



# Vertical vibration tests to study the effect of foundation geometry and embedment on the non-linear response of block foundations

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## Abstract

The nonlinear behavior of the block foundation soil oscillator system for surface and embedded foundations can significantly impact the foundation's response, and understanding these effects is crucial for designing stable and resilient structures. Therefore, the present study specifically and explicitly aims to determine the effect of geometrical parameters on nonlinear frequency amplitude responses of surface and embedded block foundations under rotating machine-induced vertical vibrations by experimental and theoretical investigations. To accomplish these objectives, block vibration tests under vertical dynamic loading were performed in the field on three different foundations of aspect ratio ( $L/B$ ) = 1, 1.25, and 1.5. The frequency-amplitude responses are measured for different excitation forces ( $W_e = 0.221, 0.868, 1.450, \text{ and } 1.944 \text{ Nm}$ ) under two static loads ( $W_s$ ) of 6.6 kN and 8.6 kN. Another aspect of the present study is estimating dynamic foundation-soil stiffness and damping with the frequency-dependent and independent impedance characteristics using various theories. A comparison has been made between the observed and theoretical results to understand the efficacy of the various approaches in predicting the nonlinear response of the block foundations. In addition, the effect of the eccentric moment, aspect ratio, static load, and depth of embedment on the frequency amplitude response of the soil-foundation system was also determined. Comparing the theoretical and experimental response curves, a variation of about 5 to 25% in the resonant frequency is observed for the parameters considered, whereas an overestimation of damping value is observed from the different theories. Therefore, in conclusion, the frequency-dependent theories are more applicable than elastic half-space theories for predicting the response of block foundations subjected to higher dynamic forces where the nonlinearity of the soil is predominant. Also, analyzing the measured and predicted responses and investigating the influence of these parameters, the study provides practical insights that can be applied to optimize the design of block foundations.

**Keywords** Block foundation · Vertical vibration · Aspect ratio · Embedment ratio · Soil-foundation

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## Introduction

Machines with different rotating or oscillating masses require foundation support to transfer static and dynamic loads to the subsoil. With the accelerating speed of industrial growth in the country, a considerable number of machine foundations are being built. The design and analysis of foundations subjected to dynamic loads are more complicated than those that support only static loads. In addition to static loads, the dynamic forces caused by the working of the machine must be considered in the design of a machine foundation. Machine foundations thus form an integral and expensive part of an industrial complex. However, with the recent advances in the design methodology, the design of machine foundations has gradually improved and been established for a group of machine foundations. The basic

concept in designing foundations subjected to machine-induced vibrations is to limit the vibrational amplitudes such that the machine operations are satisfactory and no adverse effects are produced on the machines, foundations, structures, and people working in the immediate vicinity. To produce efficient and economical designs, it has become imperative to know the design methodology, various aspects of the design analysis, and dynamic response of a dynamically loaded foundation.

The frequency amplitude response of dynamically loaded machine foundations supported on blocks, mats or rafts, frames, and piles has been reported in the literature for the last 50 years. However, in the last few decades, several analytical and experimental investigations have been carried out, and the vibration of soils and foundations has led to the development of fundamental methods for formulating soil-foundation systems. To obtain the dynamic amplitude response of a surface foundation purely by analytical means, Rayleigh (1885) and Lamb (1904) carried out the study of the propagation of waves through elastic solids. Lamb (1904) investigated the surface of the soil medium for a vertical concentrated harmonic force and gave an integral representation for the vertical and radial displacements. In the 1950s, several authors studied the footing response to dynamic loading for different vibration modes. Quinlan (1953) and Sung (1953) studied vertical vibration for rigid base distribution. Arnold et al. (1955) and Bycroft (1956) extended Reissner's and Sung's theory for other modes of vibration for weighted average displacement conditions. Aggarwal and Ablow (1967) have given integral solutions to a class of three-dimensional pulse propagation problems. Johnson et al. (1974) and Apsel (1979) used Green's function to solve Lamb's problem and later formulated the procedure for layered media. Among a few authors, a definitive solution for the dynamic response of layered media was reported by Kausel (1981). Veletsos and Verbic (1973) proposed a finite element approach to determine the frequency-dependent dynamic impedances, i.e., stiffness and damping of the shallow foundations resting on the surface of a homogeneous half-space deposit. Novak and Beredugo (1972a) proposed an approximate solution for the analysis of the vertical vibration of embedded foundations. The frequency-dependent stiffness and damping were calculated for a series of thin independent elastic layered soil systems overlying above the subsoil, assuming an elastic half-space.

Many investigators have reported experimental studies on the dynamic behavior of foundations resting on the surface and partially or fully embedded in the ground in the early 1900s. Most of these studies addressed the vertical mode of vibration for surface and embedded foundations (Bycroft 1959; Novak 1970; Kanai and Yoshizawa 1961; Barkan 1962; Awojobi 1964; Beredugo 1971; Gupta 1972; Stokoe 1972; Maccalden and Matthiesen 1973; Erden 1974; Varadhi

and Saxena 1980). Experimental studies on small and rigid model foundations on shaking table (Hadjian et al. 1975; Luo et al. 2005; Hamidzadeh and Grootenhuis 1981) or centrifuge (Morris 1979, 1981) were reported to examine the effect of foundation geometry, viz., size and shape, mass and moments of inertia, engineering, and dynamic soil parameters, depth of foundation embedment on the response of machine foundations and to simulate an elastic soil medium represented as half-space subjected to dynamic loading. The effect of layer thickness was analyzed experimentally by Sridharan et al. (1990), Baidya and Muralikrishna (2000, 2001), and Baidya and Mandal (2006). Later, Baidya, and Mandal (2006) conducted block vibration tests using a Lazan-type mechanical oscillator on two and three-layered soil systems. It was found that both the soil layers' position and thickness significantly influence the foundations' dynamic response under different modes of vibration. A parametric study on the geometrical effects, centrifugal acceleration, and moment of inertia of the block foundation system was also performed by Baidya and Mandal (2006) for the fundamental mode of rocking-sliding vibration.

Gazetas and Stokoe (1991) conducted field experiments to compute the dynamic amplitude response of the surface and embedded block foundations. Luco and Wong (1992) performed in-situ testing to determine various properties of dynamically loaded soils, such as shear-wave velocities, material damping ratios, and Poisson's ratios from forced vibration tests. It was observed that impedance functions depend on the soil's elastic and dissipative characteristics and the known geometry of the foundation. It was also found that the impedance functions are independent of their inertial properties or the location of the external forces acting on the foundation.

Jafarzadeh and Asadinik (2008) conducted physical model block vibration tests on circular, square, and rectangular foundations resting on a sand layer to investigate the dynamic behavior and impedance function of foundations for various shapes and embedment for footing. Prathap et al. (2010) experimentally analyzed the effect of saturation on the foundation-soil system, and it was observed that stiffness reduces due to saturation, and consequently, resonant frequency reduces, and maximum resonant amplitude increases. Fattah et al. (2016, 2017) experimentally investigated the effect of harmonic vertical vibration on the dynamic amplitude response of machine-loaded foundations placed in saturated soil. The model response was validated and then simulated for the prototype conditions by numerical modeling using the three-dimensional finite element method in the prototype. It was observed that increasing the embedment depth reduces the dynamic response up to a certain depth, beyond which the effect becomes less pronounced. As the soil strength increases, the effect of embedment depth in reducing dynamic response diminishes. They observed

a reduction of 46%, 37.5, and 40% for loose, medium, and dense sand, respectively, when increasing the embedment depth from 0.5 to 1 m. A reduction of 45%, 38%, and 31% was observed for loose, medium, and dense sands when the embedment increased from 1 to 1.5 m, respectively. Furthermore, when the embedment depth was increased from 1.5 to 2 m, the vertical displacements decreased by 42%, 36%, and 18% for loose, medium, and dense sand, respectively. Mbawala et al. (2017) experimentally evaluated the effect of embedment on the dynamic response of a foundation subjected to vertical vibration. It was observed from the field tests that the resonant amplitude decreased with increasing embedment and the reduction is more significant at low embedment depths. This indicates a significant reduction in amplitude can be observed even at small levels of embedment. However, the increase in damping ratio with increasing embedment signifies that even small levels of embedment can enhance the performance of a machine-foundation system.

Abdul Kareem et al. (2020) experimentally investigated the effect of different footing shapes on dry sand subjected to dynamic loading from a machine. A laboratory simulation was carried out using nine different models with varying frequencies and foundations (circular, square, and rectangular). It was observed that the shape of the footing significantly influenced the bearing capacity of the supporting soil under dynamic loading. Specifically, under the same conditions, the strain, amplitude displacement, and stress in the dry sand were significantly higher under circular footings (by approximately 41%, 17%, and 12%, respectively) compared to square and rectangular footings. Abdulrasool et al. (2021) experimentally simulated forced vibrations with harmonic loading at various operating frequencies based on serviceability limit state performance considerations rather than strength requirements. They investigated the influence of embedment depth and load frequency on model footings on soft clay soil. Also, the stresses and excess pore water pressures at different depths below the footing were measured. It was concluded that the maximum displacement increases with the higher operating frequencies, and there is also an increase in excess pore water pressure with higher dynamic loads. Das et al. (2021) carried out experimental and numerical studies on an isolated square shallow foundation subjected to vertical periodic machine vibrations. The parameters affecting the dynamic responses of the foundation were studied such as resonant amplitude, natural frequency of the soil, and damping ratio. Fattah et al. (2022) examined the behavior of sandy soils with different densities as well as the influence of a single impulsive load. The authors investigated the variation of damping with different parameters, including footing embedment, depth-to-diameter ratios, the diameter of the impact plate, applied energy, soil density (ranging

from loose to dense), and moisture content (dry or saturated sands). It was concluded that as the soil density increased from loose to dense, the damping ratio also increased by approximately 20–80%.

The broad view of the literature review shows an enlarged scenario of the developments made in analytical methods. The systematic experimental studies to examine the dynamic nonlinear response of foundation-soil systems subjected to machine-induced vibrations reported in the literature are still scanty. More experimental data is needed to check the efficacy and applicability of various theories concerning the effect of various influential parameters such as excitation intensity, aspect ratio, static load, and embedment depth on the dynamic nonlinear response of block foundations.

Therefore, an attempt has been made in the present study to provide a good number of experimental results to examine and counter the effect of dynamic excitation intensity, aspect ratio, static load, and the depth of foundation embedment on the nonlinear frequency amplitude response of block foundation soil oscillator system. The dynamic field tests were conducted on the block foundations of three aspect ratios ( $L/B = 1, 1.25, \text{ and } 1.5$ ) under four excitation moments. Dynamic tests were also conducted for three embedded conditions ( $h/H = 0, 0.5, \text{ and } 1$ ). Furthermore, this study includes the dynamic response behavior of foundations using the theory proposed by Reissner's frequency-dependent displacement functions, Lysmer's Analog and Veletsos and Wei (1971), Gazetas impedance method (Gazetas and Stokoe 1991) for surface foundations, and Novak and Beredugo (1972b), and Gazetas impedance method (Gazetas and Stokoe 1991) for embedded foundations, respectively. A detailed comparative study between measured and predicted response is carried out to investigate the effect of various influencing parameters, viz., excitation intensity, static weight, aspect ratio, and embedment ratio, on the nonlinear response of the block foundation.

