

# Real-Time Inertia Compensation for Multi-Axis CNC Machine Tools

Sungchul Jee<sup>1#</sup> and Jungseung Lee<sup>1</sup>

<sup>1</sup> Department of Mechanical Engineering, Dankook University, 126 Jukjeon-dong, Suji-gu, Yongin-si, Gyeonggi-do, South Korea, 448-701  
# Corresponding Author / E-mail: scjee@dku.edu, TEL: +82-31-8005-3504, FAX: +82-31-8005-4007

KEYWORDS: CNC machine tools, Load disturbance, Effective moment of inertia, Inertia compensation, Contouring accuracy

*Variation of the effective moment of inertia due to load disturbance in a CNC feed drive system may increase control error for each axis of motion and thereby degrading total precision including contouring accuracy. Although some investigations have been made on estimation and control of variation of the effective moment of inertia, most of them are for individual axis control: there has been no research that considered the improvement of contouring accuracy in CNC machine tools under load variation. In this paper, a real-time inertia compensation method is proposed including a newly defined contour error model that represents error caused by load disturbance. Position error due to load variation is calculated based on the estimated disturbance torque by a full-order disturbance observer for each drive axis. The position error is then used to establish the new contour error model considering the effect of load variation. The proposed compensation method is realized by adjusting velocity loop gains in the control system using the least square method such that the effect of variation of the effective moment of inertia on the contour error is minimized. The proposed method is evaluated with a 3-axis open CNC testbed.*

Manuscript received: January 19, 2012 / Accepted: March 19, 2012

## 1. Introduction

Recently, interest in high-end machine tools has increased in industry to produce a wide variety of precise products and it has been strived in various ways to improve both precision and productivity of machine tools. Especially, efforts have been continuously made to improve contouring accuracy of multi-axis CNC systems.<sup>1-3</sup> The contouring accuracy of CNC machine tools affects directly on machining precision.

During the operation of CNC machine tools, load variation may occur in feed drive systems due to change in cutting force and workpiece weight. This load variation causes variation of the effective moment of inertia for each drive axis and increases control error, which may degrade total precision including contouring accuracy. Although investigations have been made on estimation and control of variation of the effective moment of inertia, most of the previous research has focused on single axis control.<sup>4,5</sup> To our knowledge, there has been no research that has considered the improvement of contouring accuracy for multi-axis CNC systems in the presence of load variation in the axes of motion.

This paper proposes a real-time inertia compensation method for multi-axis CNC machine tools including a newly defined

contour error model. This model represents contour error caused by load variation in feed drive systems. Position error due to load variation can be calculated based on estimated disturbance torque of each drive axis using a full-order disturbance observer. The load variation is regarded as being caused by load disturbance which should be compensated for. Then, the resulting contour error is deduced from the above position error estimation.

In the proposed compensation method, the velocity loop gains for axes of motion are adjusted in real time, which is realizable in commercial servo drive systems. The least square method is used such that the effect of the variation of the effective moment of inertia on the contour error is minimized. The proposed inertia compensation method was implemented on a 3-axis CNC testbed. The experimental results show that the proposed method can improve the contouring accuracy under load disturbance in feed drive systems.

## 2. Estimation of effective moment of inertia

The effective moment of inertia for each drive axis needs to be estimated for inertia compensation. Variation of moment of inertia

due to load variation can be found by system variables such as acceleration and disturbance torque. These variables, however, are difficult to measure. In this paper, a general full-order observer is utilized to estimate the acceleration and disturbance torque for each axis. A method to estimate the effective moment of inertia of a feed drive system is described below based on the estimated load disturbance and acceleration.

