Local geoid model of the Western Desert in Egypt using terrestrial gravity data and global geopotential models

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Abstract

The local hybrid geoid model for the western desert in Egypt is computed by recent data, including the global geopotential models, gravimetric measurements, and a high-resolution digital topographic model. The research points out the importance of accurate local geoid computation not only as a survey datum but also to figure out the crustal structure. Detailed crustal structure patterns could reveal the origin of artesian aquifers and geothermal activities of the selected region. In this research, recent global geopotential models deduced from recent satellite missions have been used. The oases' area of the western desert in Egypt is characterized by an irregular topographic pattern suggesting a complicated geoid model. Six recent global geopotential models (GGMs) have been evaluated for long and medium wavelengths. GGM accuracy for each model has been computed using GPS leveling data. EIGEN-6C model with spherical harmonic expansion to degree 250 gives the smallest error with a standard deviation equal to 0.019mGal. Also, a high-resolution digital topographic model with 3arcsec is used for terrain correction (Tc). The resulting geoid has been evaluated using GPS leveling in the selected region. Due to the recent geopotential models with high accuracy, the random distribution and low terrestrial gravity data have not become a problem in calculating the local geoid model.

Keywords GGM · Western Desert Oases · Local geoid · GPS

Introduction

The Western Desert (WD) is distinguished by the presence of a set of oases. They bear high potential groundwater, which is very important in this arid region. Besides, they are rich in many economic minerals and show promising geothermal energy (Zaher et al. 2018). On the other hand, the natural topography of the study area "oases region" is significantly rough as most of the oases' area is less than the sea level datum. On the

other hand, the presence of these oases leads to a high-gradient topographic pattern, although the oases can be considered one of the cornerstones of development in Egypt.

Until now, Egypt does not have a precise local geoid model due to the lack of data, particularly terrestrial gravity data. The current study focuses on the oases' region of the western desert in Egypt. All available updated data and recent global geopotential models are used to compute a precise local geoid model. Nowadays, the calculation of a precise local geoid model, which is considered an important surveying datum, is required to add a high value of accuracy to the construction of the high efficient and less expensive height system using GPS. Thus, an accurate local geoid model for the oases' region is highly appreciated. The geoid surface is a reference datum to determine the terrain heights and ocean depths (Manandhar and Shanker 2018). Modern satellites have significantly improved data of the Earth's gravity field (Barthelmes 2016). The current study aims to compute a local geoid model utilizing recent GGM and terrestrial data. Computation has been done by the remove, compute, restore technique and least-square collocation method (Forsberg and Sideris 1993).

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Fig. 1 The study area is located in the Western Desert (Western Desert Oases)

Fig. 2 The flow chart shows the methodology used for the computation of the local geoid model

Fig. 3 Selected GGMs and their spherical harmonic (SH) expansion (degree n) and standard division (SD) in mGal
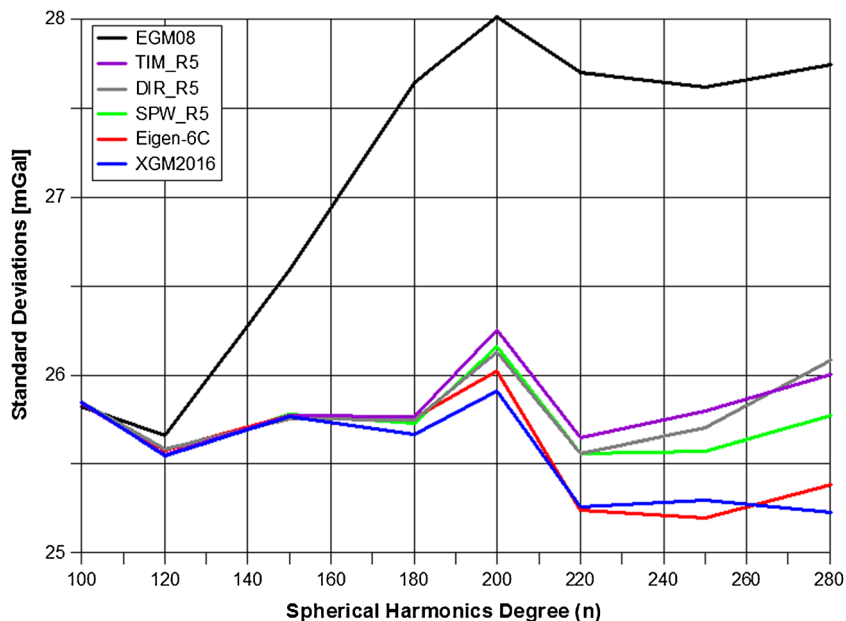

Study region

There is a sequence of five depressions in the Western Desert of Egypt, i.e., Dakhla, Baharia, Kharga, Farafra, and Siwa oases, which are most expected to be structurally controlled and illustrate significant geomorphologic features. These oases are located in the lowest area in Egypt, suggesting scientific interest in a local geoid

