DOI: 10.1007/s12541-012-0099-y

A Comparison Among Neo-Hookean Model, Mooney-Rivlin Model, and Ogden Model for Chloroprene Rubber

Beomkeun Kim[1], Seong Beom Lee[1,#], Jayone Lee[2], Sehyun Cho[2], Hyungmin Park[2], Sanghoon Yeom[2] and Sung Han Park[3]

1 Department of Mechanical and Automotive Engineering, High Safety Vehicle Core Technology Research Center, Inje University, Eobang-dong, Gimhae, South Korea, 621-749
2 Department of Mechanical Engineering, Inje University, Eobang-dong, Gimhae, South Korea, 621-749
3 Agency for Defense Development, 111 Sunam-dong, Yuseong-gu, Daejeon, South Korea, 305-152
Corresponding Author / E-mail: mechlsb@inje.ac.kr, TEL: +82-55-320-3667, FAX: +82-55-324-1723

KEYWORDS: Hyperelastic model, Neo-Hookean model, Mooney-Rivlin model, Ogden model, Chloroprene rubber, Uniaxial test, Biaxial test, Planar test

Neo-Hookean model and Mooney-Rivlin model are hyperelastic material models where the strain energy density function is made from invariants of the left Cauchy-Green deformation tensor. Even though Ogden model is a hyperelastic material model, its strain energy density function is expressed by principal stretch ratio. These three models have been widely used in industries. Recently, Ogden model, especially Ogden 3rd model, shows better agreement with the test data than others. In spite of the limitations to describe particular stress states, it is known that reasonable results using these models can be obtained for various structural components. In this research, three kinds of models are considered for Chloroprene rubber. Three kinds of tests (Uniaxial tension test, Biaxial tension test, and Planar shear test) are performed for Chloroprene specimen and through four kinds of test combinations (Uni+Bi, Uni+Pl, Bi+Pl, Uni+Bi+Pl), numerical simulations are carried out for Neo-Hookean model, Mooney-Rivlin model, and Ogden model. It is shown that Mooney-Rivlin model and Ogden model can be used for Chloroprene Rubber in the specific ranges for Isotropic Hyperelastic model.

Manuscript received: September 27, 2011 / Accepted: December 27, 2011

1. Introduction

When load is applied and then removed, general materials show elastic behavior returning to its original state within the range where the relationship between load and deformation is linear. However, rubber-like materials show hyperelastic characteristics representing elastic behavior in the range of large deformation showing nonlinear relationship between load and deformation.[1] In general, the behavior of rubber-like materials can be represented as a strain energy density function. Thus, many attempts have been made to reproduce theoretically the stress-strain curves obtained from experiments on the deformation of highly elastic rubber-like materials. However, since the deformations, which rubber-like materials undergo, are too large and the aspect of behavior shows a significant difference depending on the materials, it is difficult to decide a stress energy density function which adequately represents the stress-strain relation from experiments.[2]

Accordingly, the researches related to accurate behavior's prediction of hyperelastic materials have been actively performed these days. As one of similar research cases, there is a research performed by Jang et al. which dealt with finite element analysis of weatherstrip made of EMPM and TPE.[3] Jang et al. carried out Uniaxial and Biaxial tension tests to obtain the material constants and stress-strain curves and the results demonstrated that Finite element method can be used to predict the behavior of TPE and EPDM components in weatherstripping.

In this research, chloroprene rubber is used for hyperelastic material modeling. Chloroprene rubber, also widely known as Neoprene, is one of the first oil resistant synthetic rubbers. It can be considered as a good general purpose rubber with an excellent balance of physical and chemical properties. In addition, it has better chemical, oil, ozone, and heat resistance than natural rubber. Chloroprene rubber is widely used because of its wide range of useful properties and reasonable price. Typical applications include belting, coated fabrics, calve jackets, seals, and gaiters.

In this research, chloroprene rubber's finite element analysis results using ABAQUS of each hyperelastic model were compared and analyzed with the experiments data, and it was utilized to understand the stress-strain relationship of hyperelastic materials.