**2.1 Modeling of a servo system**

The compensation in this paper is for a velocity control system shown in Fig. 1 which employs a PI control and is widely used in industry. The system variables are listed in Table 1. The load disturbance  $T_d$  affects the system in the form of acceleration variation. This system, however, is unobservable, and instead, a servo motor in the velocity control system shown in Fig. 2 is considered which describes the relationship between acceleration and output. The corresponding state equations can be represented as follows.

$$\frac{d}{dt} \begin{bmatrix} \theta_o \\ \omega_o \\ T_d \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_o \\ \omega_o \\ T_d \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} T_r \quad (1)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} \theta_o \\ \omega_o \\ T_d \end{bmatrix} \quad (2)$$

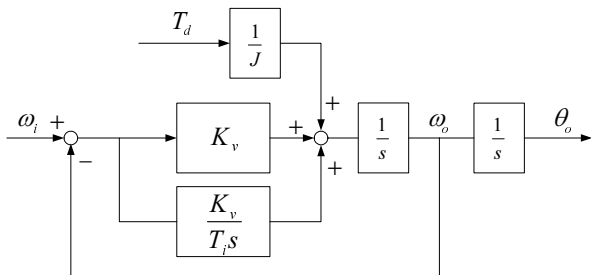


Fig. 1 Velocity control loop

Table 1 System variables

Variables	Description
$\omega_i$	Velocity input
$\omega_o$	Velocity output
$\theta_o$	Position output
$T_d$	Disturbance torque
$K_v$	Velocity gain
$T_i$	Integral time constant
$J$	Moment of inertia

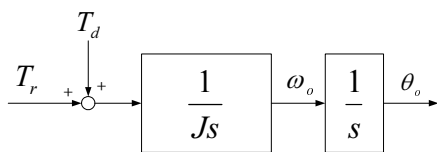


Fig. 2 Servo motor

This system is observable and therefore a full-order observer can be designed.

**2.2 Design of a full-order observer**

The observation system shown in Fig. 3 can be expressed by

$$\frac{d}{dt} \hat{x}(t) = A\hat{x}(t) + BT_r(t) \quad (3)$$

where  $\hat{x}(t)$  is an estimated value of the state vector  $x(t) = [\theta_o \ \omega_o \ T_d]^T$ . The observer gains  $K_1$ ,  $K_2$  and  $K_3$  can be determined using pole placement. Based on the information on the input and output of the servo motor, the disturbance torque can be observed.

**2.3 Estimation of effective moment of inertia**

While the moment of inertia of the motor  $J$  is constant, that of the axis of motion may vary according to various load conditions and this is called effective moment of inertia. The effective moment of inertia  $\hat{J}$  affects the velocity control system, making the acceleration change as represented in the following equation.

$$s\omega_o(s) = (\omega_i(s) - \omega_o(s)) \left( K_v + \frac{K_v}{T_i s} \right) + \frac{T_d(s)}{\hat{J}} \quad (4)$$

From Eq. (4), the effective moment of inertia for the velocity control system can be given by

$$\hat{J} = \frac{\int T_d(t) \dot{\omega}_i(t) dt}{\int (\dot{\omega}_i(t))^2 dt} \quad (5)$$

which indicates that the effective moment acting on the velocity control system can be found using the observed disturbance torque and input acceleration without additional hardware.

**3. Real-time inertia compensation**

The contour error defined as deviation from the desired tool path directly affects the dimensional accuracy of a machined part. In this paper, a real-time inertia compensation method is proposed to improve contouring accuracy in the presence of load disturbance. The proposed method is based on Jee's contour error model<sup>3</sup> and

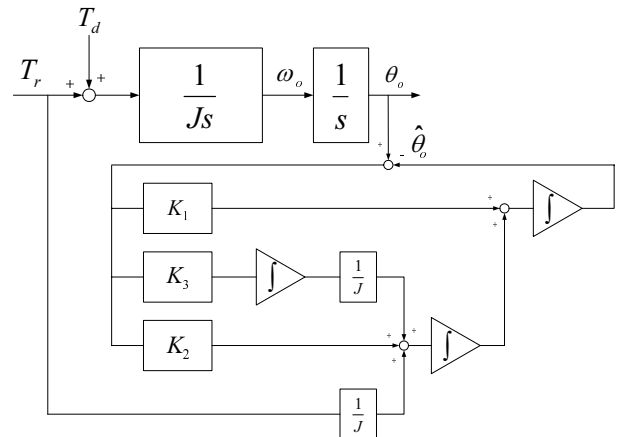


Fig. 3 Full-order disturbance observer

compensates for the change in the contour error caused by load disturbance and its variation.

