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Estimation and evaluation of GPS geoid heights using an artificial neural network model

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Abstract The aim of the present study is the employment of the artificial neural network (ANN) model in the estimation and evaluation of geoid heights. For that reason, a number of control points with known orthometric heights in the region of North Greece were measured by GPS and used to test the presented algorithms. The derived ANN geoid heights are compared to those produced by a well-known conventional method through a combined gravimetric geoid. In order to evaluate the computed heights, numerical tests were carried out for points distributed inside and at the borders of the study area. The obtained results show that the ANN model is a competitive approach with certain advantages.

Keywords Artificial neural network · Geoid height ·
GPS leveling · Gravimetric geoid

Introduction

One of the major tasks in most Global Positioning System (GPS) projects is the transformation of geometric GPS heights (h) into orthometric heights (H). The difference between them is called geoid undulation or geoid height (N) given by the simple relationship $N=h-H$. For many geodetic and surveying applications, geoid heights must be determined with an accuracy of a few centimeters. In general, the estimation of GPS geoid heights, where h comes from GPS

and H from geometric and/or trigonometric leveling on a number of control points, has been carried out using polynomial surface fitting and least squares collocation (e.g., Andritsanos et al. 1999; Andritsanos et al. 2004). Moreover, there are choices that have been successfully used as the least squares collocation (LSC) with gravimetric data (e.g., Arabelos 1980; Arabelos and Tscherning 1998; Andritsanos et al. 2000) and the combined solutions (e.g., Tziavos 1984; Fotiou et al. 2008). It must be pointed out that the LSC approach in geodesy (e.g., Moritz 1980; Dermanis 1987) is the most favorite in geoidal studies for precise geoid determination. Some recently developed methods like the second-generation wavelets (Soltanpour et al. 2006) and the artificial neural network (ANN; Hu et al. 2004; Kavzoglu and Saka 2005; Stopar et al. 2006; Sheng-Lin 2007) have been effactually applied providing reliable results.

The ANN, or simply neural network (NN), represents a class of distinct mathematical models originally motivated by the information processing in biological neural networks. In general, NN models can be characterized as a forecasting technique. The NN's implementation has the skill to learn from experience via sample data and apply this taught in solving practical problems underlying by very difficult mathematical models. The same situation holds for the geoid determination process because in most of the cases, its shape is very complex and the approximation by simple mathematical models is almost rare.

The aim of the present study is to develop and evaluate a neural methodology as ANN's ability to produce accurate results is similar or even better than what other methods have. The primary inputs in the NN process are the measured GPS geometric heights and the orthometric heights for the control points. The basic characteristics of the NN computational model, the test data, and the

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efficiency of the applied algorithm are described and discussed in the following.
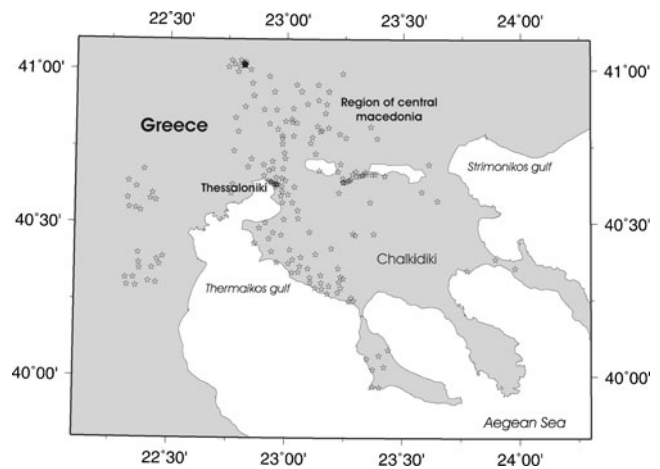
GPS leveling data

A total number of 234 points with known orthometric heights (control points, benchmarks) belonging to the Greek national geodetic datum and/or to the vertical datum were measured by GPS in the study area of about 100×90 km. It should be underlined that the accuracy of these heights is not homogenous and varies from few millimeters to few centimeters as heights from the vertical datum are significantly accurate than heights from the horizontal datum derived by trigonometric leveling. Moreover, there are some inconsistencies in the vertical datum due to the long period passed since its last measurement.

