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# Evaluation of Stress Life of Aluminum Alloy Using Reliability Based Approach

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*In the present investigation the characteristic fatigue life prediction of A 356.2 T6 aluminum alloy has been statistically analyzed by log normal distribution. Fatigue tests were conducted on aluminum alloy specimen on rotary bending fatigue testing machine at six different stress levels. A step wise procedure is outlined to determine the number of specimen required at predetermined stress amplitudes. Details of generation of S-N curve for A 356.2 T6 aluminum alloy using a regression analysis is delineated. ANOVA is performed in order to check the significance of regression equation. The adequate sample size required for evaluating the average fatigue life of A 356.2 T6 with an acceptable error at 50 % probability and 90 % confidence level using log normal distribution is established from this study. The experimental results are presented in the form of R-S-N curves, which are helpful for designers.*

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**NOMENCLATURE**

s = standard deviation of log fatigue life
$\bar{x}$ = sample mean of log fatigue life
t = student '*t*' value
n = sample size
u = normal deviate
N = Average Fatigue Log life in Cycles

## 1. Introduction

An efficient structural element must have three primary attributes; namely, the ability to perform its intended function, adequate service life, and the capability of being produced at reasonable cost. Most of the structures such as nuclear containments, reactor vessels, aerospace structures, ship hulls, automotive components and offshore structures are required to operate under stringent controllable operating conditions. The environment may also be variable, regardless of the operating regime. During the service, the above structures, in general are subjected to fatigue loading.

In recent years, there is an extensive use of aluminum alloy materials for structural applications. One of the main applications of aluminum alloys is in automobile industry, especially for the manufacturing of wheels and many other light weight components. The wheels are one of the most critical parts of automobiles, which must perform their intended function to human safety. During the service, the wheels are subjected to either constant amplitude fatigue load or variable amplitude fatigue load.

In the present scenario, the interest to researchers is to understand the fatigue fracture behavior of these alloys under fatigue loading. Statistical evaluations[1] are important because of different distributions of the test results in aluminum samples. For safe and reliable applications of the materials in industry, their fatigue data must be known well. The statistical properties used in general, are related to distribution in mean strength. Log normal distribution[1] is a widely used statistical model than other distributions for evaluation of fatigue data in terms of important variables, endurance life and strength. Hence, log normal distribution is used for estimation of fatigue life of components used in aerospace, electronics and automotive industries.

Recent advances in Weibull theory have also created numerous specialized Weibull applications. Modern computing technology has made many of these techniques accessible across the engineering spectrum. A detail review of Weibull distribution models including Monte Carlo method is available in the ref.[2]

Belmonte et al.[3] proposed a Weibull based methodology for assessing the condition of pipes based on strength characteristics obtained from small samples. Khandaker et al.[4] applied a modified Weibull failure theory to biomaterial specimen under thermal loading.

Ramamurthyraju et al.[14,18] generated S-N curve for aluminum alloy A356.2-T6 and estimated fatigue life using two parameter Weibull distribution under radial fatigue load. Safety factor was suggested for reliable fatigue life estimation by conducting a parametric finite element studies. Zhao et al.[15] carried out a statistical investigation of 23 groups of fatigue life data on Q235 steel-welded joints in terms of linear regression analysis and observed that the three-parameter Weibull distribution may give misleading results in fatigue reliability analysis because the shape parameter is often less than unity. Schijve[16] made a comparison between three statistical distribution functions, namely, (i) the log (N)-normal distribution function, (ii) the 3-parameter Weibull distribution function and (iii) the log ($N-N_0$)-normal distribution function. It was observed that second and third functions gave a good data fit of the results of 30 similar tests with a skew distribution, but it still has to be recognized that the distribution function is actually unknown.

Many researchers[5-9,26] developed graphical and analytical methods to evaluate the fatigue life or strength and S-N curve from a limited amount of data. Most of the analytical methods are generally based on either normal or log normal distribution. As the fatigue test is time consuming and costly, setting of minimum sample size required to extract the statistical information is of great importance. Gope and others[10-13] presented a methodology for determination of sample size to estimate the fatigue life, confidence level and maximum acceptable error.

