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COMMENT

Revisit the rational method for flood estimation in the Saudi arid environment

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Abstract

Arid regions are suffering from the availability of streamflow data for reliable design of hydraulic structures. The rational method is a common method for peak discharge (Q_p) estimation in the design of water structures since it is easy and needs a few parameters. However, the method is restricted to small basins of size less than 5 km^2. Concerning such method, many objectives have been investigated in the current study. Two of the main targets are the validity of the application of the rational method for large watersheds in dry regions. Moreover, the method is extended to calculate the runoff volume (V). It has been found that the method can be applied for large basins that vary between 170 and 4930 km^2. First-order-second moment sensitivity analysis is utilized to derive analytical expressions to relate the variability in Q_p and V as a function of the variability of the runoff coefficient, rainfall intensity or depth, and basin area. Five basins with 19 subbasins in the southwestern of Saudi Arabia are analyzed with 160 storms recorded in the period (1984–1987). The design duration of the rainfall intensity that is used in the rational formula should be estimated based on the time of concentration (*tc*) calculated from the equation developed by (Albishi et al., Arab J Geosci 10:1–13, 2017) and not from (Kirpich, Civ Eng 10:362, 1940) as commonly used in the literature and Kingdom of Saudi Arabia flood studies. The former provides a minimum RMSE of 87.66 m^3/s, while the latter has a RMSE of 168.41 m^3/s. This suggests the use of (Albishi et al., Arab J Geosci 10:1–13, 2017) *tc* equation for a safer design, especially in this region and regions alike. The log-normal distribution fits well the hydrological variables based on the Kolmogorov–Smirnov test. Therefore, it can be utilized for the flood uncertainty analysis of these basins and similar ones. The first-order analysis shows quite reliable results since the variability of Q_p (σ_{lnQ}) and V (σ_{lnV}) of the data (1.21 and 1.24), respectively, are pretty close to the developed expressions (1.3 and 1.24).

Keywords Rational method · Peak discharge · Arid · Saudi Arabia · Flood · Time of concentration

Introduction

The frequency of floods in the Kingdom of Saudi Arabia (KSA) has increased due to climate change and was proven by Almazroui (2013). Such change led to huge floods resulted in heavy losses in property and lives. This prompted the government to impose flood studies as a pre and basic requirement for any construction work, regardless of its size.

On the other hand, in response to the 2030 Plan in KSA, the vast, rapid, and intense development in the kingdom is currently ongoing. New, vast expansion areas developed. New roads have been constructed. Transferring flood water in safe ways across these new roads entails the need for design flood mitigation works, particularly, hydraulic road crossing structures.

Moreover, Saudi Arabia lies in arid regions. It is well known that arid area suffers from variability in rain and losses, reduction in vegetation, and high erosion rate. Zero flows dominate in stream networks during most of the days in the year (Farquharson et al. 1992; Cordery and Fraser 2000). Infrequent floods usually occur as a result of storms of high intensity over a smaller part of the catchment (Elfeki et al. 2014). The variability of the

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floods is enormously changing from year to year and from site to site. Consequently, these regions undergo a shortage of high-quality streamflow data and lack of broad flood estimation methods (Nemec and Rodier 1979; McMahon 1979). Thus, the use of simple design methods such as rational method for estimating floods became a necessity. The use of design methods that requires a large amount of data is not feasible and unpracticable (Pilgrim et al. 1988). Many studies use the rational method, due to its simplicity and its dependence on little data. Only the runoff coefficient and the storm intensity are the parameters used in such method.

However, this method is known to be used only for small watershed areas (Chow et al. 1988). It is only estimated the maximum peak discharge and does not give any idea about the volume of the flood. When applying this method in ungauged areas, the values of the formula parameters' were extracted from textbooks, which is frequently derived from the humid regions of the world due to the accessibility of recorded runoff data with accompanying spatial coverage and temporal distributions. In arid regions, the availability of such data is usually uncommon. Pilgrim and Cordery (1993) investigated the applicability of the rational method for design purposes. Design values of the key parameter of the rational formula (runoff coefficient) were derived for 105 small agricultural catchments in and around southeast Queensland, Australia. The derived values were significantly diverse from conventional textbook values; the latter gave inaccurate estimates of design runoff and flood peaks. The extent to which design values of the key parameters represent the conditions of KSA and the similar dry regions has not been investigated.

Therefore, reconsider the rational method for flood estimation application in the arid area is of utmost importance. The main aims of this study are to answer the following research questions:

- Is the rational method valid for large watersheds in dry regions? and to what extent is valid for the application?
- How can one develop the method to calculate the volume of the flood?
- How long is the storm duration suitable for use? What is the relationship between the time of concentration and the duration of the storm?
- What is the best equation to estimate the time of concentration to validate the peak flow?
- What are the appropriate statistical distributions for the variables of the equation? What is the degree of sensitivity of these variables to the output of the equation?
- How far is the actual rainfall intensity of storm event from the design intensity–duration–frequency curves (IDF) in a certain region? What is the corresponding return period of the actual events that caused floods under different estimation methods of the time of concentration?

Relying on the measured rainfall and runoff events in the Saudi arid environment, the method is reevaluated. 158 rainfall-runoff storms, recorded in the period (1984–1987) at 19 subbasins in the southwestern of Saudi Arabia for five big catchments, have been used to re-assess the rational method.

Results show that the method can be applied for large watersheds vary between 170 and 4930 km^2. The method is not restricted to estimate peak discharge (Q_p) but also extended to estimate the runoff volume (V) as well. The variability in Q_p and V as a function of the variability of the runoff coefficient, rainfall intensity or depth, and basin area was derived via a first-order-second moment (FOSM) sensitivity analysis. A new appropriate design duration of the rainfall depth has been defined. The log-normal distribution was found that fits well the hydrological variables based on the Kolmogorov–Smirnov (K-S) test.

The results of this study will save a lot of cash being spent on overestimated flood protection works. A very high factor of safety usually has been chosen because of a lack of a reliable factor of safety that is derived from typical equations dedicated for arid conditions of Saudi Arabia. They will be of great importance to all workers in flood control works either in the governmental and/or private organizations and agencies in Saudi Arabia and similar regions.

Study area

The KSA climate is classified as arid desert, that is hot in most of the country and equatorial desert in the northern part (Kottek et al. 2006). In the day times, the temperature is very high in summer (> 45 °C) and encompasses around > 25 °C in winter. During the night, the temperature descents abruptly to < 20 °C. throughout the year. Rainfall is scarce and irregular, either in space or in time (Sharon 1972). So, rainfall–runoff measurements of good quality for a long period were not inaccessible as frequently occurred in most arid and semi-arid regions.

Though, five wadis in the southwestern Saudi Arabia are considered as representative catchments for collecting extensive data by the Ministry of Environment, Water, and Agriculture (MEWA), Riyadh, in 1983. Saudi Arabian Dames and Moore company was asked to carry intensive hydrological study, surface and ground water, for 4 years. The data of this study was collected in Saudi Arabian Dames and Moore (1988) reports. The reports include measurement of all hydrological parameters, rainfall, surface, soils, and groundwater. From these data, we are interested, in this paper, to consider the data for the rainfall storms that caused runoff hydrographs.

Extensive rain gauges (100 gauge) and water level recorders (19) are installed. The number of rain gauges per basin varies from 10 gauges in Wadi Liyyah to 33 in Wadi Habawnah (Wheater et al. 1991a). Spacings between rain gauges range between 8 and 20 km. Water level recorders are used to calculate runoff at their locations. The number of gages per sub-basins ranges between 4 and 6 recorders.

