



# Auto-characterization of naturally fractured reservoirs drilled by horizontal well using multi-output least squares support vector regression

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## Abstract

Pressure transient response (PTR) of horizontal well in naturally fractured reservoirs (NFR) has a particular characteristic shape. This PTR is often used to estimate parameters of NFRs and detect their wellbore and boundary regimes. Interporosity flow coefficient ( $\lambda$ ) and storativity ratio ( $\omega$ ) are two important parameters of the NFR that often estimated by matching process on the PTR. Since the matching techniques' results are not often unique, in this study, the multi-output least squares support vector regression (MLS-SVR) is employed for simultaneous estimation of  $\lambda$  and  $\omega$ . A databank of 500 PTRs for horizontal wells in naturally fractured reservoirs is generated by the finite element method, converted to the pressure derivative (PD) curves, and then used to develop and evaluate this auto-characterization paradigm. The predictive accuracy of the model is checked and validated by both smooth and noisy PTRs. The proposed model predicts  $\omega$  and  $\lambda$  with overall absolute average relative deviations (AARD) of 0.186% and 3.754%, respectively. The correlation coefficients ( $R^2$ ) of 1 and 0.99992 are obtained for the prediction of  $\omega$  and  $\lambda$ , respectively. The Leverage outlier detection technique justified that only less than 6% of the predictions are within the suspect region. This MLS-SVR model can be simply integrated with commercial pressure transient analysis (PTA) packages for accurate prediction of  $\omega$  and  $\lambda$  even from the noisy PTRs.

**Keywords** Horizontal wells · Naturally fractured reservoirs · Storativity ratio · Interporosity flow coefficient · MLS-SVR

## Introduction

Pressure transient signals of underground porous media, including oil, gas, gas condensate, and water reservoirs, are valuable information sources. Despite the high importance of pressure transient response (PTRs), the characterization of fluid flow in porous media from the PTR is challenging. Like all other actual signals, the PTR is often poisoned by noisy

data that are difficult to either be distinguished or removed (Moosavi et al. 2018a; Su et al. 2019; Chen et al. 2021). The primary objectives of reservoir fluid flow characterization are to improve the reservoir modeling and simulation accuracy, enhance oil recovery, provide production-performance predictions of different reservoir scenarios, and help determine remaining recoverable oil reserves (Cheng et al. 2016). Generic reservoir characterization utilizes multiple data sources, including well logging, PTR, geology, core analysis, and seismic. Such information helps to elucidate the drive mechanisms, reservoir and wellbore geometry, fluid types, rock properties, saturations, and dominant flow directions (Landa et al. 2000; Abdel-Fattah et al. 2015; Khan et al. 2016; Ciftci 2018; Yang et al. 2020). Some researchers focused on different aspects of facilities used to transport oil and gas from the reservoir to consumers (He et al. 2018; Liu et al. 2019b). A multi-dimensional mixed-integer nonlinear optimization model based on modified particle swarm optimization (MPSO) algorithm is proposed for a large-scale oil and gas gathering system (Liu et al. 2019b).

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Pressure transient signals make a key contribution to overall reservoir characterization.

This signal typically represents the reservoir pressure drop during changes the flow rates over specified periods. The PTR is long established as one of the best techniques for predicting well and reservoir properties by matching observed pressure responses with some ideal reservoir flow models (Huang et al. 2018; Nategh et al. 2019).

