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ORIGINAL PAPER

A novel variable-order fractional damage creep model for sandstone

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Abstract

In this paper, based on the variable-order fractional derivative with convolution form and the effect of relaxation time, a modified variable-order fractional damage creep model was proposed. The varying-order function related to relaxation time was given and applied in the proposed damage creep model. To describe the triaxial creep deformation of rock material based on the modified variable-order fractional damage creep model, the triaxial variable-order fractional damage creep model was derived. The applicability and validation of the derived triaxial variable-order fractional damage creep were verified by the creep experimental data of sandstone. Then, a comparative study between the triaxial variable-order fractional damage creep model and the current triaxial constant-order fractional damage creep model was performed to highlight the advantage of the derived triaxial variable-order fractional damage creep model. Finally, the influence of mechanical parameters on the creep deformation was revealed in depth.

Keywords Variable-order fractional derivative · Relaxation time · Rock material · Triaxial variable-order fractional damage creep model · Model verification

Introduction

Rheological deformation of rock material is common in underground engineering, whose time-dependence and abruptness are of great significance for controlling and predicting the deformation of rock mass (Sun 1999). It is important to establish an effective and accurate theoretical model to reflect creep deformation under complex stress statuses. Much effort has been put into the construction of the theological model, and numerous achievements also have been realized (Maranini and Yamaguchi 2001; Adeli et al. 2020, 2017; Chen et al. 2014). Maranini and Yamaguchi proposed a nonassociated constitutive equation to describe the elastic and viscoplastic properties in the compressive deformation of granite rock. Adeli used the Bayesian method to identify the parameters of the viscoplastic-damage model, which is a stochastic simulation technique that generates artificial data. Chen et al. used the fractional calculus theory and damage variables to construct time-dependent damage creep constitutive model to depict the accelerating creep deformation of marble. Xia et al. proposed a unified rheological model that including 15 elements and the identifications of parameters also have been performed (Xia et al. 2009). Xu et al. presented a modified creep model that called Hohai model, which can depict three stages of creep deformation (Xu et al. 2007). Liu et al. gave the modified Burgers model under triaxial stress status, and the total creep deformation can be well reflected (Liu et al. 2017). Based on the above mentioned, we can see that numerous element models were constructed to describe creep mechanical behaviors of rock materials. Although those previous models can describe the total creep deformation of rock materials, in the applications of those previous models, those have many parameters with unclear physical and mechanical meaning. Especially, in the description of the accelerating creep deformation, its strong nonlinearity characteristics and obvious time dependence cannot be well described, and it is lack of a reasonable and clear damage model. Hence, it is necessary to construct a

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nonlinear damage creep model with less and clear parameters that also can be applied in depiction of the three-stage creep deformation.

It Is well known that due to its strong history and time-memory, fractional calculus has become more popular in characterization of time-dependent behavior of material (Almeida et al. 2018). During the description of creep mechanical behavior, fractional calculus has played a significant role and obtained much progress. Based on Riemann–Liouville (R–L) fractional calculus, Koeller presented a fractional viscoelastic dashpot that named as Koeller dashpot, which can reflect viscoelastic behavior of material (Koeller 1984). Zhou et al. (Zhou et al. 2011, 2018) used the Scott-Blair dashpot to replace traditional Newtonian dashpot and construct a fractional damage creep model. Yin et al. (Yin et al. 2012) proposed a modified fractional Bingham model, and the validations of proposed model were verified by the creep experimental data of clay. Considering the damage evolution of rock material, a fractional damage creep model was presented to characterize the accelerated creep deformation of salt rock (Wu et al. 2019). Based on Caputo fractional derivative, Liu et al. established a fractional damage creep model with time-varying viscoelasticity to depict the creep behavior of sandstone (Liu et al. 2020, 2021). And, in the identification of the viscoplastic-damage model, some progress has been obtained based on the mathematical method (Adeli and Matthies 2019). Adeli and Matthies applied the transitional Markov chain Monte Carlo method to estimate the viscoplastic-damage model parameters in the Bayesian setting, and the efficiency of the identification was given.

Based on the above mentioned, it is demonstrated that constant-order fractional (COF) calculus has achieved much progress, but there are still defections in the application of COF calculus. During the characterization of time-varying viscoelasticity within creep deformation, compared to COF calculus, variable-order fractional (VOF) calculus has attracted much attention due to its varying-order function, and the order can be assumed as one function related to time and space (Sun et al. 2011; Ingman and Suzdalnitsky 2005). Theoretically, the evolution of the mechanical property of material within deformation can be reflected by VOF calculus. Lorenzo, Coimbra, and Ingman et al. (Ingman et al. 2000; Lorenzo and Hartley 2002; Coimbra 2010) gave the definition of VOF derivative and a VOF constitutive model to describe the viscoelastic behavior of composite material. Sun et al. (Sun et al. 2009) applied the VOF derivative in the description of abnormal diffusion behavior of the fluid. For the application of VOF derivative in reflection of mechanical behavior of rock material, Wu et al. proposed a VOF creep model based on the short memory of deformation (Wu et al. 2020a, b). A fractional damage creep model was constructed by Tang et al. (Tang et al. 2018) based on the continuous damage mechanic, whose validations were verified by the creep experimental data of sandstone. However, how to determine an accurate varying-order function within the VOF creep model and construct a triaxial VOF damage creep model with less parameter and clear physical meaning is valuable in the application of VOF derivative in rock engineering.