The five basins originate along the Asir escarpment (up to 3000 m a.s.l. elevation) and cover a wide range of altitude. Their areas range between 456 (Wadi Liyyah) and 4930 km^2 (Wadi Habawnah). Two of these wadis drain to the interior, towards the Rub al Khali, or "empty quarter," and three (A1 Lith, Yiba, and Liyyah) drain towards the Red Sea. The gradient of the basins elevations coincides with the gradient in annual rainfall, from 450 mm at elevations above 2000 m a.s.l. to 30–100 mm on the Red Sea coastal plain (Wheater et al. 1991a). Locations of the five representative basins and sub-catchments are shown in Fig. 1. Extensive data regarding the basins such as the catchment areas, recorded events period and date, no. of records, and maximum peak discharge (Q_p) are given by Ewea et al. (2020).

Features of the data

Rainfall occurs regionally in all months. Three main periods had the highest frequency of occurrence, in spring (April/May), in autumn (September), and in winter (November/December). The rainfall displays the features of convective storms pattern: short duration, high intensities, and a high degree of spatial variability. Such type is consistent with the properties of thunderstorms as investigated in detail by Eagleson et al. (1987).

Continually, rainfall is highly localized. The majority of point rainfall duration had 1-h duration or less (Wheater et al. 1991b). Occasionally individual gauges typically record 1 or 2 h of rainfall. Very occasionally (once or twice per year) more widespread rainfall happens, observed particularly in Wadi Habawnah.

Two types of data were used in the current study: rainfall-runoff data of the five representative catchments shown in Fig. 1 and historical records of autographic rain gauges set up by MEWA in Table 1.

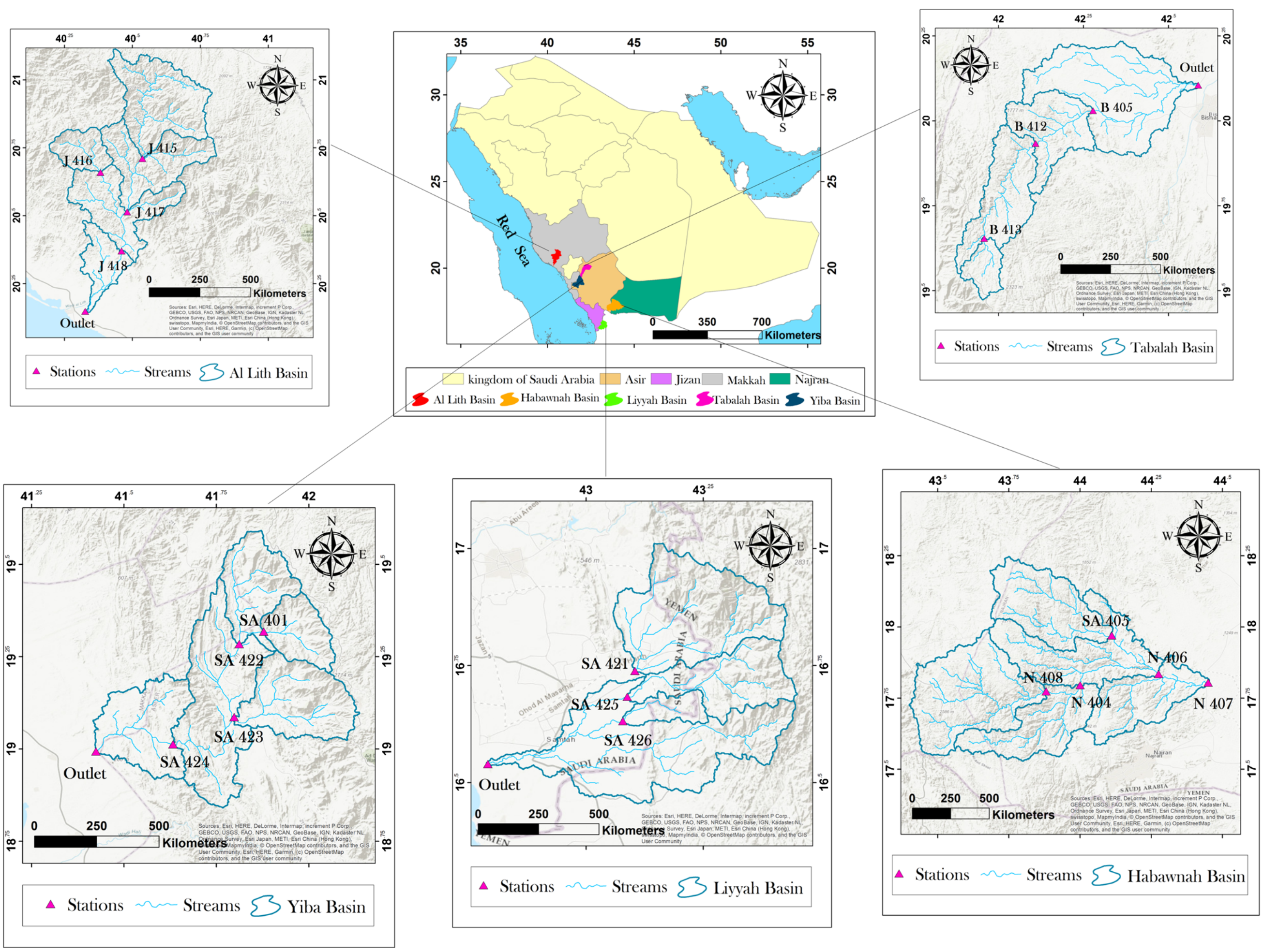

Fig. 1 Map of the location of the study area, the basins, and their subbasins at the runoff stations

Table 1 Rainfall stations form the Ministry of Environment, Water and Agriculture located (MEWA) in the vicinity of the study basins

Zone	Station number	Station name	Station symbol	Recorded storms		Coordinates		Total number of storms
				From	To	Longitude	Latitude	
Tabalah	67	BIiljorshi	B 007	1975	2002	41°33′00″	19°52′00″	27
Habwanah	405	Najran	N 001	1975	1999	44°1539″	17°34′00″	23
Liyyah	496	Malaki	SA 001	1975	2003	42°57′00″	17°03′00″	23
Yiba	498	Kwash	SA 003	1975	2003	41°53′00″	19°00′00″	22
Lith	625	Hema Saysid	TA 002	1975	2000	40°30′00″	21°18′00″	27
Total								122

Measurements of the storms in the representative basins include rainfall storms and its resulted runoff. The location of the 19 flow measuring stations on the five representative basins is shown in Fig. 1. Measurements of (a) rainfall depth in mm, (b) average rainfall intensity over the storm duration in mm/h, (c) peak discharge in m^3/s, and (d) runoff volume in Mm^3 for the period (1984–1986) are shown in Figs. 2 and 3. Although the five representative basins have only 4 years of data, 160 records have been recorded.

Another type of historic rainfall records that lies in the vicinity of the five representatives is used. Storms' number varied between 15 and 28 storms/stations. Locations of rainfall stations are shown in Fig. 4. Annual maximum rainfall depths from 1975 to 2003 with time intervals (10, 20, 30, 60, 120 min, etc.) were used to develop IDF curves. IDF equations developed by Ewea et al. (2016) are shown in Table 2.

Methodology

In this section, the equations used to achieve the goals of this research are presented. The rational method equation in terms of discharge is presented; then, the derivation of the rational method in terms of volume is obtained. The application of the first-order-second moment sensitivity analysis of the rational equations is derived. The equations for the estimation of the time of concentration are also presented.

The rational method for peak discharge and runoff volume

The traditional rational method is given by the formula (Chow et al. 1988).

$$Q_p = Ci_T A \tag{1}$$

where

Qp is the peak discharge,
C is the runoff coefficient,
i_T is the rainfall intensity at return period T, and
A is the basin area.