Naturally and hydraulic fracture reservoirs pose some additional challenges of the uneven distribution and orientation of their fractures and typically more significant heterogeneity than homogeneous reservoirs (Li et al. 2020). Consequently, although such reservoirs are known to contain significant in-place oil resources worldwide (Saidi 1983), their oil recovery factors tend to be small. This circumstance highlights the need for better reservoir characterization to improve fluid flow simulation models leading to more efficient drainage of these complex reservoirs. Variation of fracture propagation by the in situ stress, natural fissure development, temporary plugging, and treatment parameters was studied (Guo et al. 2020b). Fluid flow theory was first applied to naturally fractured reservoirs (NFR) by two research groups (Barenblatt and Zheltov 1960; Warren and Root 1963). Their studies develop a dual-porosity model for fractured reservoirs with two different key parts, i.e., a fracture with high fluid transmissibility and low storage capacity, and the matrix with low fluid transmissibility and high storage capacity. These specific characteristics of dual-porosity reservoirs can be quantified by deriving two useful metrics: interporosity flow coefficient ( $\lambda$ ) and storativity ratio ( $\omega$ ). It is an earlier measure interaction between matrix and fractured sections, while the latter represents an amount of fluid stored in the fractured section relative to the total fluid held in the whole reservoir (Egya et al. 2017).

Although the different values of  $\lambda$  and  $\omega$  produce no unique characteristic shapes in the PTR, they produce entirely different patterns in the pressure derivative (PD) graphs. This feature of pressure derivative plots makes it possible to apply various machine learning algorithms to estimate  $\lambda$  and  $\omega$  using the characteristic shapes of such plots. Similar to wide applications of machine learning techniques in different fields of science and engineering (Zhang et al. 2018; Yang et al. 2019; Shi et al. 2020), they are also employed to predict reservoir underlying models (Vaferi et al. 2011, 2016; Ghaffarian et al. 2014), while some researches focused on predicting specific values for key reservoir parameters (Eslamloueyan et al. 2010; Şahin and Çiftçi 2016).

Most models proposed to predict reservoir fluid flow metrics using machine learning algorithms have applied artificial neural networks (ANN). Alajmi and Ertekin developed an ANN model applied to NFR (Alajmi and Ertekin 2007). Their model involved a 4th-degree polynomial fit to the semi-log relationship between well-test pressure and time data derived via a simulation model (Sierra 1986). Well-test data

combined with rock properties and fluid compositions are considered the input variables for their ANN model. Well-test pressure derivative graphs show much greater sensitivity to  $\lambda$  and  $\omega$  variability for dual-porosity NFR. Eslamloueyan et al. developed their ANN model so that the digitized pressure derivative graph roles as input for characterization of NFR system through prediction both  $\lambda$  and  $\omega$  (Eslamloueyan et al. 2010). Deep convolutional neural network (Xu et al. 2018; Li et al. 2019; Chen et al. 2020a; Lv and Qiao 2020; Qian et al. 2020a, b) as a new generation of artificial intelligence techniques is recently applied for reservoir characterization from well-testing data (Daolun et al. 2020; Liu et al. 2020).

This study's main objective is to develop a powerful smart strategy, named multi-output least squares support vector regression (MLS-SVR), for simultaneous prediction of  $\lambda$  and  $\omega$  from digitized PD graphs. For this purpose, a dataset including 500 simulated pressure-time patterns is generated for a NFR system drilled by a horizontal well with the infinite acting condition. This dataset is converted to PD graphs and then used to train a multi-output least squares support vector regression model to predict  $\lambda$  and  $\omega$ . The proposed method does not rely upon any graphical or correlation between variables and requires no prior information about the NFR system and its associated flow regimes. Both simplicity and accuracy of the designed MLS-SVR model are among key advantages of this technique. It saves time in analyzing of pressure response of NFR systems drilled by the horizontal well.

## Methods

### Pressure transient responses of NFR

In a drawdown test, the flow rate is held approximately constant while the bottom-hole pressure is continuously measured during the production period. Figure 1 depicts a typical pressure transient curve for a NFR system drilled with a horizontal well for a drawdown test. This pressure transient data is generated through a solution of the governing equation for the NFR drilled with a horizontal well by the finite element method. The NFR governing equations have been developed using the Warren-Root approach with some basic assumptions applied, including the uniform thickness of the reservoir with impermeable lower and upper boundaries, radial flow, isothermal, single-phase, and slightly compressible fluid with constant viscosity and rock properties. In this model, only fracture-fed wells are considered, and it is further assumed that each continuum (fracture and matrix) is homogenous and separate. Moreover, as mentioned, constant rock and fluid properties are also assumed.