The outline of this paper is illustrated as follows. The "VOF damage creep model" section presents the VOF damage creep model, which is based on the Caputo VOF derivative and relaxation time. The proposed VOF damage creep model is extended to the triaxial VOF damage creep model. In the "Verification and analysis of the proposed model" section, based on the triaxial creep experimental data of sandstone, the applicability of the proposed triaxial VOF damage creep model is verified. Then, the effect of mechanical parameters on creep behavior is also analyzed. Finally, several conclusions are drawn in detail.

VOF damage creep model

Modified VOF Maxwell model

Due to its strong history-memory characteristics, fractional calculus has been recognized as an effective tool in the description of time-dependent mechanical behavior and achieved many applications in rheological deformation. In 1978, Koeller proposed a fractional Koeller dashpot to replace the traditional Newtonian dashpot to depict the viscoelastic behavior of the material, whose expression is expressed as follows (Koeller 1984):

$$\sigma(t) = E\tau^{\alpha} D^{\alpha}\varepsilon(t) \tag{1}$$

where $\sigma(t)$, $\varepsilon(t)$, E, and τ represent stress, strain, elastic modulus, and relaxation time, respectively. The D^{α} denotes Riemann–Liouville (R–L) fractional calculus (Zhou et al. 2011). When $\alpha = 0$, Eq. (1) describes pure elastic solid and when $\alpha = 1$, Eq. (1) characterizes pure viscous fluid. If $0 < \alpha < 1$, Eq. (1) can depict the viscoelasticity.

Meanwhile, R–L fractional calculus is COF calculus, and the fractional-order α is a constant value within describing the mechanical behavior of the material. However, it is well-known that the mechanical behavior within the material is varying with time in deformation, and for better exhibiting these dynamic mechanical behaviors, based on COF calculus, Lorenzo and Hartley proposed a VOF derivative by employing convolution transform, whose expression is shown (Lorenzo and Hartley 2002)

$${}_0{}^1D^{-\alpha(t)}f(t) = \frac{1}{\Gamma[\alpha(t)]}\int_0^t \frac{f(\mu)}{(t-\mu)^{1-\alpha(t)}}d\mu \tag{2}$$

where $\alpha(t)$ is varying-order function related to time and Γ is Gamma function (Almeida et al. 2018) ($\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^t dt$).

The varying-order function within Eq. (2) has been defined as a variable related to time with clear physical meaning, and the historical memory of itself is considered in the depiction of the deformation behavior of the material. Hence, by employing Eq. (2) in the COF Koeller dashpot, the VOF Koeller dashpot can be obtained,

$$\sigma(t) = E\tau^{\alpha(t)} D^{\alpha(t)} \varepsilon(t) \tag{3}$$

In conjunction of Eq. (2), the Eq. (3) can be derived as an integral form.

$$D^{-\alpha(t)} \sigma(t) = E\tau^{\alpha(t)} \varepsilon(t) \tag{4}$$

$$E\tau^{\alpha(t)} \varepsilon(t) = \frac{1}{\Gamma[\alpha(t)]} \int_0^t \frac{\sigma(\mu)}{(t-\mu)^{1-\alpha(t)}} d\mu \tag{5}$$

Considering to stress within creep deformation is a constant value, i.e., $\sigma(t) = \sigma_0$, the creep response of the VOF Koeller dashpot can be deduced,

$$\varepsilon(t) = \frac{\sigma_0 t^{\alpha(t)}}{E\tau^{\alpha(t)} \Gamma[1 + \alpha(t)]} \tag{6}$$

where σ_0 is a constant stress in creep deformation.

Before applying the VOF derivative in the construction of the creep model, it is necessary to determine a reasonable varying-order function to exhibit the dynamic viscoelasticity within the creep deformation of the material. As mentioned in previous research (Liu and Li 2020), relaxation time is a controlling parameter in the evolution of viscoelasticity in rheological deformation, and with an increase in relaxation time, the ability to resist deformation of material will become stronger. Considering the significance of relaxation time in rheological deformation, a new varying-order function related to relaxation time with exponential form is presented, i.e., $\alpha(t) = \exp(-t/\tau)$, which transforms from 0 to 1 and characterizes the continuity of variations of mechanical property of the material.

Figure 1 presents that when creep stress is 40MPa and relaxation times are 800, 1000, 2000, 4000, and 8000 h, respectively, the creep response of the VOF Koeller dashpot rises up with the development of time. With relaxation time approaching infinity, the viscosity of material will increase, and the ability to resist to deformation of material will also become great, which indicates the applicability of the presented VOF Koeller dashpot in the description of viscoelasticity within the process of creep.