By analyzing the measured and predicted responses and investigating the influence of various parameters, the study provides practical insights that can be applied to optimize the design of block foundations. This information is valuable for engineers and practitioners in the field of geotechnical and structural engineering.

In conclusion, the novelty of this study lies in its thorough and systematic investigation of the nonlinear frequency amplitude responses of surface and embedded block foundation soil oscillator systems under rotating machine-induced vertical vibrations. The consideration of various influencing parameters, comparison of theoretical approaches, and recognition of soil nonlinearity contribute to its significance and potential impact on the field of geotechnical engineering and foundation design. The findings from this study can aid engineers in designing more resilient and efficient foundations for various practical applications.

## Experimental investigations

A natural soil deposit of area  $6\text{ m} \times 6\text{ m}$  between Block II and Block III of the Indian Institute of Technology, Delhi, Hauz Khas, New Delhi, India, was selected for experimental investigations. The soil profile was established from both the in situ and field tests. The forced vibration tests were performed on the foundations constructed in the field under vertical dynamic load to measure the frequency versus translational amplitude responses of different embedded foundation conditions for different excitation intensities.

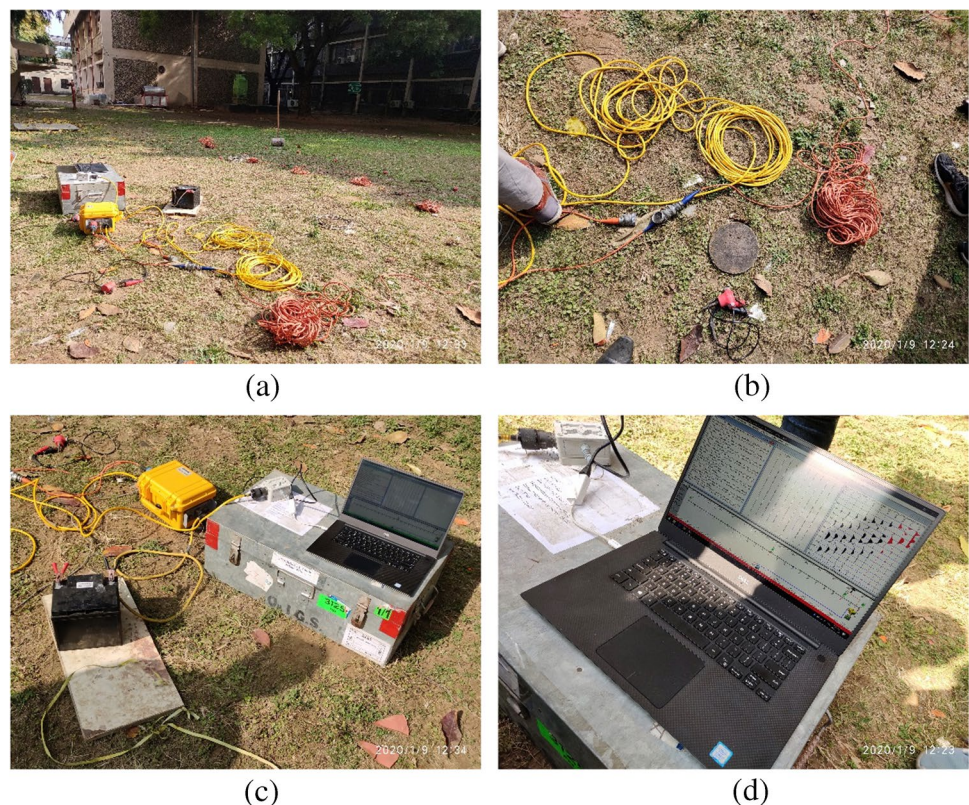
## Site characterization

Both in situ and laboratory tests were carried out to characterize the test site soil conditions. Two types of in situ dynamic tests were conducted, i.e., standard penetration tests (SPT) as per IS 2131(1981a) and multichannel analysis of surface waves (MASW). The standard penetration tests were conducted in three different boreholes at 1 m intervals up to 5 m. The disturbed and undisturbed soil samples collected from the boreholes during SPT for laboratory testing of the soil were used to obtain the engineering properties of the soil media. It consisted of bulk density, natural water content (IS 2720 Part 2, 1973), particle size distribution (IS 2720 Part 4, 1985a), and Atterberg limit (IS 2720 Part 5, 1985b)

tests. The vertical soil profile was determined based on the field and laboratory results, and it was found that soil comprises a homogenous soil layer of inorganic clayey silt with low plasticity (ML-CL), extended up to a depth of 5.25 m.

The seismic surface wave method known as multichannel analysis of surface wave tests (MASW) was carried out to estimate the ground stiffness one and two-dimensional shear wave velocity profiles in the field by spreading multiple geophones. MASW tests at three different locations in the field were conducted to obtain the shear wave velocity ( $V_s$ ) of the subsoil layer. The experimental setup for the MASW test is shown in Fig. 1. In total, seven geophones of natural frequency 4.5 Hz were used in the test, as shown in Fig. 1a, with a spacing of 3 m between two geophones, making the total test length 21 m. The offset distances, i.e., the source to the receiver, were placed at 3 m, and the distance between the source and the nearest geophone was fixed at 1.5 m. A 2D versus  $V_s$  map was constructed from the acquisition of multichannel seismic data using the seven channels signal enhancement seismograph. A wooden hammer of 11 kg weight manually impacted on the circular aluminum plate of  $165\text{ cm}^2$  area was used for creating the energy (Fig. 1b). Seismic waves were generated at an offset distance of 1.5 m from the nearest geophone). Fixed source-receiver configuration was adopted in the study; i.e., geophones were kept at an equal distance of 1.5 m from each other, and hence, the survey depth was 5.25 m (half of the survey length).

**Fig. 1** Experimental MASW test setup: **a** location of geophones; **b** aluminum plate; **c** digital recorder; **d** data processing using SeisImager



The recording was initialized using a trigger geophone. The generated Rayleigh wave data were digitally recorded and saved in the equipment at all shot points (Fig. 1c). The acquired surface wave data is transferred to the computer to obtain 1D and 2D MASW shear wave velocity profiles. It is processed using SeisImager/SW software through spectral inversion (Fig. 1d). The engineering soil properties obtained from the in situ dynamic tests and laboratory tests are summarized in Table 1. The vertical cross-section of the soil layers at different depths with SPT-N values and shear-wave velocities are shown in Fig. 2a and b, respectively.

### Field vibration testing setup and test procedure

The present research investigates the effects of various influencing parameters on the dynamic response of the block-foundation soil oscillator system. This requires studying different combinations of blocks with various parameters, such as the magnitude of dynamic force, embedment depth, and aspect ratio of foundations. In this study, a total of three blocks with aspect ( $L/B$ ) ratios of 1, 1.25, and 1.5 were considered by varying only the length of the blocks. The width and height of the block considered in the investigation are kept constant, i.e., 0.6 m and 0.5 m, respectively. So, the three blocks of size: (a) 0.6 m × 0.6 m × 0.5 m, (b) 0.75 m × 0.6 m × 0.5 m and, (c) 0.9 m × 0.6 m × 0.5 m were constructed in situ using M30 grade concrete. The reinforcements were placed in all three directions per the IS 2974 part 4 (2000). The reinforcement used in the block consists of 20 mm bars kept at 250 mm c/c spacing in both longitudinal and vertical directions and 16 mm bars in the lateral directions. The complete test cases considered in the analysis are summarized in Table 2, and the details are given below:

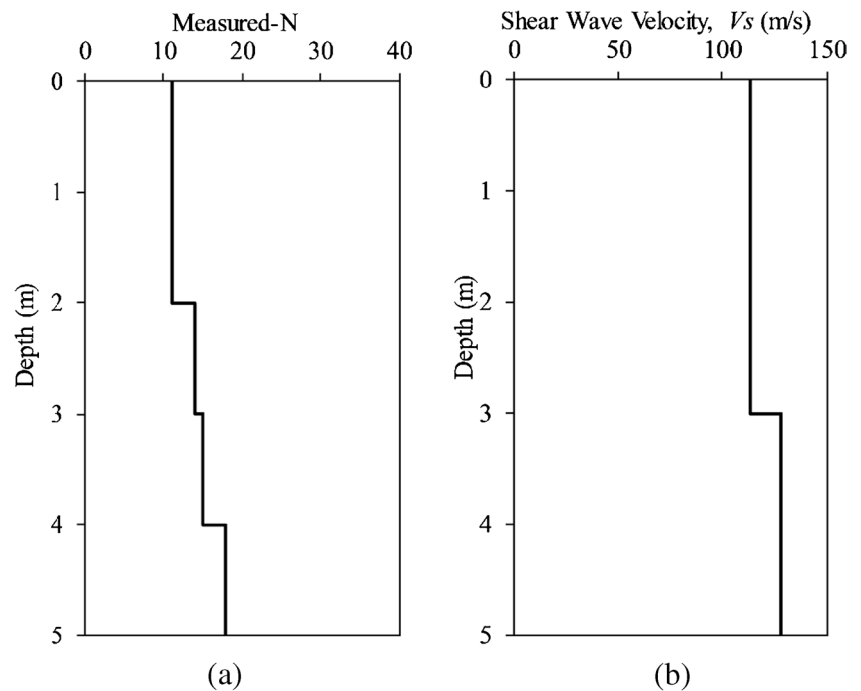
- Case 1:  $h/H=0$ ;  $L/B=1, 1.25, 1.5$  for  $W_s=6.6$  kN
- Case 2:  $h/H=0, 0.5, 1.0$ ;  $L/B=1.5$  for  $W_s=6.6$  kN
- Case 3:  $h/H=0$ ;  $L/B=1.5$  for  $W_s=6.6$  and 8.6 kN

The dynamic field tests were conducted on rigid concrete blocks under rotating machine-induced vertical vibrations for different excitation intensities to measure the translational amplitude responses of the block-foundation soil oscillator system. The tests were conducted by inducing vertical dynamic force on the block along the C.G. axis line with the help of a Lazan type mechanical oscillator. The mechanical oscillator induced purely vertical dynamic load on the block foundation as the vertical dynamic load was applied through the C.G. axis line, as shown in Fig. 3. The magnitude of the induced dynamic excitation force was controlled by adjusting the eccentricity ( $\theta$ ) of the rotating masses. The dynamic excitation force ( $P$ ) in Newton corresponding to the rotating mass eccentricity,  $\theta$  at any frequency can be expressed as

**Table 1** Laboratory and field test results for different depths of soil

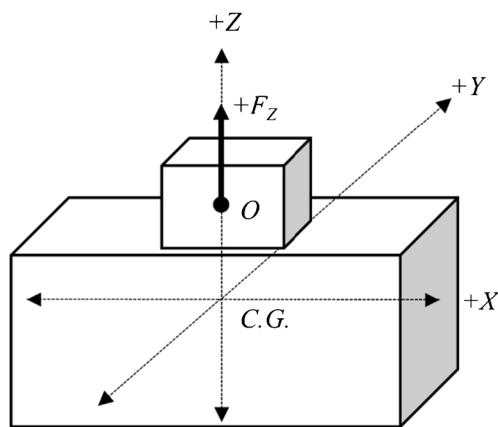
Depth (m)	Water content (%)	Wet unit weight (kN/m <sup>3</sup> )	Dry unit weight (kN/m <sup>3</sup> )	L.L. (%)	P.L. (%)	Specific gravity	Grain size distribution (%)	In situ SPT-N	$V_s$ (m/s)	Shear strength parameters
1	3.85	15.84	15.23	-	-	2.63	<b>Sand = 80, Silt = 18, Clay = 2</b>	11	113.36	-
2	4.00	15.98	15.34	-	-	-	-	14	113.36	-
3	4.17	16.97	16.27	24.2	20.2	-	-	15	113.36	$c=14$ kPa, $\phi=24.2^\circ$
4	4.23	17.13	17.08	-	-	-	-	18	128.21	-
5	4.54	17.25	17.20	-	-	-	-	18	128.21	-

