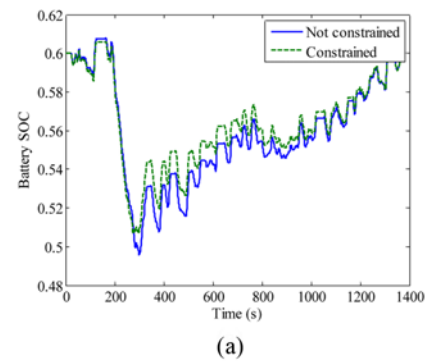


Fig. 6 (a) Comparison of optimal battery SOC trajectories, (b) Comparison of optimal costate trajectories (strategies reviewed in section 3 and introduced in subsection 4.1)

introduced in the previous research^{4,21} is used.

Equation (16) is still true when the new cost function S is added if the battery OCV and internal resistance are not dependent on the battery SOC when the battery operates. Because S is only related to P_{fcs} as shown in equation (17) and there is no state variable other than the battery SOC. Thus, the PMP-based power management strategy, which considers the battery SOC constraint with the cost function S , can serve as a global optimal solution (DP). Meanwhile, the PMP-based power management strategy can save much time compared to DP approach. In our example with the FTP72 urban driving cycle, the elapsed time is 3 hours 23 minutes 14 seconds for DP approach and is just around 7 seconds for the PMP-based strategy in a computer simulation environment.

4.2 PMP-based power management strategy considering fuel cell system lifetime

In section 3, it was assumed that there were no limits on the fuel cell system power changing rate. However, the power changing rate of a fuel cell system is limited in reality because of the slow dynamic of its air circuit.^{9,22} Besides, frequent and rapid changes of the dynamic load shorten the fuel cell system lifetime.²³ Hence, these changes should be avoided in order to improve the fuel cell system durability and prolong the fuel cell system lifetime. In this subsection, a new cost function is defined for the fuel cell system lifetime factor and is introduced to the PMP-based optimal control problem. The objective of the optimal control problem here is to minimize the fuel consumption while considering the fuel cell system lifetime.

The state equation is also the same with equation (9), as there is also one state variable in this optimal control problem. The new cost function L which is related to the objective of prolonging the fuel cell

system lifetime is defined as follows:

$$L(P_{fcs}(t)) = a \cdot (P_{fcs}(t) - P_{fcs}(t-b))^2 \quad (21)$$

Here, a is a tuning parameter, t represents a time step, and $t-b$ represents its previous time step. b is the duration of one time step. This new cost function is related to the power changing rate of the fuel cell system. The forms of performance measure, Hamiltonian, and necessary conditions are the same with those in subsection 4.1, as the new cost function L is only dependent on the fuel cell system net power and there is no other state variable except the battery SOC. The cost function L replaces S in this case. This new cost function makes the optimal trajectory of the fuel cell system net power smooth through the third necessary condition.

Fig. 8, Fig. 9, and Fig. 10 illustrate the simulation results on the FTP72 urban driving cycle, NEDC 2000, and Japan 1015 driving cycle, respectively. These figures indicate that the optimal trajectory of the fuel cell system net power becomes smooth through the reformulation of the PMP-based optimal control problem introduced in this subsection. The power changing rate of the fuel cell system also becomes smaller for the case of reformulation. The initial battery SOC and the final SOC are both 0.6 here. Table 3 shows comparisons of the simulation results illustrated in the three figures. It can be observed that the mean power changing rate of the fuel cell system is reduced through the reformulation. Thus, the fuel cell system lifetime can be increased.

Equation (16) is still true in this subsection under the assumption that the battery OCV and internal resistance are not dependent on the battery SOC during the battery operation, as the new cost function L is

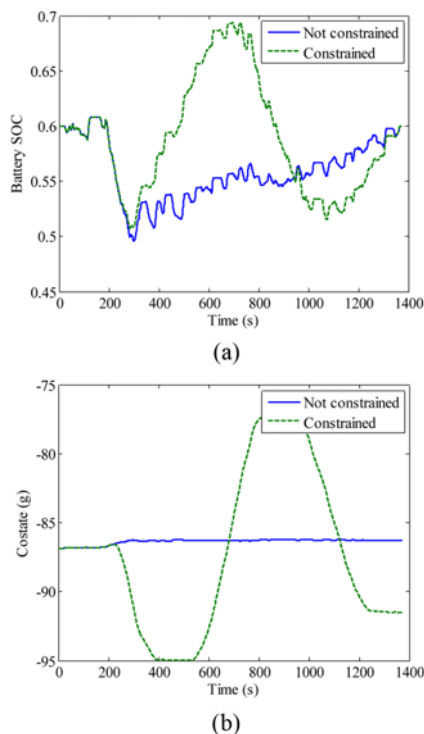


Fig. 7 (a) Comparison of optimal battery SOC trajectories, (b) Comparison of optimal costate trajectories (strategies reviewed in section 3 and introduced in the previous research)