2. General Hyperelastic models

2.1 Neo-Hookean model

A Neo-Hookean model is a hyperelastic material model that can be used for predicting the stress-strain behavior of materials, and the model is similar to Hooke's law.[4] For general materials, the relationship between applied stress and strain is initially linear, but at a certain point the stress-strain curve changes to nonlinear. A Neo Hookean model is one of the simple models and the strain energy density function for an incompressible Neo-Hookean material is as follows:

$$W = C_1(\bar{I}_1 - 3) \tag{1}$$

where C_1 is a material constant, and $\bar{I}_1$ is the first invariant of the left Cauchy-Green deformation tensor.

The Neo-Hookean model is based on the statistical thermodynamics of cross-linked polymer chains and is possible to use for rubber-like materials for initial linear range. Cross-linked polymers act in a Neo-Hookean manner in the linear states. However, at a certain point, the polymer chains will be stretched to the maximum point that the covalent cross links will allow and this will cause a dramatic increase in the elastic modulus of the material. It is generally known that Neo-Hookean material model does not predict accurate phenomena at large strains.

2.2 Mooney-Rivlin model

A Mooney-Rivlin model, which was introduced by Melvin Mooney and Ronald Rivlin,[5] is a hyperelastic material model, where the strain energy density function W is a linear combination of two invariants of the left Cauchy-Green deformation tensor B.[6] The strain energy density function for an incompressible Mooney-Rivlin material is as follows:

$$W = C_1(\bar{I}_1 - 3) + C_2(\bar{I}_2 - 3) \tag{2}$$

where C_1 and C_2 are empirically determined material constants, and $\bar{I}_1$ and $\bar{I}_2$ are the first and the second invariant of the deviatoric component of the left Cauchy-Green deformation tensor.

It is confirmed that material constants of Mooney-Rivlin model are related to the linear elastic shear modulus G[7] and G can be expressed as follows:

$$G = 2(C_1 + C_2) \tag{3}$$

Mooney-Rivlin model allows a simple definition of the quasi-static temperature dependency and suffices to define shear modulus G as a function of the temperature and is widely used for rubber-like materials up to now. In spite of the known limitations to describe particular stress states, it is known that these models can be used for various structural components with local values of the strains up to about 200%.[8]

2.3 Ogden model

An Ogden model is a hyperelastic material model that can be used for predicting the nonlinear stress-strain behavior of materials such as rubber or polymer. Ogden model was introduced by Ogden in 1972, and the strain energy density function for an Ogden material is as follows:[9]

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) \tag{4}$$

where λ_j, $(j = 1,2,3)$ is the principal stretch ratio, and μ_i and α_i are empirically determined material constants.

Ogden model is the most widely used model up to now, and it has been frequently used for the analysis of rubber components like O-ring and seal. Ogden model is different from the other models (Neo-Hookean model, Mooney-Rivlin model) which are expressed by invariants. In addition, it has the advantages that the test data can be directly used, and it shows good agreement with the test data up to 700% of the tensile test results.

In the analysis of the behavior of a rubber component, Ogden model, especially Ogden 3rd model, well describes the test data than Mooney-Rivlin model which is mentioned before. Since the stretch ratio's exponents of Ogden model are composed of actual numbers. In contrast, the stretch ratio's exponents of Mooney-Rivlin model are composed of integer. Therefore, Ogden model has better flexibility in describing the curve than Mooney-Rivlin model.

3. Material Testing Method

To define the input requirements of hyperelastic material models, several experiments are required. Even though the experiments are carried out separately and the strain states are different, the result from all of the individual experiments is utilized as a set. It means that the specimens used for each of the experiments should be of the same material.

There are several standards for the testing of elastomers in tension. However, the experimental requirements for analysis are somewhat different for the most standardized test methods. The appropriate experiments are not yet clearly defined by national or international standards organizations. In this research, to define and to satisfy input requirements of hyperelastic material, Uniaxial tension test, Biaxial tension test, and Planar shear test were carried out.

3.1 Uniaxial tension test

a. Deformation state:

$$\lambda_2 = \lambda = L/L_0, \quad \lambda_1 = \lambda_3 = 1/\sqrt{\lambda} \tag{5}$$

where λ_j $(j = 1,2,3)$ is the principal stretch ratio. Also, A and L, respectively, mean the cross-sectional area and length of a specimen.

b. Stress state:

$$\sigma_2 = \sigma = P/A_0, \quad \sigma_1 = \sigma_3 = 0 \tag{6}$$

where P is the load, and σ_j, $(j = 1,2,3)$ is the axial stress.