**Fig. 2** SPT-N values and shear wave velocities of soil layers: **a** depth vs measured-*N* values; **b** depth vs shear wave velocity (*V<sub>s</sub>*)



**Table 2** Details of the block vibration tests under vertical vibration

Block	Block dimension ( <i>L</i> × <i>B</i> × <i>H</i> )	Aspect ratio ( <i>L/B</i> ) (Case 1)	Embedment ratio ( <i>h/H</i> ) (Case 2)	Static load (kN) (Case 3)	Eccentric moment (Nm)
<i>B<sub>1</sub></i>	0.60 m×0.60 m×0.50 m	1.00	0	6.6	0.221
<i>B<sub>2</sub></i>	0.75 m×0.60 m×0.50 m	1.25	0	6.6	0.868
<i>B<sub>3</sub></i>	0.90 m×0.60 m×0.50 m	1.50	0	6.6	1.450
			0.5	8.6	1.944
			1.0		



**Fig. 3** Unbalanced force induced by the mechanical oscillator (*O*) on the block foundation subjected to single dynamic load under vertical vibration

$$P = m_e \cdot e \cdot \omega^2 = \frac{2.59 \sin(\theta/2) \cdot \omega^2}{g} N \tag{1}$$

where *W* and *m<sub>e</sub>* are the weight and mass of eccentric rotating parts of the oscillator, respectively; *e* is the eccentric distance of the rotating masses; *g* is the acceleration due to gravity; and  $\omega$  is the circular frequency of vibration. The eccentric moment can be represented as

$$m_e \cdot e = \frac{W}{g} \cdot e = \frac{2.59 \sin(\theta/2)}{g} N \text{sec}^2 \tag{2}$$

The mechanical oscillator and the steel plates, each weighing 500 N (8 numbers) used to simulate the weight of the machine, were placed on the top of the block foundation sandwiched between two steel plates. The assembly was tightly fixed to a mild steel plate (0.8 m×0.8 m×0.025 m) welded on the top of the block foundation through vertical steel rods. The vibrating mass of the block foundation soil-oscillator system was adjusted using a test body attached

to the concrete block with a steel plate. The oscillator was then mounted over the steel plate connected to the concrete block. The concrete block with a static weight of  $W_s = 6.6$  kN (including the weight of mild steel plates and oscillator) was loaded for about 15 days for proper seating on the soil. The center of gravity of the foundation-oscillator assembly was maintained along the same vertical axis line to avoid any rotation. The complete setup was connected in such a way that it acts as a single unit. The oscillator was connected using a flexible shaft with a 10 HP DC motor, and the speed of the mechanical oscillator was increased gradually through the speed control unit. The oscillator induces a dynamic excitation force at the base in the vertical direction. The vibration measuring instrument consisted of two accelerometers connected with a data acquisition system. One accelerometer ( $A_1$ ) was mounted at the center axis on top of the loading assembly, and another accelerometer ( $A_2$ ) was mounted at the base of the loading system to check the vertical vibration at the top of the loading system to measure the vibration transferred at the foundation level. Two more accelerometers ( $A_3$  and  $A_4$ ) were placed horizontally to C.G of the block foundation soil-oscillator system to monitor any lateral motion of the foundation. The complete schematic diagram and experimental setup of the vertical vibration testing setup of the machine foundation system for vertical vibration of block foundation are shown in Fig. 4a and b, respectively.

The dynamic force was applied on the foundation through the oscillator at desired values of the eccentric moment ( $W.e$ ). In this investigation,  $W.e = 0.221, 0.868, 1.450,$  and  $1.944$  Nm were used. The frequency of the oscillator was measured using the sensor attached to the motor. A data acquisition system consisted of a Windows 10 laptop with Data Acquisition Software "CatmanEasy." The multichannel digital carrier frequency amplifier system connected with four accelerometers was used to observe the time-acceleration responses of excitation. The tests were carried out at the ground surface for all three blocks ( $L/B = 1, 1.25,$  and  $1.5$ ). The steady-state frequency displacement amplitude response of the block foundation soil-oscillator system under different frequencies and excitation intensities was measured for three cases (Case 1, Case 2, and Case 3).

The typical acceleration response for different frequencies is given in Fig. 5 for the surface foundation and Fig. 6 for the embedded block foundation. It can be observed from Figs. 5 and 6 that the acceleration response is purely sinusoidal for various frequencies. The amplitude of the acceleration varies with the frequency. The displacement amplitude at any frequency is calculated by dividing the acceleration value by the square of the frequency at which the response is recorded.

Furthermore, the plot of displacement amplitudes with respect to excitation frequency was obtained for

different dynamic excitation forces using the time acceleration responses. The responses were also measured from the accelerometers ( $A_3$  and  $A_4$ ) placed horizontally at C.G, and it was found that the magnitude of the accelerometer was too small. Therefore, it confirms that no significant lateral vibrations occurred during testing; hence, the vibration was only in the vertical direction.

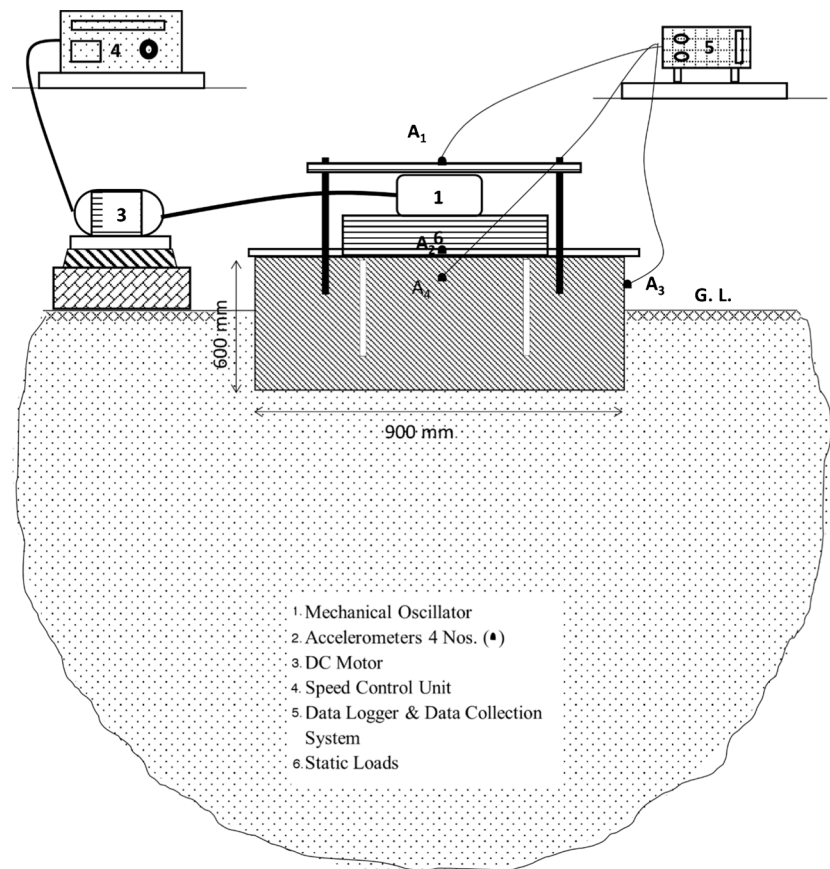
Finally, the maximum peak displacement  $A_z$  and resonant frequency,  $f_{n1}$ , was determined from the frequency amplitude responses for different excitation forces and embedment depths. It was found that both the accelerometers ( $A_1$  and  $A_2$ ) gave the same results, indicating that the vibration at the top of the oscillator is the same as the vibration at the footing level. Frequency versus vertical amplitude response of block foundations ( $L/B = 1.5$ ) for two embedment conditions  $h/H = 0$  and  $h/H = 1.0$  under different excitation intensities are presented in Fig. 7a and b, respectively. It is found that the response curves show one peak up to the range of frequency considered. It can also be observed from the response curves that the resonant frequencies decrease, and resonant translational amplitudes increase with increasing excitation forces. This decrease in resonant frequency and the disproportionality of the translational amplitudes correspond to the nonlinear behavior of the block foundation soil oscillator system.

The effects of static loads on the dynamic amplitude response are also studied and presented in Fig. 8, and it is observed that both the resonant frequency and translational amplitude decrease with the increasing static load. The variation of resonant frequency and amplitudes were observed for partial and full embedment of the foundations, as shown in Fig. 9, and it is found that as the depth of embedment ratio increases, resonant frequency increases, and the resonant amplitude decreases.

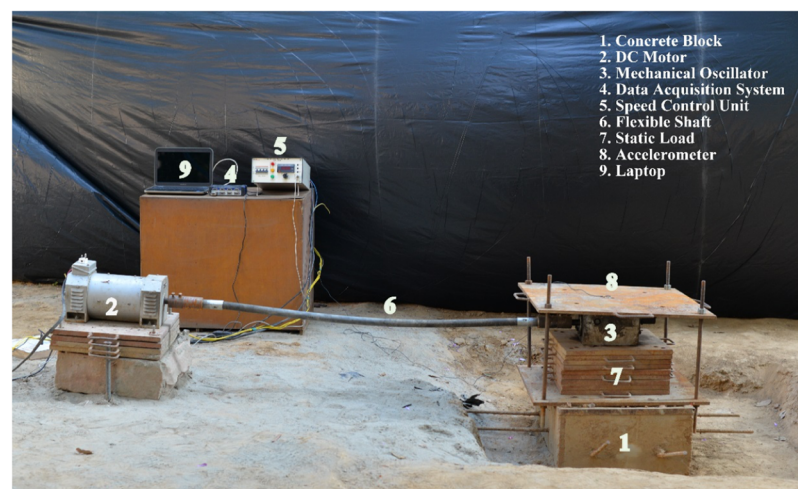
## Theoretical investigations

The dynamic response of the block foundation soil system is primarily affected by the geotechnical and geometrical properties, viz., surrounding soil and half-space parameters and foundation geometry (shape, size, depth of embedment). These parameters define the static and dynamic stiffness of the soil-foundation system and generate damping through energy radiation. The present study investigates the frequency-independent and frequency-dependent dynamic stiffness and damping functions of the block foundation soil-oscillator system subjected to vertical dynamic load on the surface and partially or fully embedded foundations using the available theories summarized in Table 3. The dynamic vertical stiffness and damping constants of the block foundation-soil oscillator system are also evaluated using the

**Fig. 4 a** Schematic representation of block foundation subjected to vertical vibration. **b** Complete experimental Test Set up of Block foundation subjected to vertical vibration at  $h/H=0.5$



(a)



(b)

DYNA 6 (El. Naggar et al. 2011) program for surface and embedded foundations.