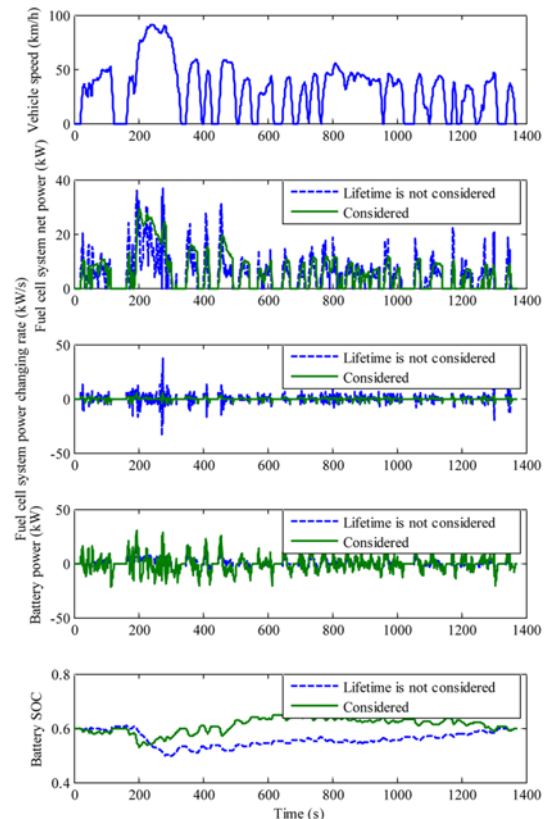


Fig. 8 Optimal trajectories on the FTP72 urban driving cycle

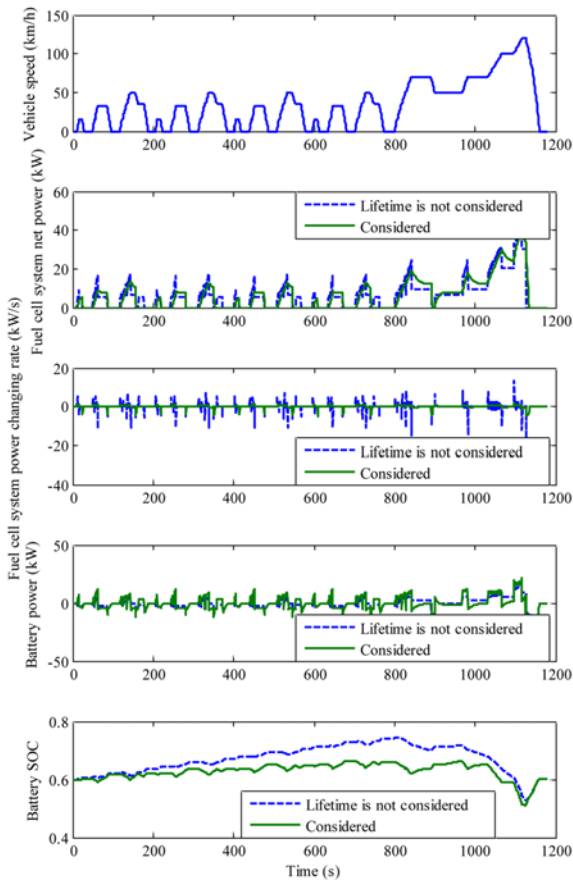


Fig. 9 Optimal trajectories on the NEDC 2000

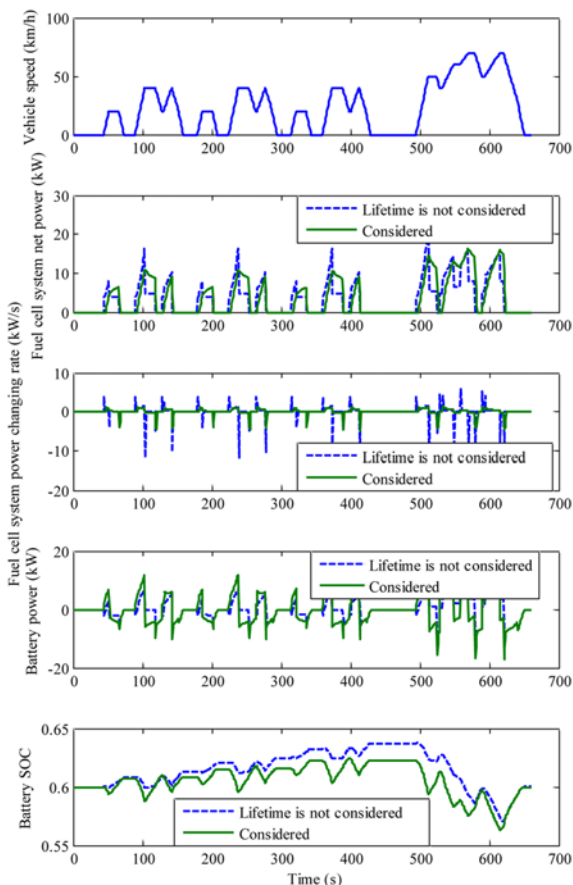


Fig. 10 Optimal trajectories on the Japan 1015 driving cycle

only dependent on the fuel cell system net power and there is no other state variable except the battery SOC. Thus, the PMP-based power management strategy can serve as a global optimal solution when the fuel cell system lifetime is taken into account by the cost function L . In the meantime, the PMP-based power management strategy can save much time compared to DP approach. In our example with the NEDC 2000, the elapsed time is 7 hours 38 minutes 39 seconds for DP approach and is just around 5 seconds for the PMP-based strategy in a computer simulation environment.