First, the change in the velocity caused by load variation due to disturbance for each axis of motion can be expressed by

$$\Delta\omega_{d,i} = \alpha_i T_{d,i} \quad (i = x, y, z) \quad (6)$$

where  $\alpha$  is a constant related to the servo motor.<sup>6</sup> By integrating Eq. (6), the change in the position due to the variation in the effective moment of inertia can be obtained.

$$\Delta\theta_{d,i} = \alpha \int T_{d,i} dt \quad (7)$$

The current actual position  $\theta_i$  can be divided into two parts, one independent of and the other dependent on load variation:

$$\theta_i = \theta'_i + \Delta\theta_{d,i} \quad (8)$$

where  $\theta'_i$  is a component independent of load variation. If the ballscrew pitch of the machine is  $l$ , Eq. (8) can be rewritten in machine coordinates as below

$$P_i = P'_i + \Delta P_{d,i} \quad (9)$$

where  $\Delta P_{d,i} = (l/2\pi)\Delta\theta_{d,i}$ . Fig. 4 illustrates the contour error model used in the proposed compensation. The contour error is denoted by  $\varepsilon$  and its magnitude can be given by

$$\varepsilon = \sqrt{\sum_i (P_i - K_{gi})^2} \quad (10)$$

where  $K_g$  is the nearest point on the desired path from the actual tool position. On the other hand, by introducing  $\Delta P_d$ , the magnitude of the possible contour error  $\varepsilon'$  without the load variation can be represented as

$$\varepsilon' = \sqrt{\sum_i (P'_i - K_{gi})^2} \quad (11)$$

If we define an objective function such as given by (12) which indicates the square sum of the difference in the magnitude of the contour errors  $\varepsilon$  and  $\varepsilon'$  for three consecutive time steps  $k-2$ ,  $k-1$  and  $k$ ,

$$S = \sum_{j=k-2}^k \left[ \varepsilon'_j - \left( \sqrt{\sum_i (P_i + A_i \Delta P_{di} - K_{gi})^2} \right)_j \right]^2 \quad (12)$$

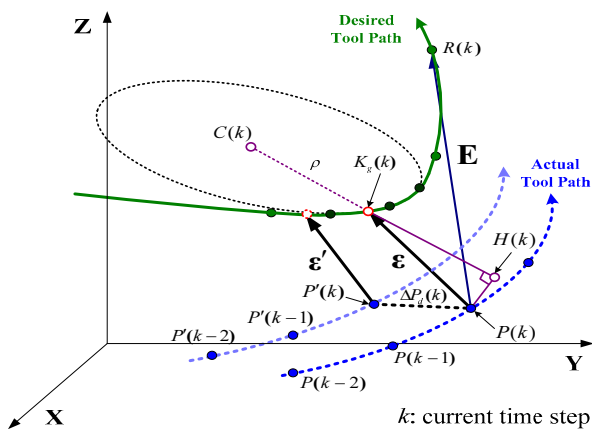


Fig. 4 Contour error model for inertia compensation

the coefficients  $A_i$  can be determined by the least square method such that the objective function is minimized. Accordingly, the contour error caused by load variation can be minimized with proper selection of  $A_i$ .

In the meantime, the dominant time constant for the second order control system shown in Fig. 1 can be approximated as follows

$$\tau_{system} \approx \frac{2}{K_v} \quad (13)$$

whereas the time constant for the servo motor system is

$$\tau_{motor} = \alpha J \quad (14)$$

Assuming a motor gain of 1, the velocity gain needs to be

$$K_v \approx \frac{2}{\alpha J} \quad (15)$$

so that the response of the two systems is approximately identical. Eq. (15) shows that  $K_v$  is necessarily inversely proportional to  $J$ .