The traditional rational method formula can be extended to read a more general formulation as (see Fig. 5 for illustration of the conceptual model),

$$Q(t) = Ci(t)A \tag{2}$$

where

$Q(t)$ is the discharge at any time t on the hydrograph, and
$i(t)$ is the rainfall intensity that corresponds to Q(t).

since $i(t)$ can be expressed mathematically as

$$i(t) = \frac{dr(t)}{dt} \tag{3}$$

where

$r(t)$ is the rainfall depth at time t in the storm.

Therefore, we can write

$$Q(t) = C\frac{dr(t)}{dt}A \tag{4}$$

Consequently,

$$\int_0^{T_B} Q(t)dt = C\int_0^{R} drA \tag{5}$$

where

R is the total rainfall of the storm, and
T_B is the time base of the hydrograph.

By definition of the runoff volume, V, that is equal to the area under the hydrograph, one may replace the right-hand side of the integral by V to get the final version of the formula in terms of runoff volume as

$$V = CRA \tag{6}$$

This leads us to the conclusion that the rational method can be used to estimate runoff volume and not only peak discharge.

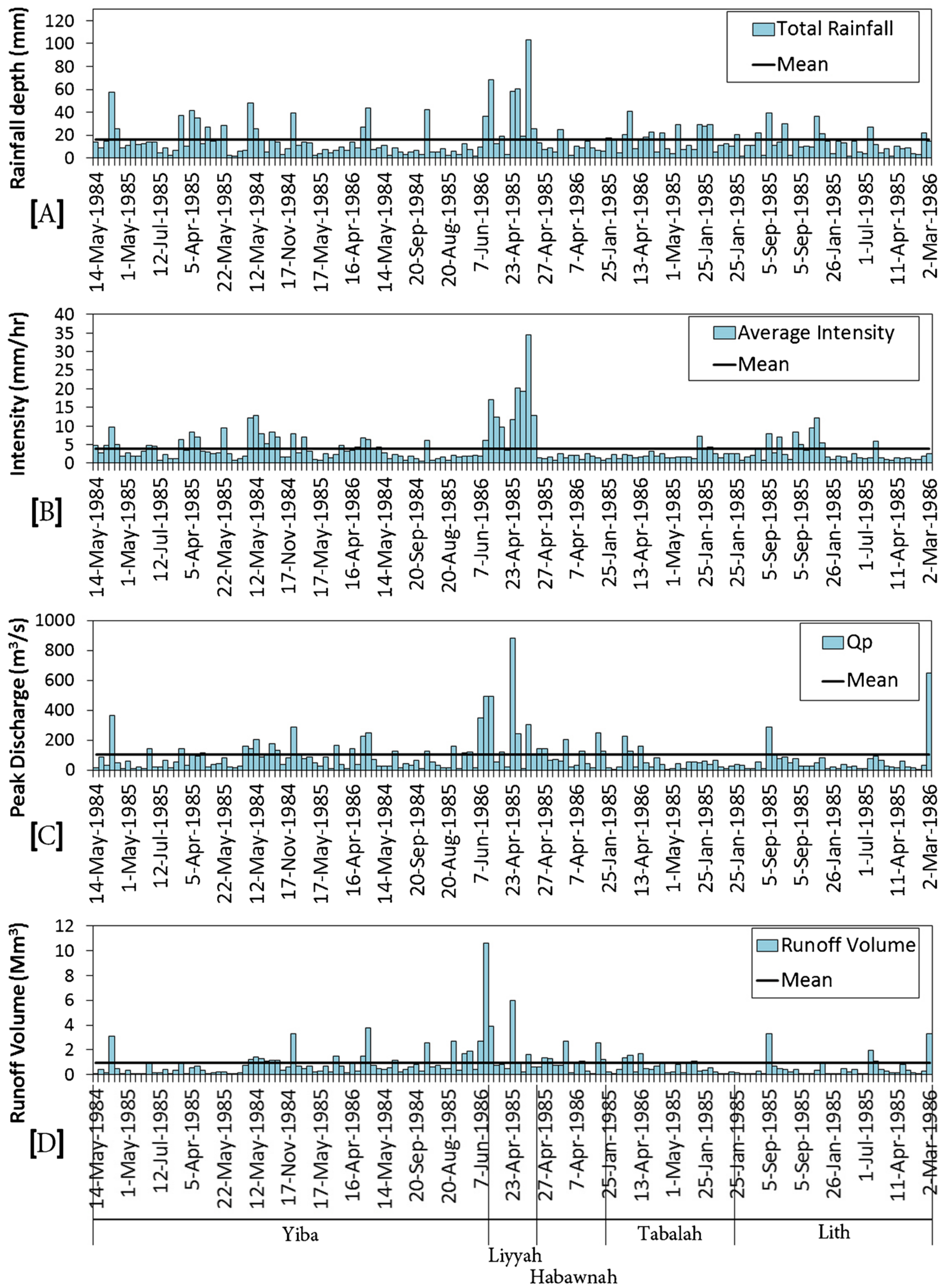

Fig. 2 Measurements of the storms in the basins (1984–1986): **a** rainfall depth in mm, **b** average rainfall intensity over the storm duration in mm/h, **c** peak discharge in m^3/s, and **d** runoff volume in Mm^3. Note that an extreme value of 3219.65 m^3/s in Yiba catchment is omitted from the graph and its corresponding values to be able to visualize other data

First-order-second moment sensitivity analysis of the equations of the rational method

The first-order second-moment (FOSM) method is widely used in uncertainty analysis (Maskey 2003). This method uses a linearization of the function that relates the input variables and parameters to the output variables.

Since the variability of the parameters of the rational method is relatively high (see Figs. 2 and 3), it is recommended to work with the logarithms of the parameters. Therefore, the rational method equation can be written in a functional relationship based on the logarithms as

$$lnQ = f(lnC, lni, lnA) = lnC + lni + lnA \tag{7}$$

Using Tylor series expansion, one may write the equation in terms of the mean value of $\langle lnQ \rangle = f(\langle lnC \rangle, \langle lni \rangle, \langle lnA \rangle)$ as

$$\begin{aligned} lnQ = {} & f(\langle lnC \rangle, \langle lni \rangle, \langle lnA \rangle) + \frac{\partial f}{\partial lnC}(lnC - \langle lnC \rangle) + \frac{\partial f}{\partial lni}(lni - \langle lni \rangle) + \frac{\partial f}{\partial lnA}(lnA - \langle lnA \rangle) \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial lnC \partial lni}(lnC - \langle lnC \rangle)(lni - \langle lni \rangle) + \frac{1}{2}\frac{\partial^2 f}{\partial lnC \partial lnA}(lnC - \langle lnC \rangle)(lnA - \langle lnA \rangle) \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial lnA \partial lni}(lnA - \langle lnA \rangle)(lni - \langle lni \rangle) + h.o.t \end{aligned} \tag{8}$$

