



# DEM simulation of sandy pebble soil based on polyhedral particles

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## Abstract

Discrete element method (DEM) was used to evaluate the geometric characteristics of sandy pebble soil. The accuracy of the grain size distribution in the sandy pebble soil was ensured using the cutting ratio ( $\zeta$ ) and a gradation correction formula, which revised the normal vector of the polyhedron particle cutting surface for a single unit and the space volume of polyhedral particles. The modelling method was verified by conducting direct shear experiment on sandy pebble soil. The feasibility of polyhedral particles of sandy pebble soil for discrete element simulation was verified in comparison with that of the spherical particle model.

**Keywords** Sandy pebble soil · Discrete element model (DEM) · Polyhedral particles · Shear test

## Introduction

Sandy pebble soil is a type of coarse-grained soil (Li 2019), which is formed due to shock caused by flash floods and water transportation. When a flood, carrying a large amount of sediment, flows downstream in a river, its flow rate decreases due to the gentle terrain slopes. The decrease in flow rate results in the gradual deposit of the sediment on the riverbed to form sandy pebble soil. The composition of sandy pebble soil is unique because it consists of large-size block stones and fine particles, and their physical and mechanical properties are significantly different. Its internal structure is complex. The shape, content, arrangement, spatial distribution, and contact

form of large-size block stones directly affect the mechanical properties of the soil (Zhang et al. 2020). Previous researches based on indoor and outdoor experiments (Zou 2018; Zheng 2018; Yang et al. 2018) have evaluated the internal features of sandy pebble soil (stone content, stone distribution). However, randomness was observed during the sampling. Theoretical and numerical methods are required for the analysis of sandy pebble soil due to the expensive cost of tests, poor repeatability of the experiments, structure complexity, and uncertainty related to sandy pebble soil.

The numerical simulation of coarse-grained soil is divided into two different approaches. The first approach is based on numerical simulation using finite element method, whereas the second approach is based on numerical simulation using discrete element method. The former approach considers soil as a continuous medium, whereas the latter approach divides soil into a finite number of particles, and the macroscopic characteristics of the soil are evaluated by simulating the interaction and relative motion between granular units. Sandy pebble soil is composed of a limited number of debris particles, conforming to the concept of discrete element method. In the discrete element method, the irregular geometrical shapes of the granule media are developed using three methods. The first is the direct definition method, in which shapes such as ellipse, cylinder, and polyhedron unit are described by an equation except for non-circular or spherical particles (Zhao et al. 2018; Nougouier-lehon et al. 2003). The second method considers the particles as regular elements with a bonding-breaking function (Fan 2007; Zhang et al. 2017). The irregular elements are formed by the breaking and bonding of regular

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elements. The third method includes developing the irregular elements by imbuing a few regular elements into irregular elements (Shao-Hui et al. 2017; Danda et al. 2008; Ying and Shunying 2009).

The elements used in the discrete element simulation of direct shear test in Tejchman (2005); Cividini and Gioda 1992) were almost spherical or disc-shaped. However, this method neglects the effect of the shape of coarse-grained soil on the shear strength. Therefore, the shear properties of irregular particles cannot be evaluated. Sandy pebble soil has different sizes and geometrical shapes under natural conditions. Therefore, a polyhedron particle model was developed to simulate the direct shear test of sandy pebble soil based on the concept of constructing irregular combinatorial elements. Additionally, the feasibility of using the polyhedron particle model was analysed in comparison with that of the spherical particle model.

### The simulation using discrete element model based on spherical particles

The gradation of the sandy pebble model was selected based on the direct shear test of sandy pebble soil, as shown in Table 1. The particles of the shear box were generated using the radius expansion method to obtain the designed sizes. Based on the research conducted by Ming (Ming 1997), and due to the limitation of the number of particles and the computational efficiency, the smallest particle size used for the model was 10 mm.

Initially, the volume percentage (mass percent) of different sized particles was set. Subsequently, the spherical particle model of sandy pebble soil was developed by a cycle of random function. The model is shown in Fig. 1. The normal stiffness was  $1.0 \times 10^6$  N/m and the stiffness ratio was 2.5.