Fig. 4 Free-air gravity anomaly map of the study area (NRIAG and SRI)
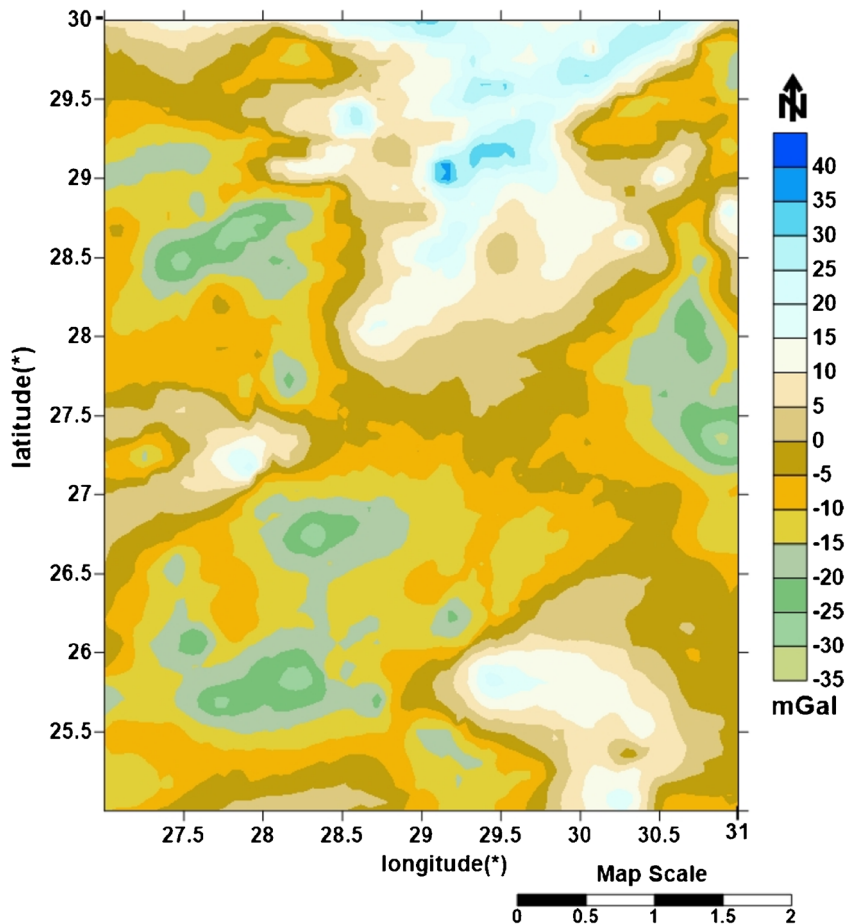
Fig. 5 Topographic high-resolution (3 arcsec, which equals 90m) digital model (DTM) is used for terrain correction (Tc)
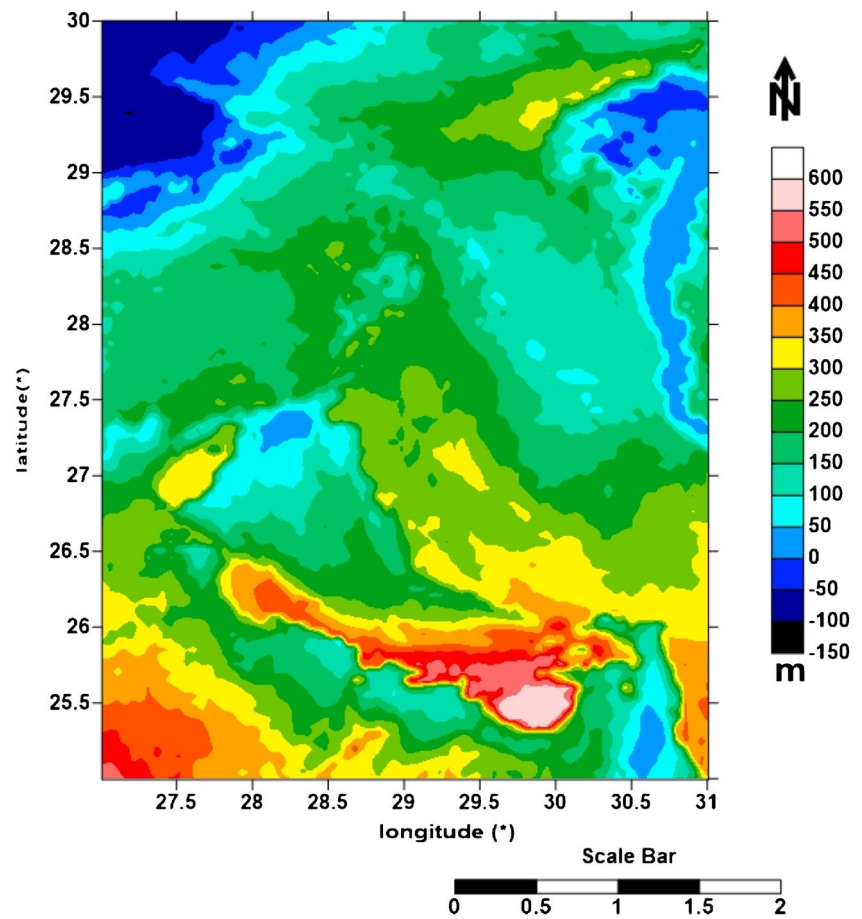
determination (Mohamed and EHARA 2009). From an economic point of view, it has a high water reservation potential leading to high population activities (Elbehiry et al. 2019).

On the other side, the area shows geothermal activities in different locations. The computation of the local geoid model will be enabled to evaluate the geothermal activities, particularly the hot springs in this area. As the main two factors are affected, the distribution of the hot springs is the subsurface structures and the thinned lithosphere (Ranalli and Rybach 2005). The study area (Fig. 1) extends between latitude 25–30 degrees north and longitude 27–31 degrees east. Nowadays, significant progress has been achieved on the computation of local geoids based on new satellite missions and recent geoid computation methodology.