Ravi et al.[20] used Analysis Of Variance (ANOVA) method to predict the fatigue life of High Strength Low Alloy (HSLA) steel welds using regression analysis. Balasubramanian et al.[21] developed a mathematical model to predict the fatigue life of Shielded Metal Arc Welded (SMAW) cruciform joints failing from toe region. ANOVA technique was applied to find out the significant factors. Mahagaonkar et al.[22] employed Design of Experiment (DOE) technique in carrying out test, using an air blast type shot peening machine. An ANOVA was carried out to identify significant peening parameters.

Reliability is defined as the probability of a device performing its purpose for the period intended under the given operating conditions. The starting point in reliability analysis is the evaluation or estimation of the reliability of a device or a component. This is generally done from the available failure data of the component. Albeit there are many objectives for conductive life test, the main objectives are to study the exact behavior of the component or device under normal working environment and to generate data to evaluate life. The important aspect to be considered while performing life test is sample size, which is defined as the number of test specimen required for the test. It is very important to ensure that we do the test with a statistically significant sample size. The required minimum sample size can be calculated using a well established statistical distributions for a given limit of acceptable error.[24]

From the above it can be summarized that the studies on determination of sample size for estimation of the fatigue life of aluminum alloy A356.2-T6 using log normal distribution are limited.[14,18] Since this alloy has been extensively used in automobile wheel applications and undergo fatigue loading, it is necessary to determine the sample size. The present work focuses on sample size determination of Aluminum alloy A356.2-T6 to evaluate the characteristic fatigue life at the desired probability and confidence level. Details of generation of S-N curve and R-S-N curves for aluminum alloy A356.2-T6 have been presented.

## 2. Experimental Set up

To predict the fatigue properties of aluminum alloy A356.2-T6, at actual manufacturing conditions, a rotary bending fatigue test, according to BIS: 5075-1985,[17] was conducted at different predetermined stress amplitudes on various machined specimen taken from the spokes of alloy wheels. According to machine specifications, when 1, 2, 3, 4, 5, and 6 kg loads are applied the stresses induced in the specimen are 88, 117, 146, 176, 205 and 234 MPa, respectively. Generally a low pressure die casting process followed by T6 heat treatment process is used for manufacturing of aluminum car wheels for passenger vehicles. The molten aluminum kept in a gas tight heat insulated container flows under a mild pressure of approximately 70-100 kPa via a standpipe to escape through vent-holes and enters the die without turbulence. After solidification of the material in the die, the container is depressurized and the molten contents of the standpipe flow back into the container. Finally the wheel is then machined according to

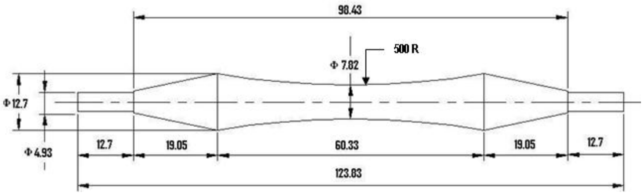


Fig. 1 Rotating bending fatigue test specimen as per BIS 5075-1985 (All dimensions are in mm)

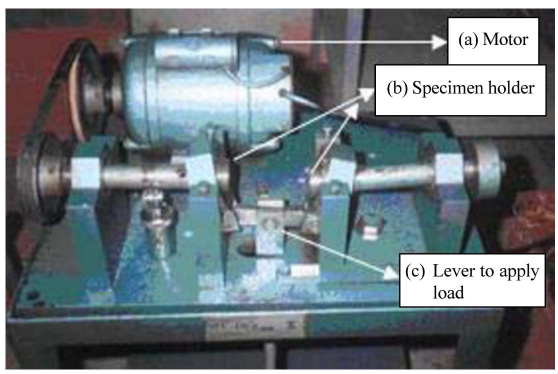


Fig. 2 Typical test set-up for conducting rotating bending fatigue test

the specifications of the sample used for testing as shown in Fig. 1. The machined specimen is then tested on the rotary bending fatigue test set up as shown in Fig. 2. The chemical composition of A356.2-T6 is shown in Table 1.

The following is monotonic material data for the specimens taken from finished wheels.

Ultimate Tensile Strength (Su): 250 MPa
Yield Strength (Sy): 230 MPa
Elongation (e): 5%
Hardness (HB): 90

The results obtained from the rotary bending fatigue test are plotted as shown in Fig. 3.