It can be observed in Fig. 2 that the test and simulation results had the same curvilinear trend under different axial compressions, and the shear displacements increased with an increase in the shear stress. However, the displacements obtained during the simulation were larger than that of the experiments, which indicated that the values of shear strength were lower in the case of the experiments. The simulation defects were not resolved even after adjusting the microscopic parameters. Research on rock (Potyondy and Cundall 2004; Cho et al. 2007; Hoek and Brown 1998) and coarse-grained soil (Geng et al. 2011) conducted by a few scholars demonstrated that the shear strength values of

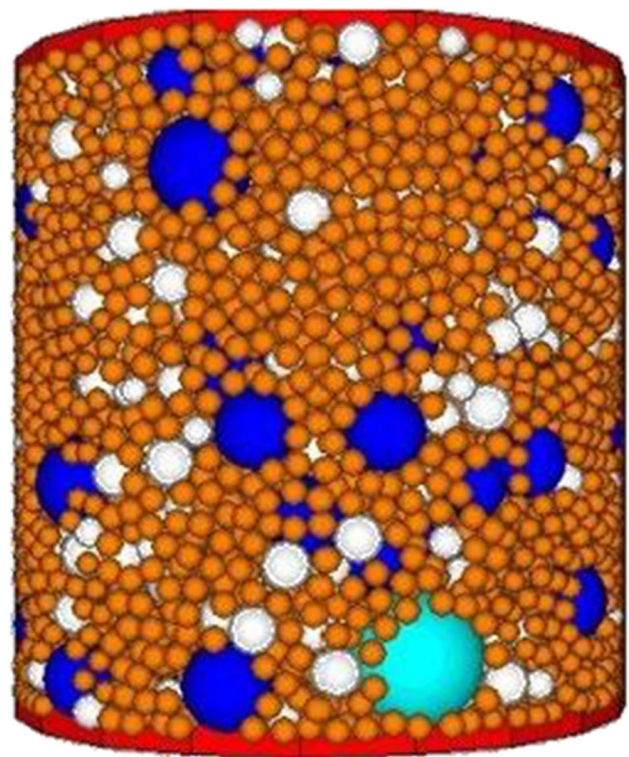


Fig. 1 Representation of sandy pebble soil model using spherical particles

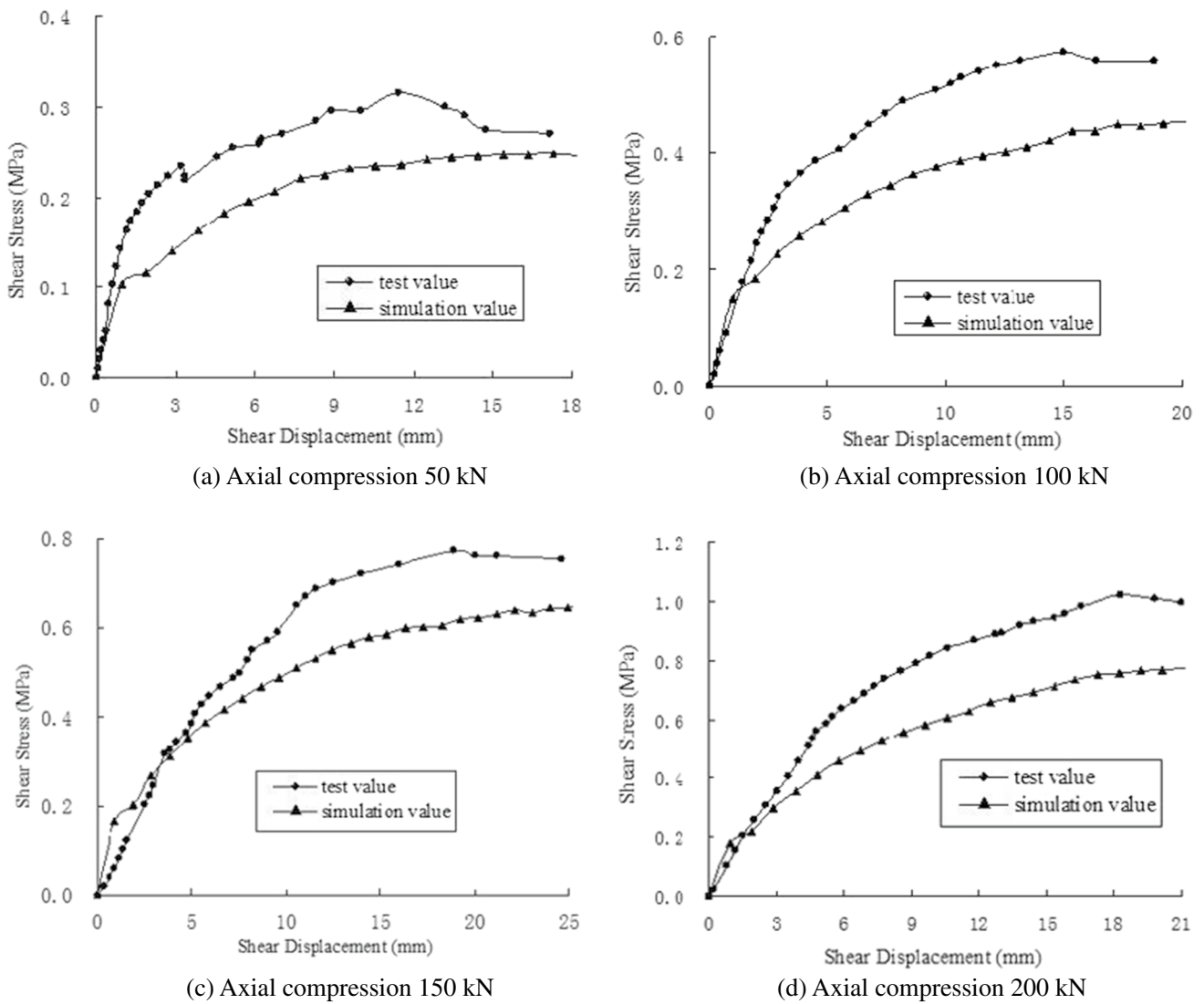
spherical particles during the three-axial numerical simulation were lower than that of the test values, which is an insurmountable disadvantage of conventional spherical particles. A study was conducted in which it was ensured that the simulation results (Jian et al. 2000) agree with that of the test results by modifying the particle size, distribution, and properties. However, the maximum coefficient of friction was 80, which is not possible in practical situations. Additionally, this value does not accurately represent the frictional force contributing to soil strength.