Local geoid determination

The accuracy of the geoid model depends mainly on the terrestrial gravity data so, if the terrestrial gravity data have low accuracy, this will lead to a geoid model with low accuracy (Jalal et al. 2019). Therefore, long and medium local geoid disturbances lead

to a significant change from the global geoid model to the local geoid model. Therefore, any local geoid model must consider the local gravity model (Abdulrahman 2020). Besides the geodetic importance of the local geoid computation as an essential vertical datum, it would also shade more light onto the lithospheric thickness of the study area. Geoid determination has been the primary research in geodesy for decades. Many geopotential models (earth models) have been developed to give the geoid height in the global sense (Abd-Elmotaal 2015). Orthometric height can directly be calculated for any point on earth surface from the recent GGM models with accuracy varying from 30 cm to a few meters (Chen and Yang 2001; Rapp 1997). Since a geoid is an extension of the mean sea level, it represents the actual figure of the Earth. Calculation of geoid height includes the acquisition of gravity data, which is linked to geoid surface variations. So, low and random distribution of the terrestrial gravity data can significantly influence geoid height accuracy (Sjöberg and Featherstone 2004).

Methodology

Accurate local geoid model determination is the main task of the current research. Local geoid determination includes both

**Fig. 6** The resulting gravity residual anomalies (Δg RES)
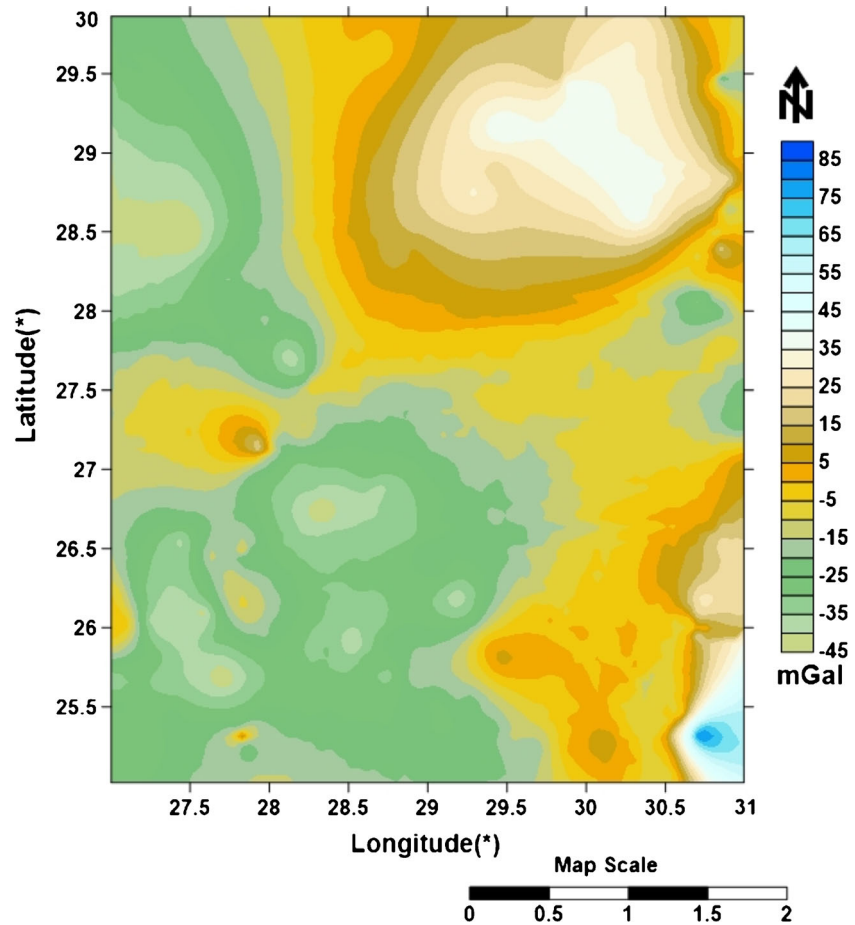
long- and short-wavelength gravity information. Long wavelengths are represented by the global geopotential model and short wavelengths by the ground observations. In the current research, evaluation has been carried out to the most recent global geopotential models. Evaluation has been done using existing GPS leveling locations. The local geoid model has been computed using least-square collocation due to the random distribution of terrestrial ground data (Godah and Krynski 2015). Residual geoid model deduced from the difference between the computed gravimetric geoid and

Fig. 7 The resulting relation between covariance [mGal2] and spherical cap distance (degree)

geometric geoid model, which has been interpreted in terms of oases rough topographic location. The methodology can be utilized to achieve the current research objective through the following chart below in Fig. 2.