## 3. Method for determination of minimum sample size

From the literature[5-8] it was observed that graphical and analytical methods were used to evaluate the fatigue life with limited experimental observations. Further, it was noted that some of these methods can not be used for fatigue life prediction or strength at higher levels of probability. It is known that reliability and functionality are two of the most important requirements of engineering structures and components. It is an important requirement to find out the minimum sample size as the fatigue testing is time consuming and costly. In general, S-N plots are based on limited test data ranging from 6 to 10 specimens. It is mainly due to the availability of specimens, test time, and also on the actual number needed to plot S-N curve. The aspects of testing are largely a matter of subjective choice and accumulated experience.[19] A stepwise procedure is outlined to determine the number of specimens required at predetermined stress amplitude to estimate the average fatigue life within an acceptable error at 50% probability and various confidence levels. Fig. 4 shows the flow chart followed for determination of minimum sample size for A356.2-T6 aluminum alloy.

Initially tests were conducted on minimum of three samples.[5] Corresponding to 50% probability of failure the equation (1)

Table 1 Chemical composition of A356.2-T6 aluminum alloy in wt. %

| Si | Fe | Cu | Mn | Mg | Zn | Ti |
|---|---|---|---|---|---|---|
| 6.5-7.5 | 0.15 | 0.03 | 0.10 | 0.3-0.45 | 0.07 | 0.1-0.18 |

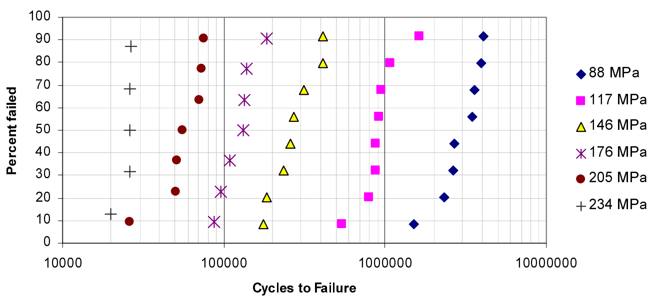


Fig. 3 Scattered points of fatigue life at different stress levels

reduces to equation (2). Likewise tests were further conducted at each stress levels until the error estimation converges to an acceptable level. If the fatigue life, N, follows normal distribution then x = Log (N) will follow log normal distribution. Then the percentage of error can be estimated using equation (2).[10]

$$R_N = \frac{t.s\sqrt{\left(\frac{1}{n} + u^2(\psi^2 - 1)\right)}}{\bar{x} + u \cdot \psi \cdot s} \tag{1}$$

Where,

$$\psi = \sqrt{\frac{n-1}{2}}\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$$

=Correction factor for sample standard deviation.

$$R_N = t \cdot \left(\frac{s}{\bar{x}}\right)\sqrt{\frac{1}{n}} \tag{2}$$

## 4. Results and Discussions

### 4.1 Determination of minimum sample size

The error is estimated using equation (2) at various confidence levels (90%, 95% and 99%) for various predetermined stress amplitudes 88, 117, 146, 176, 205 and 234 MPa. For the evaluation of fatigue life of A356.2-T6 aluminum alloy, an acceptable error of