### The establishment of discrete element model based on polyhedral particles

A spherical original clump comprised of multiple spheres, which overlapped each other, was constructed to simulate the basic mechanical properties of the irregular particle body. Subsequently, a polyhedral clump was obtained by cutting

**Table 1** Percentage of quality of sandy pebble soil gradation

	Accumulative mass percentage (%)								
Grain size	60	50	40	20	10	5	2	0.5	0.075
Gradation	100	95	90	70	52	40	25	17	7



**Fig. 2** The test and simulation shear stress-displacement curves of sandy pebble soil under different axial compressions

the original clump with multiple reference planes. The final clump obtained was used as the model for sandy pebble soil particle for the simulation of the direct shear tests. The contact force between the particles was absent due to the unlimited overlap, which indicated that the clump cannot break into smaller particles under the effect of an external force. Hence, the polyhedral clump was considered as a rigid body.

**Geometric parameters of the polyhedral clump element**

A point on the plane and the outward normal direction of the plane can determine a reference plane. The direction of the outward normal consists of three randomly generated numbers ( $n_x, n_y, n_z$ ) expressed as:

$$\begin{aligned}
 n_x &= \cos(2\pi \times rand()) \\
 n_y &= \cos(2\pi \times rand()) \\
 n_z &= \cos(2\pi \times rand())
 \end{aligned}
 \tag{1}$$

$rand()$  represents a random number in the range of 0.0–1.0.

$$\begin{aligned}
 x'_c &= x_c + R(1 - \zeta) \times \frac{n_x}{\sqrt{n_x^2 + n_y^2 + n_z^2}} \\
 y'_c &= y_c + R(1 - \zeta) \times \frac{n_y}{\sqrt{n_x^2 + n_y^2 + n_z^2}} \\
 z'_c &= z_c + R(1 - \zeta) \times \frac{n_z}{\sqrt{n_x^2 + n_y^2 + n_z^2}}
 \end{aligned}
 \tag{2}$$

The point on the reference plane  $O' (x_c', y_c', z_c')$  is determined using Eq. (2).  $n_x, n_y,$  and  $n_z$  are three components of the outward normal direction.  $\zeta$  is the cutting ratio.

$$\zeta = 1 - (1 - l_{\min}) \times rand() \tag{3}$$

$l_{\min}$  is the minimum value of  $\zeta$ , which is set by the user to determine the particle shape. For example, the percentage of flat particles increases with an increase in the value of  $l_{\min}$ , and vice versa (Yu 2011).

It can be observed from Eq. (2) that the randomly constructed point  $O' (x_c', y_c', z_c')$  should be inside or on the surface of the spherical clump with a centre  $O (x_c, y_c, z_c)$  and radius  $R$ . It can be inferred that the randomly constructed reference plane should intersect or should be in contact with the sphere while the normal vector direction is deviated from the centre.