where

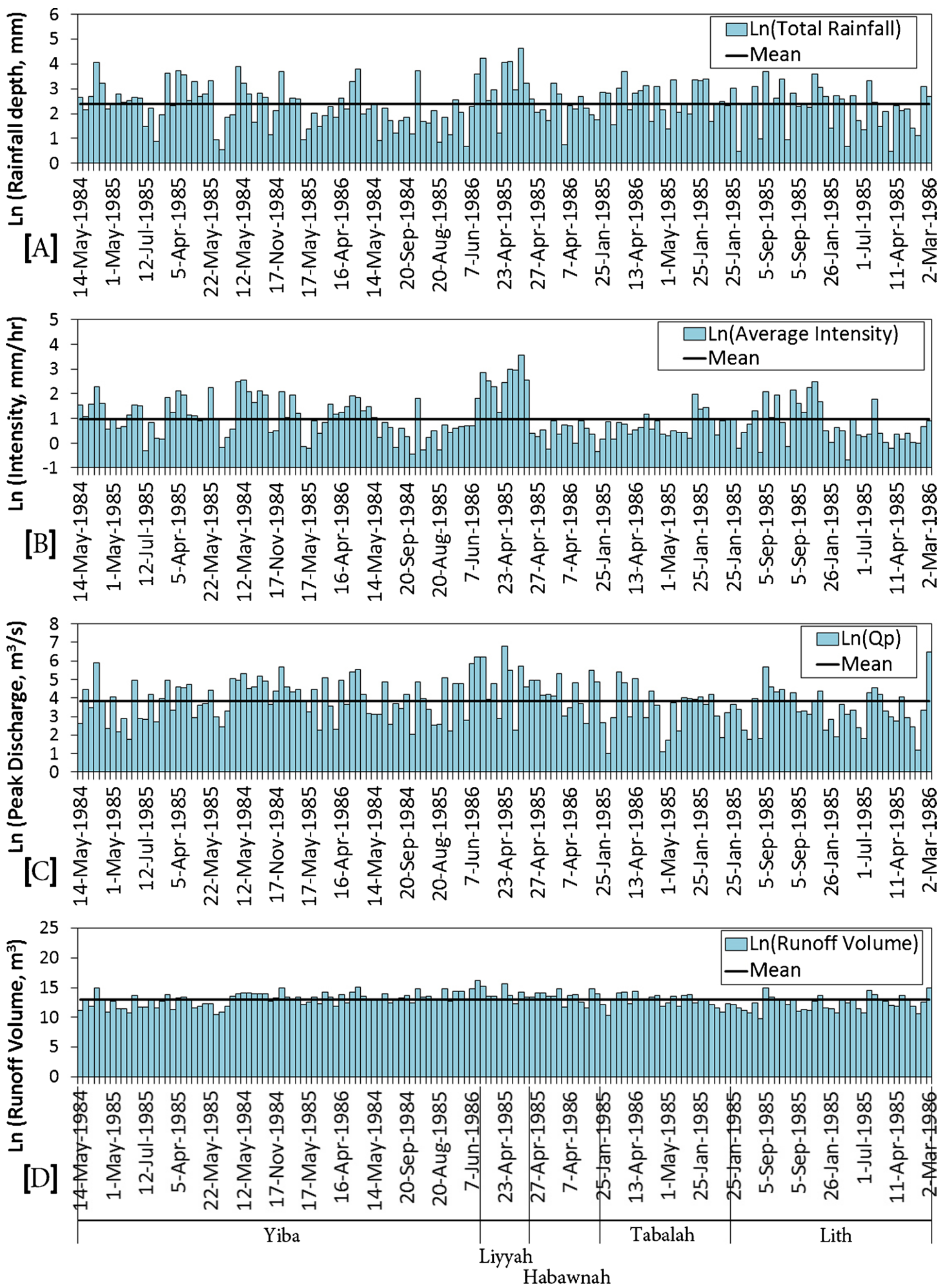

Fig. 3 Logarithms of the measurements of the storms in the basins: **a** rainfall depth in millimeter, **b** average rainfall intensity over the storm duration in millimeter per hour, (c) peak discharge in cubic meter per second, and (d) runoff volume in cubic meter

$\langle lnC \rangle$ is the angle bracket used to describe the mean of the parameter, and

h.o.t represents the higher-order terms in the Tylor series.

Neglecting the h.o.t. and rearranging, one may obtain

$$\begin{aligned} lnQ - \langle lnQ \rangle = {} & \frac{\partial f}{\partial lnC}(lnC - \langle lnC \rangle) + \frac{\partial f}{\partial lni}(lni - \langle lni \rangle) \\ & + \frac{\partial f}{\partial lnA}(lnA - \langle lnA \rangle) + \frac{1}{2}\frac{\partial^2 f}{\partial lnC \partial lni}(lnC - \langle lnC \rangle)(lni - \langle lni \rangle) \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial lnC \partial lnA}(lnC - \langle lnC \rangle)(lnA - \langle lnA \rangle) \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial lnA \partial lni}(lnA - \langle lnA \rangle)(lni - \langle lni \rangle) \end{aligned} \tag{9}$$

The variance of the logarithm of the discharge, σ^2_{lnQ}, is expressed as

$$\sigma^2_{lnQ} = E\left[(lnQ - \langle lnQ \rangle)^2\right] \tag{10}$$

where

E[] is the expected value operator.

Substituting Eq. 9 into Eq. 10 leads to

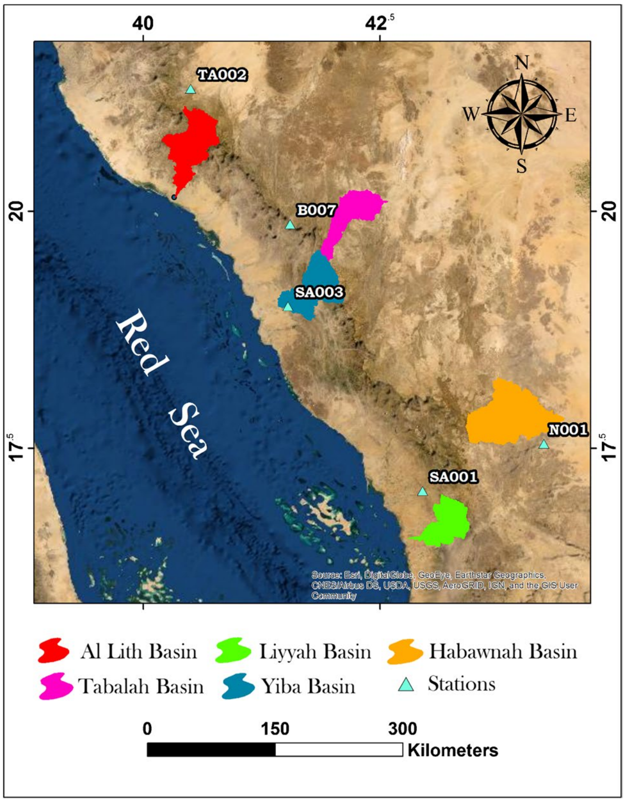

Fig. 4 Locations of the rainfall stations form the Ministry of Environment, Water and Agriculture for IDF curves (Ewea et al. 2016) in the vicinity of the study basins

$$\sigma^2_{\ln Q} = E\left[\left\{\begin{array}{l} \frac{\partial f}{\partial lnC}(ln\,C - \langle ln\,C\rangle) + \frac{\partial f}{\partial\,ln\,i}(ln\,i - \langle ln\,i\rangle) \\ + \frac{\partial f}{\partial lnA}(ln\,A - \langle ln\,A\rangle) + \frac{1}{2}\frac{\partial^2 f}{\partial\,ln\,A\partial\,ln\,i}(ln\,C - \langle ln\,C\rangle)(ln\,i - \langle ln\,i\rangle) \\ + \frac{1}{2}\frac{\partial^2 f}{\partial ln\,C\partial\,ln\,A}(ln\,C - \langle ln\,C\rangle)(ln\,A - \langle ln\,A\rangle) \\ + \frac{1}{2}\frac{\partial^2 f}{\partial\,ln\,A\partial\,ln\,i}(ln\,A - \langle ln\,A\rangle)(ln\,i - \langle ln\,i\rangle) \end{array}\right\}\right] \quad (11)$$

Assuming that C, i, and A are not independent, one may write Eq. 11 into

$$\sigma^2_{\ln Q} = \left(\frac{\partial E[f]}{\partial lnC}\right)^2 \sigma^2_{lnC} + \left(\frac{\partial E[f]}{\partial lni}\right)^2 \sigma^2_{lni} + \left(\frac{\partial E[f]}{\partial lnA}\right)^2 \sigma^2_{lnA} + \left(\frac{\partial^2 E[f]}{\partial lnC\partial lni}\right)^2 \sigma_{lnClni} + \left(\frac{\partial^2 E[f]}{\partial lnC\partial lnA}\right)^2 \sigma_{lnClnA} + \left(\frac{\partial^2 E[f]}{\partial lnA\partial lni}\right)^2 \sigma^2_{lnAlni} \quad (12)$$