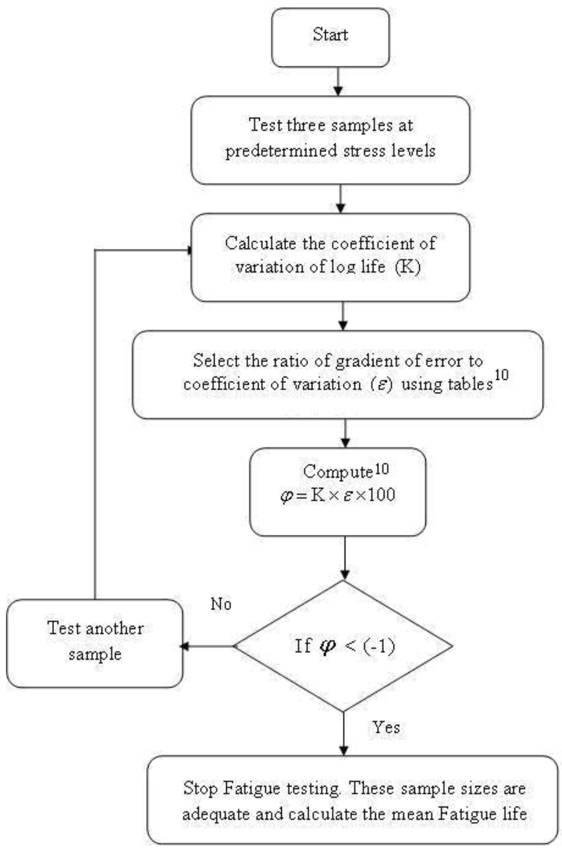


Fig. 4 Flow chart for determination of minimum sample size

5% and 50% probability are considered. The variation of error with sample size for 88 MPa stress level is plotted at various confidence levels and is shown in Fig. 5. It is observed that at 90% confidence level the error is with in an acceptable value, which can be seen in Fig. 5. Similar procedure is adopted for the remaining stress levels and it is observed that 90% confidence level is most reliable in evaluating the fatigue life.

The gradient of error[5] ($\varphi$) at 50% probability and 90% confidence level with an acceptable error for the fatigue life data at various stress levels is calculated and it is observed that the gradient of error ($\varphi$) for 88 MPa stress level is greater than –1.0 at n = 5, which is the minimum required sample size to be tested for evaluating the fatigue life of aluminum alloy. Similar statistical procedure was adopted for various predetermined stress levels to determine the minimum sample size. The variation of gradient of error with sample size at 50% probability and 90% confidence level for different stress levels is plotted in Fig. 6 and it is observed that gradient of error is independent of sample size when its value is greater than -1.0.

### 4.2 Generation of S-N Curve

From the above study, it is observed that minimum sample size is obtained as n = 5, 4, 5, 5, 4 and 4 at 88, 117, 146, 175, 205 and 234 MPa, respectively which are used in evaluating the fatigue life of aluminum alloy. The fatigue life of aluminum alloy is determined by taking the average life values of minimum number of samples using log normal distribution. When the error values obtained in log normal distribution are compared with those obtained in Weibull distribution,[18] it was found that the error values are less in log normal distribution. S-N curves generated using log normal distribution are superior compared to those obtained using Weibull distribution. Similar trend was observed in reference.[11]

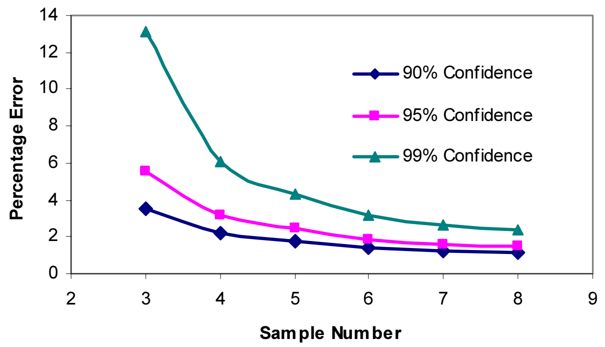


Fig. 5 Variation of error with sample size at 50% probability for 88 MPa stress level

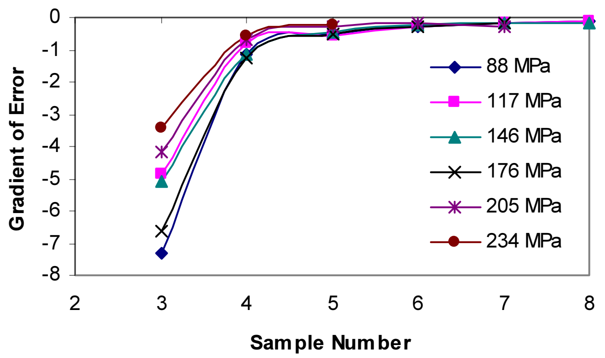


Fig. 6 Variation of gradient of error with sample size at 90% confidence level

Linear and non-linear (second degree) regression analyses were performed. The R-square values obtained in linear and non-linear regression analyses are 0.986 and 0.991, respectively. Since the accuracy is more in non-linear analysis than that in linear, polynomial of second order was considered. S-N Curve is generated by taking the average values of fatigue log life on x-axis and stress levels on y-axis as shown in Fig. 7 and a curve fitting is performed by taking a second degree polynomial using regression analysis.