Based on the presented VOF Koeller dashpot, as displayed in Fig. 2, the modified VOF Maxwell model is obtained by combining the spring element with VOF Koeller dashpot in series. When stress is a constant value, the creep response of the modified VOF Maxwell model is expressed as follows:

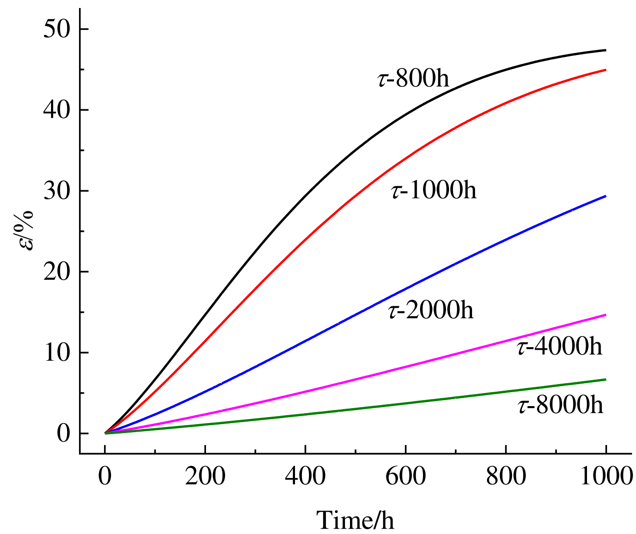

Fig. 1 Creep response of variable-order fractional Koeller dashpot

$$\varepsilon(t) = \varepsilon_1(t) + \varepsilon_2(t) \tag{7}$$

where $\varepsilon_1(t)$ is creep response of spring element and $\varepsilon_2(t)$ is creep response of the presented VOF Koeller dashpot.

$$\varepsilon(t) = \frac{\sigma_0}{E} + \frac{\sigma_0 t^{\alpha(t)}}{E\tau^{\alpha(t)} \Gamma(1 + \alpha(t))} \tag{8}$$

As shown in Eq. (8), compared to the COF Maxwell model, the modified VOF Maxwell has relatively fewer parameters and more clear physical meaning, which will furtherly and effectively reflect the evolution of viscoelasticity within creep deformation. Its corresponding verifications will be performed in next section.

A damage factor considering the effect of relaxation time

When creep stress exceeds the long-term strength of the sample, creep deformation will enter the accelerated creep stage, and the cracks within the sample will develop fast and damage will accumulate until failing to sample. Hence, before constructing the damage creep model, it is necessary to present a clear damage factor with exact physical meaning. Considering the varying-order function characterizing the evolution of viscoelasticity and the damage factor describing plastic deformation of the accelerated creep stage, it is assumed that the damage factor can be expressed by employing the varying-order function, which is denoted,

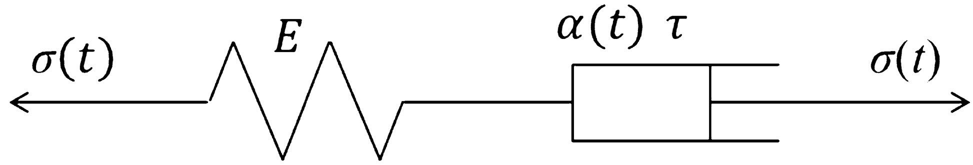

Fig. 2 VOF Maxwell model

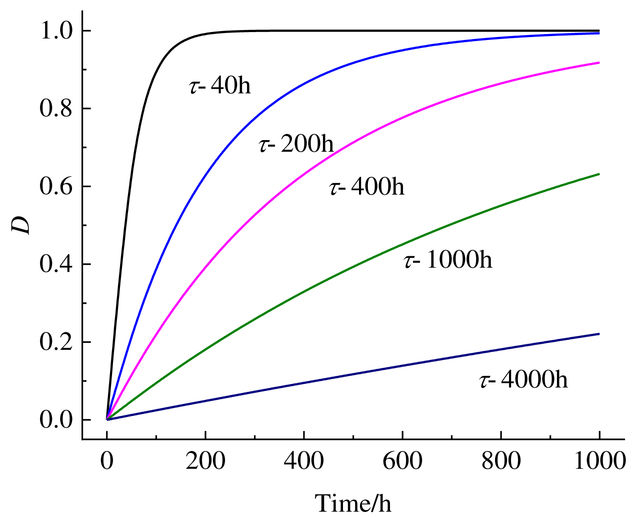

Fig. 3 Evolution of damage factor under various relaxation time

$$D = 1 - \alpha(t) = 1 - \exp\left(\frac{-t}{\tau}\right) \tag{9}$$

Compared to other previous research (Zhou et al. 2018), the presented damage factor has clear expression and physical meaning, and it can divide the viscoelasticity and damage behavior by utilizing the particularity of varying-order function that transform from 0 to 1. By introducing the relaxation time in the damage factor, it is convenient to understand the mechanical mechanism of creep from the view of relaxation time.

It can be seen from Fig. 3 that with relaxation time tending to 0, the sample will arrive at failure fast at the initial deformation, and mechanical property will be weakened within a short time. Hence, based on the damage factor related to relaxation time, the viscosity efficiency of material M is supposed that decays with exponential form (Zhou et al. 2018).

$$M = M_0(1 - D) \tag{10}$$

where M and M_0 represent the viscosity efficiency and initial viscosity coefficient of the material. Based on the above assumption, when creeping stress σ_0 exceeds the long-term strength of the sample σ_s, the constitutive model of the damaged body is defined,

$$\frac{\sigma_0 - \sigma_s}{M_0(1 - D)} = \frac{d\varepsilon(t)}{dt} \tag{11}$$

where σ_s is the long-term strength of sample.