According to the abovementioned method for the establishment of reference plane, multiple planes can be established under the cycle control of C language. The normal vector  $n_x, n_y,$  and  $n_z$  can be revised to the following forms to avoid superposition of the normal vector during cycling:

$$\begin{aligned} n_x &= \cos(2\pi \times rand() + num \times 0.1 \times \pi) \\ n_y &= \cos(2\pi \times rand() + num \times 0.1 \times \pi) \\ n_z &= \cos(2\pi \times rand() + num \times 0.1 \times \pi) \end{aligned} \tag{4}$$

The  $num$  represents an artificial changing number that is used to avoid the generation of the same outer normal vector. The polyhedron particle shapes were changed by varying  $\zeta$  as shown in Fig. 3.

### Correction of the gradation of polyhedral particle model

Based on the generation principle of polyhedral particle model, a single polyhedron particle is formed by cutting

the spherical particle with respect to multiple reference planes. Therefore, the single polyhedron particle volume is less than that of the volume of the corresponding spherical particle. Hence, the gradation of the polyhedron particles model is changed.

A few parameters were adjusted while modelling to ensure the accuracy of gradation of the sandy pebble soil used in the polyhedron particle model. Moreover, the sphere was regarded as a circle with a centre  $O$  and radius  $R$  to simplify the analysis process, as shown in Fig. 4. The straight line  $BD$  was the reference plane.

During the development of the polyhedral particle model of the sandy pebble soil, the parameter  $\zeta$  was randomly selected as 0.3, 0.4, or 0.5. Additionally, the number of the reference planes was set as 3. The average length of the section  $OF$  was calculated to be  $0.6 R$ . Hence, the angle  $\alpha$  in Fig. 4 can be calculated using the following equation:

$$\sin \alpha = \frac{|OF|}{R} \tag{5}$$

The area of the quadrilateral  $OFDE$  can be expressed as:

$$\begin{aligned} S_{OFDE} &= S_{OFD} + S_{ODE} \\ S_{OFD} &= \frac{1}{2}|OF|R \sin \beta \\ S_{ODE} &= \frac{1}{2}\alpha R^2 \end{aligned} \tag{6}$$

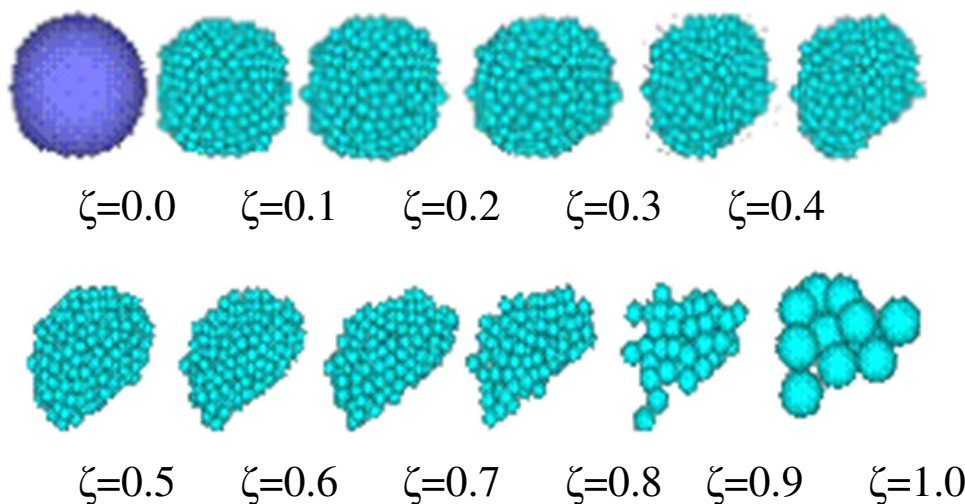
$S_{OFD}$  is the area of triangle  $\Delta OFD$ ;  $S_{ODE}$  is the area of the sector  $ODE$ ,