Uniaxial tension experiments are very common for elastomers. The most significant requirement is that the specimen should be

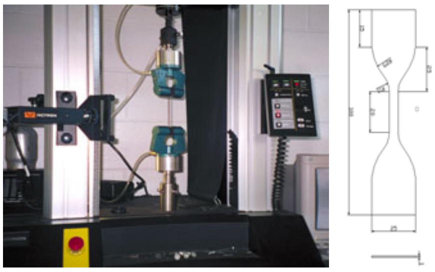

Fig. 1 Uniaxial tension test equipment and specimen

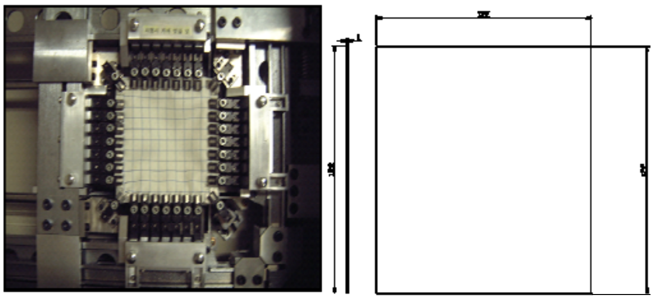

Fig. 2 Biaxial tension test equipment and specimen

very long compared with the width and thickness in order to achieve a state of pure tensile strain. The objective is to create an experiment where there is no lateral constraint to specimen thinning. The results of this analysis show that the specimen needs to be at least 10 times longer than the width or thickness. Since the experiment is not intended to fail the specimen, there is no need to use a dumbbell shaped specimen that is commonly used to prevent specimen failure in the clamps. Also, there is not an absolute specimen size requirement. The equipment and specimen for Uniaxial tension test are shown in Fig. 1.

3.2 Biaxial tension test

a. Deformation state:

$$\lambda_1 = \lambda_2 = \lambda = L/L_0,\ \ \lambda_3 = 1/\lambda^2 \tag{7}$$

where λ_j $(j = 1,2,3)$ is the principal stretch ratio. Also, L and t, respectively, mean the length and thickness of a specimen.

b. Stress state:

$$\sigma_1 = \sigma_2 = \sigma,\ \ \sigma_3 = 0 \tag{8}$$

where σ_j, $(j = 1,2,3)$ is the axial stress.

The equal biaxial strain state may also be achieved by radial stretching a square sheet as shown in Fig. 2. The nominal equibiaxial stress contained inside the specimen calculated as: $\sigma = P/A_0$ where $A_0 = Wt_0$, and W is the width and height of the specimen, P is the average of the forces normal to the width and height of the specimen and t_0 is the original thickness.

3.3 Planar shear test

a. Deformation state:

$$\lambda_1 = 1,\ \ \lambda_2 = \lambda = L/L_0,\ \ \lambda_3 = 1/\lambda \tag{9}$$

where λ_j $(j = 1,2,3)$ is the principal stretch ratio. Also, L and t, respectively, mean the length and thickness of a specimen.

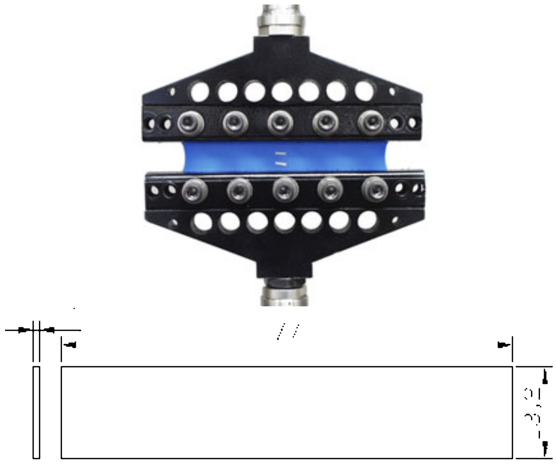

Fig. 3 Planar shear test equipment and specimen

b. Stress state:

$$\sigma_1 \neq 0,\ \ \sigma_2 = \sigma,\ \ \sigma_3 = 0 \tag{10}$$

where σ_j $(j = 1,2,3)$ is the axial stress.