By applying the Laplace transform in Eq. (11), when stress is constant, the creep response of the damaged body is derived as follows:

$$\varepsilon(t) = \frac{\sigma_0 - \sigma_s}{M_0}\tau exp\left(\frac{t}{\tau}\right) \tag{12}$$

Uniaxial VOF damage creep model

It is well-known that creep deformation of the material is divided into decaying, steady, and accelerated creep stages. For constructing an effective and reasonable fractional damage creep model, in this study, by connecting the modified VOF Maxwell model and damage body related to relaxation time in series, a novel uniaxial VOF damage creep model is given, whose expression of elements is shown in Fig. 4.

As said in the "A damage factor considering the effect of relaxation time" section, when the sample arrives at the accelerated creep stage, the creep response of the damaged body is obtained as follows:

$$\varepsilon_3(t) = \frac{\sigma_0 - \sigma_s}{M_0}\tau\exp\left(\frac{t}{\tau}\right) \tag{13}$$

where $\varepsilon_3(t)$ represents creep response of damage model.

Based on the Boltzmann strain superposition principle (Wu et al. 2020a, b), uniaxial VOF damage creep model is deduced,

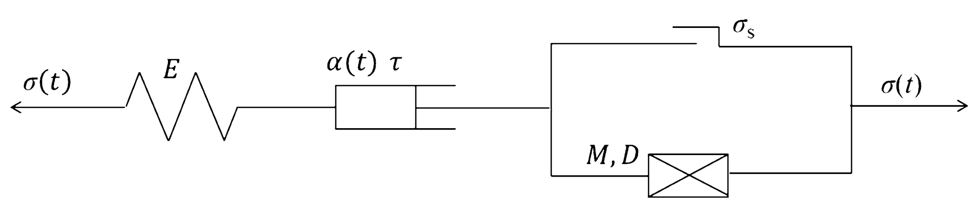

Fig. 4 VOF damage creep model

$$\varepsilon(t) = \begin{cases} \varepsilon_1(t) + \varepsilon_2(t), & \sigma_0 < \sigma_s \\ \varepsilon_1(t) + \varepsilon_2(t) + \varepsilon_3(t), & \sigma_0 \geq \sigma_s \end{cases} \tag{14}$$

where $\varepsilon_1(t) + \varepsilon_2(t) = \frac{\sigma_0}{E} + \frac{\sigma_0 t^{\alpha(t)}}{E\tau^{\alpha(t)}\Gamma(1+\alpha(t))}$.

Triaxial VOF damage creep model

During the actual underground engineering, rock material is often in complex three-dimensional stress status, and the creep behaviors under a triaxial stress environment cannot be depicted accurately by the above proposed uniaxial VOF model, hence, it is valuable to establish a triaxial VOF damage creep model.

Based on the proposed uniaxial VOF damage creep model, when rock mass is in an actual stress environment, the total strain of rock can be expressed as follows:

$$\varepsilon_{ij}(t) = \varepsilon_{ij}^{e}(t) + \varepsilon_{ij}^{VFK}(t) + \varepsilon_{ij}^{D}(t) \tag{15}$$

where stress tensor σ_{ij} is composed of spherical stress tensor σ_m and deviator stress tensor S_{ij}. Strain tensor ε_{ij} is combined by spherical strain tensor ε_m and deviator strain tensor e_{ij}, whose expression is shown,

$$\begin{cases} \sigma_{ij} = S_{ij} + \sigma_m \delta_{ij} \\ \varepsilon_{ij} = e_{ij} + \varepsilon_m \delta_{ij} \end{cases} \tag{16}$$

where δ_{ij} is the Krobecker function (Zheng and Kong 2005).

Since spherical stress tensor only causes volume deformation and has no effect on shape of material. The deviator stress tensor only induces changes of shape and has no effect on volume deformation. So, the strain of elastic body is shown,

$$\varepsilon_{ij}^{e}(t) = \frac{S_{ij}}{2G_0} + \frac{\sigma_m \delta_{ij}}{3K} \tag{17}$$

where $\varepsilon_{ij}^{e}(t)$ is the strain deviator, G_0 is the shear modulus of elastic body, and K is the bulk modulus of elastic body.

It can be known that the deformation induced by spherical tensor is elastic deformation and the deviator tensor only causes viscoelastic deformation, which is expressed,

$$\varepsilon_{ij}^{VFK}(t) = \frac{S_{ij} t^{\alpha(t)}}{2G_1 \Gamma[1+\alpha(t)]} \tag{18}$$

where G_1 is the shear modulus of viscoelastic body and ε_{ij}^{VFK} is the creep response of the presented VOF Koeller dashpot.

For the viscoplastic body, based on the generalized plastic mechanic (Zheng and Kong 2005), the triaxial creep equation of the viscoplastic body is shown as Eq. (19).

$$\varepsilon_{ij}^{D}(t) = \frac{1}{M_0(1-D)} \tau \langle \phi\left(\frac{F}{F_0}\right) \rangle \frac{\partial f}{\partial \sigma_{ij}} t \tag{19}$$

where F is the yield function, F_0 is the initial value of yield function of rock, ϕ is the power function, and $\phi = 1$, and ε_{ij}^{D} is the creep response of damage body under triaxial stress status. f is the plastic potential function based on the associated flow rule (Zheng and Kong 2005), $F = f$.