If we define the velocity loop gain as  $K_{vp}$  which is one of few real-time adjustable gain parameters in commercial servo drives, the relationship between the velocity gain and the velocity loop gain is as follows.

$$K_{vp} = K_v / J \quad (16)$$

Using this relation, inertia compensation is conducted to minimize the contour error. The estimated effective moment of inertia  $\hat{J}$  is considered in the adjustment of the velocity loop gain  $K_{vp}$  such that the time constant is maintained in the presence of load variation. In the compensation, the coefficients  $A_i$  are also considered to coordinate the direction of adjustment. Consequently, the velocity loop gain is adjusted in real time as given below.

$$K_{vp,i} = A_i K_{v0} / \hat{J}_i \quad (17)$$

where  $K_{v0}$  is the velocity gain without load disturbance. Finally, the calculated velocity loop gain is fed into a loss-pass filter to reduce the effect of high frequency noise from the observer and the system. The schematic of the inertia compensation is depicted in Fig. 5.

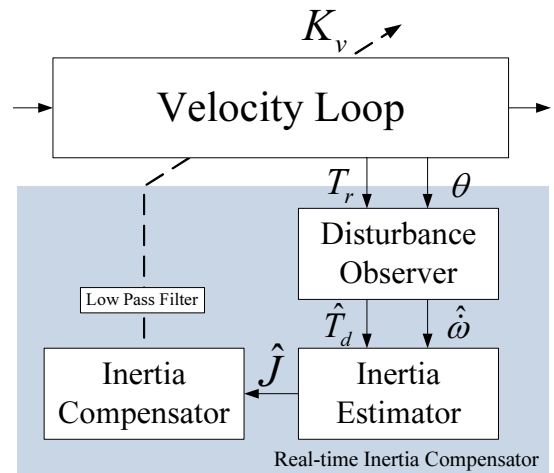


Fig. 5 Schematic of real-time inertia compensation

### 4. Experimental Results

In order to evaluate the proposed real-time inertia compensation method, a 3-axis open CNC testbed was built as shown in Fig. 6. The X and Y driving axes are connected through couplings to the load motors which generate load torque and provide the effects of varying effective moment of inertia to the system. The three axes of motion are operated by high-speed serial communication between the host controller (i.e., PC) and servo drives while the load motors by analog communication.

The proposed compensation method was implemented on the testbed for 3-dimensional linear and circular contouring motions. In both cases, the load motors were coordinated to exert load torque on the control system in the direction normal to the desired path at all times in order to maximize the contour error. The experimental conditions are listed in Table 2 and the reference paths and the load torque are illustrated in Figs. 7 through 10. The contour errors with

the proposed compensation method are compared with those without compensation and those without load disturbance in Figs. 11 and 12. The experimental results are summarized in Tables 3 and 4. From the results, it can be seen that the proposed inertia compensation method is effective and substantially reduces the contour errors due to load disturbance and its variation. The root mean square (RMS) error was reduced approximately by factors of 3 and 2 respectively for linear and circular motion compared with the results without compensation. The maximum errors were also reduced by a factor of 3 in both cases.

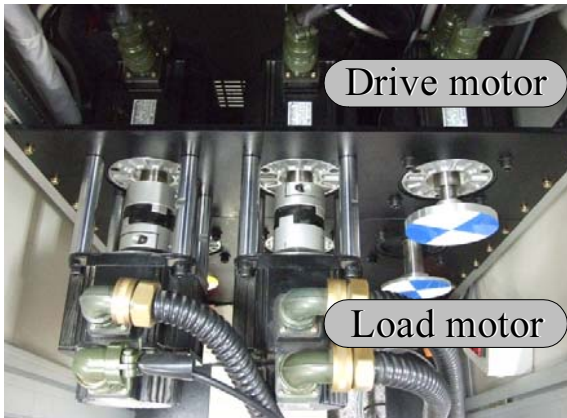


Fig. 6 Three-axis CNC testbed

Table 2 Experimental conditions

Parameters	Values
Sampling time [ms]	1
Position resolution [ $\mu\text{m}$ ]	0.1
Feedrate [m/min]	1
Initial velocity loop gain [Hz]	40
Integral time constant [ms]	20
Position control gain	100
Maximum load torque [Nm]	2.5