Radial change in the reservoir pressure results in a fluid movement towards the production well. During the early

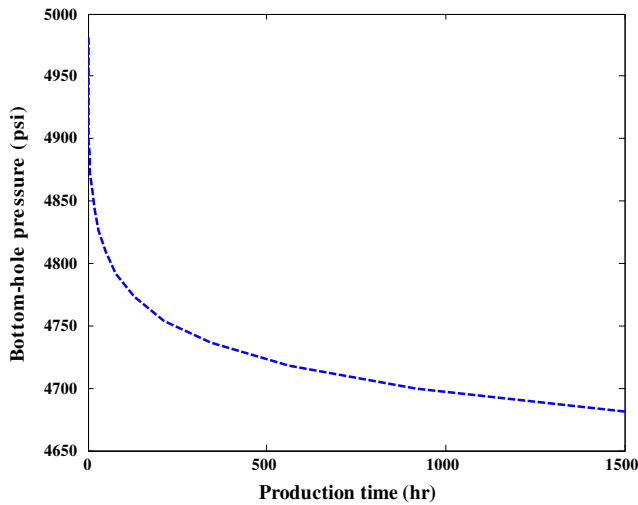


Fig. 1 Schematic of variation of pressure a horizontal well drilled in a naturally fractured reservoir during production period

production period, the bottom-hole pressure changes rapidly due to wellbore storage effects and the limited radius of investigation. Subsequently, as time progresses and the larger volume of the reservoir contributes to production, the bottom-hole pressure changes more slowly. Finally, the bottom-hole pressure represents the behavior of the outer boundaries.

### Naturally fractured reservoirs

Naturally fractured reservoirs have two key parameters more than homogeneous ones. Therefore, it is useful to detail how these two parameters, i.e., interporosity flow coefficient and storativity ratio, influence on characteristic shapes of the NRFs. It is evident that they have an essential role in distinguishing pressure transient responses of the NRF from homogeneous reservoirs.

### Storativity ratio

The storativity ratio is defined as the fraction of the total pore volume associated with one of the two porosities in a dual-porosity reservoir. Specifically, in NRF,  $\omega$  refers to the volume fraction of total reserves contained within the fractures. It is defined as follows (Moosavi et al. 2018b):

$$\omega = \frac{(\varphi V c_i)_f}{(\varphi V c_i)_f + (\varphi V c_i)_m} \tag{1}$$

where  $\varphi$  and  $c$  represent porosity and compressibility factor, respectively. Subscript f and m stand for fracture and matrix, respectively.

The storativity ratio has a significant effect on the short-term fluid production deliverability of a reservoir. The ratio is typically between 0.01 and 0.1, and dual-porosity reservoir

analysis is used in conjunction with the interporosity flow coefficient.

### Interporosity flow coefficient

Interporosity flow coefficient is directly correlated with the permeability of the matrix segment of the reservoir, but inversely correlated with the permeability of the fractured segment of the reservoir. For dual-porosity reservoirs,  $\lambda$  typically exists in the value range  $10^{-4}$  to  $10^{-8}$ . This parameter can be mathematically expressed by Eq. (2).

$$\lambda = \alpha \frac{k_m}{k_f} L^2 \tag{2}$$

where  $k$  is the permeability,  $\alpha$  shows the interporosity shape factor related to the block's matrix of the NRF, and  $L$  stands for half-length of the horizontal well.

### Semi-log graph for NRF

As Fig. 2 shows, the dual-porosity components of a NRF generate two parallel straight lines on a semi-log graph. The semi-log is made up in the detail of three distinct components: (1) transient radial flow period, an early straight line component representing the homogeneous flow of the fractured segment of the reservoir before the matrix segment of the reservoir makes a noticeable contribution, (2) interporosity flow onset period—a transition component joining the two straight line components reflecting the first noticeable contribution from the matrix segment of the reservoir, and (3) composite homogeneous flow period—a later stage straight line component reflecting the combined contributions from the fractured segment and the matrix segment, beginning when fluids in the fracture segment are consistently replenished by fluids flowing from the matrix segment of the reservoir.