Most of the points are concrete pillars, and their heights have been determined through geometric/spirit leveling and/or trigonometric techniques carried out in previous years by the Hellenic Military Geographical Service (HMGS). On the other hand, GPS measurements were made during the last years using dual-frequency GPS receivers, mainly the Leica 300, 500, and 1200 systems. GPS data were collected in static positioning mode, the occupation time was selected from one to several hours with a 15-s observation rate and a $15°$ cut of angle. GPS data were processed by the Bernese software v5.0 (Dach et al. 2007) using the IGS precise ephemerides. All baseline ambiguities were successfully fixed to their integer values applying the sigma strategy. AUT1 GPS station, which is located at Thessaloniki city and participates in the EUREF permanent network (Bruyninx 2004), was selected as the fixed point in order to refer all station coordinates to the reference frame ITRF2005 (Fotiou and Pikridas 2006). Concerning the accuracy of the GPS estimated position, it is at the order of 1–2 cm for the horizontal position and up to 3 cm for the vertical position.

Figure 1 depicts the distribution of the observed GPS points, noting that the distribution is not sufficiently uniform (clusters of gathered points), and this situation needs improvement by adding more points. Nevertheless, it is a kind of challenge for the various techniques used in that area of the present study.

Geoid height computations

In the present paper, two different methods are applied for the geoid height computation. The first one is a combined classical gravimetric geoid estimation, and the second method follows an artificial neural network model that is evaluated compared to the other method.

Fig. 1 Distribution of the GPS leveling points

The gravimetric geoid

A combined classical gravimetric geoid estimation is derived from gravity data available in the $40 < \varphi < 41.1$, $22.1 < \lambda < 24.3$ coverage area. A high resolution 90×90 m GINA 2004 (Geographic Information Network of Alaska) digital terrain model in land and marine areas is also used. The gravity data include two and a half thousand point gravity anomalies given by the HMGS databank referred to the IGSN71 datum. The horizontal datum for the gravity points is the global ellipsoid of GRS80.

The geoid computation was based on the remove–restore technique. The contribution of the EGM08 geopotential model as well as the terrain effects contribution was subtracted from the original data. Both contributions were restored at the output procedure, and a final geoid solution was derived. A more detailed presentation of the specific solution is given in Andritsanos et al. 2004. A transformation procedure was applied based on the GPS/leveling data of 234 benchmarks in the study area. The used model is the four-parameter polynomial,

$$H = h_{GPS} - N_{grav} - b_0 - b_1 \cos \varphi \cos \lambda$$
$$- b_2 \cos \varphi \sin \lambda - b_3 \sin \varphi \qquad (1)$$

where H is the given orthometric height, h_{gps} is the geometric/ellipsoidal height derived from GPS measure-

Table 1 Statistics for the two applied geoid solutions

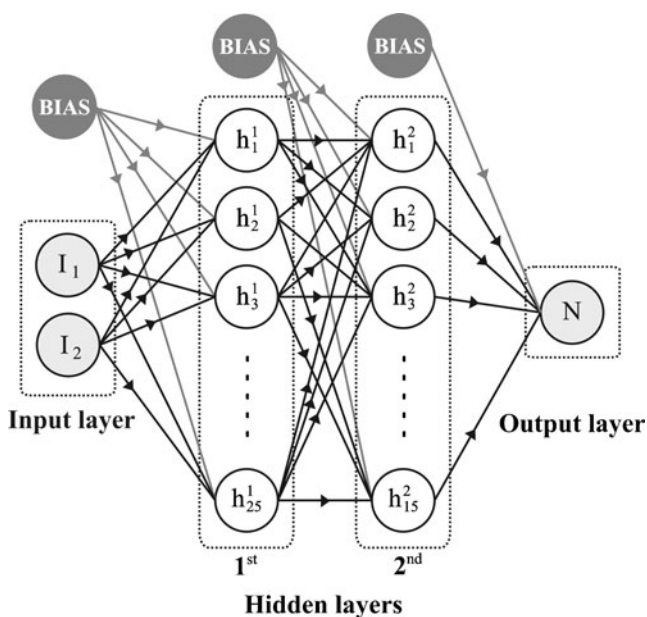| Geoid model	Residuals and RMSE (cm)		
	Min	Max	RMSE
Combined gravimetric	−10.0	10.89	9.6
Neural network	−13.9	13.48	8.4

Fig. 2 The architecture of the applied multi layer neural network

ments, N_{grav} is the predicted gravimetric geoid height, and b_n, $n=0$, 1, 2, 3, the parameters of the model to be estimated. Thirty-four GPS points showed large residuals and were excluded from the process mainly due to inconsistencies in the vertical datum between new and old traverse solutions and the less accurate trigonometric leveling for many of the measured points. The summarized basic statistical measures for the residual differences $[(h_{GPS}-H)-N_{grav}]$ resulted from the 200 "clean" benchmarks after the transformation are given in Table 1. In addition, a graphical representation of these differences is shown in Fig. 3.