Equation (20) is yield criterion of rock.

$$\phi\left(\frac{F}{F_0}\right) = \begin{cases} 0, & F < 0 \\ \frac{F}{F_0}, & F \geq 0 \end{cases} \tag{20}$$

During the creep deformation of rock, deviator stress tensor will occupy main role in creep deformation, and spherical stress tensor has little effect on creep deformation. Hence, the Mises function (Cai et al. 2009; Zheng and Kong 2005) is selected, which is illustrated,

$$F = \sqrt{J_2} - \frac{\sigma_s}{\sqrt{3}} \tag{21}$$

where J_2 is the second invariant of stress deviator.

The normal triaxial creep experiments are conducted in pseudo-triaxial state (i.e., $\sigma_1 > \sigma_2 = \sigma_3$).

$$\begin{cases} \sigma_m = \frac{\sigma_1 + 2\sigma_3}{3} \\ S_{11} = \sigma_1 - \sigma_m = \frac{2(\sigma_1 - \sigma_3)}{3} \\ \sqrt{J_2} = \frac{\sigma_1 - \sigma_3}{\sqrt{3}} \end{cases} \tag{22}$$

When the Eqs. (20–22) are substituted into Eq. (15) and Eq. (19), we can deduce the creep deformation of visco-plastic body and total creep equation of rock under triaxial stress status.

$$\varepsilon_{ij}^{D}(t) = \frac{1}{M_0(1-D)} \tau \frac{(\sigma_1 - \sigma_3) - \sigma_s}{3} t \tag{23}$$

$$\varepsilon_{11} = \begin{cases} \frac{(\sigma_1 - \sigma_3)}{3G_0} + \frac{\sigma_1 + 2\sigma_3}{9K}\delta_{ij} + \frac{(\sigma_1 - \sigma_3) t^{\alpha(t)}}{3G_1\Gamma[1+\alpha(t)]} \\ \quad , \sigma_1 - \sigma_3 < \sigma_s \\ \frac{(\sigma_1 - \sigma_3)}{3G_0} + \frac{\sigma_1 + 2\sigma_3}{9K}\delta_{ij} + \frac{(\sigma_1 - \sigma_3) t^{\alpha(t)}}{3G_1\Gamma[1+\alpha(t)]} \\ \quad + \frac{(\sigma_1 - \sigma_3) - \sigma_s}{3M_0(1-D)/\tau} t, \sigma_1 - \sigma_3 \geq \sigma_s \end{cases} \tag{24}$$

where ε_{11} is the creep strain of rock in vertical direction.

Verification and analysis of the proposed model

Model validation and comparison

For verifying the validation and applicability of the proposed triaxial VOF damage creep model, the creep experimental data of sandstone in previous research is selected to validate and identify the proposed model (Liu et al. 2017). Creep experimental data is obtained under the confining pressure of 10 MPa and six kinds of vertical pressure, and the experimental creep curves are displayed in Fig. 5. In order to exhibit the advantages of VOF calculus in depiction of creep mechanical behavior, the COF damage model with same mechanical elements is selected as comparative example. It is different from the proposed VOF damage creep model that the Koeller dashpot within COF damage creep model is COF Koeller dashpot, whose order is a constant value.

As illustrated in Fig. 5(a–e), the decaying and steady creep deformation can be well described by the proposed VOF damage creep model, and experimental data is well agreement with the proposed VOF damage creep model. For better highlighting superiority of the proposed VOF damage creep model, COF creep damage model with same