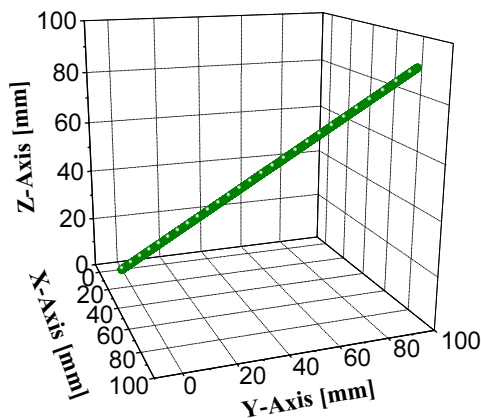


Fig. 7 Linear contour

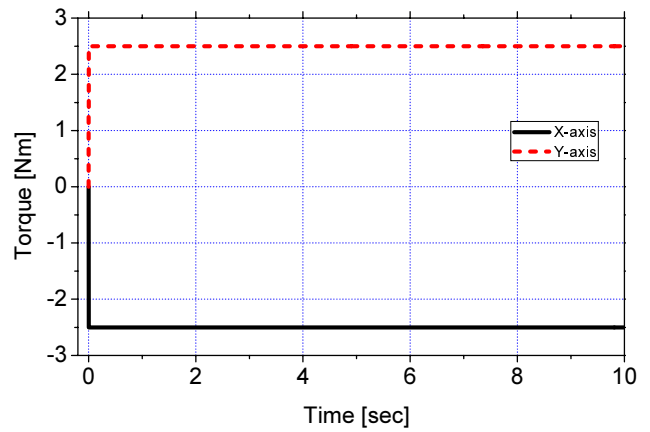


Fig. 8 Load torque for linear motion

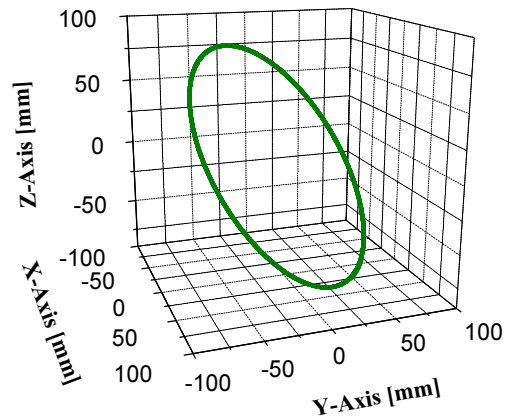


Fig. 9 Circular contour

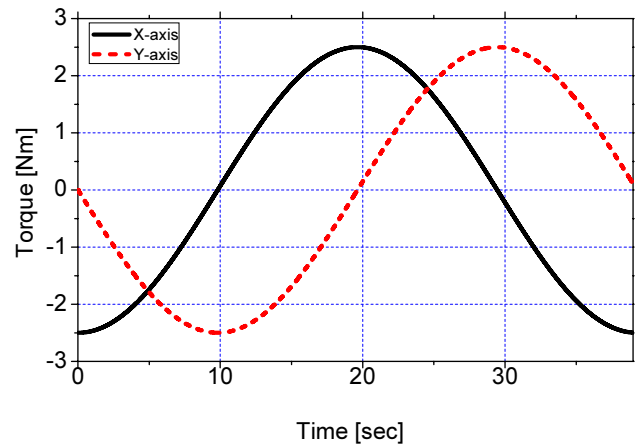


Fig. 10 Load torque for circular motion

## 5. Conclusions

In this paper, a real-time inertia compensation method was proposed to improve the contouring accuracy of multi-axis CNC machine tools in the presence of load variation. A full-order observer was designed based on the input and output of the control system to estimate disturbance torque. By the use of the observed torque and the acceleration of the motor, the effective moment of inertia for each drive axis was estimated. A new contour error model was defined which includes position deviation due to load variation, based on the estimated disturbance torque. In the proposed compensation method, the velocity loop gains for axes of motion were simultaneously adjusted in real time in the direction

that the contour error due to load disturbance is minimized.