The experiment appears to be nothing more than a very wide tensile test. However, because the material is nearly incompressible, a state of planar shear exists in the specimen at a 45 degree angle to the stretching direction. The most significant aspect of the specimen is that the length of stretching is much shorter than that of that of width. The objective is to create an experiment where the specimen is perfectly constrained in the lateral direction such that all specimens thinning occur in the thickness direction. This requires that the specimen should be at least 10 times wider than the length in the stretching direction.[10] Fig. 3 shows the equipment and specimen for Planar shear test.

4. Results

In this research, as mentioned before, three kinds of tests were carried out; Uniaxial tension test, Biaxial tension test, and Planar shear test. In addition, three models (Neo-Hookean model, Mooney-Rivlin model, Ogden model) were used for the numerical simulations with the commercial software (ABAQUS). That is to say, the results of Neo-Hookean model, Mooney-Rivlin model, and Ogden model using ABAQUS were compared with the experimental data. The comparisons are shown in the following figures and all figures show the experimental result and numerical simulations using four kinds of combinations (1. Uniaxial and Biaxial test, 2. Uniaxial and Planar test, 3. Biaxial and Planar test, 4. Uniaxial, Biaxial, and Planar test).

4.1 Uniaxial tension test results

Fig. 4 shows Uniaxial tension test results for Neo-Hookean model.

As shown in Fig. 4, Biaxial and Planar test results and Uniaxial and Biaxial and Planar test results are matched to the experimental results in the below 100% strain range. These results can be used for pre-test results for Hyperelastic modeling.