### Block foundations resting on surface

Several theories are available for the foundations resting on the surface, subjected to a single vertical dynamic load. The

surface foundations subjected to a single vertical dynamic load are analyzed using the theories proposed by Reissner (1936) — VVS 1, Lysmer and Richart (1966) — VVS 2, and Impedance methods proposed by Gazetas and Stokoe (1991) — VVS 3. The half-space stiffness and damping constants are considered frequency-dependent and are obtained using the theories incorporated in the computer program DYNA



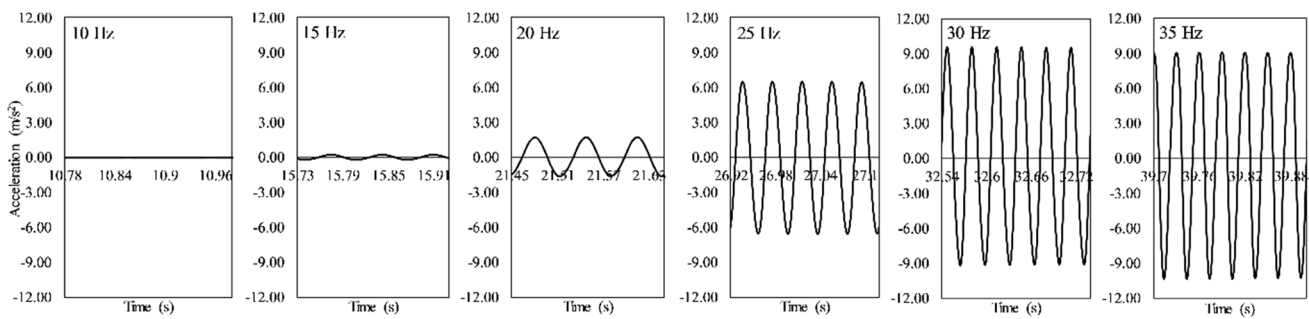


Fig. 5 Typical time vs acceleration response of the surface ( $h/H=0$ ) block foundation with aspect ratio  $L/B=1.5$

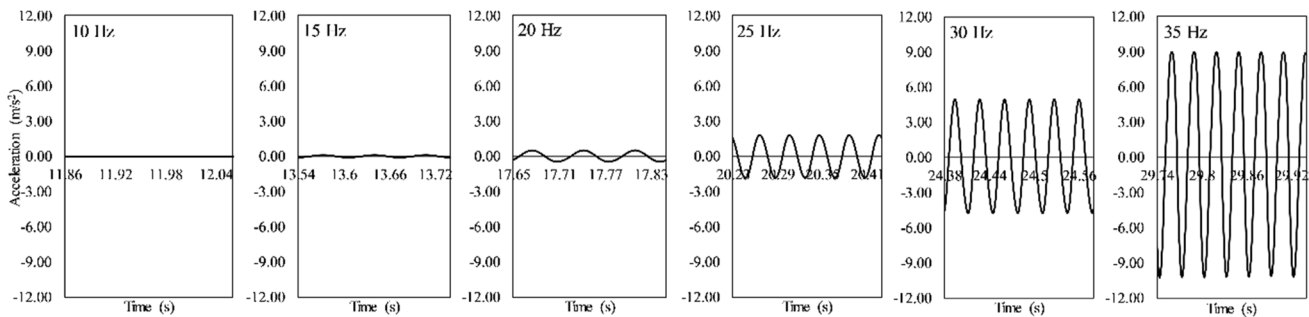


Fig. 6 Typical time vs acceleration response of the embedded ( $h/H=1$ ) block foundation with aspect ratio  $L/B=1.5$

6, proposed by Veletsos and Verbic (1973); and Kausel and Ushijima (1979) — VVS 4.

VVS 1: Reissner’s theory involves elastic half-space theory, and the frequency amplitude responses are calculated using Reissner’s displacement functions (Reissner 1936) for foundations subjected to frequency-dependent excitation. Reissner (1936) assumed that the soil medium on which the footing rests is a semi-infinite homogeneous, isotropic, and elastic body. The frequency-dependent amplitude for the soil-foundation system is calculated using the displacement functions  $f_1$  and  $f_2$  related to Poisson’s ratio of the subsoil medium, shear wave velocity, frequency of the exciting force, soil density, and footing size.

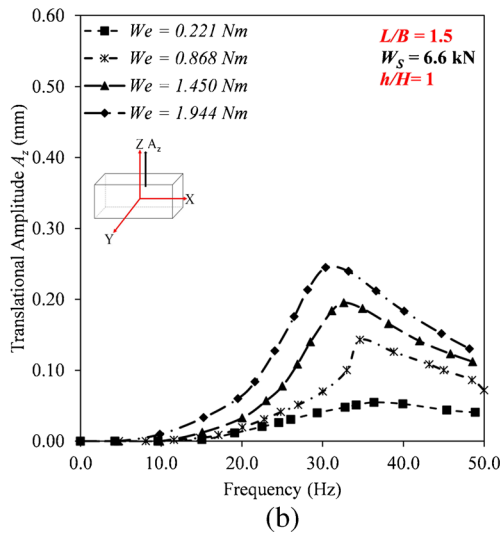
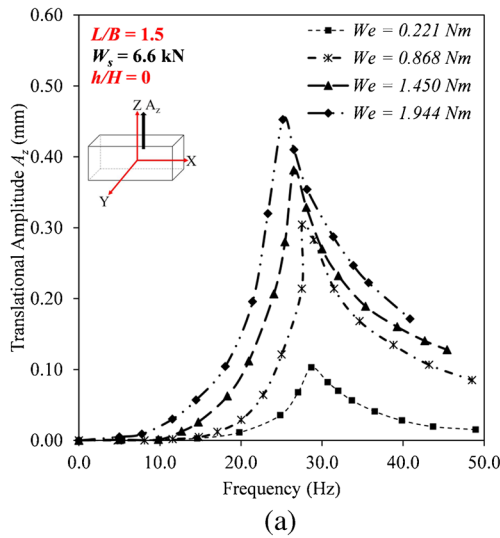
VVS 2: The theory proposed by Lysmer and Richart (1966) includes the frequency-independent analog solution based on a mass-spring dashpot model for a single degree of freedom system for a surface foundation. Lysmer studied the effect of frequency ratio on the variation in damping and spring factors. This theory was developed for constant values of effective damping and spring factors and the vertical displacement amplitude of motion at any frequency for rotating mass-type excitations. All effects contributing to the stiffness are lumped together in one stiffness parameter ( $k_z$ ) and represented by a spring. And all effects contributing to damping are lumped together in one parameter  $D_z(\xi_z)$  and represented by dashpot. The spring and dashpot coefficients  $k_z$  and  $D_z(\xi_z)$  are frequency-dependent parameters.

VVS 3: The dynamic vertical stiffness and damping of the soil foundation systems were computed by the formula and charts proposed by Gazetas and Stokoe (1991) for foundations resting on the surface. The dynamic stiffness depends on the geometrical parameters of the footing, Poisson’s ratio, and the excitation frequency. The damping coefficient is also a function of forcing frequency and does not include hysteretic damping.

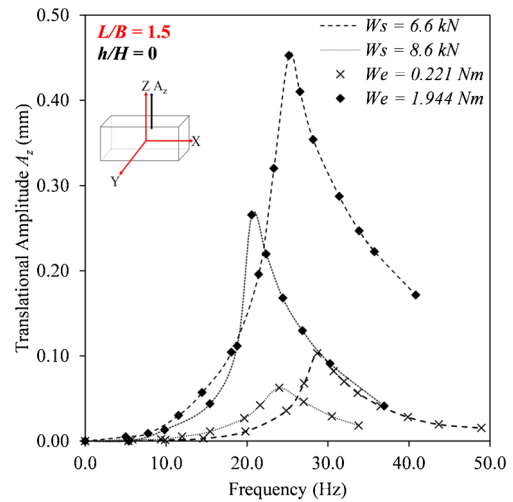
VVS 4: Veletsos and Verbic (1973) proposed a finite element approach for foundations resting on the surface of a homogeneous deposit considered a half-space. An approximate solution for the steady-state response of rigid disk with mass is presented. The rigid disk is supported at the surface of a viscoelastic half-space excited by a harmonically varying vertical force to determine the frequency-dependent dynamic stiffness and damping of the base of the foundation.

**Block foundations embedded in ground**

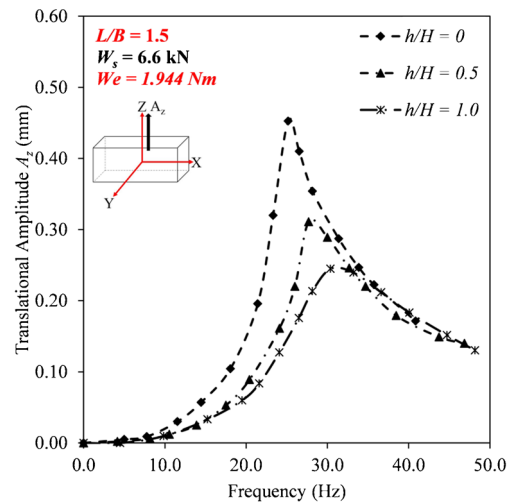
The frequency amplitude response for the embedded block foundations under vertical vibrations was obtained using the impedance coefficients, i.e., effective dynamic stiffnesses and dashpot coefficients in a homogeneous half-space proposed by Gazetas and Stokoe (1991) — VVE 1. The available approximate analytical procedures based on an elastic half-space theory proposed by Beredugo and Novak (1972) — VVE 2 was also used to



**Fig. 7** Experimental frequency vs vertical resonant amplitude responses of the block foundation for  $L/B=1.5$ : **a**  $h/H=0$ ; **b**  $h/H=1.0$



**Fig. 8** Experimental resonant amplitudes versus frequency for different static loads ( $L/B=1.5$ ,  $h/H=0$ )



**Fig. 9** Experimental resonant amplitudes versus frequency for different embedment ratio ( $L/B=1.5$ ,  $W_s=6.6$  kN,  $W_e=1.944$  Nm)

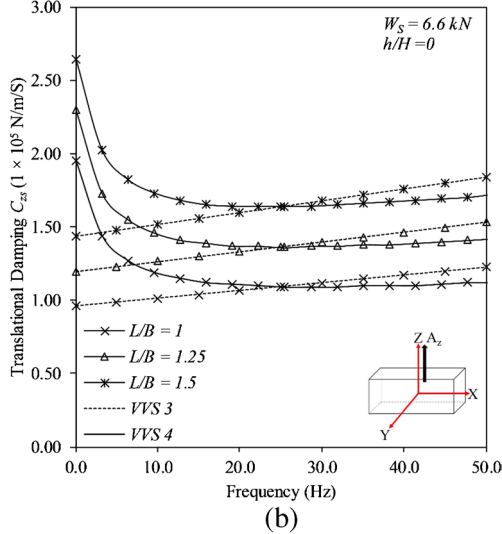
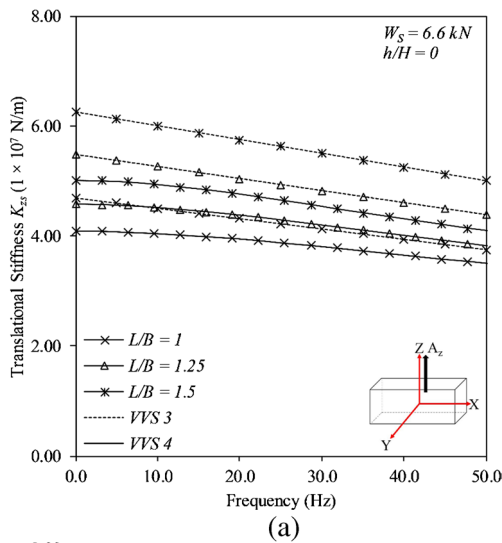
determine the response of an embedded foundation. The theory involves simple approximate solutions to cater to the embedment effect of soil by assuming the weak zone around the soil foundation. The frequency-dependent stiffness and damping were calculated, assuming the overlying soil was a series of thin independent elastic layers and subsoil as an elastic half-space. The properties of the side layer overlying the half-space zone, mass participation, as proposed by Beredugo and Novak (1972), are applied to the weak embedment zone. The approximate method is incorporated in the commercially available software DYNA6 (Novak et al. 1999), a computer program to calculate foundation response to dynamic loads.