Since $E[f]$ can be expressed in terms of the mean of the parameters, then substituting of the function of the mean and calculating the derivatives in Eq. 12, one may obtain

$$\sigma^2_{lnQ} = \sigma^2_{lnC} + \sigma^2_{lni} + \sigma^2_{lnA} + 2\left(\sigma^2_{lnClni} + \sigma^2_{lnClnA} + \sigma^2_{lnAlni}\right) \quad (13)$$

Similarly, one may obtain the variance for the logarithms of the volume as

$$\sigma^2_{lnV} = \sigma^2_{lnC} + \sigma^2_{lnR} + \sigma^2_{lnA} + 2\left(\sigma^2_{lnClnR} + \sigma^2_{lnClnA} + \sigma^2_{lnAlnR}\right) \quad (14)$$

It can be noticed from Eqs. 13 and 15 that the variance of the logarithm of the discharge or the logarithm of the volume is the sum of the variance of the logarithms of the parameters C, i, R, and A and their covariances.

Estimation of storm duration based on the time of concentration

Kirpich (1940) developed an equation for the time of concentration (t_c) based on geomorphological parameters of the basin. This equation is given by

$$t_c = 0.0663\left(\frac{L}{\sqrt{S}}\right)^{0.77} \quad (15)$$

where

t_c is in hours,
L is the length of the channel from headwater to the outlet in kilometer, and
S is the average watershed slope.

Albishi et al. (2017) developed an equation for the time of concentration from field measurements (Ari-Zo model) for two watersheds in the Makkah region (Al-Lith and Yiba basins and their sub-basins) in the form given by

$$t_c = \frac{L^{0.09}}{S^{0.11}} \quad (16)$$

Both equations are used in the current analysis to see the suitability of these equations to estimate the storm duration needed for the calculation of rainfall intensity and therefore

Table 2 The derived IDF equations from the rainfall stations of the Ministry of Environment, Water and Agriculture (Ewea et al. 2016)

Station symbol	IDF Eqn. parameters			
	a	R^2	b	R^2
B 007	$a = 208.37\ln(T_r) + 263.53$	0.9985	$b = -5\text{E}-04\ln(T_r) - 0.6927$	0.7479
N 001	$a = 100.6\ln(T_r) + 147.79$	0.9987	$b = 0.005\ln(T_r) - 0.7662$	0.8324
SA 001	$a = 192.33\ln(T_r) + 389.38$	0.9986	$b = -0.002\ln(T_r) - 0.7576$	0.8971
SA 003	$a = 132.83\ln(T_r) + 775.06$	0.9994	$b = 0.0147\ln(T_r) - 0.8473$	0.9238
TA 002	$a = 151.02\ln(T_r) + 263.5$	0.9989	$b = 0.0071\ln(T_r) - 0.743$	0.8368

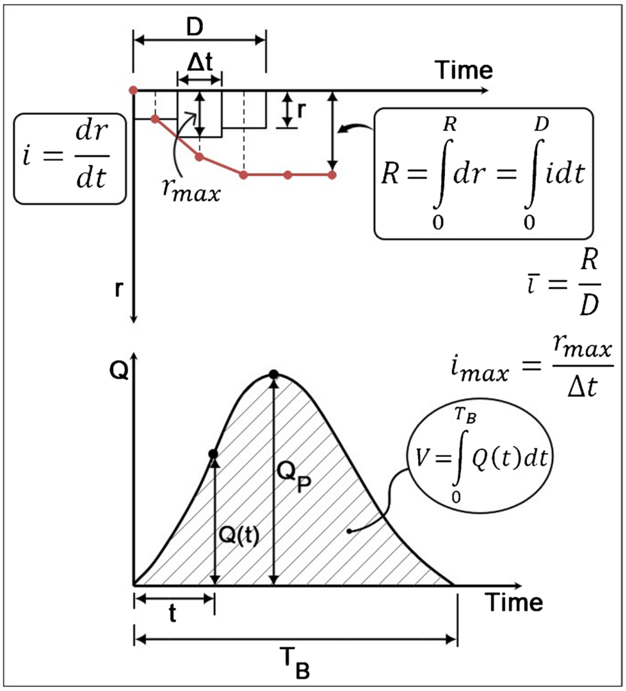

Fig. 5 Conceptual model of the rainfall depth, rainfall intensity, discharge, and runoff volume in the discharge-based and the volume-based rational methods

for application in the rational equation to evaluate the peak flow and runoff volume.

Results and discussions

The following sections discuss the results of the current study. These sections are as follows: (1) "Statistical analysis of the rational method parameters," (2) "Results of the first-order-second moment sensitivity analysis," (3) "Comparison of estimation methods of rainfall intensity for the rational method," and (4) "Comparison between calculated and observed peak flows."

Statistical analysis of the rational method parameters

Table 3 describes the summary of the statistics of the parameters in the rational method equation. The table shows the general statistical descriptors such as the mean, standard deviation (SD), coefficient of variation (CV), skewness, and kurtosis (kurt). The results show that the maximum CV is from the peak discharge (CV = 2.65), while the minimum CV is for the storm duration (CV = 0.64). However, in general, the CV is relatively high (> 0.5). This indicates high variability in the parameters of the rational method. Therefore, it is recommended to study the variability of the transformed parameters. The logarithmic transformation is recommended. It has been applied in the first-order sensitivity analysis in the methodology section. The table shows high skewness coefficients of the parameters. The skewness coefficient is positive (skew > 0) indicating that the peak of the distribution is oriented towards the left side (i.e., towards the low parameter value and having a long tail). The maximum skewness is for the peak discharge (9.58), while the minimum skewness is for the area of the basins (1.39). In terms of kurtosis, all the parameters are leptokurtic (kurt > 3) except the area which is platykurtic (kurt < 3). The maximum kurt is 105.95 for the peak discharge indicating a highly peaked distribution, while the minimum is for the basin area which is 2.17 for flat distribution. The aforementioned statistical descriptors provide a quick quantitative assessment of the distributions of the rational method parameters and could provide some indication about the shape of the distribution. However, for testing the best distribution that could fit the data, a test statistic is needed. Therefore, Fig. 6 and 7 show the fitting of different probability density functions (pdf) and the cumulative distribution function (CDF) to the rational method parameters. The common tested distributions are Gaussian, log-normal, gamma, beta, exponential, and Gumbel distributions. The Kolmogorov–Smirnov (K-S) test (Smirnov 1948) is used to test the best distribution to the data. The results of the test

Table 3 Summary of the statistical parameters of the subbasins

Parameter	Mean	SD	CV	Skew	Kurt
Area (km^2)	1233.03	957.91	0.78	1.39	2.17
Runoff coefficient, *C*	0.07	0.07	0.99	2.12	5.51
Rainfall depth, *R* (mm)	1.18	3.01	2.55	8.43	83.95
Rainfall intensity, *i* (mm/h)	3.95	4.81	1.22	3.56	16.59
Storm duration, *D* (h)	4.96	3.17	0.64	1.93	5.61
Peak discharge, Q_p (m^3/s)	103.93	275.40	2.65	9.58	105.95
Runoff volume, *V* (m^3)	923,985.78	1,920,805.42	2.08	7.11	62.86

SD is the standard deviation, CV is the coefficient of variation, skew is the skewness coefficient, and kurt is the kurtosis coefficient