The proposed method was implemented on a 3-axis open CNC testbed and was evaluated in terms of contouring accuracy. The experimental results show that the proposed inertia compensation method substantially improves the contouring accuracy in the presence of load disturbance and its variation.

## ACKNOWLEDGEMENT

The present research was conducted by the research fund of Dankook University in 2010.

## REFERENCES

1. Koren, Y. and Lo, C. C., "Variable-Gain Cross-Coupling Controller for Contouring," *Annals of the CIRP*, Vol. 40, No. 1, pp. 371-374, 1991.
2. Chiu, G. T.-C. and Tomizuka, M., "Coordinated Position Control of Multi-Axis Mechanical Systems," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 120, No. 3, pp. 389-393, 1998.
3. Jee, S. and Lee, H., "An Integrated Approach to the Analysis and Design of a Three-Axis Cross-Coupling Control System," *Int. J. Precis. Eng. Manuf.*, Vol. 8, No. 2, pp. 59-63, 2007.
4. Awaya, I., Kato, Y., Miyake, I., and Ito, M., "New Motion Control with Inertia Identification Function Using Disturbance Observer," *Proceedings of the IECON'92*, Vol. 1, pp. 77-81, 1992.
5. Kim, N.-J., Moon, S.-H., and Hyun, D.-S., "Inertia Identification for the Speed Observer of the Low Speed Control of Induction Machines," *IEEE Transactions on Industry Applications*, Vol. 32, No. 6, pp. 1371-1379, 1996.
6. Koren, Y., "Computer Control of Manufacturing Systems," McGraw-Hill, New York, 1983.

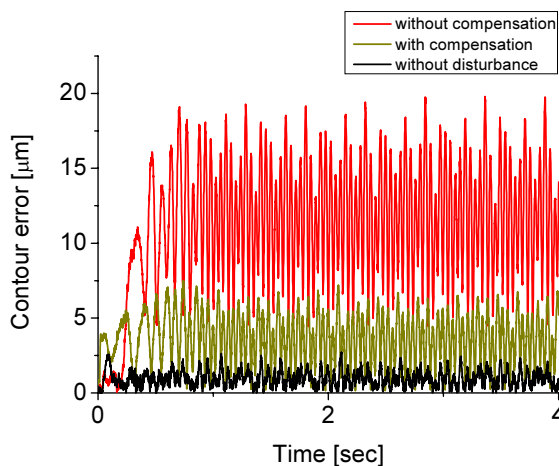


Fig. 11 Contour error for linear motion

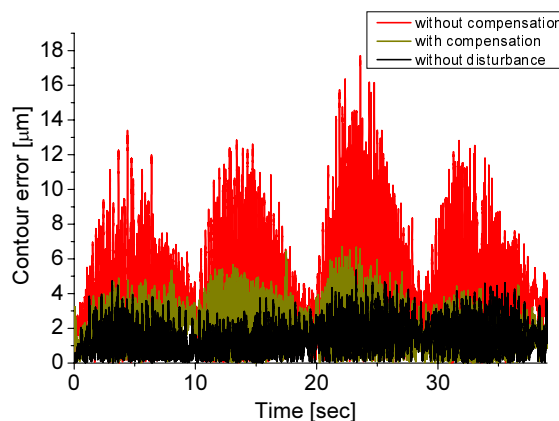


Fig. 12 Contour error for circular motion

Table 3 Comparison of contour errors for linear motion

Contour error [ $\mu\text{m}$ ]		RMS	Max.
With load disturbance	Without compensation	12.14	19.78
	With compensation	3.98	7.48
Without load disturbance		1.11	2.86

Table 4 Comparison of contour errors for circular motion

Contour error [ $\mu\text{m}$ ]		RMS	Max.
With load disturbance	Without compensation	4.51	17.70
	With compensation	2.45	6.71
Without load disturbance		1.41	5.35