### Dynamic stiffness and damping

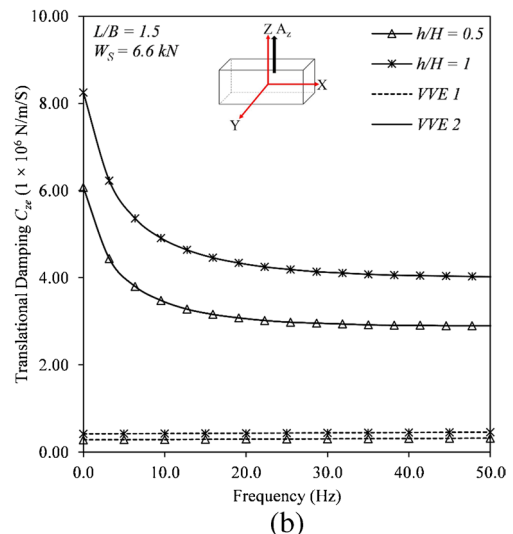
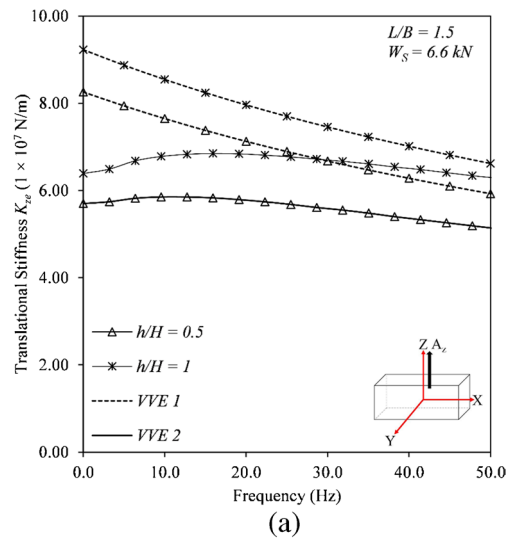
The typical frequency variations of the dynamic stiffness and damping of the foundation are shown in Figs. 10 and 11, respectively. It can be observed from Figs. 10 and 11 that as the frequency increases, the dynamic stiffness and damping of the soil foundation system increase. It has been found from Fig. 10a and b that as the aspect ratio increases from  $L/B=1$  to  $L/B=1.5$ , the dynamic stiffness and damping increase with increasing frequency. The stiffness and damping variation with frequency for various embedded conditions are shown in Fig. 11a and b. It can be observed from Fig. 11a and b that as the embedment depth,  $h$ , increases,

**Table 3** Theories adopted for surface and embedded foundations subjected to vertical vibration

Notations Used for comparison	Method/ Theory/Model	Reference
<b>Surface foundations</b>		
VVS 1	Elastic half-space method (frequency independent)	Reissner (1936)
VVS 2	Mass-spring dashpot model (frequency independent)	Lysmer and Richart (1966)
VVS 3	Impedance method (frequency dependent)	Gazetas and Stokoe (1991)
VVS 4	Elastic half-space theory (frequency dependent)	Veletsos and Verbic (1973), Kausel and Ushijima (1979)
		Incorporated in DYNA 6
<b>Embedded foundations</b>		
VVE 1	Impedance method (frequency dependent)	Gazetas and Stokoe (1991)
VVE 2	Elastic half-space method (frequency dependent)	Beredugo and Novak (1972)
		Incorporated in DYNA 6



**Fig. 10** Variation of stiffness and damping with the frequency for surface foundations of different block sizes: **a** translational stiffness  $Kze$ ; **b** translational damping  $Cze$



**Fig. 11** Variation of stiffness and damping with the frequency for surface foundations for different embedment depths: **a** translational stiffness  $Kze$ ; **b** translational damping  $Cze$

**Table 4** Comparison of static stiffness and damping values of surface block foundations

Aspect ratio (L/B)	Elastic half-space theory (VVS 1 and VVS 2)	Gazetas and Stokoe (1991) (VVS 3)	Veletos and Verbic (1973) (VVS 4)
Static Stiffness $K_{zs}$ (N/m)			
1.0	$4.02 \times 10^7$	$4.69 \times 10^7$	$4.09 \times 10^7$
1.25	$4.49 \times 10^7$	$5.47 \times 10^7$	$4.58 \times 10^7$
1.5	$4.92 \times 10^7$	$6.25 \times 10^7$	$5.01 \times 10^7$
Static damping $C_{zs}$ (N s m <sup>-1</sup> )			
1.0	$3.38 \times 10^4$	$9.58 \times 10^4$	$1.95 \times 10^5$
1.25	$4.23 \times 10^4$	$11.97 \times 10^4$	$2.30 \times 10^5$
1.5	$5.07 \times 10^4$	$14.37 \times 10^4$	$2.64 \times 10^5$

**Table 5** Comparison of static stiffness and damping values of embedded block foundations

Embedment ratio (h/H)	Gazetas and Stokoe (1991) (VVE 1)	DYNA 6 (VVE 2)
Static Stiffness $K_{ze}$ (N/m)		
0.5	$8.18 \times 10^7$	$5.70 \times 10^7$
1.0	$9.14 \times 10^7$	$6.35 \times 10^7$
Static damping $C_{ze}$ (N s m <sup>-1</sup> )		
0.5	$0.28 \times 10^6$	$1.69 \times 10^6$
1.0	$0.42 \times 10^6$	$7.36 \times 10^6$

both the vertical stiffness and damping increase, which leads to increasing the resonant frequency and decreasing the resonant translational amplitude of the soil-foundation system under the vertical mode of vibration. This characteristic is primarily due to the sidewall-soil contact and the areas and moments of inertia of the sidewall in embedded foundations, which may affect the increase in damping and stiffness of the soil foundation system.

The theories used in the present study (VVS 1, VVS 2) are based on the assumptions of elastic half-space and apply only to the cases where foundations undergo low vibration amplitudes. The theories (VVS 3, VVE 1) proposed by Gazetas and Stokoe (1991) and theories incorporated in DYNA 6 (VVS 4, VVE 2) present the frequency-dependent stiffness and damping values, whereas the stiffness and damping values obtained from other theories (VVS 1 and VVS 2) are frequency independent. The VVS 4 theory underestimated the stiffness and damping values compared to the values obtained from VVS 3 (Gazetas and Stokoe 1991), as the hysteretic frequency-independent material damping is considered by Veletos (VVS 4). This results in higher equivalent viscous damping, and the rate of increase is very high as the frequency decreases. Similarly, from Fig. 11a and b, it can be observed that the theory VVE 1 overestimates the dynamic stiffness, whereas the damping values are underestimated

with the increasing depth of embedment as compared to the theory VVE 2.

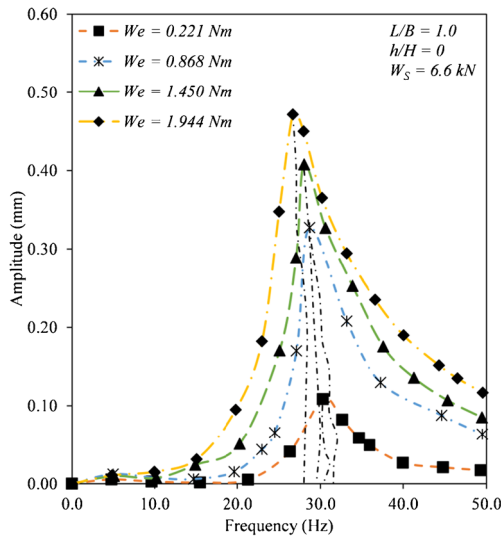
The stiffness and damping values for the static condition, i.e.,  $a_o = 0$  ( $f = 0.0$  Hz), are presented in Tables 4 and 5. The average values of stiffness and damping obtained from the frequency-dependent theories VVS 3 and VVS 4 are close to the standard theories based on elastic half-space (VVS 1 and VVS 2). For example, the average of the stiffness value, as seen in Fig. 6 for the maximum and minimum frequency obtained from the theory VVS 3, is  $3.9 \times 10^7$  N/m, and from VVS 4, it is  $4.345 \times 10^7$  N/m. The stiffness value obtained from VVS 1 and VVS 2 is  $4.02 \times 10^7$  N/m which is close to the values obtained from VVS 3 and VVS 4. The percentage increase for stiffness values obtained from different theories is found to be 8 to 15%, and damping is in the range of 10 to 25%. The stiffness and damping values obtained for embedded foundations using VVE 1 (Gazetas and Stokoe 1991) theory are much higher than those obtained from approximate solutions using the elastic half-space approach (VVE 2, Novak and Beredugo 1972a). The change in stiffness and damping varied between 20 and 45% for  $h/H = 0$  to 0.5 and between 25 and 35% for  $h/H = 0.5$  to 1. In the field, stiffness and damping of the soil-foundation system vary with frequency due to the nonlinear behavior of the soil. The nonlinearity of the soil depends on the frequency of the vibration and the dynamic stresses. Therefore, the frequency-dependent theories (VVS 3, VVS 4, VVE 2, and VVE 3) should preferably be used for predicting the response of block foundations subjected to higher dynamic forces where the nonlinearity of the soil is predominant.

## Evaluation of vertical nonlinear response

A typical feature of nonlinear vibration is the resonant frequency and amplitude change with different eccentric moments. The physical properties, such as stiffness and damping of the elastic base of the foundation, are determined from the experimental frequency amplitude response curves. The nonlinear response curves are back-calculated from the measured response curves using the theory of nonlinear vibration. The reduction of natural frequency is recognized using the backbone curve established on the measured response curve. The calculated backbone curves describe the variation of the undamped natural frequencies with translational amplitudes. The undamped natural frequency is calculated, assuming the restoring force to be nonlinear and the damping force as linearly viscous.

$$\Omega = \sqrt{\omega_1 \omega_2} \quad (3)$$

where  $\omega_1$  and  $\omega_2$  are the frequencies corresponding to the interaction points between the response curve and a line passing through the origin.