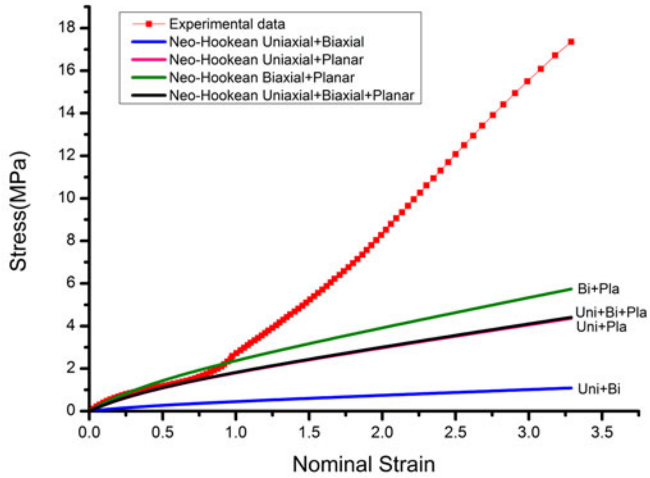

Fig. 4 Uniaxial tension test results for Neo-Hookean model

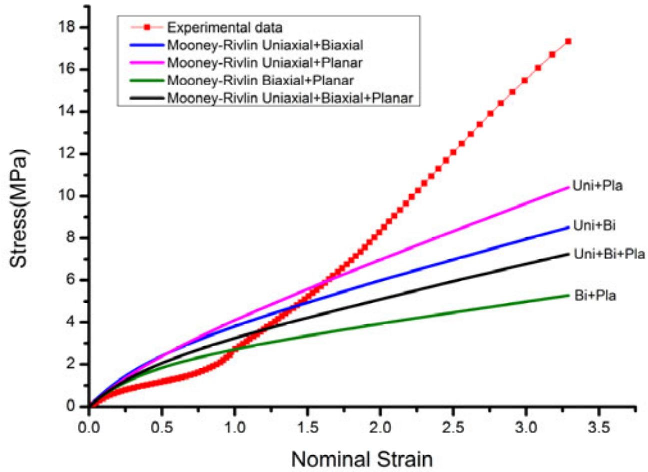

Fig. 5 Uniaxial tension test results for Mooney-Rivlin model

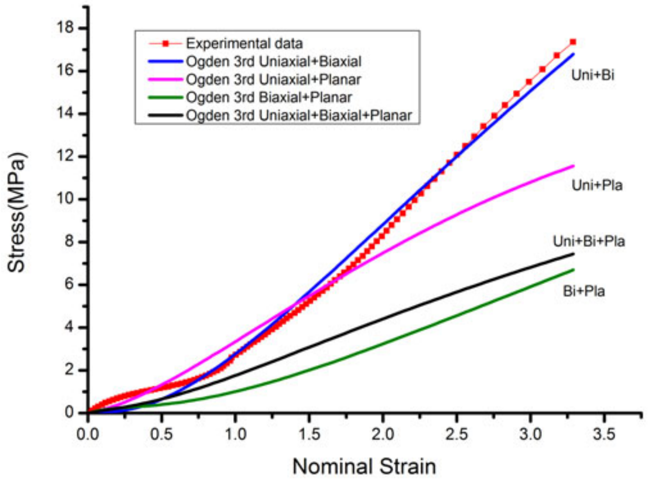

Fig. 6 Uniaxial tension test results for Ogden 3rd model

Fig. 5 shows Uniaxial tension test results for Mooney-Rivlin model.

As shown in Fig. 5, Mooney-Rivlin model cannot catch up the experimental test results.

Lastly, Fig. 6 shows Uniaxial tension test results for Ogden 3rd model.

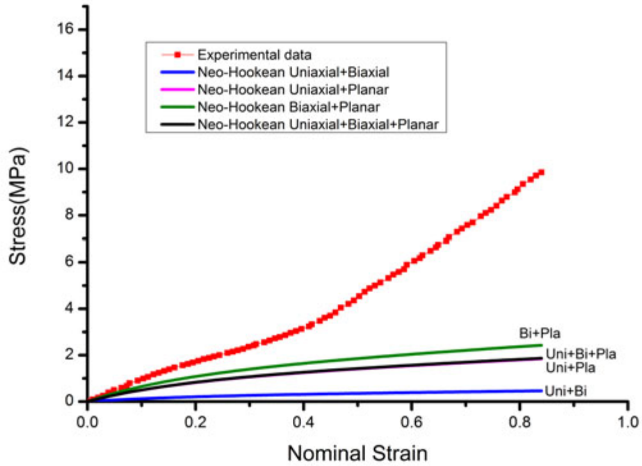

Fig. 7 Biaxial tension test results for Neo-Hookean model

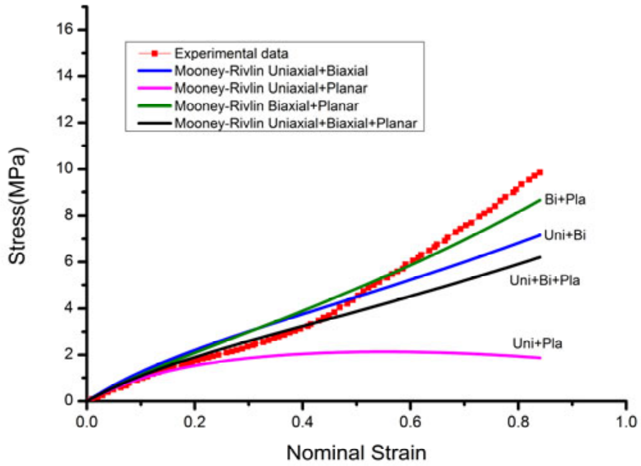

Fig. 8 Biaxial tension test results for Mooney-Rivlin model

As shown in Fig. 6, Uniaxial and Planar test results are approximately matched up to the experimental results in the below 200% strain range. And, Uniaxial and Biaxial tension test results are very good agreement with the experimental results in all the experimental test ranges.

Therefore, Neo-Hookean model and Ogden model can be used for Uniaxial tension test in the specific strain ranges.

4.2 Biaxial tension test results

Fig. 7 shows Biaxial tension test results for Neo-Hookean model.

It is shown that Neo-Hookean model is not adequate to Biaxial tension test.

Fig. 8 shows Biaxial tension test results for Mooney-Rivlin model.