The ANN model

In brief, an ANN model is a complex network scheme of a large number of processors which are called neurons or nodes and are connected to one another (Callan 1999). These characteristics are primary known from the biological neural

systems that enable human brain to learn through training. An array of neurons is named layer, and a network can be constituted by a number of layers. The first layer is an input layer, where input data (signals) are introduced. The last layer is an output layer, where the predicted results are obtained. The input and output layers are separated by one or more intermediate layers named the hidden layers. The nodes in adjacent layers are usually fully connected by acyclic edge arcs starting from the lower to the higher layer.

The performance of a neural network is critically dependent on the training data. Specifically, there are cases where the training procedure fails if the input data are not normalized. Usually, this happens due to numerical calculation problems. Therefore, in our study, the sample data (D_i) are transformed to input data I_i according to the general simple formula (linear scaling):

$$I_i = I_{min} + (I_{max} - I_{min}) \frac{D_i - D_{min}}{D_{max} - D_{min}}, \; i = 1, 2 \quad (2)$$

In our study, Eq. (2) is written as

$$I_1 = 2 \frac{x - x_{min}}{x_{max} - x_{min}} - 1, \; I_2 = 2 \frac{y - y_{min}}{y_{max} - y_{min}} - 1 \quad (3)$$

where I_{min} and I_{max} are the so-called network range usually set from −1 to +1. The terms I_1 and I_2 are the transformed input values obviously derived from the (x, y) plane coordinates of the control points. The output value will be considered here the geoid height of each point.

Another meaningful choice is the selection of the activation function. The contribution of this function is the estimation of the relationship between inputs and outputs of a node. In our algorithm, the most popular function, the bipolar sigmoid, was used (Zhang et al. 1998), given by

$$f(x) = \frac{2}{1 + e^{-x}} - 1 \quad (4)$$

The training process is, in general, an optimization problem. The applied algorithm minimizes the error that is

Fig. 3 Geoid height differences at the GPS/leveling points between each solution and GPS geoid
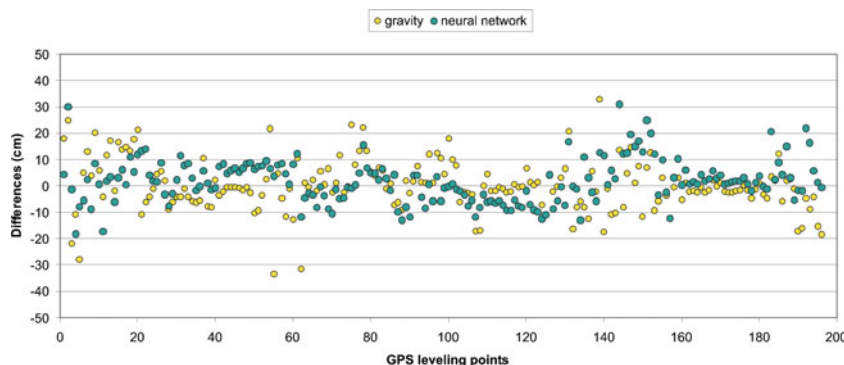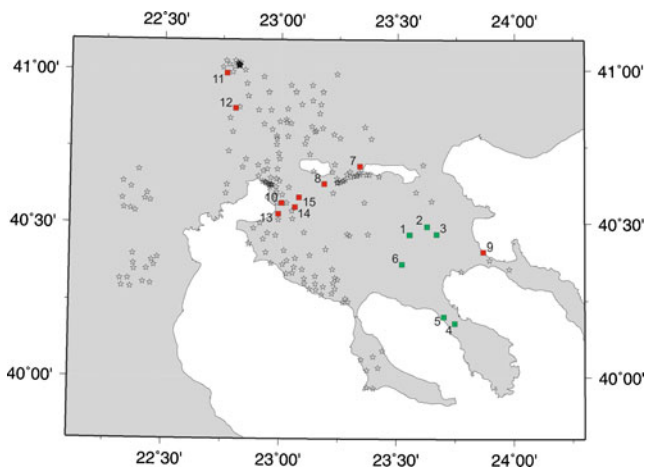
Fig. 4 GPS/leveling and test points distribution

the sum of the differences between the real output (given geoid heights) and the computed output values (estimated geoid heights).