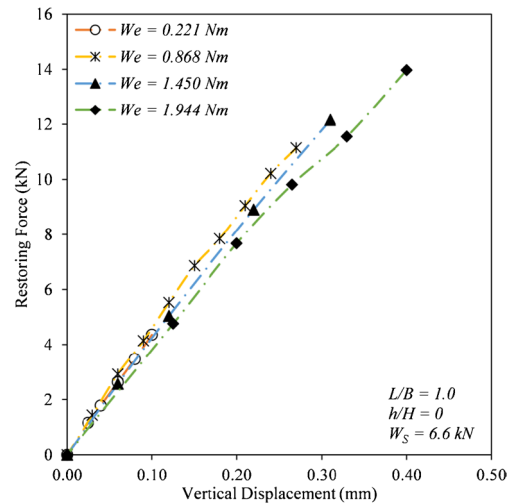


**Fig. 12** Experimental and back-calculated response curves of block foundation ( $L/B = 1.0$ ) under vertical vibration

The present study considers the nonlinear frequency-amplitude response obtained from dynamic field tests ( $L/B = 1.0$  and  $h/H = 0$ ) for a static load of 6.6 kN. The backbone curve  $\Omega(A)$  is constructed for each measured dynamic response curve by intersecting the curves by a trace of lines, as shown in Fig. 12. It is observed from each response curve that the dynamic stiffness characteristic of the block foundation soil-oscillator system varies with the level of dynamic excitation intensity. The results reported are first approximation accuracy based on the equivalent linearization of damping and restoring forces. The dynamic excitation by a steady-state harmonic force is considered whose amplitude is proportional to the square of the frequency. The stiffness characteristics for every steady-state amplitude  $A$  can be expressed, assuming the restoring force  $F(A)$  is nonlinear. The damping and effective mass are determined (Novak 1971) using the geometric properties of the nonlinear response curves. The apparent additional mass can be expressed in terms of the mass coefficient as

$$\xi = \frac{m_{\text{eff}} - m_s}{m_s} \tag{4}$$

The effective mass ( $m_{\text{eff}}$ ) is found to be greater than the mass of the foundation loading system ( $m_s = W_{st}/g$ ;  $W_{st}$  is the total mass of the foundation, including steel plates, ingots, and the mechanical oscillator) as the surrounding soil participates in the dynamic analysis of the foundation. The restoring force characteristics are also determined from the backbone curve  $\Omega$  and the calculated values of the effective mass to each response curve. The



**Fig. 13** Nonlinear force characteristic curve ( $L/B = 1.0$ ,  $W_S = 6.6$  kN, and  $h/H = 0$ )

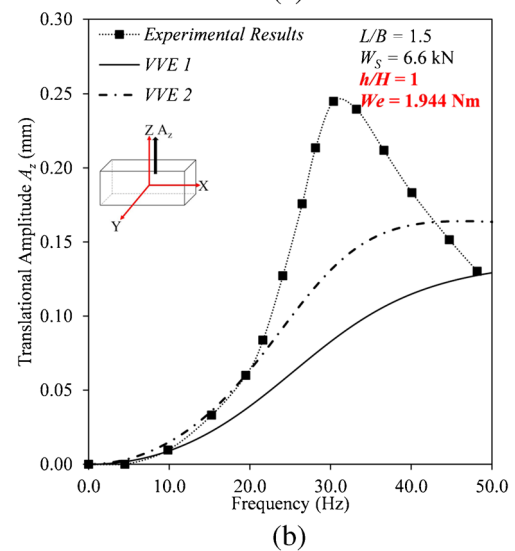
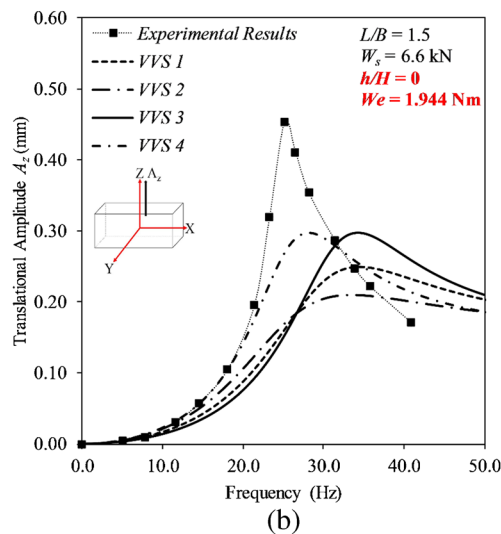
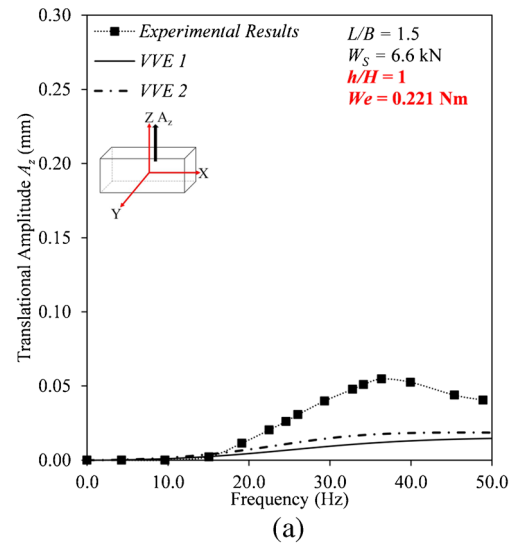
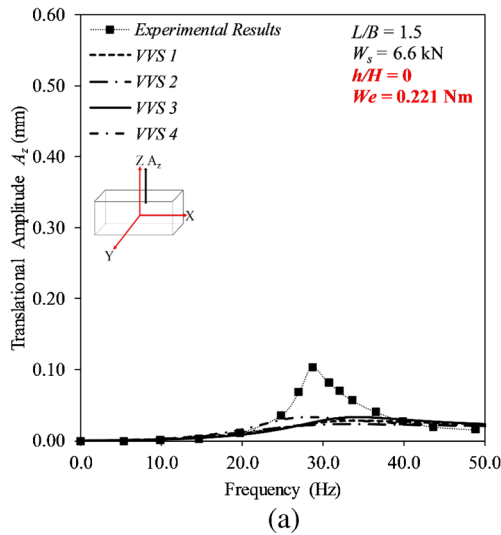
typical restoring force versus displacement characteristics is shown in Fig. 13. It can be observed from Fig. 13 that the stiffness of the soil-foundation oscillator system decreases with an increase in the excitation force. The calculated values of effective mass, stiffness, and damping for different eccentric moments are given in Table 6. It can be seen from Table 6 that the damping values of the soil medium are increased with the increase in dynamic excitation moment. However, the stiffness values and the effective mass are reduced with higher eccentric moments. The frequency amplitude curves are back-calculated using the nonlinear theory with the calculated soil-foundation oscillator system mass, damping, and restoring force characteristics. The calculated response curves using the inverse problem are then compared with the dynamic field test results, as shown in Fig. 12. The theoretical back-calculated response curves match well with the test data obtained from the forced vibration tests under vertical oscillations. So, it can be concluded that the nonlinear parameters obtained from the theoretical analysis for surface and partially or fully embedded foundations (Novak 1971) are accurate enough to be considered in the response analysis.

### Comparison between theoretical and experimental results

The dynamic field test results from the experimental investigations are compared with the theoretical approaches, as mentioned in Table 3. A typical comparison of frequency,  $f$ , vs. translational amplitude,  $A_z$ , response curves for surface and embedded foundations ( $L/B = 1.5$ ,  $W_s = 6.6$  kN) for

**Table 6** Nonlinear parameter of block foundation soil-oscillator system under vertical vibration

Exciting moment (Nm)	Mass of the soil-foundation system, $m_s$ (kg)	Mass $m_{eff}$ (kg)	Mass coefficient, $\xi$	Damping ratio	Stiffness (N/m)
0.221	1008	1182.56	0.173	0.147	$4.33 \times 10^7$
0.868	1008	1173.65	0.164	0.135	$4.00 \times 10^7$
1.450	1008	1165.51	0.156	0.136	$3.97 \times 10^7$
1.944	1008	1124.96	0.116	0.138	$3.42 \times 10^7$

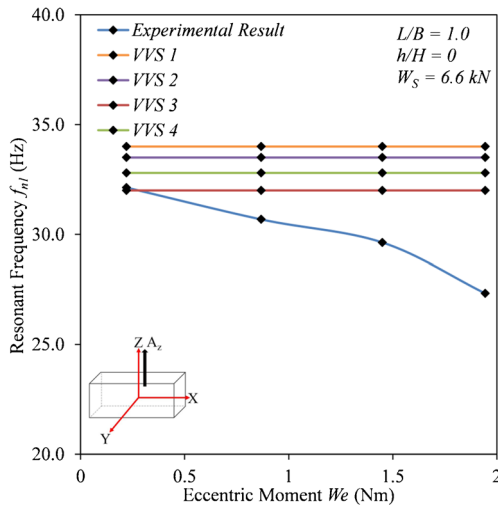


**Fig. 14** Comparison of experimental and theoretical response curves for surface foundation under vertical vibration ( $L/B=1.5$ ,  $h/H=0$ ,  $W_s=6.6$  kN): **a**  $We=0.221$  Nm; **b**  $We=1.944$  Nm

**Fig. 15** Comparison of experimental and theoretical response curves for fully embedded foundation under vertical vibration ( $L/B=1.5$ ,  $h/H=1$ ,  $W_s=6.6$  kN): **a**  $We=0.221$  Nm; **b**  $We=1.944$  Nm

$We=0.221$  Nm and  $We=1.944$  Nm are shown in Figs. 14a, b and 15a, b, respectively. It can be seen from Figs. 14 and 15 that the experimental and theoretical response curves show a single resonant peak responding to block foundations subjected to vertical vibration. The nonlinear behavior of the

soil-foundation system is observed from the experimental response curves, whereas the theoretical resonant frequencies are found to be linear with the increasing excitation force values (Fig. 16). The figures also represent that the resonant frequencies obtained from the theoretical approaches



**Fig. 16** Comparison of experimental and theoretical resonant frequency ( $f_{n1}$ ) versus eccentric moment ( $L/B=1.0$ ,  $W_s=6.6$  kN,  $h/H=0$ )

for different eccentric moments are higher than the experimental field test results. On the other hand, the maximum resonant amplitudes obtained from theoretical approaches are much lower than the experimental results.