Global geopotential models

GGM has been one of the geodetic activities to represent the global geoid. Recently, there is a great improvement of satellite data and ground-based data, leading to the development of GGM with a higher degree and order (Heliani 2016). Nowadays, many GGMs have been computed by different authors that show high accuracy, suggesting the importance of appropriate GGM for the selected area (Abd-Elmotaal 2015). So, selecting the most suitable GGM has a significant influence on the geoid calculation (Abd-Elmotaal 2015; Merry 2007). The resolution and accuracy of the GGM differ locally and globally. Actual resolution in any region is based on the density of the data and its quality (Gerstl 2008). Six recent global geopotential models have been evaluated over the study area (EGM008, DIR5, EIGEN-6C, SPW5, TIM5, and XGM2016) with different spherical harmonic (SH)
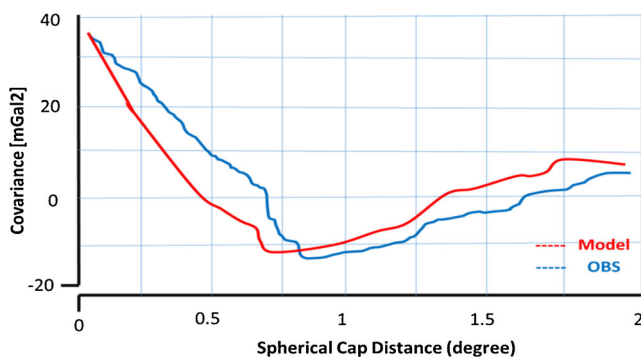
Fig. 8 The Eigen-6C4 global geoid undulations
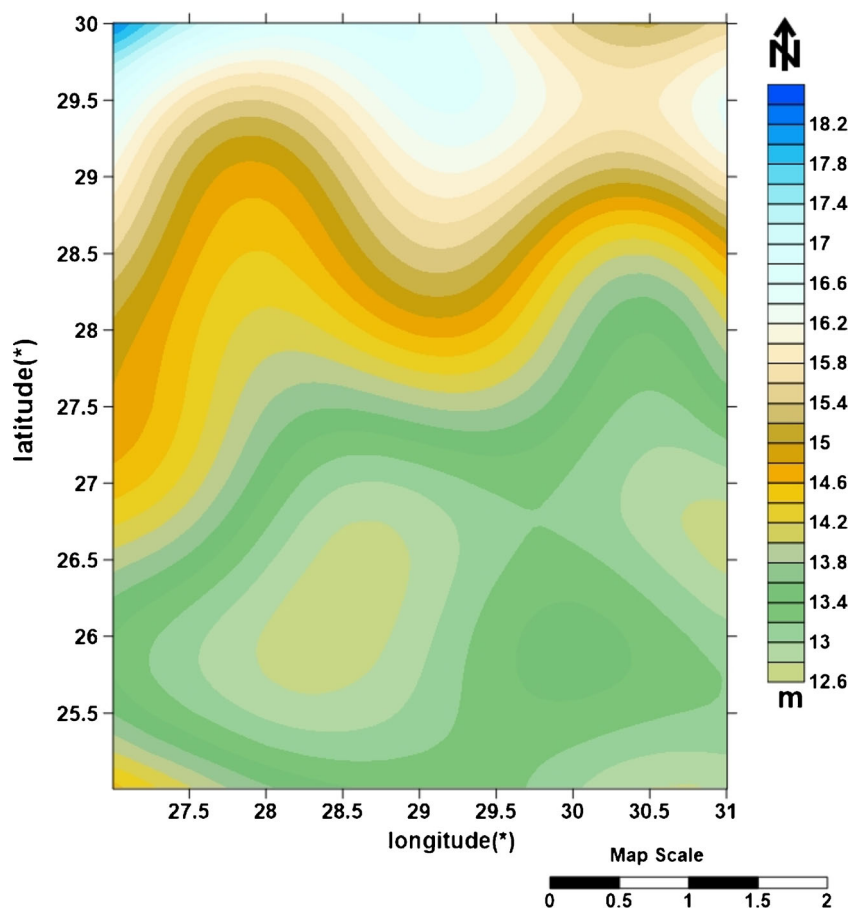


expansion (e.g., degree 100, 110, 120…………250) in order to select the most suitable model (Fig. 3). GGM accuracy of each model was compared with the GPS-leveling data value. Figure 3 shows that EIGEN-6C has the lowest standard deviation. The development of EIGEN-6C is using terrestrial datasets and satellite data to give a combined model (EIGEN-6C) (Barthelmes 2016; Ince et al. 2019). Systematic errors are calculated for each model separately and then combined. Accuracy assessment of the resulting model can be done through different methods (e.g., externally and internally). Calculation of geoid undulations usually uses a technique to compare to external datasets such as GNSS/leveling, resulting in orthometric height. However, the resulting geoid undulations give the same results from GNSS leveling data. GGMs, ground gravity data, and topography are usually used to calculate the gravimetric geoid at regional and local scales (e.g., Smith and Roman 2001; Denker et al. 2000).

Terrestrial gravity data

All available data has been selected. Ground-based gravity data is the primary basis for precise local geoid

determination. The used gravity data are from different sources; the gravity laboratory of the NRIAG geodetic department has collected gravity data in the selected region; other groups of data are chosen from SRI and BGI. The mean standard deviation of gravity data is approximately equal to 0.015 mGal. Free-air gravity anomaly data has been computed, adjusted for the ellipsoid height to obtain free-air gravity disturbances, and the resulting data have been subjected to a statistical smoothing filter to remove data with high noise. Available randomly sampled gravity data has been interpolated in a regular grid (Strykowski and Forsberg 1998; Heiskanen 1967; Wichiencharoen 1982). The used points do not have differences between the observed and computed gravity data of more than 0.034 mGal. Other points have been removed. Most types of combining survey measurements are made using the least-square adjustment method (Anderson 1998). Verification of gravity of different sources has been carried, tying the data to absolute gravity reference. After the validation process, 21245 points have been selected, and about 120 points were rejected. Data after the interpolation process shows standard deviation and mean differences between the predicted data values and observed data equal to about 0.19 mGal and 0.6