During the training/learning process, the biases of the network (errors) are iteratively adjusted by means of the back propagation algorithm (Callan 1999) in order to minimize a suitable performance function which could be the root mean squared error (RMSE). Another noteworthy feature is the learning rate selection which generally starts from small values less than 1. In our case, many trials with rates of 0.08, 0.8, 0.2, and 0.1 were applied in order to find an optimum solution, i.e., a small RMSE value for the best rate selection. The best performance occurred with a rate of 0.1 after 12,000 iterations for the same used 200 points.

Our network consists of one input layer, two hidden layers, and one output layer. The choice for two hidden layers is found to be easier for our net learning process. The first hidden layer has 25 nodes and the second one, 15 nodes. The architecture of the created neural network is shown in Fig. 2.

The NN algorithm was applied by means of a proper software developed by the authors in C sharp programming language. The program was based on the open source software library "AForge Neural Net Library" given by Kirillov 2006. The results are presented in Table 1 and Fig. 3 where it is interesting to note that the two methods give almost the same results.
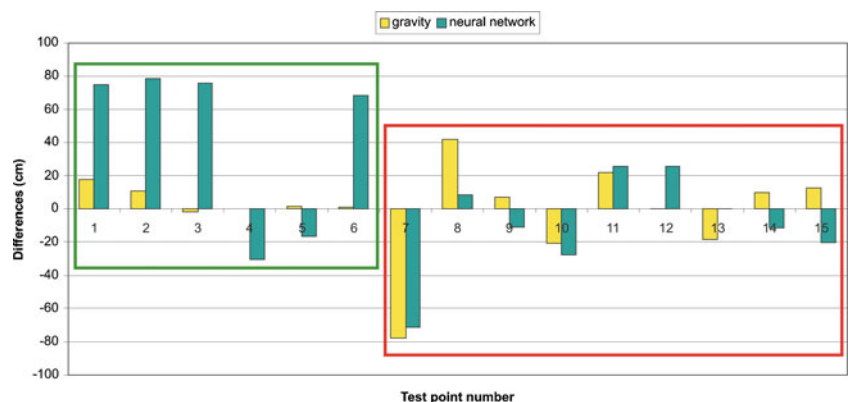
Evaluation of the ANN model

The developed NN algorithm for geoid heights computation is evaluated by an external reliability testing procedure. Predicted geoid heights are derived, via the NN algorithm, at 15 new points with known orthometric heights (H) from few meters to 1 km. Therefore, geoid heights ($N = h_{GPS} - H$) are compared to the predicted values. In addition, predicted geoid values were also calculated at the same points according to the combined gravimetric solution.

In Fig. 4, points marked with a square represent the 15 test points, while a star symbol indicates the 200 GPS/ benchmarks participating to the two geoid solutions. Moreover, for testing reasons, the group of the 15 points is further divided into a group of 6 points (1, 2, 3, 4, 5, 6) indicated by green color and located at the edge of the measured area and a group of 9 points (7, 8, 9, 10, 11, 12, 13, 14, 15) indicated by red color and located inside the area.

In Fig. 5, geoid height differences between each one of the two presented methods and the known values at the 15 test points are presented. From Fig. 5, it is obvious that for the first group of the six test points, the differences for the ANN method are large enough (almost 80 cm), while the combined gravimetric solution gives small differences. The reason for large differences and therefore the failure of the ANN model may be asked in the location of the six points, i.e., the six points are located somehow at the edge or outside the measured area noting that there is not around sufficient information (measured GPS/benchmarks) for the desired functional relationship during the learning process. In contrast, for the second group of the nine test points, the differences are mostly small and of the same order for each

Fig. 5 Geoid height differences at 15 test points

one of the two methods, and therefore, the ANN proves to be a satisfactory technique.

Conclusions

In this study, the presented ANN model for GPS geoid height estimation proved to be satisfactory compared to a classical accurate method as it is the combined gravimetric geoid solution. The gravimetric GPS geoid results are based on rigorous methods, like the so-called remove–restore procedure, where a spherical harmonic model and residual terrain effects are subtracted and later added back.

The ANN approach does not require different data sources like gravity anomalies and digital terrain values, some of which may be considered as classified. The ANN technique uses only the information extracted from GPS leveling benchmarks, and as a flexible computational scheme, is advantageous in case no precise geoid solution exists in an area.

ANN technique can be used as an alternative method for geoid height computations. Also, one of the author's future goals, which may increase the ANN reliability, is the introduction of one more neuron, including point gravity anomalies, in the input layer.

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