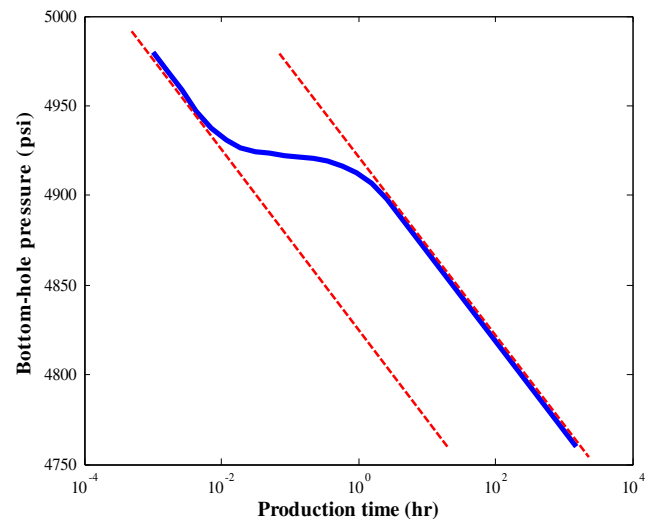


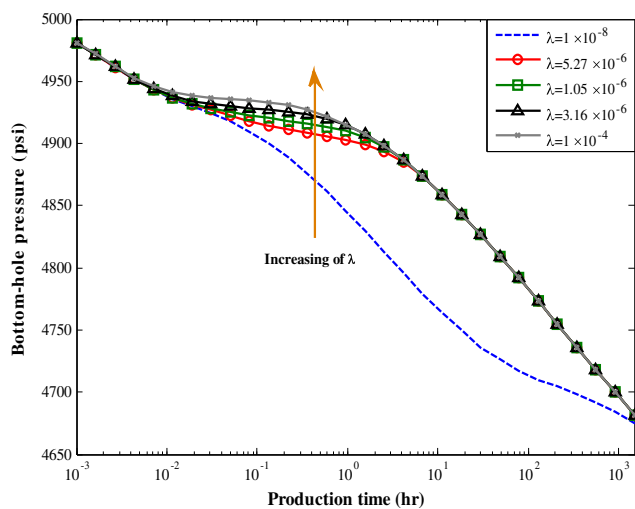
Fig. 2 Drawdown semi-log plot for a horizontal well drilled in a NRF

Effects of variation of interporosity flow coefficient and storativity ratio on a semi-log plot of a NFR drilled by a horizontal well are shown in Figs. 3 and 4, respectively. In Fig. 3, it is noticeable that as the interporosity flow coefficient increases, the first straight line component becomes shorter, and the transition component occurs sooner. Since the second zone on a semi-log plot (i.e., the transition component) involves some fluid flow from the matrix segment to the fractured segment of the NFR, any decrease in the permeability of the fractured segment or any increase in the permeability of the matrix segment will cause the fracture storage contribution to flow to be depleted more rapidly and the interporosity flow onset period to commence earlier.

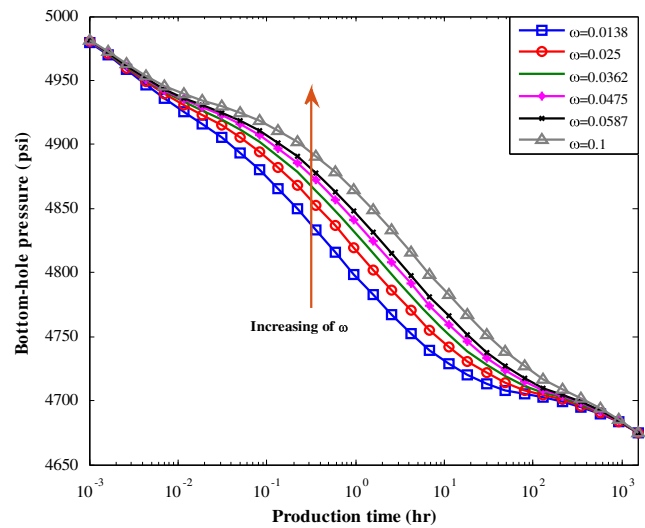
As the storativity ratio increases, the first straight line component becomes shorter, the transition component commences earlier, and the transition to the composite homogeneous flow period also commences earlier.