**Fig. 9** GPS stations with an orthometric height from the "HARN" network
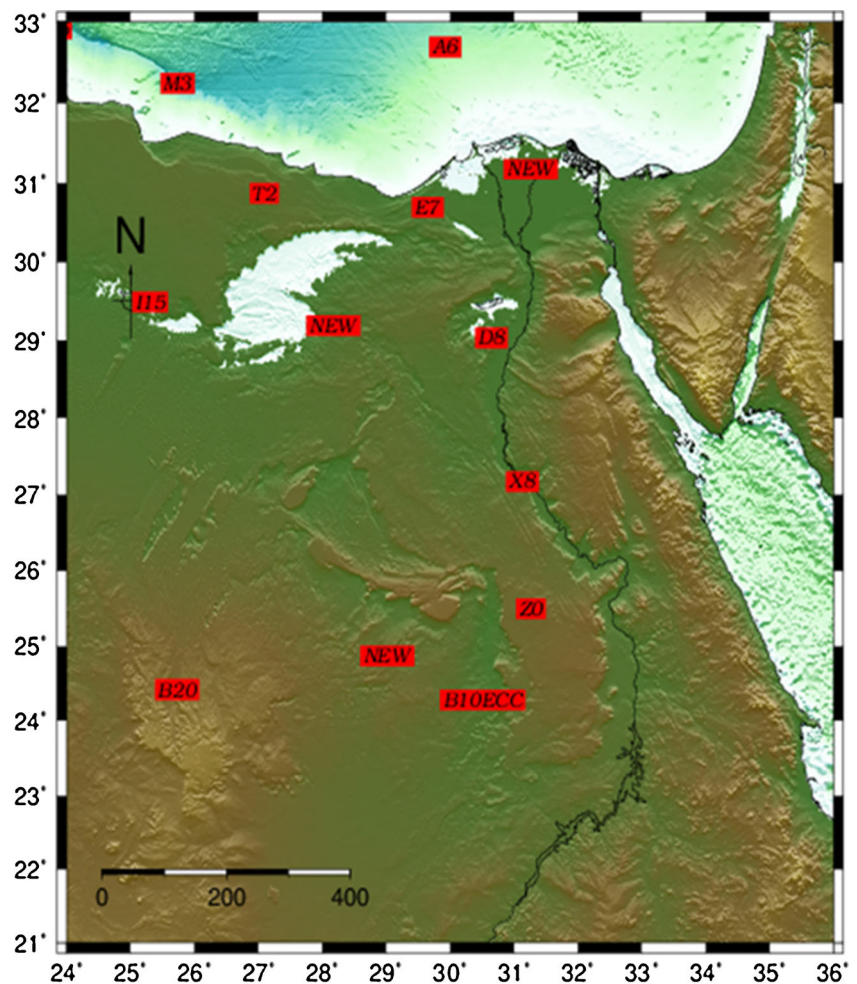
mGal. Free-air anomaly map and its data distribution through the study area are shown in Fig. 4. The map shows that free-air anomaly varies from −29 to 55 mGal, and the mean value is equal to −10 mGal. Computation of local geoid using available data has been done with Gravsoft, a Fortran package program with a Python interface used to evaluate gravity modeling (Forsberg and Tscherning 2008; Tscherning 1992). The least-square collocation, analytic modeling, and empirical covariance have been computed using Gravsoft subroutines. EIGEN-6C model is used to remove deeper earth effects, namely "very long wavelength" (>4000 km), which refers to variations of density under the lithosphere. The degree and order of the EIGEN-6C model are 2190, but after evaluation, we used up to degree and order 250. This is the first step in Gravsoft "GEOEGM" (Tscherning 1992). Lithospheric anomalies affect "the greater earth effects," which are removed through the long-wavelength components by geopotential model EIGEN-6C, resulting in a residual map. At this phase, removing the long-wavelength effect is the first attempt to produce a residual geoidal map.

Digital terrain model

For accurate geoid computation, the effect of topographic masses, which affect the potential field, should be removed. Complete Bouguer anomaly map results from the correction of terrain effect. Also, the terrain correction gives an isostasy effect (Chambers et al. 2005). All available elevation data for the study area are used in digital terrain model (DTM): tracking bathymetric gridded topography (from SRTM project "Shuttle Radar Topography Mission"). Topographic data used for terrain correction are from the database of the USGS with a 3" × 3" mesh size. The accuracy of the resulting map is based mainly on the resolution and the accuracy of the topographic model used in terrain correction (Kenney 2010). The used grid of SRTM is shown in (Fig. 5). The figure shows a high rough topography due to the presence of oases—elevation of the studied area ranging from 570 to −100 m. Most of the study area is rougher and lower than the surrounding and has a lot of depressions. Local orthometric height is compared to SRTM in order to verify the topographic data accuracy, which has a standard deviation of 2.4 m and a mean of −12.3 m. By this step, the long-wavelength was removed using GGM (EIGEN-

Fig. 10 Gravimetric regional quasi-geoid from individual contributions of EIGEN, RES, and RTM quasi-geoid
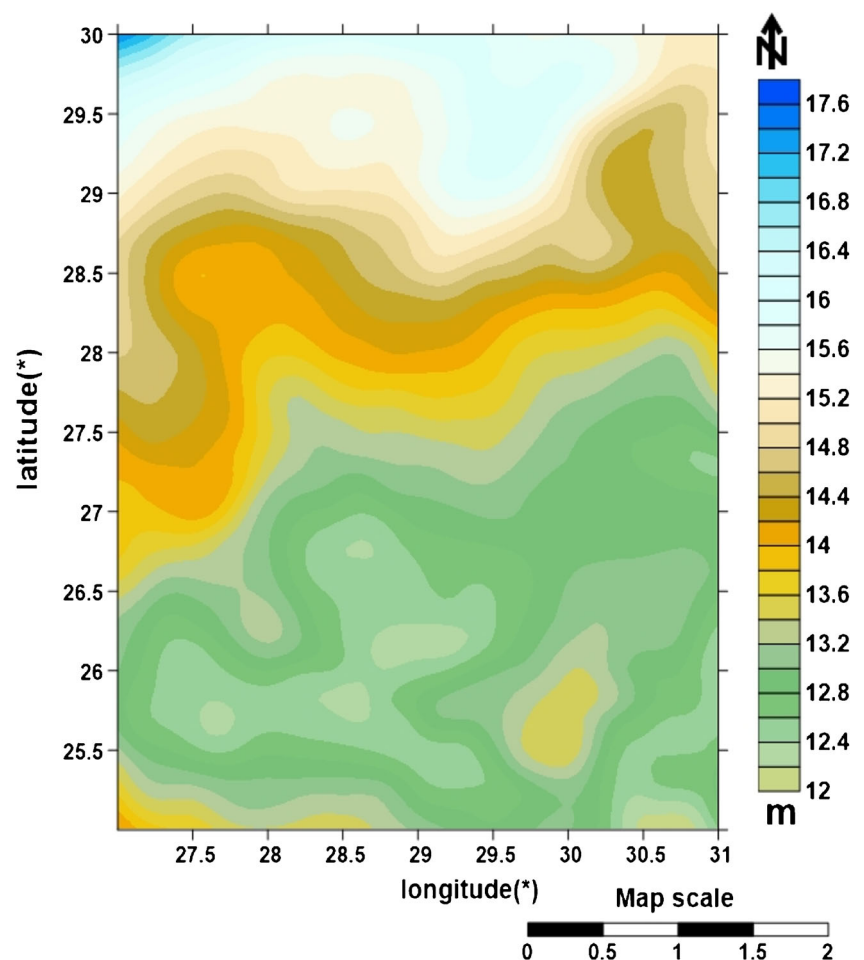
6C) and the residual wavelength (Terrain correction) using (SRTM with three arcsec grid) (Fig. 6).