As shown in Fig. 8, most combinations are acceptable within 20% strain ranges and Uniaxial and Biaxial and Planar test results are possible to use within 40% strain ranges. It is shown that Mooney-Rivlin model is adequate to Biaxial tension test in the small deformation ranges with Uniaxial and Biaxial and Planar test results.

Fig. 9 shows Biaxial tension test results for Ogden 3rd model and it is shown that Ogden model is not adequate to Biaxial tension test.

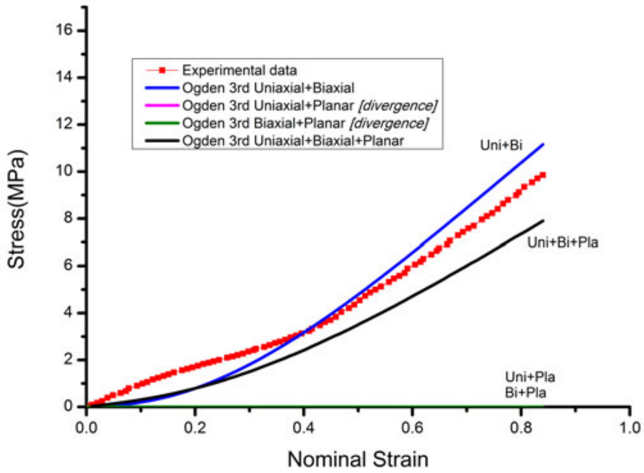

Fig. 9 Biaxial tension test results for Ogden 3rd model

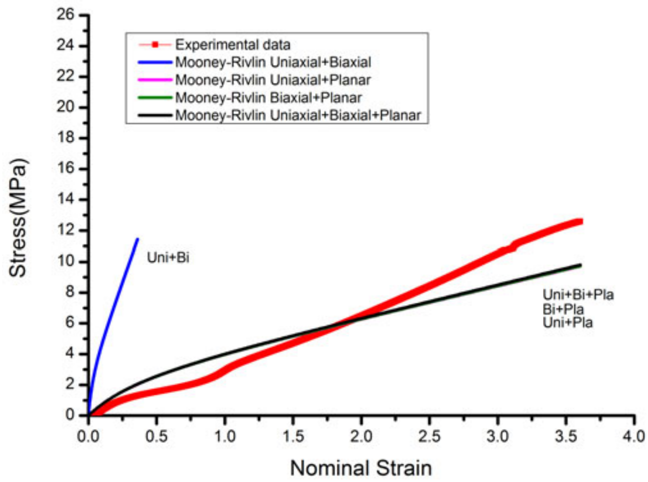

Fig. 11 Planar shear test results for Mooney-Rivlin model

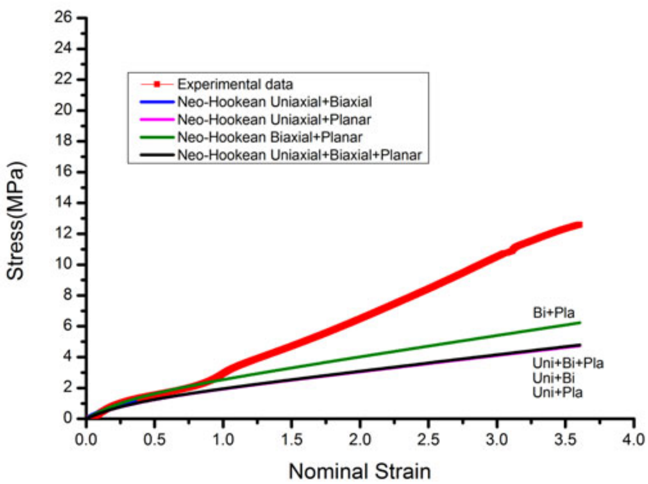

Fig. 10 Planar shear test results for Neo-Hookean model

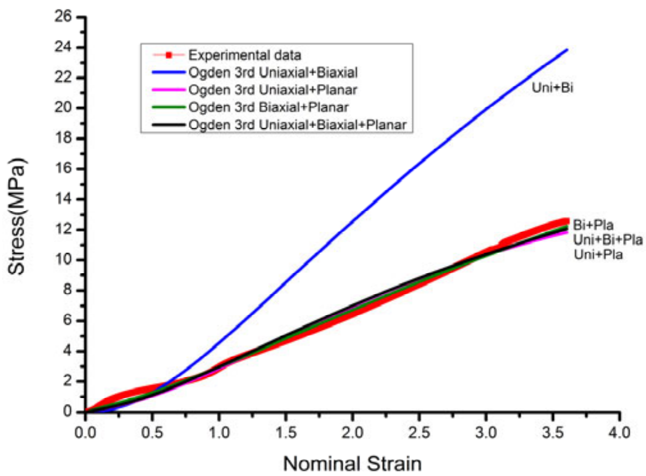

Fig. 12 Planar shear test results for Ogden 3rd model

4.3 Planar Shear Test Results

Fig. 10 shows Planar shear test results for Neo-Hookean model, and it is shown that Neo-Hookean model with Biaxial and Planar shear test results can be used for within 100% strain range.

Fig. 11 shows Planar shear test results for Mooney-Rivlin model, and it is shown that Mooney-Rivlin model is not adequate to Planar shear test.

Lastly, Fig. 12 shows Planar shear test results for Ogden 3rd model.

As shown in Fig. 12, Uniaxial and Planar, Biaxial and Planar, Uniaxial and Biaxial and Planar test results are matched to the experimental results in all the experimental test ranges. These results show that Ogden model can be used for Planar shear test if Planar test results are included in the Combinations.

5. Conclusions

Although Mooney-Rivlin model and Neo-Hookean model are easy to handle for analysis, both models have limitations for large deformation of chloroprene rubber. To obtain specific results for large deformation in rubber-like materials, more advanced hyperelastic models are recommended. For Ogden 3rd model cases, it is shown that Ogden 3rd model can be used for Chloroprene rubber. It means that Ogden model can be applied to Numerical analysis in the general working deformation ranges. Hence, for Chloroprene rubber, Ogden 3rd model is recommended for analysis. Lastly, in spite of inadequacy for Mooney-Rivlin model and Neo-Hookean model, these two models can be used for simple check in small strain ranges and pre-study for hyperelastic materials.