$$\sin^2 \alpha + \sin^2 \beta = 1 \tag{7}$$

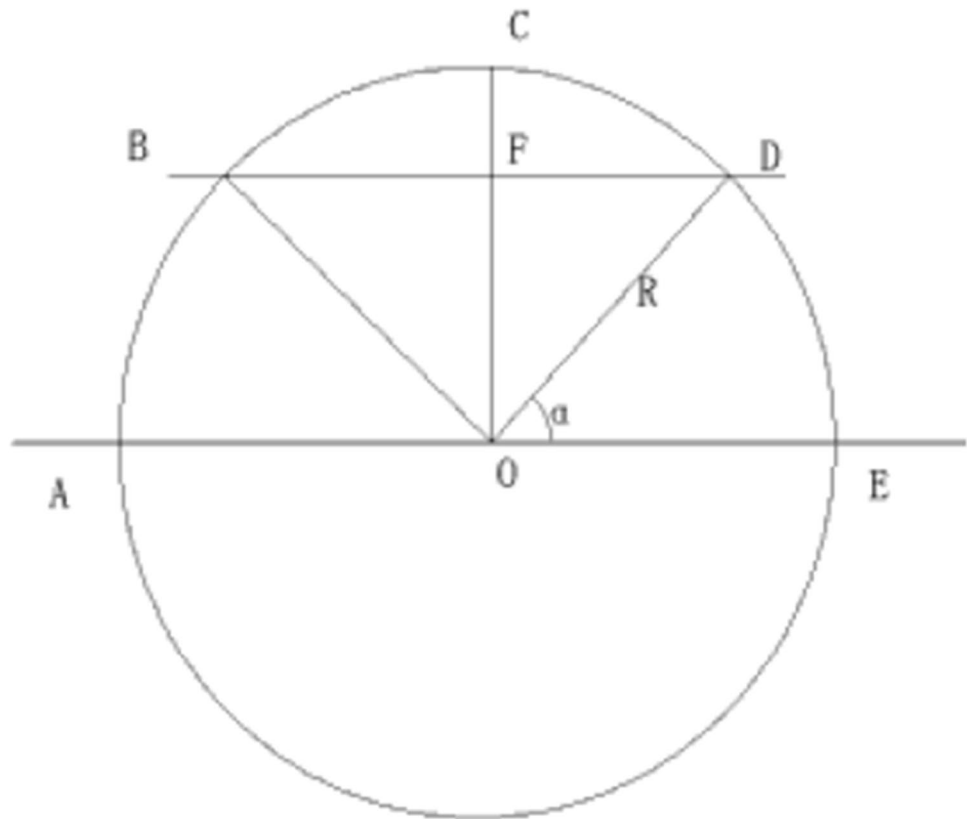
In two-dimension, the area  $S_{BCDF}$  can be expressed as:

$$S_{BCDF} = \frac{1}{2}\pi R^2 - 2S_{OFDE} \tag{8}$$

Fig. 3 Polyhedral particle shape changed by  $\zeta$



**Fig. 4** The grading ball and reference plane (2D)



The proportion of  $S_{BCDF}$  can be expressed as area percentage  $P_{CUT}$ .

$$P_{CUT} = \frac{S_{BCDF}}{\pi R^2} \times 100\% \tag{9}$$

However, in the real three-dimensional model, the portion removed by the reference plane is a body, rather than a plane, as shown in Fig. 5. According to the mathematical formula:

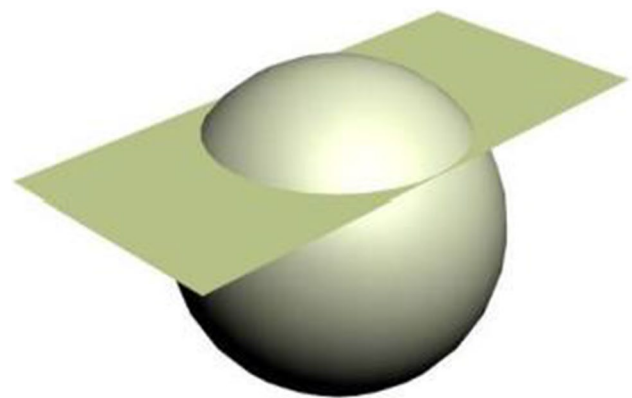
$$\frac{i}{j} = \frac{i+i}{j+j} = \frac{i+i+i}{j+j+j} = \dots = \frac{n \cdot i}{n \cdot j} = \frac{i}{j} \tag{10}$$

The percentage of the volume removed from the total spherical volume is represented by  $P_{CUT}$ .

Since three planes were considered during the polyhedral particle modelling process, the percentage of volume removed by the three reference planes from the total volume of the spherical region can be written as:

$$P_{ALL} = 3 \cdot P_{CUT} \tag{11}$$

However, the cut regions might overlap with each other because the reference planes are random. It can be observed from Fig. 4 that the probability of the second and the third reference planes cutting the same region with the first reference



**Fig. 5** The grading ball and reference plane (3D)

plane is  $P_1$ . The probability of the third and the second reference plane cutting the same area is  $P_2$ . Therefore, the probability a non-overlapping region is  $P_3$ .

$$\begin{aligned} P_3 &= P_1 P_2 \\ P_1 &= \frac{2\alpha + \pi}{2\pi} \\ P_2 &= \frac{4\alpha}{2\pi} \end{aligned} \tag{12}$$

According to Eq. (12),  $P_3$  is 28.87%, that is, the probability of the three reference planes not overlapping each other is 28.87% and the probability of obtaining an overlapping region is 71.13%. Therefore, the value of  $P_{ALL}$  in Eq. (11) requires correction. A correction coefficient (0.8) was considered after constant debugging and analysis. It was observed that 20% of the same area was present in the three reference plane cutting areas. The percentage of the total volume cut by three reference planes in the total spherical area is shown as follows:

$$P = 0.8P_{ALL} \quad (13)$$

### The simulation using discrete element model based on polyhedral particles

This study considers polyhedral particle units for the simulation of direct shear tests of sandy pebble soil under different values of normal stress, which verifies the reliability of discrete element method based on polyhedral particles for the simulation of mechanical behaviour of irregular particles.