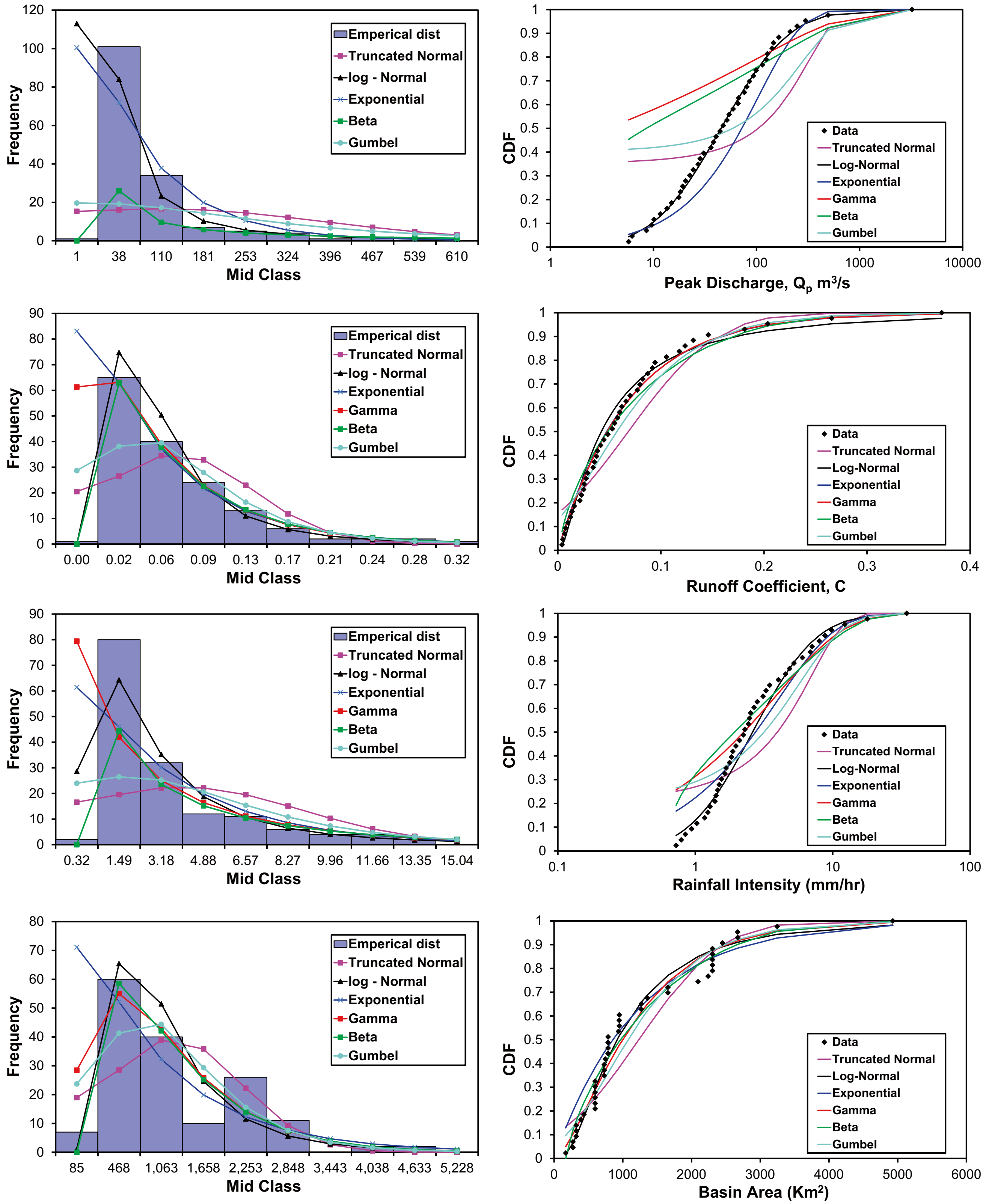

Fig. 6 Frequency histograms (left column) and cumulative distributions (right column) of peak discharge, Q_p; runoff coefficient, C; rainfall intensity; and basin area, A

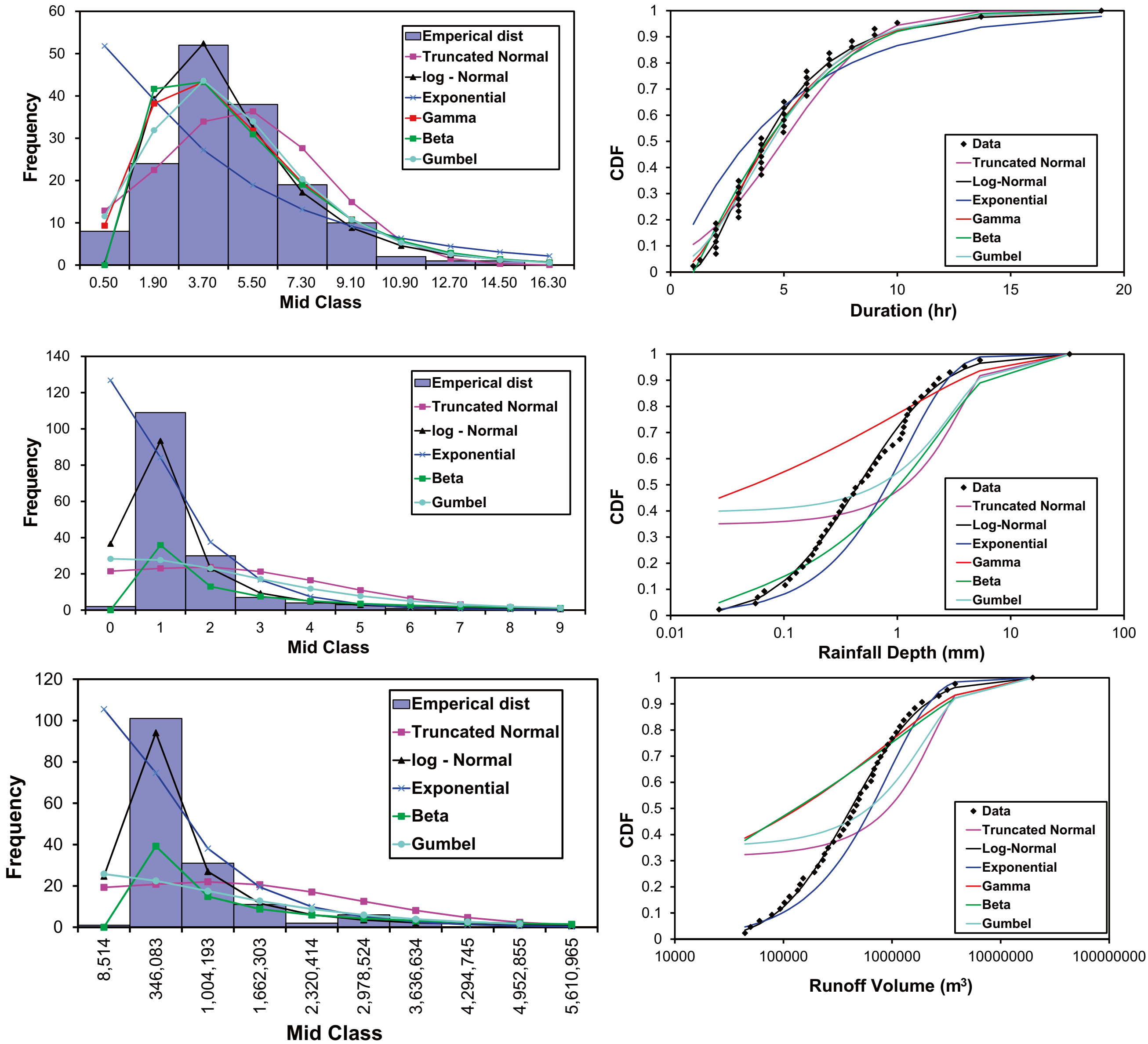

Fig. 7 Frequency histograms (left column) and cumulative distributions, CDF, (right column) of storm duration, rainfall depth, and runoff volume

are presented in Table 4. It is found out that the log-normal distribution is the best to fit the parameters of the rational method. However, for runoff coefficient, other distributions could also fit (exponential, gamma, beta, and Gumbel). The lowest K-S value is for gamma which could be a better distribution to fit the data than the log-normal. For storm duration, other distributions like gamma and Gumbel may also fit the data. However, the log-normal seems to be suited to all the parameters, and therefore, it is recommended to be used in further analysis. This result supports the use of log-transformation of the rational method equation as presented in the methodology section.