**Traditional methods for estimation of  $\omega$  and  $\lambda$**

Mathematical models are widely used to interpret PTR to establish reservoir characterization purposes. A traditional approach employs semi-log and log-log plots to provide a detailed analysis of pressure transient information. To establish meaningfully accurate reservoir interpretations, these traditional methods need to be constrained in certain ways. For example, distinct flow regimes applicable to specific time intervals must be clearly delineated on both pressure and pressure derivative plots. If the flow regimes applicable to the specific time elapsed intervals are unknown, type curve matching methods fail to provide unique solutions. In such situations, a trial-and-error approach is required to apply various flow regimes to find the best fits to the semi-log curves. The direct synthesis technique avoids type curve matching to derive reservoir metrics from pressure transient data (Tiab



**Fig. 3** Effect of interporosity flow coefficient on semi-log response of a horizontal well in a NFR ( $\omega=0.055$ )



**Fig. 4** Effect of storativity ratio on semi-log response of a horizontal well in a NFR ( $\lambda=1 \times 10^{-8}$ )

1989). It simultaneously uses pressure and PD curves to analyze vertically (Tiab 1994) and naturally fractured reservoirs (Engler and Tiab 1996) by exploiting analytical and empirical correlations.

**Pressure derivative for NFR system**

Remarkably, the pressure derivative technique is a powerful and well-established interpretation tool in providing meaningful analysis of pressure transient signals (Bourdet et al. 1989; Escobar et al. 2018). As expressed by Eq. (3), this technique uses three points (i.e., pressure drop versus superposition time) for calculation of pressure derivative at a given point.

$$(PD)_k = \frac{\frac{(\Delta p)_k - (\Delta p)_{k-1}}{\ln(\Delta t)_k - \ln(\Delta t)_{k-1}} [\ln(\Delta t)_{k+1} - \ln(\Delta t)_k] + \frac{(\Delta p)_{k+1} - (\Delta p)_k}{\ln(\Delta t)_{k+1} - \ln(\Delta t)_k} [\ln(\Delta t)_k - \ln(\Delta t)_{k-1}]}{\ln(\Delta t)_{k+1} - \ln(\Delta t)_{k-1}} \tag{3}$$

These PD plots are more useful for parameter estimation and model detection than traditional ones (Eslamloueyan et al. 2010; Vaferi et al. 2011; Wang 2016). The PD technique specifies the real interpretation model of fluid using pattern matching of observed signal with some standardized type curves. After that, it is possible to quantify reservoir properties' values (Bourdet et al. 1989; Bourdet 2002; Tiab and Donaldson 2015).

Theoretically, the dual-porosity behavior of NFR usually appears as three distinct flow regimes in pressure derivative graphs: (1) transient radial flow period, an early straight line component associated with production from the fractured segment of the reservoir, (2) interporosity flow onset period, commencing in the middle period of the test representing the first noticeable production contribution from the matrix segment of

the reservoir into the fractured segment, and (3) composite homogeneous flow period, the final test period during which simultaneous production from matrix segment and the fractured segment of the reservoir both contribute. The transition phase from the first flow regime to the second one appears as a distinct hump on the pressure derivative curve.

Figure 5 highlights the effect of changing the storativity ratio on the pressure derivative curves of NFR having horizontal well. As Fig. 5 clearly shows, by decreasing the storativity ratio, the first flow region finishes earlier. The transitional hump between first and second flow regimes moves to the left on the log-log plot. Also, the trough representing part of flow regime 2 progressively appears earlier as  $\omega$  decreases. This results in the third flow regime commencing earlier as  $\omega$  decreases.