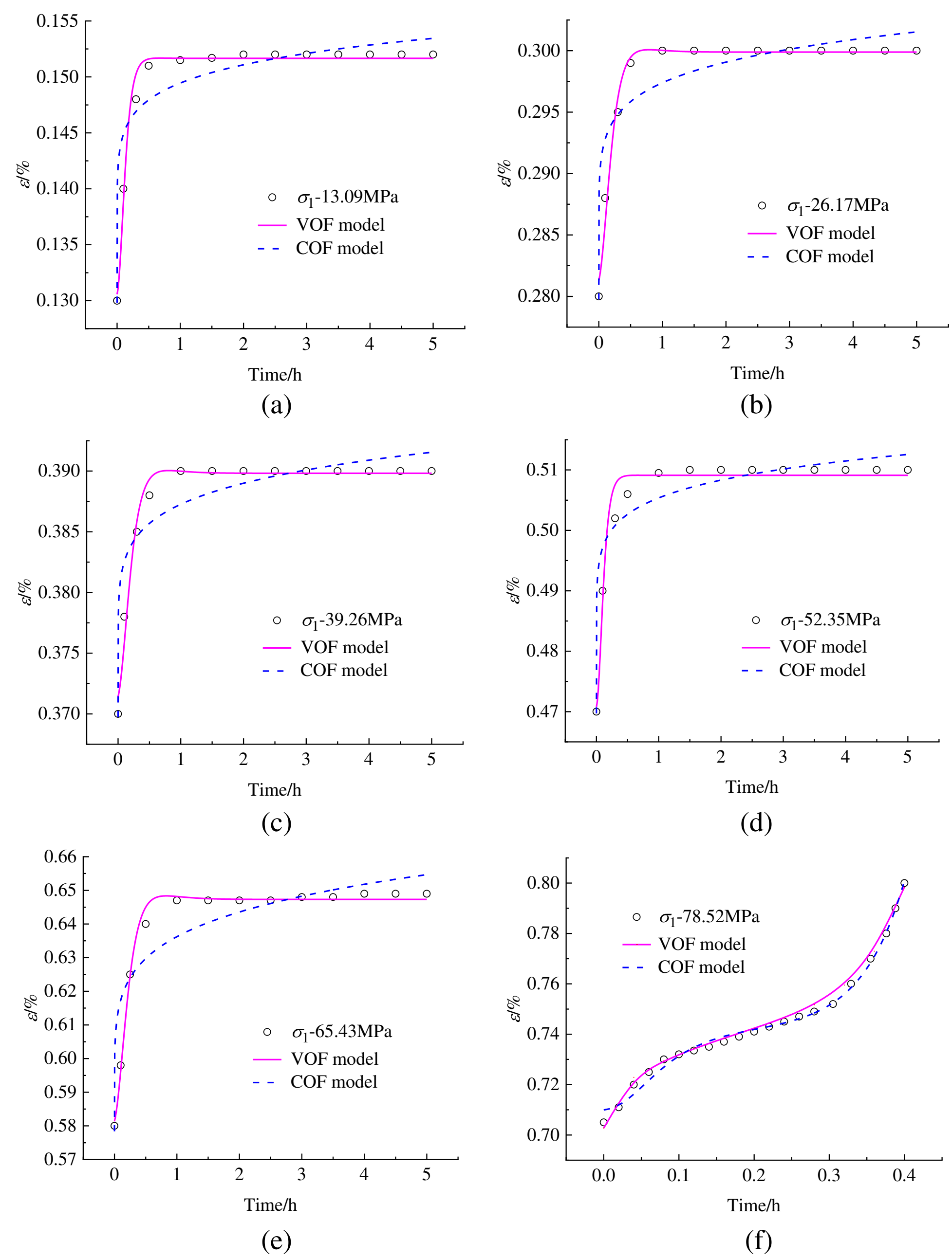

Fig. 5 Comparisons among the proposed VOF creep model, the COF creep damage model, and the creep experimental data of sandstone

Table 1 Fitting parameters of VOF damage creep model under triaxial status

σ_1/MPa	13.09	26.17	39.26	52.35	65.43	78.52
G_0/GPa	2.6260	3.1782	3.5564	3.9587	4.2563	4.9850
K/GPa	2.8157	2.5485	2.9900	3.5542	3.7280	4.4760
G_1/GPa • h^α	24.8424	28.0058	25.3360	26.8270	28.0104	35.040
τ/h	0.1292	0.2311	0.2430	0.1077	0.2669	0.0545
M_0/GPa • h^α	-	-	-	-	-	6.7661

mechanical elements is selected to compare, and it can be seen that COF creep damage model cannot well depict mechanical behavior of decaying and steady creep stage. Comparing with COF damage creep model, the proposed VOF damage creep model has obvious advantages, which also verifies the applicability of VOF in construction of creep model. In Fig. 5(f), when we concentrate on the accelerated creep stage, the VOF damage creep model and COF damage creep model both can well reflect the mechanical behavior, which may be same damage factor. But for the decaying creep stage, the proposed VOF damage creep model has more obvious advantages than COF damage creep model, which furtherly indicates the superiority of the proposed VOF damage creep model in description of creep behavior.

Analysis of sensitivity of mechanical parameters

As demonstrated in Eq. (20), the shear modulus G_0 and bulk modulus K of elastic body, shear modulus G_1 and relaxation time τ of dashpot are significant parameters in the description of the mechanical behavior of creep, which is shown in Table 1. To deeply reflect the influence of parameters on creep behavior, the experimental curve of sixth loading deformation is selected as the analyzed example, in this section, a series of analyses of the sensitivity of parameters will be conducted, whose results are displayed in Figs. 6, 7, 8, and 9.

It can be seen from Figs. 6 and 7, when corresponding to sixth creep experimental curve of sample, the shear modulus and bulk modulus are 4.985 and 4.476 GPa. With an increase in shear modulus and bulk modulus, the initial creep deformation of the sample will gradually rise down, which can be interpreted as the deformation induced by variations of shear modulus and bulk modulus following the linear stress–strain relationship (Wu et al. 2020a, b). The variations of both only change the initial point of the creep curve of the sample and have little influence on various tendencies of the creep curve. The creep curve controlled by shear modulus and bulk modulus of the elastic body is well consistent with experimental data.

Figure 8 presents that with rising of shear modulus G_1, the initial point of creep curve is still constant, and the total variations of creep gradually decrease. The variations of decaying and steady creep exhibit decreasing trend with an increase in shear modulus G_1, and that of the accelerated creep stage have none noticeable changes, which can be account for the viscoelastic deformation is the main part in decaying and steady creep deformation. It can be concluded