The mathematical relationship between stress levels and fatigue life is established and is given by equation 3. The significance of the quadratic polynomial is determined by carrying out an ANOVA for the equation 3. The R Square is 0.991, which shows that 99.1% of the observed variability, which meant that the correlation coefficient between the stress level and average fatigue value based on the regression model is high. The analysis indicated that the regression model is highly significant because the P value is zero ($P < 0.05$). The results of ANOVA are tabulated in Table 2.

$$\text{Stress} = 10.68\,N^2 - 189.0\,N + 861.2 \quad (3)$$

Where,

N = Average Fatigue Log life in Cycles

### 4.3 Generation of reliability stress life curves (R-S-N Curves)

Generally the S-N curves specified by various authors in literature refer to the 50% probability of failure unless other wise specified. Because of the scatter of fatigue life data at any given stress level, it must be recognized that there is not only one S-N curve for a given material, but a family of S-N curves with probability of failure as the parameter. These curves are called as R-

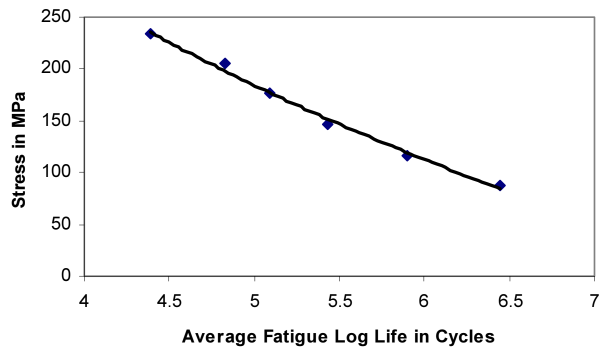


Fig. 7 S-N Curve for Aluminum A356.2-T6 alloy

Table 2 Results of ANOVA for regression model

| Standard Deviation = 5.16261<br>R-Sq = 99.5% R-Sq (adj) = 99.1% | | | | | |
|---|---|---|---|---|---|
| Source | Degree of freedom | Sum of Squares | Mean Square | F-ratio | P value |
| Regression | 2 | 14900.0 | 7450.02 | 279.52 | 0.000 |
| Error | 3 | 80.0 | 26.65 | | |
| Total | 5 | 14980.0 | | | |

S-N curves or curves of constant probability of failure on stress versus life plot. As fatigue lives usually correspond to either log-normal or Weibull distribution, linear transformations are performed to make least square method amenable to these distributions. The concept of median rank is used to determine the probability of failure for the test data. The test data is first arranged in ascending order of lives and median ranks taken from standard statistical tables[19] are assigned in ascending order as given in table 3. Regression analysis was performed using linear transformations and relationship between probability of failure and fatigue lives were determined. The R-S-N curves are generated using these relationships.

In case of log normal distribution for a set of data the relationship between a dependent and independent variable can be expressed as

Table 3 Number of cycles to failure corresponding median ranks

| S. No. | Median Ranks | Stress level 1 88 MPa | Stress level 2 117 MPa | Stress level 3 146 MPa | Stress level 4 176 MPa | Stress level 5 205 MPa | Stress level 6 234 MPa |
|---|---|---|---|---|---|---|---|
| 1 | 0.1294 | 1518389 | 537516 | 177300 | 87413 | 25974 | 19932 |
| 2 | 0.3147 | 2317776 | 797731 | 183567 | 96453 | 50326 | 25974 |
| 3 | 0.5 | 2627425 | 877734 | 236452 | 108392 | 50967 | 26212 |
| 4 | 0.6853 | 2704850 | 878631 | 259141 | 132868 | 55365 | 26212 |
| 5 | 0.8706 | 3493790 | 917730 | 269709 | 134033 | 70386 | 26402 |

Table 4 Linear transformation of percent failure and fatigue lives at stress level 1

| S.No. | Percent failed (y) | Y = ln(y) | Fatigue life (x) | X = ln(x) |
|---|---|---|---|---|
| 1 | 0.1294 | -2.04485 | 1518389 | 14.2331 |
| 2 | 0.3147 | -1.15614 | 2317776 | 14.6561 |
| 3 | 0.5 | -0.69315 | 2627425 | 14.7815 |
| 4 | 0.6853 | -0.3779 | 2704850 | 14.8105 |
| 5 | 0.8706 | -0.13857 | 3493790 | 15.0665 |