Figure 6 presents the effect of variations in  $\lambda$  on the characteristic shape of the pressure derivative curve of NFR system. The earliest hump to appear in the pressure derivative curve is related to production from the fractured segment of the NFR. A decrease in  $\lambda$  can be induced by either decreasing the permeability of the matrix segment of the reservoir or increasing the permeability of the fractured segment of the reservoir. Such a decrease in  $\lambda$  will cause decreases in the pressure decline and the pressure derivative, which will lead to the earliest hump in the pressure derivative curve being more pronounced by extending over a more extended period. The second hump in the pressure derivative curve in Fig. 6 reflects fluid flow from the matrix segment to the fractured (i.e., third flow regime). An increase in  $\lambda$  can be induced by either decreasing the fracture permeability or increasing the matrix permeability. Such an increase in  $\lambda$  results in storage in the fractured segment of the reservoir being depleted earlier, leading to the third flow regime commencing earlier.

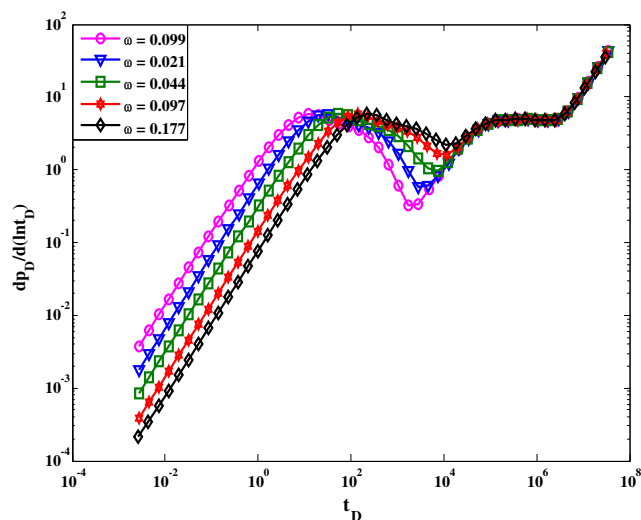


Fig. 5 Effects of variations of  $\omega$  on the pressure derivative curve of a horizontal well in a naturally fractured reservoir

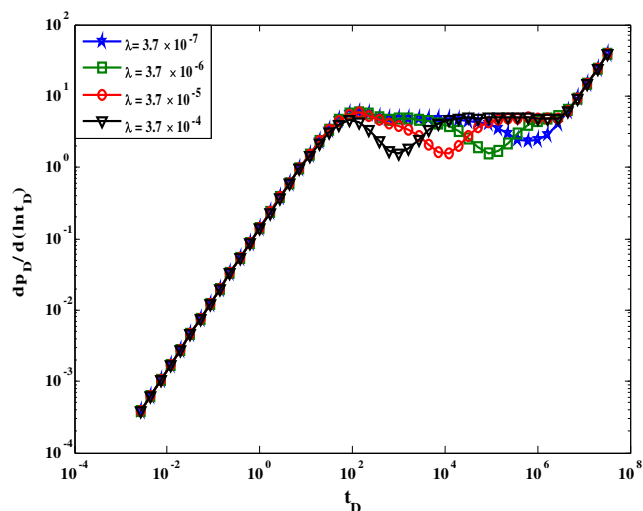


Fig. 6 Effects of variation of  $\lambda$  on pressure derivative plot of a horizontal well pressure-drawdown test in a NFR

## Results and discussion

### Interpretation model for NFR systems

In this study, the datasets, including 500 pressure transient signals for NFR system drilled by horizontal wells, are generated using an analytical solution of the governing equation of fluid flow using the finite element method. For generation these PTR, all reservoir properties except storativity ratio and interporosity flow coefficient are considered constant.