Covariance modeling

In the least-square collocation technique, covariance modeling is a crucial step based on two sequences. A fitted covariance model and an individual empirical covariance are computed for the study area. All terrestrial gravity data are used to calculate an empirical covariance and the fitted fixed covariance model (Barnard et al. 2000). Outcome statistics display the best-fitted model. Although the rough topographic border effect of the study area, the resulting corrected covariance model is acceptable for geoid collocation. After computing the empirical covariance using EMPCOV, the covariance function is calculated using COVFIT analytic. The resulting relation between covariance [mGal2] and spherical cap distance (degree) is shown in (Fig. 7). Once the degree variances are estimated, one can consistently write the covariance of Δg, N, and their cross-covariance. It depends mainly on the number of iterations until the observed and measured models are approximately fitted to each other.

Least-square collocation (geoid model)

The main objective of current research is to accurately model local geoid, which is associated with height datum values. Orthometric height (H) and ellipsoidal height (h) can be calculated directly from geoid undulations (N) or from each other (Dawod 2008; Abd-Elmotaal 2002). Geoid undulations are calculated directly from GGM to compare them with the results from the least-square collocation (Kiamehr and Sjöberg 2005). Global or long-wavelength geoid features are represented in the gravitational potential field "spherical harmonic expansion." The advanced expressions are well-known from the satellite tracking data and ground data gravity. The geoid undulations calculated from the EIGEN-6C global model are shown in (Fig. 8).

Figure 8 shows the obtained undulations of the geoid model ranging from 12.6 to 18.2m. Generally, the geoid from GGM displays long- and short-wavelength variations. In the determination of global or regional geoid undulations, this is particularly true. But in the local geoid model, the concentration on the variations comes from the short wavelengths (Ngalamo et al. 2018; Tscherning et al. 2001). The least-square collocation technique is considered the optimum

Fig. 11 Height correction
anomaly (correction added to
quasi-geoid)
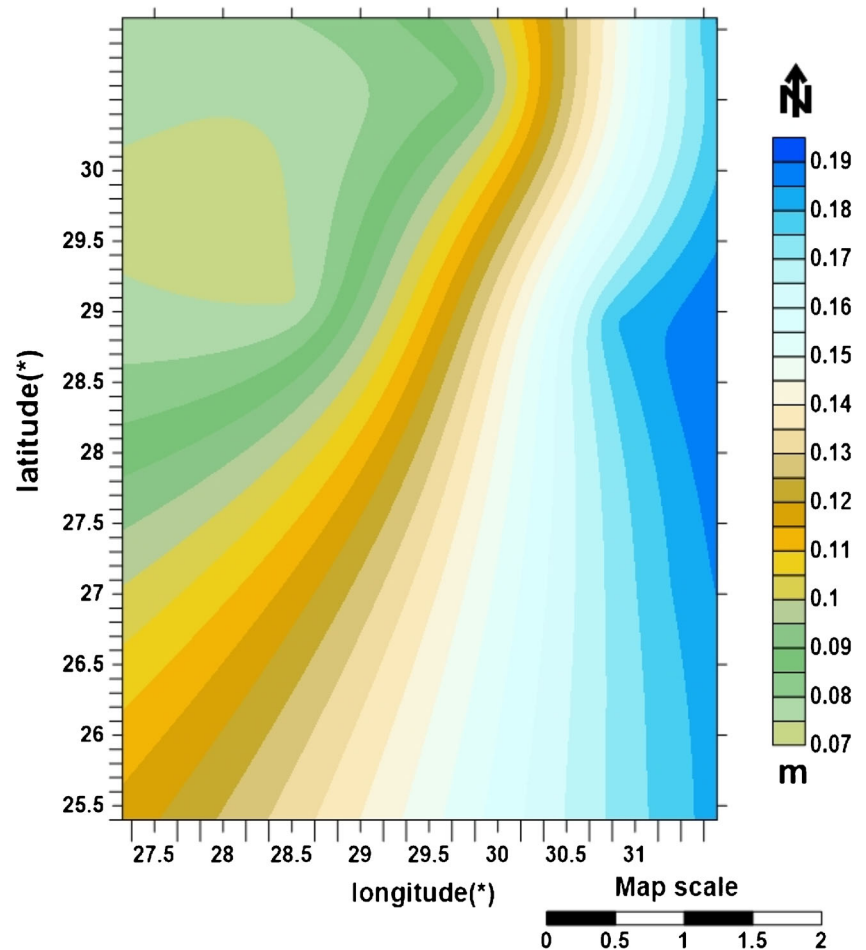
computation of a few irregulars distributed points, which enables to determine the estimated orthometric height at any other point (Hackney and Featherstone 2003; Moritz 1980). It is based on an interpolation to cover the study area with data and get a regular grid. Once the common point's number is increased, the result accuracy from this method will be improved. Error degree variance for EIGEN-6C is calculated and used for the depth to Berhamar sphere (RE-RB) and variance of gravity anomaly at zero altitudes (Var).