### The calculation parameters of polyhedral particle unit

The sampling of sandy pebble soil was in accordance with *Pebble and crushed stone for building* (GB/T 14,685–2001). The materials were prepared according to *China Test Methods of Soils for Highway Engineering* (JTG E40-2007). The compactness of the sandy pebble soil should be not less than 95%, the aggregate crushing value should not be greater than

30%, and the percentage of the long and thin particles should be less than 20%. The materials used are shown in Fig. 6. The gradation of sandy pebble soil is provided in Table 1.

A mixture of polyhedral clump particles was used to model the sandy pebble soil for the simulation of the direct shear test. Polyhedral clump particles were selected due to its irregularity, discreteness, and randomness in the distribution (Fig. 6). The density and void ratio were measured to be 2650 kg/m<sup>3</sup> and 0.35, respectively.

The interlocked phenomenon is easily realised using the polyhedral clump element during simulation. Interlocking is difficult to achieve in regular spherical particles while simulating sandy pebble soil. However, it can be achieved if the contact friction coefficient is in the range of approximately 60~80 (Jian and Yong 2004; Shunying 2007). However, the contact friction coefficient significantly exceeded the actual value.

### Parameter setting of straight shear box in discrete element simulation

The direct shear box is mainly composed of an upper and a lower shear box. The upper box was loaded with an axial compression and fixed in the horizontal direction, and the lower box was moved horizontally at a rate of 0.2 mm/s, which is faster than that in reality. Considering the simulation rate, the shear rate was acceptable considering that 1 mm is equivalent to running 10<sup>5</sup> steps at a speed of 6.777<sup>-6</sup> mm/s in the calculation by discrete element method.

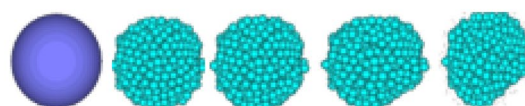
The design of shear box and its internal granular media are shown in Fig. 7. The normal stiffness, stiffness ratio,