Table 1 Fluid and rock properties and wellbore geometry assumptions applied to the reservoir flow models evaluated in this study

Parameter	Value/ range	Unit
Well radius	0.3	ft
Formation thickness	40	ft
Porosity	0.17	—
Permeability	40	md
Vertical permeability	4	md
Skin factor	1	—
Viscosity	1.2	cp
Well length	800	ft
Total compressibility	$5 \times 10^{-6}$	psi <sup>-1</sup>
Oil formation volume factor	1.1	Rb/STB
Initial pressure	5000	psi
Simulation time	1500	hr
Top boundary	Sealing	—
Bottom boundary	Sealing	—
Interporosity flow coefficient	$10^{-4} - 10^{-8}$	—
Storativity ratio	0.01 – 0.1	—



The typical properties of NFR system for fluid, rock, and wellbore geometry are listed in Table 1.

Only the two-final metrics in Table 1,  $\lambda$  and  $\omega$ , were allowed to vary while generating the PTR dataset. Although the general shape of all of the generated PTR (i.e., each with unique  $\lambda$  and  $\omega$  values) are similar, their pressure derivative curves are quite distinct. Indeed, the MLS-SVR approach discriminates among these reservoir flow systems based on the difference in the characteristic shapes of their pressure derivative curves caused by interporosity flow coefficient and storativity ratio.

### Accuracy of the MLS-SVR model

Three commonly used statistical measures of accuracy, i.e., correlation coefficients ( $R^2$ ), absolute average relative deviations (AARD%), and mean square error (MSE), were used to the numerical evaluation of the performance of the developed MLS-SVR model. These statistical indices are mathematically expressed by Eqs. (4) through (6), respectively.

$$R^2 = \frac{\sum_{i=1}^N \left( PV_i^{act.} - \overline{\Delta PV} \right)^2 - \sum_{i=1}^N \left( PV_i^{act.} - \Delta PV_i^{cal.} \right)^2}{\sum_{i=1}^N \left( PV_i^{act.} - \overline{\Delta PV} \right)^2} \quad (4)$$

$$AARD\% = \frac{100}{N} \sum_{i=1}^N \left( \left| \frac{PV_i^{act.} - PV_i^{cal.}}{PV_i^{act.}} \right| \right) \quad (5)$$

$$MSE = \frac{1}{N} \sum_{i=1}^N \left( PV_i^{act.} - PV_i^{cal.} \right)^2 \quad (6)$$

where  $N$  is the number of data records,  $PV^{act.}$  and  $PV^{cal.}$  indicate actual values of  $\omega$  or  $\lambda$  and their associated predicted values by MLS-SVR model, and  $\overline{\Delta PV}$  shows the average value of real data for  $\omega$  and  $\lambda$ .

Table 2 displays the results of sensitivity analysis for the accuracy measures relating to  $\omega$  and  $\lambda$  predictions for four different distributions of the training and testing subsets. All the divisions between training and testing subsets display high degrees of accuracy, but using the 90% of databank as training provided the best statistical indices for  $\omega$  and  $\lambda$  in terms of AARD%, MSE, and  $R^2$ . The relatively large percentage errors for the AARD% relating to  $\lambda$  are due to the very low values ( $10^{-4}$  to  $10^{-8}$ ) of that metric. Indeed, even a very small difference between actual and calculated values for  $\lambda$  tends to produce a high level of error.

For a better presentation of these results and providing visual observation, the variation of the accuracy of the MLS-SVR models with training percent for both storativity ratio and interporosity flow coefficient is illustrated in Figs. 7 and 8, respectively.

Based on the results of sensitivity analysis presented in Table 2, Figs. 7 and 8, 90% (450 PD graphs) of the available databank is selected for the training of the MLS-SVR model.

Figures 9 and 10 depict actual values of  $\omega$  and  $\lambda$  as a function of their associated predicted values by MLS-SVR model for the training and testing data subsets, respectively. The negligible difference between actual and predicted values confirms the highest degree of accuracy achieved by the MLS-SVR model for both storativity ratio and interporosity flow coefficient.

### 3.2. Performance of the developed MLS-SVR model for noisy signal

Unlike the generated PTR from a solution of governing equation which is typically smooth, signals associate with real systems typically has a level of noise and defects (Moosavi et al. 2020; Yue et al. 2020). Therefore, it is crucial to assess the performance of the MLS-SVR model when confronted with noisy data records. For this purpose, normal artificial noise was added to the ten different PD graphs of the NFR system drilled by a horizontal well. Figure 11 illustrates typical smooth as well as noisy pressure derivative records for a NFR flow regime with unique  $\omega$  and  $\lambda$  values.