They are similarly done for GPS data (GEOEGM and TC) to get the same signal with gravity anomaly, which will be computed after getting the geoid.

HARN is the GPS leveling network which is established by the Egyptian Survey Authority to calculate the New Egyptian Datum 1995 (Fig. 9) (Dawod 1998). GPS/leveling data, precise leveling, and gravity anomaly in the least-square collocation are the basic data for the height system bias parameter and regional geoid model estimation. Adjustment of bias will minimize the variation between the standard deviation of the reduction differences and the height gravimetric geoid anomalies concerning GPS/leveling data. Additionally, bias modification is crucial for gravimetric geoid fitting, especially in

the least-square collocation techniques (Zhang 1997; Abdulrahman 2020).

The HARN network stations have been used to fit the gravimetric geoid and produce a quasi-geoid model. The gravimetric regional quasi-geoid is obtained through computing and restoring the residual quasi-geoid, RES, with EIGEN-6C and their height (Dawod and Abdel-Aziz 2020). Restoring results are displayed in Fig. 10 "quasi-geoid model." Consequently, short and medium wavelengths constitute the quasi-geoid from RES and RTM and large wavelengths from EIGEN-6C.

The gravimetric regional quasi-geoid is less smooth than the local geoid model suggesting more local and short gravity disturbances (Saadat et al. 2018). The result quasi-geoid is combining with some GPS leveling points to obtain a surface that can be used to calculate orthometric heights at any point or directly calculate ellipsoidal heights, orthometric heights, and geoid undulations from each other. After the resulting quasi-geoid model, Helmert orthometric height correction is used to get a geoid model. The correction added to the quasi-geoid is displayed in Fig. 11, and the outcome geoid model is shown in Fig. 12.

Fig. 12 Outcome geoid model
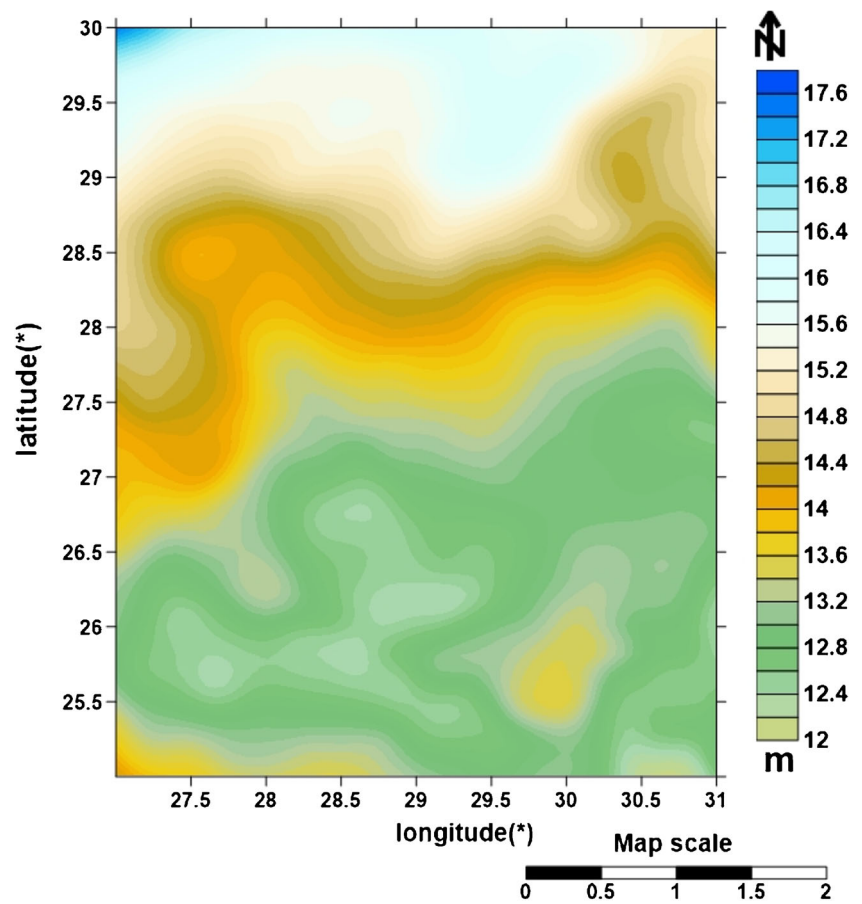
Statistics of orthometric corrections (mm) the long wavelengths using GGM (EIGEN-6C) and the residual wavelengths (terrain correction) using SRTM with three arcsec grid and adding a height correction anomaly and correction added to quasi-geoid are given in Table 1.

Figure 13 shows irregularities represented as high gradients in some locations to evaluate variations of the residual locations of the geoid model from GGM. A residual map has been produced and given in (Fig. 13). The map shows residual anomalies varying from −1.3 to 0.8 m and with a mean value equal to 0.1 m. The maximum value is equal to 0.8m which shows either an error in the geoid model computation or local features not included in the world model. The selected region consists of the oases, which have an irregular topographic pattern (Fig. 5); note that high residuals are at the same areas

as a high topographic pattern. This indicates an anomalous figure of the crust suggesting a complicated gravity pattern. Beside the oases is a source of artesian underground water, and many locations of thermal sources exist. Underground water, together with geothermal activities in this region, indicates a complicated lithosphere structure.