The comparison between theoretical and experimental resonant frequencies and resonant amplitudes for surface foundations is given in Table 7. It can be seen from the test results presented in Table 7 that the resonant frequency is within reasonable ranges with 5–10% variations compared with the standard theories. For example, the maximum experimental resonant frequency of the block foundation ( $L/B = 1.0$ ,  $W.e = 0.221$  Nm) is 32.13 Hz, whereas the maximum theoretical resonant frequency is 34 Hz (VVS 2). Similarly, the theoretical resonant amplitudes (VVS 1 and VVS 2) are much higher than the test results, whereas the results obtained from VVS 3 and VVS 4 are found to be close to the test results. For example, the maximum resonant amplitude ( $L/B = 1.0$ ,  $W.e = 1.944$  Nm) obtained from the theories VVS 3 and VVS 4 are

**Table 7** Comparison of experimental field results and theoretical results of block foundations subjected to vertical vibration for different aspect ratios and eccentric moments ( $h/H=0$ ,  $W_s=6.6$  kN)

Eccentric moment (Nm)	Resonant frequency $f_{n1}$ (Hz)	Resonant amplitude $A_z$ (mm)	L/B = 1.0		L/B = 1.25		L/B = 1.5	
			$f_{n1}$ (Hz)	$A_z$ (mm)	$f_{n1}$ (Hz)	$A_z$ (mm)	$f_{n1}$ (Hz)	Resonant amplitude $A_z$ (mm)
<b>Experimental results</b>								
0.221	32.13	0.110	30.26	0.105	28.68	0.102		
0.868	30.68	0.341	28.68	0.327	27.53	0.304		
1.450	29.63	0.424	28.03	0.407	26.52	0.381		
1.944	27.32	0.494	26.68	0.471	25.16	0.452		
<b>VVS 1 — Reissner’s solution (1936)</b>								
0.221	33.5	0.029	33.0	0.026	32.5	0.023		
0.868	33.5	0.116	33.0	0.104	32.5	0.093		
1.450	33.5	0.194	33.0	0.173	32.5	0.156		
1.944	33.5	0.260	33.0	0.232	32.5	0.209		
<b>VVS 2 — Lysmer’s Analog solution (1967)</b>								
0.221	34.0	0.041	33.5	0.033	33.0	0.028		
0.868	34.0	0.162	33.5	0.132	33.0	0.111		
1.450	34.0	0.271	33.5	0.221	33.0	0.185		
1.944	34.0	0.363	33.5	0.296	33.0	0.249		
<b>VVS 3 — Impedance Method Gazetas (1991)</b>								
0.221	32.0	0.047	33.25	0.039	34.25	0.034		
0.868	32.0	0.187	33.25	0.155	34.25	0.133		
1.450	32.0	0.313	33.25	0.259	34.25	0.222		
1.944	32.0	0.419	33.25	0.347	34.25	0.297		
<b>VVS 4 — Velestos and Verbic (1973)</b>								
0.221	32.8	0.050	30.6	0.040	28.4	0.036		
0.868	32.8	0.198	30.6	0.158	28.4	0.133		
1.450	32.8	0.331	30.6	0.264	28.4	0.221		
1.944	32.8	0.444	30.6	0.354	28.4	0.287		

**Table 8** Comparison of experimental field results and theoretical results of block foundations for different embedment ratios subjected to vertical vibration ( $L/B=1.5$ ,  $W_s=6.6$  kN)

Eccentric Moment (Nm)	Resonant Frequency $f_{nl}$ (Hz)	Resonant Amplitude $A_z$ (mm)	Resonant Frequency $f_{nl}$ (Hz)	Resonant Amplitude $A_z$ (mm)	Resonant Frequency $f_{nl}$ (Hz)	Resonant Amplitude $A_z$ (mm)
	$h/H=0$		$h/H=0.5$		$h/H=1.0$	
Experimental Results						
0.221	28.68	0.102	33.53	0.076	34.15	0.058
0.868	27.53	0.304	31.75	0.201	32.38	0.143
1.450	26.52	0.381	29.47	0.253	31.15	0.195
1.944	25.16	0.452	27.7	0.311	30.38	0.244
VVE 1 — Impedance method Gazetas (1991)						
0.221	34.25	0.0338	45	0.018	49.95	0.014
0.868	34.25	0.133	45	0.074	49.95	0.057
1.450	34.25	0.222	45	0.124	49.95	0.096
1.944	34.25	0.297	45	0.167	49.95	0.129
VVE 2 — Beredugo and Novak (1972)						
0.221	28.4	0.036	35.1	0.023	47	0.019
0.868	28.4	0.133	35.1	0.089	47	0.073
1.450	28.4	0.221	35.1	0.148	47	0.122
1.944	28.4	0.287	35.1	0.195	47	0.164

0.419 mm and 0.444 mm, respectively, which are reasonably close to the test results value of 0.494 mm.

The comparison between theoretical and experimental resonant frequencies and resonant amplitudes for embedded foundations is given in Table 8. Figure 15 shows that the resonant frequency obtained from dynamic field test results and other theoretical approaches is close for lower excitation moments. However, a significant difference can be observed between theoretical and experimental results for higher excitation moments. It can also be observed from Table 8 that experimental resonant frequency matches well with the theoretical values obtained from VVE 2 for smaller embedment depths, whereas a significant difference is observed for the fully embedded foundations. For example, the experimental resonant frequency is 33.53 Hz, and VVE 2 is 35.1 Hz for  $h/H=0.5$ . Similarly, for  $h/H=1$ , the experimental resonant frequency value is 34.15 Hz, and VVE 2 is 47 Hz. The experimental resonant amplitude values differ significantly from the theoretical values obtained from VVE 1 and VVE 2 for various embedment depths.

Therefore, it can be concluded that the experimental resonant amplitudes are much higher than the theoretical values obtained from VVS 1, VVS 2, and VVE 1. However, the theoretical resonant amplitudes obtained from VVS 3, VVS 4, VVE 2, and VVE 3 match reasonably well with the experimental results.

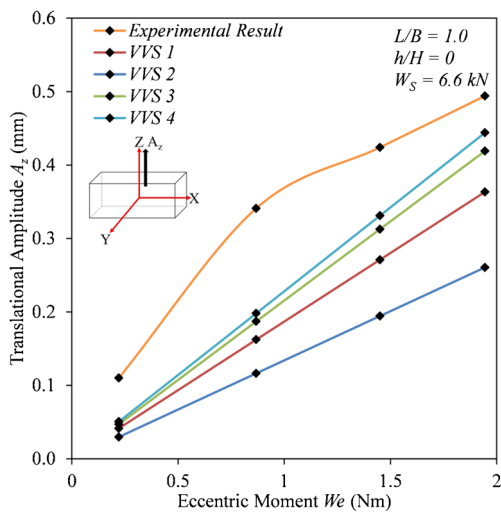
## Results and discussions

The effect of various parameters on the dynamic response of machine-loaded block foundations is studied using the test results and various theories for three aspect ratios ( $L/B=1, 1.25$ , and  $1.5$ ), three embedment ratios ( $h/H=0, 0.5, 1$ ), four eccentric moments ( $W.e=0.221$  Nm,  $0.868$  Nm,  $1.450$  Nm, and  $1.944$  Nm), and two different static loads ( $W_s=6.6$  kN and  $8.6$  kN). The test results are presented in terms of maximum resonant frequency and amplitudes and compared with theoretical studies.

### Effect of excitation force

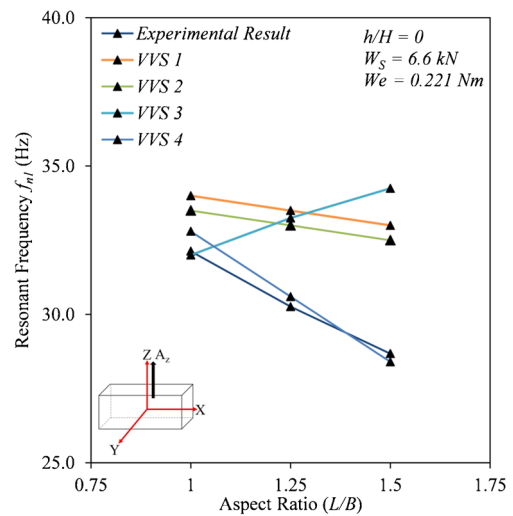
To identify the nonlinearity of soil-foundation systems, the experimental resonant amplitudes and frequency are compared with the theories such as mass-spring dash-pot analog (VVS 1), frequency-dependent displacement amplitudes (VVS 2), Gazetas impedance formulas (VVS 3), and Velestos and Verbic (VVS 4). The comparison of the experimental results with other theories is represented in the form of variation of resonant vertical amplitude and frequency with eccentric moments in Figs. 16 and 17, respectively. The nonlinearity of foundations subjected to dynamic loads is manifested in two ways: (1) the resonant





**Fig. 17** Comparison of experimental and theoretical resonant amplitudes ( $A_z$ ) versus eccentric moment ( $L/B = 1.0$ ,  $W_s = 6.6$  kN,  $h/H = 0$ )

frequency,  $f_{n1}$ , decreases with the increase in dynamic excitation force and (2) resonant amplitudes,  $A_z$ , of the excitation forces are not proportional to the amplitude steady oscillations. It is evident from the figures that as the dynamic excitation intensity increases, the resonant amplitudes increase and resonant frequencies decline under the vertical mode of vibration. It is also observed from Table 7 that with the increase in dynamic excitation intensity, the resonant frequency obtained from experiments decreases, which indicates nonlinearity in the soil, whereas the established theories show linear response. The resonant frequencies obtained from the experimental results decreased in the range of 4 to 7.5% as the excitation intensity ( $W.e$ ) increased from 0.221 to 1.944 Nm for all embedded conditions of the block foundation. The theoretical approach shows that the increase in excitation intensity is proportional to the increasing resonant amplitudes, whereas the experimental results show a significant difference in the amplitude values. It is observed from Tables 7 and 8 that the experimental resonant amplitude values are higher than the theoretical values. In contrast, the experimental resonant frequencies are lower than theoretical values obtained from the frequency-independent theories (VVS 1, VVS 2, and VVE 1), and these differences are predominant at higher excitation forces. However, the resonant frequency and amplitude values match with good agreement for frequency-dependent (VVS 3, VVS 4, VVE 2, and VVE 3) theories. Therefore, applying the half-space or mass-spring analogy theory often leads to considerable overestimating of resonant frequency and underestimating resonant amplitudes of foundations subjected to vertical vibrations. Therefore, frequency-dependent theories are more applicable and the results emphasize the advantages

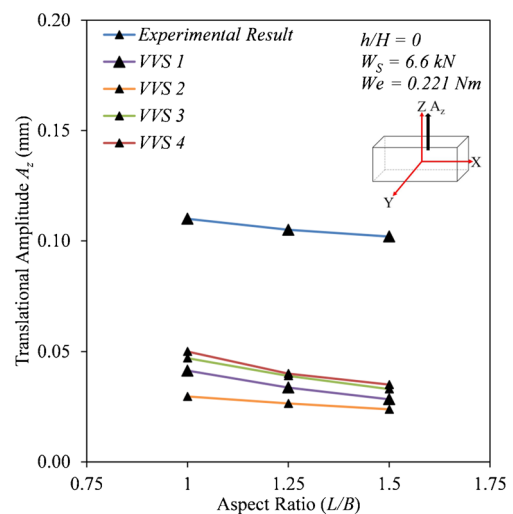


**Fig. 18** Comparison of experimental and theoretical resonant frequency ( $f_{n1}$ ) versus aspect ratio ( $W_s = 6.6$  kN,  $h/H = 0$ ,  $W.e = 0.221$  Nm)

of using frequency-dependent theories in predicting the response of foundations subjected to higher dynamic forces.