5. Discussions

In section 4, we introduced the PMP-based power management strategy considering two important factors in fuel cell hybrid vehicles. These two factors were separately considered, and now the case, in which the two factors are all taken into account, can also be discussed.

In this case, there are one state variable and three cost functions, and the state equation is identical to equation (9). The Hamiltonian here can be defined as follows:

$$H(SOC(t), P_{fcs}(t), p(t)) = \dot{m}_{h_2}(P_{fcs}(t)) + S(P_{fcs}(t)) + L(P_{fcs}(t)) + p(t) \cdot F(SOC(t), P_{fcs}(t)) \quad (22)$$

The form of necessary conditions is the same with the one in subsection 4.1, as both cost functions S and L are only dependent on the fuel cell system net power and there is no other state variable except the battery SOC. The optimal trajectories will be determined according to the necessary conditions here. Equation (16) is still true under reasonable battery assumptions in this case. Thus, the PMP-based power management strategy here can also guarantee the global optimality.

In subsection 4.1, the tuning parameter α is a negative constant. This tuning parameter is required to prevent the battery SOC reaching its lower limit. Similarly, the tuning parameter β is a positive constant and it has an effect of avoiding the battery SOC from reaching its upper limit.

Table 3 Comparison of the simulation results of the PMP-based power management strategies

Driving cycle	Fuel consumption (kg/100 km)		Mean power changing rate of fuel cell system (kW/s)	
	Basic one	Considering fuel cell system lifetime	Basic one	Considering fuel cell system lifetime
FTP72 urban	1.154	1.204	2.974	1.009
NEDC 2000	1.282	1.310	2.396	0.696
Japan 1015	1.137	1.170	1.681	0.841

Table 4 Influence of the tuning parameter

Tuning parameter	Fuel consumption (kg/100 km)	Mean power changing rate of fuel cell system (kW/s)
$a = 0.001$	1.187	1.307
$a = 0.002$	1.204	1.009
$a = 0.003$	1.216	0.836

In subsection 4.2, a is a tuning parameter. It determines the level of smoothness of optimal trajectories by affecting power distribution between power sources. Table 4 shows the influence of the tuning parameter a on the fuel consumption and on the mean power changing rate of the fuel cell system. The FTP72 urban driving cycle is used here. It can be seen that a higher value of a results in a higher fuel consumption value and a lower mean power changing rate of the fuel cell system.

It can be observed from Table 3 and Table 4 that the reformulation also increases the fuel consumption while prolonging the fuel cell system lifetime. However, we estimate that this will still be significant from an economic viewpoint because the price of the fuel cell stack is very high currently. Detailed economic improvement will need to be further investigated.

6. Conclusions

A PMP-based power management strategy for fuel cell hybrid vehicles is mathematically extended in order to consider two important factors which are limitations on the battery SOC usage and fuel cell system lifetime. These extensions are useful for realization of the PMP-based strategy, as these extensions are closer to the reality. Global optimality is discussed for the two extended cases. The following points are drawn from this research.

- (1) In order to overcome drawbacks of the existing method, which considers the battery SOC boundary but makes the battery SOC trajectory fluctuating, we introduce a new cost function to the basic PMP-based optimal control problem to consider the battery SOC constraint factor while minimizing the fuel consumption. Simulation results illustrate that the battery SOC is constrained while there is no big fluctuation in the optimal battery SOC trajectory after the reformulation.
- (2) A second cost function is defined and introduced to the basic PMP-based optimal control problem in order to take into account the fuel cell system lifetime while optimizing the fuel economy of the fuel cell hybrid vehicle. Simulation results show that the optimal trajectory of the fuel cell system net power becomes smooth through the reformulation. The power changing rate of the fuel cell system is also smaller for the case of reformulation. However, there is a fuel consumption loss when the fuel cell system lifetime is considered. Prolonging the fuel cell system lifetime by the proposed method will still be significant because of the high-priced fuel cell system. Detailed economic improvement needs to be further discussed.
- (3) The PMP-based power management strategy which considers the battery SOC constraint introduced in this research guarantees the global optimality under the assumption that the battery OCV and internal resistance are not dependent on the battery SOC during the battery operation. The PMP-based power management strategy which takes into account the fuel cell system lifetime introduced in this research also guarantees the global optimality under the same battery assumption. The time which the PMP-based strategy consumed is way shorter than DP approach's for the two cases while the PMP-based strategy guarantees the global optimality.

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