Results of the first-order-second moment sensitivity analysis

Equations 13 and 14 developed in the methodology section have been tested. These equations describe the variance of the ln (Q_p) as a function of the sum of the variance of logarithms of the intensity, area, and runoff coefficient. A similar formula is developed for the runoff volume. However, it depends on rainfall depth rather than rainfall intensity. The results of these formulae are presented in Table 5, Row 7 (in boldface italics), and Row 11 (last row). The values show the standard deviation (square root of variance) of ln (Q_p) and ln (runoff

Table 4 Results of the K-S statistical tests for the PDF of the parameters of the rational method

Parameter	Gaussian	Log-normal	Exponential	Gamma	Beta	Gumbel	Critical value (K-S)
Area (km^2)	0.22	**0.11**	0.17	0.13	0.14	0.16	0.108
Runoff Coefficient, *C*	0.15	**0.08**	**0.06**	**0.05**	**0.09**	0.13	0.108
Rainfall depth, *R* (mm)	0.33	**0.06**	0.19	0.46	0.39	0.38	0.108
Rainfall intensity, *i* (mm/h)	0.24	**0.10**	0.14	0.23	0.14	0.24	0.108
Storm duration, *D* (h)	0.15	**0.10**	0.26	**0.10**	0.12	**0.09**	0.108
Peak discharge, Q_p (m^3/s)	0.34	**0.03**	0.15	0.51	0.43	0.39	0.108
Runoff volume, *V* (m^3)	0.30	**0.04**	0.13	0.36	0.35	0.34	0.108

Boldface is the minimum value of the K-S test

Table 5 Summary statistics of the observed and calculated peak flow and first-order second-moment analysis

Statistical measure	Runoff observed (m^3/s)	Runoff coeffi-cient, *C*	Basin area (km^2)	Runoff volume (m^3)	Rainfall depth (mm)	Q_p calculated (m^3/s)			
						Average intensity (mm/h)	Max intensity (mm/h)	Intensity-based on tc from Kirpich (1940) (mm/h)	Intensity-based on tc from Albishi et al. (2017) (mm/h)
Mean	103.93	0.07	1233.03	923,985.78	15.88	63.35	135.98	45.79	120.13
SD	275.40	0.07	957.91	1,920,805.42	16.83	126.44	291.01	123.66	264.19
Max	3219.65	0.37	4930.00	19,760,337.34	123.40	1372.25	3192.23	1429.42	2832.24
Min	2.70	0.001	170.00	17,027.37	1.60	1.18	2.23	0.55	2.10
CV	2.65	0.99	0.78	2.08	1.06	2.00	2.14	2.70	2.20
Mean [ln()]	3.81	-3.16	6.82	12.95	2.38				
SD [ln()]	***1.21***	1.10	0.80	***1.24***	0.87				
CV [ln()]	0.32	-0.35	0.12	0.10	0.37				
ρ						0.93	0.95	0.97	0.95
RMSE (m^3/s)						169.91	97.03	168.41	87.66
SD based on 1st order analysis	1.30			1.24					

The symbol [ln()] means the natural logarithms of the parameter; the bold-italic face is for the SD of the parameters estimated from the data and are compared with the last row. It should be noticed that the statistics of the logarithms of the values are estimated based on the units in the table. However, they are different from the values used in the equation of the rational method since the units have to confirm with the method (i.e., rainfall depth should be in meter, intensity should be in meter per second, the area should be in square meter to get *Q* in cubic meter per second, and volume in cubic meter)

volume) of the data in Row 7 (1.21 and 1.24), respectively, and the corresponding values from Eqs. 13 and 14 in the last row (1.3 and 1.24), respectively. The results show very good agreement with the second-order sensitivity analysis and provide confidence in the use of these equations to estimate the variability of either the peak flow or the runoff volume due it the variability of the parameters in the equations.

Comparison of the estimation methods of rainfall intensity for the rational method

Figure 8 shows a comparison between the rainfall intensity of the storms recorded in the rainfall-runoff events (Dames and Moore 1988) based on the four methods: (1) the average rainfall intensity of the storms (brown circles), (2) the maximum rainfall intensity of the storms (green triangles), (3) the intensity based on tc calculated by Kirpich (1940) formula (blue diamond), and (4) the intensity based on tc calculated by Albishi et al. (2017) formula (red squares) and the intensity-duration- frequency (IDF) curves developed by Ewea et al. (2016) from the nearby stations of the Ministry of Environment, Water and Agriculture (Tables 1 and 2). These graphs show that the maximum rainfall intensity reached in Tabalah basin at 5-year return period. In Lith basin, the maximum intensity is at a return period of 10 years. In Habawnah basin, the maximum intensity is at

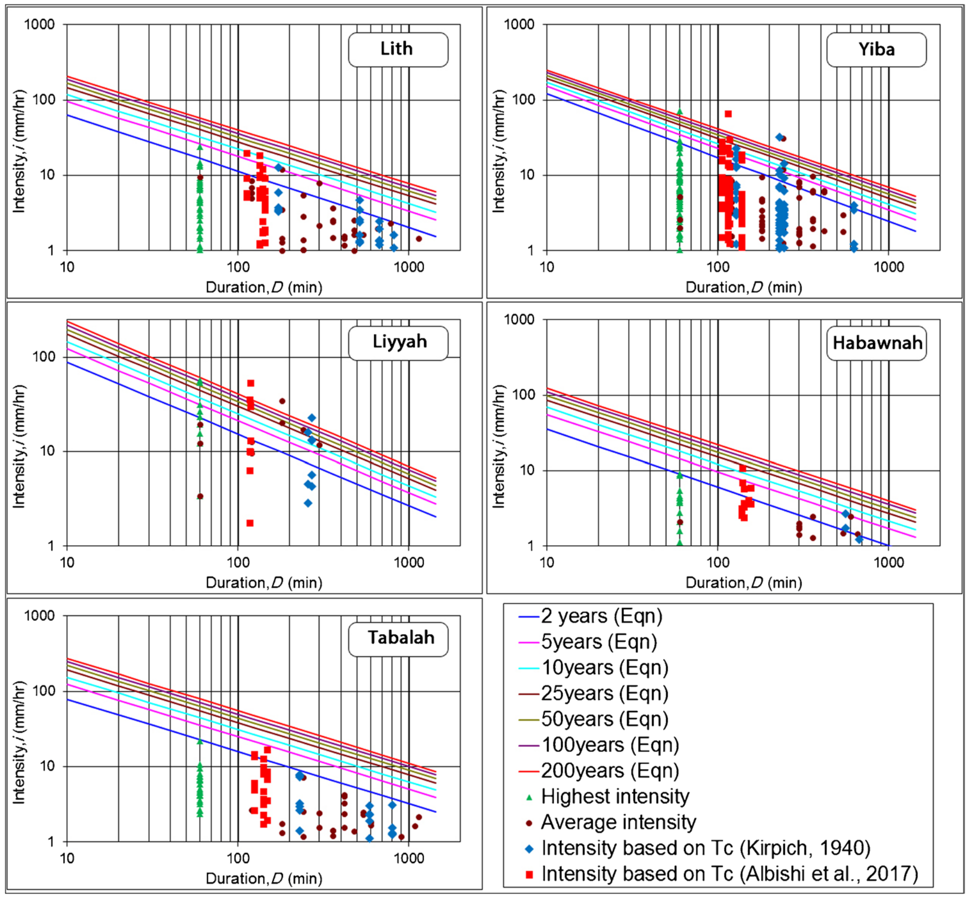

Fig. 8 Comparison between the observed intensity (both maximum and average intensities) of the storms that happened over the basins in the period (1984-1987) and registered by the local network over the basin (Dams and Moore, 1988) and the IDF derived from the stations of the Ministry of Environment, Water & Agriculture in the vicinity of the basins (Ewea, et al., 2016). See also Table 2 for the equations

a return period of 25 years. The duration of these storms is based on tc calculated by Albishi et al. (2017) formula (red squares). This indicates these storms are relatively moderate. However, in Yiba and Liyyah basins, there are extreme events that were above the 200 years return periods and their durations were estimated by the four aforementioned methods. Therefore, the hydrologists and engineers working in flood mitigation in these basins should be cautious in the design of mitigation structures in this region.