ACKNOWLEDGEMENT

Authors are gratefully acknowledging the support by Defense Acquisition Program Administration and Agency for Defense Development.

REFERENCES

1. Woo, C. S., Park, H. S., Kim, Y. G., Shin, W. G. and Joe, D. H., "Finite Element Analysis and Evaluation of Automotive Rubber Components," Proceedings of the KSAE, pp. 951-955, 2010.

2. Mooney, M., “A Theory of Large Elastic Deformation,” Journal of Applied Physics, Vol. 11, No. 9, pp. 582-591, 1940.
3. Jang, W. J., Lee, J., Woo, C. S., Kim, B. K. and Lee, S. B., “An Experimental Study and Finite Element Analysis of Weatherstrip,” Int. J. Precis. Eng. Manuf., Vol. 12, No. 1, pp. 97-104, 2011.
4. Boulanger, P. and Hayes, M., “Finite Amplitude Waves in Mooney-Rivlin and Hadamard Materials, in Topics in Finite Elasticity,” International Center for Mechanical Sciences, 2001.
5. Selvadurai, A. P. S., “Deflections of a Rubber Membrane,” Journal of the Mechanics and Physics of Solids, Vol. 54, No. 6, pp. 1093-1119, 2006.
6. Menderes, H. and Konter, A. W. A., “Advanced FE Analysis of Elastomeric Automobile Components under Realistic Loading Conditions,” Proceedings of the First European Conference on Constitutive Models for Rubber, pp. 3-12, 1999.
7. Ogden, R. W., “Non-Linear Elastic Deformations,” Dover, 1998.
8. Rivlin, R. S., “Large Elastic Deformations Isotropic Materials. IV. Further Developments of the General Theory,” Philosophical Transactions of the Royal Society A, Vol. 241, No. 835, pp. 379-397, 1948.
9. Ogden, R. W., “Large Deformation Isotropic Elasticity - On the Correlation of Theory and Experiment for Incompressible Rubberlike Solids,” Proceedings of the Royal Society A, Vol. 326, No. 1567, pp. 565-584, 1972.
10. MSC Software Co., “Nonlinear Finite Element Analysis of Elastomers,” 2001.