**Fig. 6** Sandy pebble soil used in the shear test

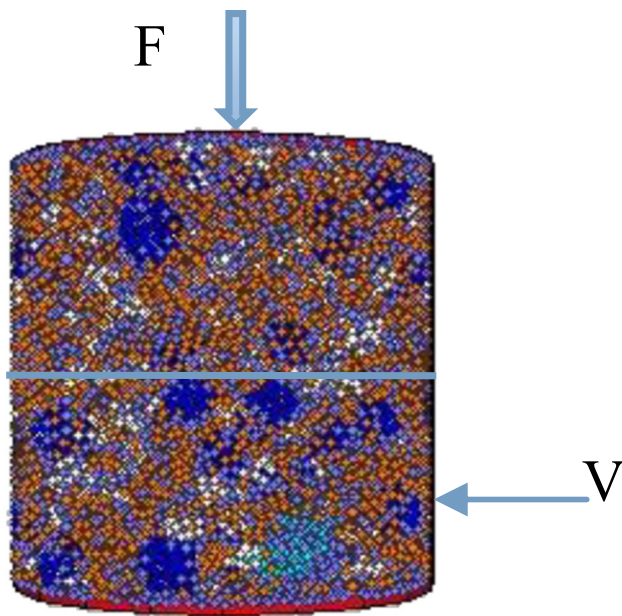


(a) Coarse aggregate

(b) Fine aggregate



$\zeta=0.0$   $\zeta=0.1$   $\zeta=0.2$   $\zeta=0.3$

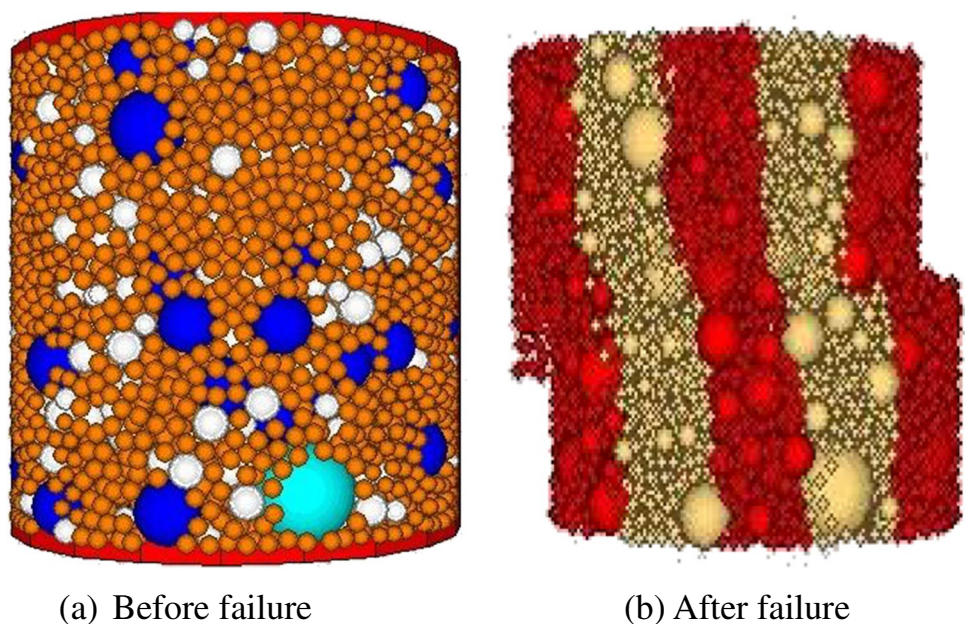


**Fig. 7** Polyhedral particles unit of PFC simulation schematic in direct shear test

and friction coefficient of the internal granular were set as  $1.0 \times 10^6$  N/m, 2.5, and 0.4, respectively.

The particle elements were randomly placed on the shear box according to the gradation during the numerical simulation of PFC. The initial particle size was set as half of the actual size to ensure an intensive initial arrangement state. The particle size was slowly increased to the required size using the radius expansion method. During the process, the particles reached an equilibrium position after colliding with each other. The abovementioned method is commonly

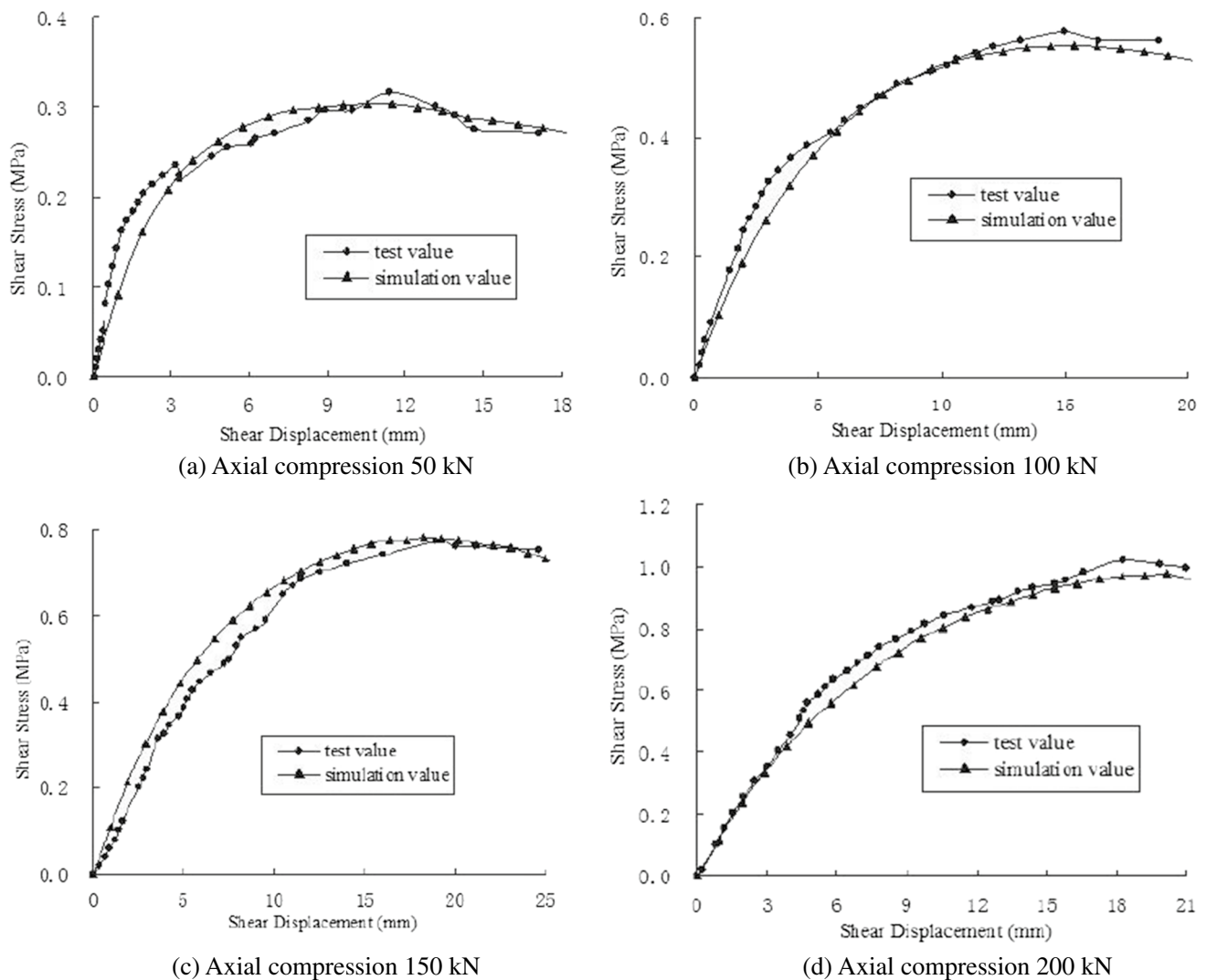
**Fig. 8** Before and after shear failure of sandy pebble soil