Comparison between calculated and observed peak flows

It is well known that the rational method assumes that the time of concentration is used to estimate the duration of the design storm that is used in the computation of the rainfall intensity (Chow et al., 1988). Therefore, a thorough investigation of this issue is made here to test the application of such a rule of thumb in the arid region. Figure 9 shows a comparison between the observed and the calculated flood peaks using the rational method equation based on estimating the rainfall intensity in various ways: (1) the average rainfall intensity of the storms (left-top), (2) the maximum rainfall intensity of the storms (right-top), (3) the intensity based on tc calculated by Kirpich (1940) formula (bottom-left), and (4) the intensity based on tc calculated by Albishi et al. (2017) formula (bottom- right). The graph shows that the use of average rainfall intensity or the intensity based on *tc* calculated by Kirpich (1940) formula both underestimates the peak flood. However, the maximum intensity overestimates the peak flood. The best graph is the estimation of the intensity using Albishi et al. (2017) formula since the points are distributed nicely around the line of a perfect fit. Table 3 i (the last four columns) shows summary statistics of the four cases. It shows the mean, standard deviation, minimum, maximum of the peak flow based on the four cases of the intensity estimation, and the observed peak flow data. It is quite clear that the estimation of the peak flow based on the intensity using Albishi et al. (2017) is close to the observed peak flow values in comparison with the rest. Although the correlation coefficient in all cases is high ≥ 0.93, the RMSE (87.66 m^3/s) is minimal for the case of the intensity estimated by using Albishi et al. (2017). This gives confidence in using Albishi et al. (2017) in estimating the storm duration from *tc* in arid regions of KSA rather than using the Kirpich formula (Kirpich, 1940).

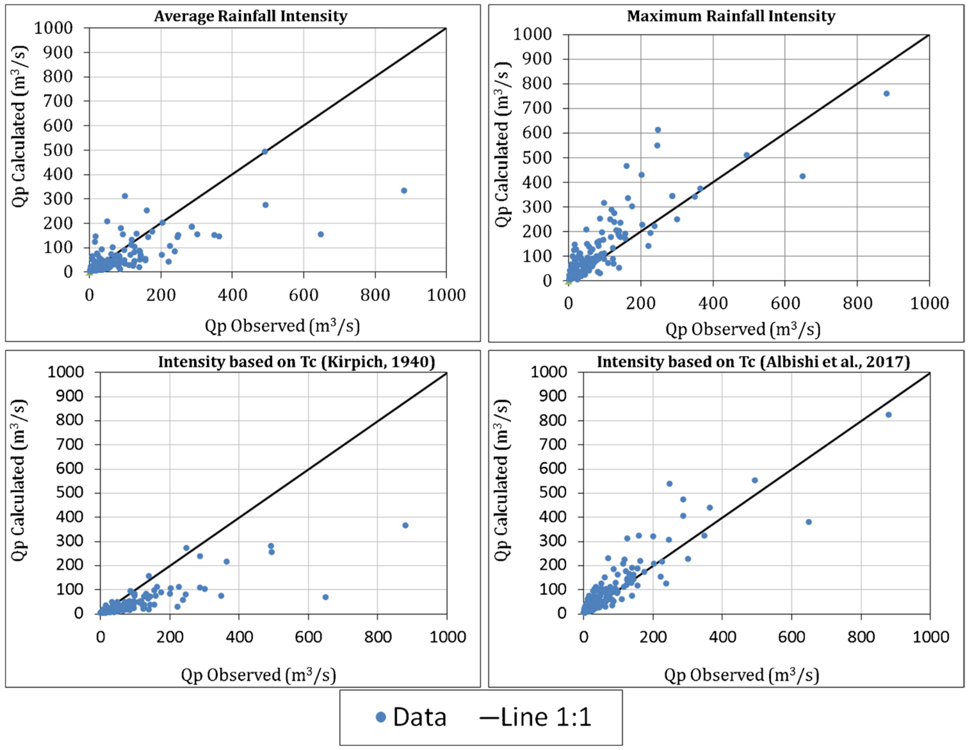

Fig. 9 Comparison between observed and calculated flood peaks using the rational method equation based on: the average rainfall intensity of the storms (left top), the maximum rainfall intensity of the storms (right-top), intensity-based to the tc calculated by Kirpich (1940) formula (bottom left), intensity-based to the tc calculated by Albishi et al. (2017) formula (bottom right)

Conclusions

The following conclusions can be drawn from this study:

It has been found that the rational method is not restricted to peak discharge estimation for the catchment areas of less than 5 km^2; it is soundly applicable to estimate peak flow and runoff volume for larger catchment areas. Although the method was applied, in the current study, on the basins' areas ranges between 170 and 4930 km^2, it produces very good agreements with observations of peak discharges and volumes, on the contrary of what is mentioned in the literature.

The first-order second-moment analysis shows reliable results since the variability (SD) of the logarithm of the peak discharge (σ_{lnQ}) and the logarithm of the runoff volume (σ_{lnV}) of the data (1.21 and 1.24, respectively) are pretty close to the developed theoretical equation given by Eqs. 13 and 14 (1.3 and 1.24, respectively). Therefore, it can be used to estimate the variability in the peak flow and volume concerning the variability of the parameters in the rational method (i.e., σ_{lnC}, σ_{lni}, σ_{lnR}, and σ_{lnA}). The results can be used for the uncertainty analysis of these basins which is rather important for the design of safety measures.

The log-normal distribution fits well the hydrological variables (rainfall depth, rainfall intensity, runoff volume, peak flow, storm duration, runoff coefficient, and basin area) based on K-S test. Therefore, it can be utilized for the flood uncertainty analysis of these basins and similar basins.

Most of the rainfall intensities of the flood storms within the recorded period of 4 years in the basins are below the 25-year return periods (those based on the average intensity, the maximum intensity, and using duration based on Kirpich, 1940) when compared with the IDF curves of the rainfall stations of the Ministry. There are some exceptional extreme events in wadi Yiba and Wadi Liyyah that are above the 200-year return periods. Therefore, hydrologists and engineers working in flood mitigation in these basins should be cautious in the design of mitigation structures.

The rainfall intensity used in the rational formula should be calculated based on the time of concentration estimated from the equation of Albishi et al. (2017) and not from Kirpich (1940). The former provides the minimum RMSE of peak flow of 87.66 m^3/s in comparison with the latter that has a RMSE of 168.41 m^3/s. This provides confidence in the use of Albishi et al. (2017) equation in flood studies. The estimation of the rainfall intensity based on duration estimated by *tc* using Albishi et al. (2017) equation is the best in the Saudi arid environment.

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Declarations

Conflict of interest The authors declare no competing interests.

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