Discussion

Local geoid determination is an important geodetic task for its wide application. Many authors computed local geoid models at different locations in Egypt. However, the accuracy of the obtained models does not meet the recent geodetic application. This is due to the lack of ground-based data and does not accurately present the long wavelength of the potential data. The selected region is a newly developed area, and an accurate local geoid model is strongly needed.

On the other hand, the topographic characteristic of the selected region includes the lowest location of Egypt as the oases' location has the significant effect of isostatic correction. The recent global geopotential model shows significant improvements but is still not able to figure out complicated local gravity and topographic features. Obtained local geoid model

Table 1 Statistics of the results after adding height correction

Number of values	21465
Sum	1337023
Minimum	0
Maximum	17.4
Mean	13.4
Standard deviation	0.0012

Fig. 13 Residual map (resulting from the subtraction of geoid model and geoid undulations)

Conclusion
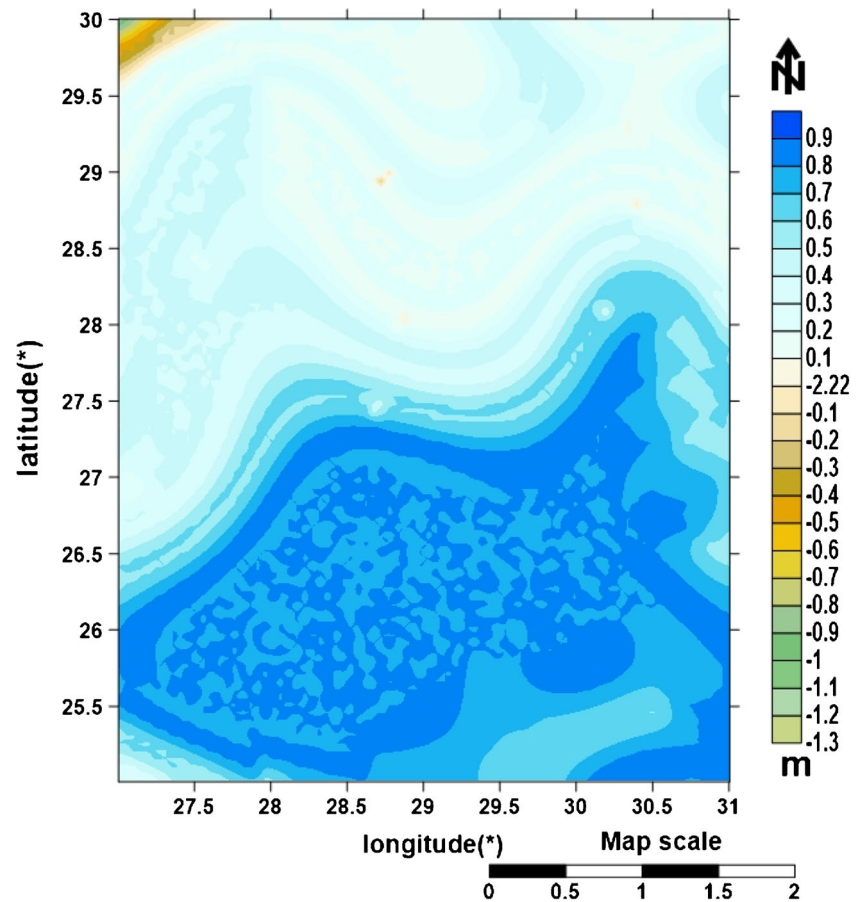
showed significant residual at the location of the oases. This suggests the important result of the high resolution of DTM and local gravity observation. The authors highly recommend the resulting geoid for geodetic purposes in the oasis region of the western desert in Egypt.

Conclusion

Based on the obtained results, the following conclusion can be figured out. GGM is widely used for different purposes as it shows remarkable significance from both resolution and accuracy. Also, it shows high resolution in some locations of complicated gravity anomaly. In these locations, dense gravity and GPS data are required if a higher accurate geoid is needed. The study suggests that EIEGN6C is representative of long and medium geoid wavelengths. Some terrestrial gravity data have been used to calculate the short wavelengths. Short and long wavelengths have been added together using RCR to produce the gravimetric geoid model. The obtained gravimetric geoid model shows some locations of anomalous behavior. Remove, compute, and restore, and least-square collocation, is the method used

because of the disseminated data that are not well distributed. EIGEN-6C, DTM (90 m), some GPS leveling points, and terrestrial gravity data are used to calculate the geoid model. Some GPS leveling data are used to compute the quasi-geoid to fit the obtained gravimetric geoid. Outcome assessment is a significant step that is based on some independent control set of obtained GPS leveling points. So, the geoid undulations of the Western desert have been checking using the other geoid undulations from geopotential satellite models and ground-based GPS/leveling data. Some GPS leveling data are used to compute the quasi-geoid to fit the obtained gravimetric geoid. The geoid undulation from the geopotential model could not be directly used. As a result, the global geoid model has been refined using ground gravimetry and surface terrain data, which leads to a more reliable local geoid model. The current research points out the importance of high-resolution gravity data and more dense GPS data. The complicated topographic pattern of the region suggests that high-resolution DTM could improve the obtained geoid model. Also, due to the great development of satellite data and terrestrial ground data, the maximum degree and order of the global geopotential model will exceed the current maximum degree and order of the geopotential

model. Consequently, the accuracy of the global geoid model will be increased.

Availability of data and material The raw/processed data required cannot be shared at this stage as the data is also part of an ongoing study.

Code availability Gravsoft program, GMT, Surfer

Declarations

Conflict of interest The authors declare that they have no competing interests.

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