adopted for particle discrete element simulation (Ying and Shunying 2009). The actual size was obtained using the magnifying coefficient, and a vertical force was exerted on the box to ensure a dense arrangement of the particles. When the particles reached a stable equilibrium state, the lower shear box was moved horizontally with a shear velocity  $V$  until the critical damage condition was observed. The simulation process of direct shear tests consisted of four parts: randomly placed, growth, loaded, and shear.

**Direct shear test and numerical simulation results**

Figure 8 represents the model of sandy pebble soil before and after failure. Figure 9 demonstrates the relationship between shear stress and shear displacement of sandy pebble soil under four different axial pressures. It can be observed from Fig. 9 that the initial trend of shear strength of sandy pebble soil obtained by numerical simulation was consistent with that of the test results. However, the shear strength was lower than that of the test value. Although the meso parameters were adjusted to a certain extent, it slightly overcame the defect. The same results were obtained from the numerical simulation of the other three graded sandy pebble soils. The research conducted by domestic and foreign scholars on rock (Nouguier-lehon et al. 2003; Fan 2007; Zhang et al. 2017) and coarse-grained soil (Zheng 2018) demonstrated that the shear strength obtained by using spherical particles in triaxial numerical simulation is lower than that of the test value, which is a defect that conventional spherical particles cannot overcome. Additionally, a few scholars have obtained simulation results which were consistent with that of the test results by modifying the particle size, properties, and



**Fig. 9** Comparison between the test and simulation results

distribution (Shao-Hui et al. 2017). However, the maximum friction coefficient in the simulation was 0.8, which is not possible in practical situations, and it cannot accurately represent the contribution of friction to soil strength.

The shear stresses and shear displacements were tested in the experiments under an axial pressure of 50, 100, 150, and 200 kN, respectively. The results were compared with that of the simulation results shown in Fig. 9. The results demonstrated that the experiment was in accordance with the simulation using polyhedral particle model.

## Discussion

The shape of sand pebble in sand pebble stratum changes randomly, and its shape will directly affect the bearing capacity, shear strength, and other parameters of sand

pebble stratum. In addition, the spatial distribution of sand pebble blocks in the stratum will also affect their strength and bearing capacity. At present, relevant scholars have carried out relevant research, but it is not deep enough. In this paper, there is no research on the spatial distribution of sand pebbles in the stratum. This problem will be a hot spot of follow-up research and worth further discussion.

## Conclusions

- (1) The normal vector formulae were revised to avoid superposition of randomly producing normal vector during cycling in the process of development of a single polyhedral particle model.



- (2) Based on the field sandy pebble soil sample, the indoor model test is carried out to obtain the discrete element numerical calculation parameters of sandy pebble soil layer. Based on the super element idea, the random polyhedral particle model of sandy pebble soil is established, and a correction method is proposed for the reduction of coarse particle size in modelling, so as to improve the model accuracy of sandy pebble soil in polyhedral particle model.
- (3) The utilisation of granular discrete element method of non-continuum mechanics for the simulation of coarse-grained soil is inevitable and it improves the mechanical behaviour of coarse-grained soil by using polyhedron particle model.

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**Data availability** Readers can access the data by email dyj820@swust.edu.cn.

## Declarations

**Conflict of interest** The authors declare no competing interests.

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