### Effect of aspect ratio

Three blocks of different  $L/B$  ratios of 1, 1.25, and 1.5 were considered for the investigations to study the effect of the aspect ratio. The experimental resonant frequency and amplitude values compared with the theoretical solutions for various aspect ratios are presented in Figs. 18 and 19, respectively. It is found from the experimental

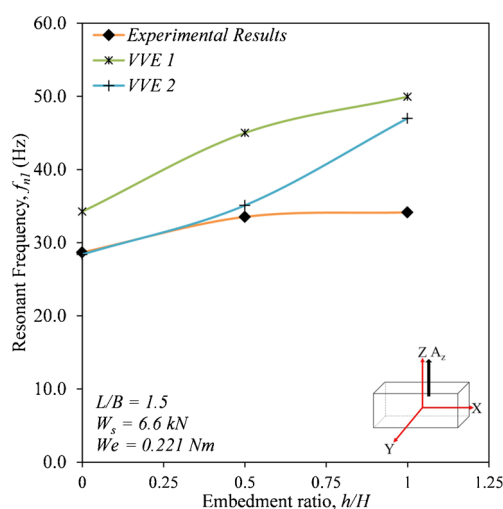


**Fig. 19** Comparison of experimental and theoretical resonant amplitudes ( $A_z$ ) versus aspect ratio ( $W_s = 6.6$  kN,  $h/H = 0$ ,  $W.e = 0.221$  Nm)

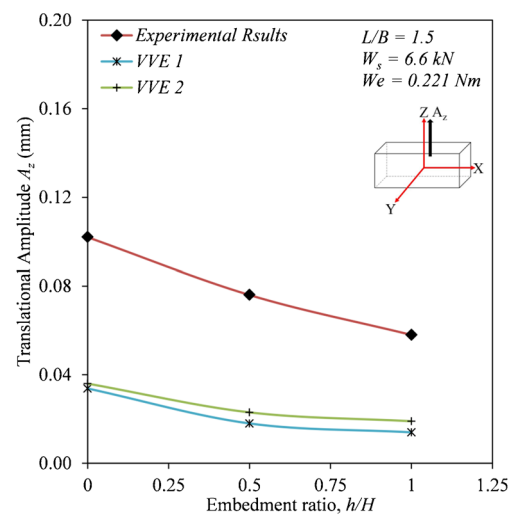
investigation that as the  $L/B$  ratio increased, both the maximum resonant frequency,  $f_{n1}$ , and the resonant amplitude,  $A_z$ , decreased. This is due to the increase in the mass of the foundation for higher  $L/B$  ratios. The theoretical approaches *VVS 1*, *VVS 2*, and *VVS 4* show a decrement in the resonant frequency, whereas the theoretical approach *VVS 3* shows an increment in the resonant frequency with the increasing aspect ratio as the length of the foundation increased from  $L=0.6$  to  $0.9$  m. The stiffness and damping functions proposed by *VVS 3* (Gazetas 1991) consider the block dimensions, shear modulus, and Poisson's ratio of soil.

It can be observed from Table 7 that the experimental resonant frequency decreased by about 2 to 20%, and the resonant amplitude reduced by about 2 to 12% as the  $L/B$  ratio increased from  $L/B = 1$  to  $L/B = 1.5$ . It is also observed that the variation of theoretical resonant frequencies and amplitudes with aspect ratios ( $L/B$ ) follows a similar trend as observed from experimental investigations. More accurate predicted values of  $f_{n1}$  and the resonant amplitude,  $A_z$ , are found with the variation of  $L/B$  for the higher eccentric moment ( $W.e = 1.944$  Nm) than the lower eccentric moment ( $W.e = 0.221$  Nm).

It can be concluded from the above results that as the aspect ratio ( $L/B$ ) increases, the resonant frequency and amplitude of block foundations decrease. The theoretical predictions and experimental results exhibit similar trends, validating the accuracy of the theoretical approaches. Additionally, the effect of the eccentric moment on the resonant frequency and amplitude is observed, with higher eccentric moments leading to more accurate predictions.



**Fig. 20** Comparison of experimental and theoretical resonant frequency ( $f_{n1}$ ) with respect to embedment ratio ( $h/H$ ) ( $L/B=1.5$ ,  $W_s=6.6$  kN,  $W.e=0.221$  Nm)



**Fig. 21** Comparison of experimental and theoretical resonant amplitudes ( $A_z$ ) with respect to embedment ratio ( $h/H$ ) ( $L/B=1.5$ ,  $W_s=6.6$  kN,  $W.e=0.221$  Nm)

### Effect of embedment depth

The effect of depth of foundation embedment ( $h$ ) on the dynamic amplitude response of block foundations is crucial as the foundations are generally embedded partially or fully in the field. The comparison of experimental resonant frequency and amplitude with the theoretical solutions for different embedment ratios for an eccentric moment of  $W.e=0.221$  Nm are shown in Figs. 20 and 21, respectively. It can be seen from the figures that as the foundation embedment depth ( $h$ ) increases, the resonant frequency,  $f_{n1}$ , increases, which indicates the increase of stiffness of the soil foundation system. On the other hand, the resonant amplitude decreases with the increasing embedment depth, increasing the equivalent damping in the sub-soil. The decrease in the maximum resonant amplitude is much more significant at shallow depths than at deeper depths. The theoretical resonant frequency increases, and the resonant amplitude decreases with the increasing embedment depth, similar to the experimental results. The experimental resonant amplitudes and frequencies are compared with theoretical values and are shown in Table 8. The approximated analytical values obtained from *VVE 1* and *VVE 2* are consistently lower than the experimental resonant amplitude and resonant frequencies by a factor of 2 to 3 in the described case. It is also observed from the experimental investigations that the maximum resonant amplitude decreased by about 25 to 50% for the change of embedment ratio ( $h/H$ ) of 0 to 0.5 and 30 to 55% for the change of embedment ratio ( $h/H$ ) of 0.5 to 1.0. In contrast, an increase in resonant frequency,  $f_{n1}$ , is found to be 5 to 25% for different depths of embedment ( $h$ ) compared to machine-loaded block foundations resting on the

surface. The depth of foundation embedment significantly influences the dynamic amplitude response of block foundations. It can be concluded from the above results that deeper embedment increases stiffness, while shallower embedment increases equivalent damping. Theoretical predictions generally match experimental results, with specific theoretical approaches showing limitations. These findings contribute to a better understanding of the behavior of block foundations with varying embedment ratios and provide practical insights for foundation design and stability considerations.

**Effect of static load**

The effect of static load on the dynamic response of the foundation ( $L/B = 1.5$  and  $h/H = 0$ ) is also studied for two different static loads of 6.6 kN and 8.6 kN. The experimental resonant amplitudes and resonant frequencies compared with the theoretical approaches for two different static loads are summarized in Table 9. It can be observed from Table 9 that as the static load increases, both the resonant frequency and resonant amplitude decrease. This phenomenon is due to the higher mass ratio of the system due to the higher

static load. The experimental resonant amplitude decreases by about 35 to 45%, and resonant frequency decreases by about 15 to 20% at different eccentric moments. The theoretical approaches *VVS 1* and *VVS 2* show a decrement of 3 to 5% in the resonant frequency and 7 to 14% in the resonant amplitude with the increase in static load from 6.6 to 8.6 kN. It is also observed that the maximum resonant frequencies obtained from the theoretical solutions are higher than the experimental values, which indicates that the theoretical stiffness of the foundation-soil system is more as compared to the actual stiffness of the soil-foundation system in the field. On the other hand, the theoretical resonant amplitudes for the different eccentric moments were lower than the experimental field test results.

In conclusion, the higher static load results in a stiffer and less oscillatory response of the foundation under dynamic loading. It can also be concluded that increasing the static load on the foundation leads to a decrease in both the resonant frequency and resonant amplitude. These findings provide valuable insights into the dynamic behavior of the foundation and contribute to improving foundation design and stability considerations.

**Table 9** Comparison of experimental field results and theoretical results of surface block foundation subjected to vertical vibration with different static loads ( $L/B = 1.5$ ,  $h/H = 0$ )

Eccentric moment (Nm)	Resonant frequency $f_{nl}$ (Hz)		Resonant amplitude $A_z$ (mm)	
	$W_s = 6.6$ kN		$W_s = 8.6$ kN	
Experimental results				
0.221	28.68	0.102	23.98	0.062
0.868	27.53	0.304	23.02	0.176
1.450	26.52	0.381	21.73	0.218
1.944	25.16	0.452	20.58	0.265
<i>VVS 1</i> — Reissner’s Solution (1936)				
0.221	32.5	0.023	31.0	0.020
0.868	32.5	0.093	31.0	0.082
1.450	32.5	0.156	31.0	0.137
1.944	32.5	0.209	31.0	0.184
<i>VVS 2</i> — Lysmer’s Analog solution (1966)				
0.221	33.0	0.028	32.0	0.026
0.868	33.0	0.111	32.0	0.103
1.450	33.0	0.185	32.0	0.172
1.944	33.0	0.249	32.0	0.231
<i>VVS 3</i> — Impedance method Gazetas (1991)				
0.221	34.25	0.034	31.8	0.031
0.868	34.25	0.133	31.8	0.123
1.450	34.25	0.222	31.8	0.206
1.944	34.25	0.297	31.8	0.276
<i>VVS 4</i> — Velastos and Verbic (1973)				
0.221	28.4	0.036	25.6	0.031
0.868	28.4	0.133	25.6	0.097
1.450	28.4	0.221	25.6	0.162
1.944	28.4	0.287	25.6	0.217

## Conclusions

This study's implications are significant for geotechnical engineering, specifically in designing block foundations under dynamic loads. Nonlinearity in soil-foundation systems is identified as resonant frequencies decrease with increasing dynamic excitation force. Comparisons with theoretical approaches reveal the limitations of linear theories for higher dynamic forces. Frequency-dependent theories show good agreement with experimental results, making them more applicable for predicting nonlinear responses. Engineers should consider dynamic excitation intensity when selecting appropriate design parameters to ensure stability and safety. Overall, this research advances the understanding of block foundation behavior, guiding more accurate and reliable design practices. The critical findings of this study, the role of various influencing parameters responsible for the nonlinear dynamic responses of the machine foundation under vertical vibrations partially or fully embedded in the soil examined through experimental tests, and theoretical approaches are summarized below:

1. The experimental response curves represent a single resonant peak under vertical vibration within the frequency range adopted for all the eccentric moments. It is also observed that with the increase in the dynamic excitation intensity, the resonant frequency decreases, and the resonant amplitude increases non-proportionally, which depicts the nonlinear behavior of the soil-foundation system. The theoretical analyses are based on linear assumptions; hence, the resonant frequencies are constant for different eccentric moments, and resonant amplitudes increase linearly with an increase in the eccentric moment.
2. The dynamic stiffness and damping of the block foundation soil-oscillator system increase with the increasing aspect ratio. With the increase in foundation embedment depth, the stiffness and damping of the foundation increase, resulting in a decrease in resonant amplitude,  $A_z$ , and an increase in the resonant frequency,  $f_{n1}$ .
3. The back-calculated response curves found that the effective mass and the stiffness values decrease, and the damping values of the soil medium increase with the increase in dynamic excitation moment. The measured nonlinear response curves determine a good agreement between the experimental and back-calculated response curves for vertical vibration.
4. The frequency-dependent theories (VVS 3, VVS 4, VVE 2, and VVE 3) can predict the nonlinear response of the block foundations more accurately as compared to the frequency-independent theories for block foundations subjected to higher dynamic forces.
5. The resonant frequencies of the soil foundation system decrease as the eccentric moment increases, and the resonant amplitudes disproportionately increase with the eccentric moments. The resonant frequency decreased from 4 to 7.5%, and resonant amplitudes increased two to three times with the increasing excitation force.
6. The foundation geometry significantly affects the dynamic response of the block foundation soil-oscillator system. A decrease of about 2 to 12% in resonant frequency and 2 to 20% in resonant amplitude is observed with the increasing aspect ratio ( $L/B=1.0$  to  $1.5$ ) under vertical vibration.
7. The resonant frequency and amplitudes decrease with the increased embedment depth of footing depicting the non-linear behavior of soil. The resonant frequency increased by 5 to 25%, and the resonant amplitude decreased by 20 to 55%, increasing the embedment ratio from  $h/H=0$  to  $1$ .
8. The experimental resonant frequencies decrease by about 15 to 20%, whereas the experimental resonant amplitudes decrease by about 35 to 45% for different eccentric moments with the increasing static load.

**Data availability** All data, models, and code generated or used during the study appear in the submitted article.

## Declarations

**Conflict of interest** The authors declare no conflict of interest.

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