Table 5 Probability of failure fatigue life at stress level 1

| S.No. | Percent failed | Y = ln(y) | X, predicted | Antilog(X) = x, life in cycles | Reliability |
|---|---|---|---|---|---|
| 1 | 0.01 | -4.605 | 13.16490 | 521728.4 | 0.99 |
| 2 | 0.1 | -2.302 | 14.12022 | 1356243 | 0.9 |
| 3 | 0.5 | -0.693 | 14.78797 | 2644449 | 0.5 |
| 4 | 0.75 | -0.287 | 14.95619 | 3128919 | 0.25 |
| 5 | 0.99 | -0.010 | 15.07138 | 3510909 | 0.01 |

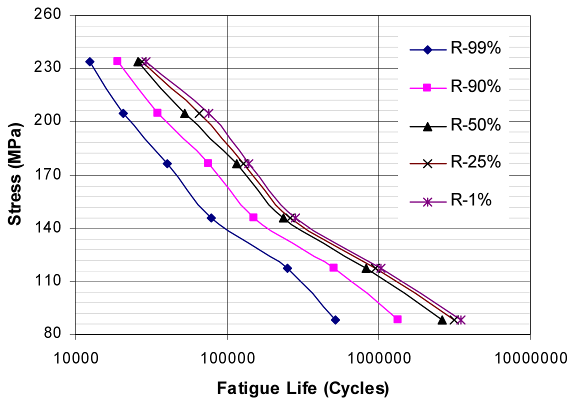


Fig. 8 R-S-N Curves for various reliabilities for A356.2-T6 Aluminum alloy

$$Y = aX + b \tag{4}$$

Where Y and X linearsied variables of y and x respectively, such that

$$Y = \ln\ y \tag{5}$$

$$X = \ln\ x. \tag{6}$$

y represent the percentage failure and x represent the fatigue life.

Table 4 shows the values of x and y after linear transformation at all six levels of testing. The constants in equation Y = aX + b were determined as follows:

Level 1: $Y_1 = a_1X_1 + b_1$; $a_1 = 2.410$, $b_1 = -36.336$

Level 2: $Y_2 = a_2X_2 + b_2$; $a_2 = 3.261$, $b_2 = -45.157$

Level 3: $Y_3 = a_3X_3 + b_3$; $a_3 = 3.593$, $b_3 = -45.140$

Level 4: $Y_4 = a_4X_4 + b_4$; $a_4 = 3.739$, $b_4 = -44.295$

Level 5: $Y_5 = a_5X_5 + b_5$; $a_5 = 3.855$, $b_5 = -42.891$

Level 6: $Y_6 = a_6X_6 + b_6$; $a_6 = 5.437$, $b_6 = -55.903$

The above equations relate the probability of failure (percent failed) to fatigue life. Based on these equations and taking anti logarithms of X and Y, fatigue lives corresponding to different probabilities of failures at each stress level have been estimated as given in Table 5. Owing to the scatter between individual results of fatigue tests, the stress-life relation cannot be expressed by a single curve but must be considered as a family of curves, each representing a definite probability of failure.[25] The Figure 8 represents the reliability stress life curves at various predetermined stress levels of the test specimen between 99% and 1% reliabilities.

## 5. Conclusions

The characteristic fatigue life of aluminum alloy A356.2-T6 is estimated by a log normal distribution model. Fatigue life for all the samples is obtained by conducting rotary bending fatigue test. Initially a minimum of three tests are conducted and calculated the error in estimation using a stepwise statistical procedure. Likewise tests are further conducted at each of stress levels until the error of estimation converges to an acceptable level and it is observed that percentage of error is increasing with increase of percent of confidence levels for a particular stress level. A quantitative method is presented for determination of sample size for evaluating the fatigue life of A356.2-T6 aluminum alloy with 50% probability, at various confidence levels and with a maximum acceptable error. Details of generation of S-N curve for aluminum alloy A356.2-T6 is presented. Reliability stress life curves presented give more valuable information of the fatigue behavior of aluminum alloy to the designer. The fatigue life corresponding to a required reliability at a particular stress level can be read off by reliability stress life curves.

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