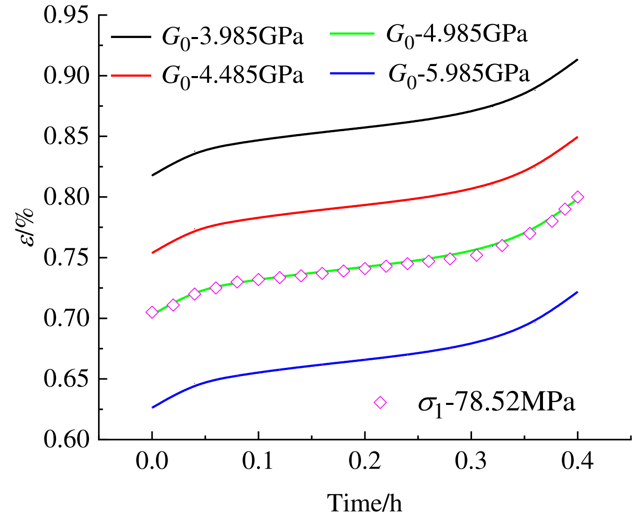

Fig. 6 Effect of shear modulus G_0 on experimental data

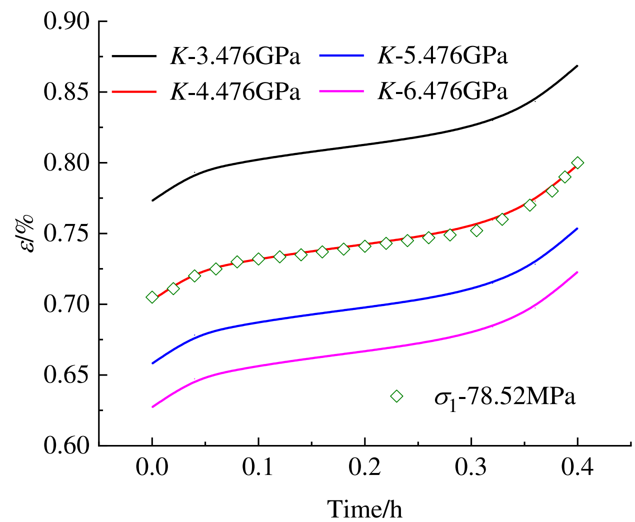

Fig. 7 Effect of volume modulus K on experimental data

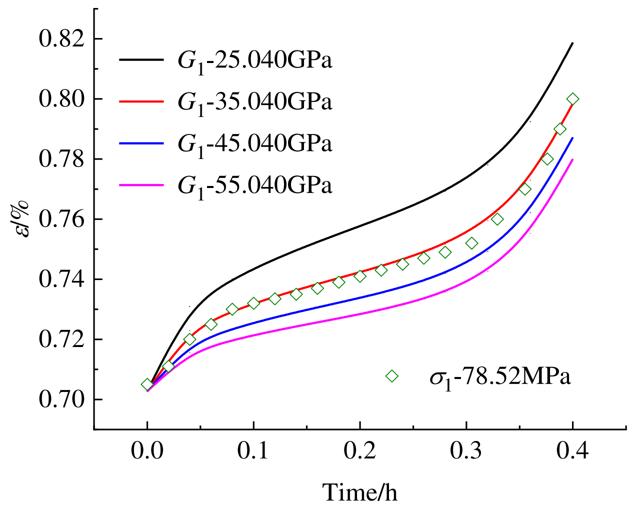

Fig. 8 Effect of shear modulus G_1 on experimental data

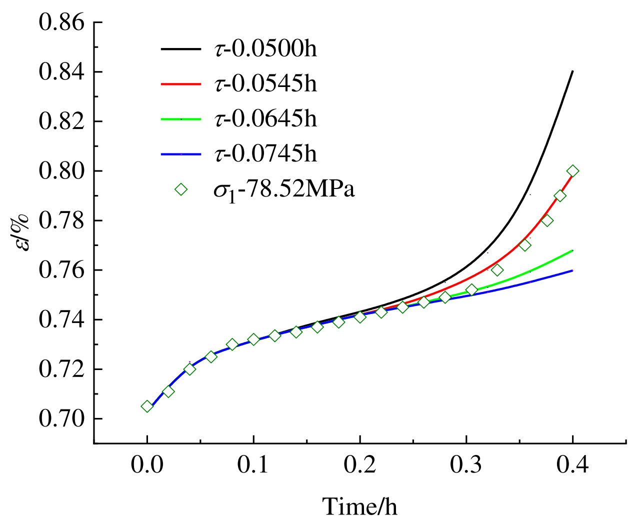

Fig. 9 Effect of relaxation time τ on experimental data

that the variation of shear modulus G_1 has no influence on development of the accelerated creep, whose role is same as non-Newtonian dashpot that playing role within element creep model.

Nevertheless, it is demonstrated in Fig. 9 that with an increase in relaxation time, the development of steady creep needs more time and the time point that the accelerated creep occurs is delayed, which is corresponding to when relaxation time approaches infinity, the creep deformation of material needs more time to consume viscosity of itself to enter the accelerated creep stage. We can see with decreasing of relaxation time, the variation of the accelerated creep gradually increases, and relaxation time is a significant parameter in controlling the development time of steady creep and sustainability time of the accelerated creep, which will provide valuable references in study of effect of relaxation time within creep deformation.

Conclusions

(1) By introducing Caputo VOF derivative with convolution form, we proposed a new VOF Koeller dashpot, and in conjunction with elastic element, the VOF Maxwell model was constructed. Based on the importance of relaxation time, varying-order function and damage factor related to relaxation time were also constructed, then a novel VOF damage creep model was presented.
(2) Based on the uniaxial VOF damage creep model, considering generalized plastic mechanics, the triaxial VOF damage creep model was set. By applying triaxial creep experimental data of sandstone, the validation of the set triaxial VOF damage creep model has obtained verification.
(3) To better highlight the superiority of VOF derivative, a comparative study between the VOF and COF damage creep model was performed, and the few parameters and clear physical interpretation were obvious. The influence of shear modulus, bulk modulus, and relaxation time on creeping mechanical behavior has been analyzed and discussed, which provides a deep understanding of the deformation mechanism of creep.

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Declarations

Conflict of interest The authors declare that they have no competing interests.

References

Adeli E, Rosi B, Matthies HG et al (2020) Comparison of Bayesian methods on parameter identification for a viscoplastic model with damage. Probab Eng Mech 62(1–2):103083
Adeli E, Matthies HG (2019) Parameter identification in viscoplasticity using transitional Markov chain Monte Carlo Method. http://arxiv.org/abs/1906.10647
Adeli E, Rosic B, Matthies HG et al (2017) Bayesian parameter identification in plasticity. Complas
Almeida R, Malinowska AB, Monteiro MT (2018) Fractional differential equations with a Caputo derivative with respect to a kernel function and their applications. Math Meth Appl Sci 41:336–352
Cai MF, He MC, Liu DY (2009) Rock mechanics and engineering. Science Press, Beijing, pp 198–219

Chen BR, Zhao XJ, Feng XT, Zhao HB, Wang SY (2014) Time dependent damage constitutive model for the marble in the Jinping II hydropower station in China. Bull Eng Geol Environ 73:499–515

Coimbra CFM (2010) Mechanics with variable-order differential operators (p692–703). Ann Phy 12(11–12):692–703

Ingman D, Suzdalnitsky J (2005) Application of differential operator with servo-order function in model of viscoelastic deformation process. Eng Mech 131(7):763–767

Ingman D, Suzdalnitsky J, Zeifman M (2000) Numerical simulations for the space-time variable order nonlinear fractional wave equation. J Appl Mech 67:383–395

Koeller RC (1984) Applications of fractional calculus to the theory of viscoelasticity. J Appl Mech 51(2):299–307

Liu XL, Li DJ (2020) A link between a variable-order fractional Zener model and non-Newtonian time-varying viscosity for viscoelastic material: relaxation time. Acta Mech 2:1–13

Liu DY, Xie LJ, Tuo XF (2017) Creep properties of sandstone under different confining pressures and research on a nonlinear viscoelasto-plastic creep model. Chine J Rock Mech Eng 36(S2):3705–3712 ((**in Chinese**))

Liu XL, Li DJ, Han C (2020) A nonlinear damage creep model for sandstone based on fractional theory. Arab J Geosci 13(6):246–254

Liu XL, Li DJ, Han C, Shao YM (2021) A Caputo variable-order fractional damage creep model for sandstone considering effect of relaxation time. Acta Geotech 168:1–15

Lorenzo CF, Hartley TT (2002) Variable order and distributed order fractional operators. Nonlinear Dyn 29(1):57–98

Maranini E, Yamaguchi T (2001) A non-associated viscoplastic model for the behavior of granite in triaxial compression. Mech Mate 33(5):283–293

Sun J (1999) Rheology and engineering application of geotechnical material. China Architecture & Building Press, Beijing

Sun HG, Chen W, Chen YQ (2009) Variable-order fractional differential operators in anomalous diffusion modeling. Journal of Physics A 338(21):4586–4592

Sun HG, Chen W, Wei H (2011) A comparative study of constant-order and variable-order fractional models in characterizing memory property of systems. Eurp Phy J Spec Topic 193(1):185–192

Tang H, Wang D, Huang R (2018) A new rock creep model based on variable-order fractional derivatives and continuum damage mechanics. Bull Eng Geol Environ 77(375):375–383

Wu F, Chen J, Zou QL (2019) A nonlinear creep damage model for salt rock[J]. Int J Damage Mech 28(5):758–771

Wu F, Gao RB, Liu J (2020a) New fractional variable-order creep model with short memory. Appl Math Comput 380:125278

Wu F, Zhang H, Zou QL, Li C, Chen J, Gao RB (2020b) Viscoelastic-plastic damage creep model for salt rock based on fractional derivative theory. Mech Mate. https://doi.org/10.1016/j.mechmat.2020.103600

Xia CC, Xu CB, Wang XD (2009) Method for parameters determination with unified rheological mechanical model. Chine J Rock Mech Eng 28(002):425–432

Xu WY, Yang SQ, Chu WJ (2007) Nonlinear viscoelasto-plastic rheological model (Hohai model) of rock and its engineering application. Chine J Rock Mech Eng 3:433–447

Yin D, Zhang W, Chen GC (2012) Fractional time dependent Bingham model for muddy clay. J Non-Newton Fluid Mech 187(188):32–35

Zheng YR, Kong L (2005) Generalized plastic mechanics and its application. Engineering Science 7(11):21–36 ((**in Chinese**))

Zhou HW, Wang CP, Han BB (2011) A creep constitutive model for salt rock based on fractional derivatives. Int J Rock Mech Min Sci 48(1):116–121

Zhou HW, Liu D, Lei G (2018) The creep-damage model of salt rock based on fractional derivative. Energies 11(9):342–349