This figure clearly shows that a relatively high level of noise is added to the original curve. The developed MLS-SVR is provided with ten noisy PD graphs, and the obtained results are reported in Table 3. This table provides a comparison between actual values of  $\omega$  and  $\lambda$  and their associated predicted values by the MLS-SVR model for noisy PD signals. This comparison reveals that noise in the data records does not significantly degrade the prediction performance of the MLS-SVR model, and it tolerates a relatively high level of uncertainty in independent variables.

### Outlier detection

The presence of outlier and defect in the data records (Chen et al. 2020b; Guo et al. 2020a) making up the dataset influences the achievable accuracy by a model. As a large dataset has been generated and analyzed to develop the MLS-SVR model, some data record error is expected, resulting in some outliers in the predicted versus actual values. A useful statistical algorithm to detect outliers in datasets is the Leverage method enabling outlier points to be detected (Rousseeuw and Leroy 2005). This method determines standardized residuals between predicted and real values for each record in the dataset.

The distribution of standard residual has a mean of 0 and a standard deviation of 1. The leverage method then creates a Hat matrix for those standardized residuals defining a leverage index determined by Eq. (7):

**Table 2** Sensitivity analysis results for proposed MLS-SVR model to training and testing subset divisions of the data records

Numbers of signals assigned to training/testing subsets	Dataset	Sensitivity analyses					
		Storativity ratio			Interporosity flow coefficient		
		AARD%	MSE	$R^2$	AARD%	MSE	$R^2$
375/125	Training	0.225	$1.38 \times 10^{-8}$	0.99996	4.246	$1.32 \times 10^{-14}$	0.99996
	Testing	0.399	$2.71 \times 10^{-8}$	1	41.188	$2.28 \times 10^{-14}$	0.99978
400/100	Training	0.243	$1.50 \times 10^{-8}$	0.99996	5.318	$1.38 \times 10^{-14}$	0.99996
	Testing	0.226	$1.82 \times 10^{-8}$	0.99958	34.951	$1.96 \times 10^{-14}$	0.99983
425/75	Training	0.182	$9.11 \times 10^{-9}$	0.99998	2.191	$6.71 \times 10^{-15}$	0.99998
	Testing	0.209	$1.28 \times 10^{-8}$	1	32.034	$8.06 \times 10^{-15}$	0.99984
<b>450/50*</b>	<b>Training</b>	<b>0.177</b>	$8.81 \times 10^{-9}$	<b>0.99998</b>	<b>2.755</b>	$6.37 \times 10^{-15}$	<b>0.99998</b>
	<b>Testing</b>	<b>0.265</b>	$1.47 \times 10^{-8}$	<b>1</b>	<b>12.743</b>	$1.10 \times 10^{-14}$	<b>0.99945</b>

\* The most accurate MLS-SVR model. Note: multiple cases were executed with different signals assigned to the training and testing subsets

$$H = X(X^T X)^{-1} X^T \tag{7}$$

$$H^* = 3(n + 1)/m \tag{8}$$

where  $X$  is a matrix of standardized residuals with  $p \times q$  elements and  $X^T$  is its transpose.  $p$  and  $q$  represent the number of data records (500 in the NFR dataset generated) and the number of model parameters (30 data points used to define each pressure derivative curve), respectively.

The diagonal elements of the  $H$  matrix establish the leverage index. A William’s diagram cross plotting the standardized residual and the leverage index calculated by Eq. (7) for each data record then reveals suspected outlying data records (Gramatica 2007).

Figures 12 and 13 display William’s plots for  $\omega$  and  $\lambda$  values obtained by the MLS-SVR model, respectively.

In these plots, the warning leverage value ( $H^*$ ) is displayed as a vertical dashed line calculated using Eq. (8):

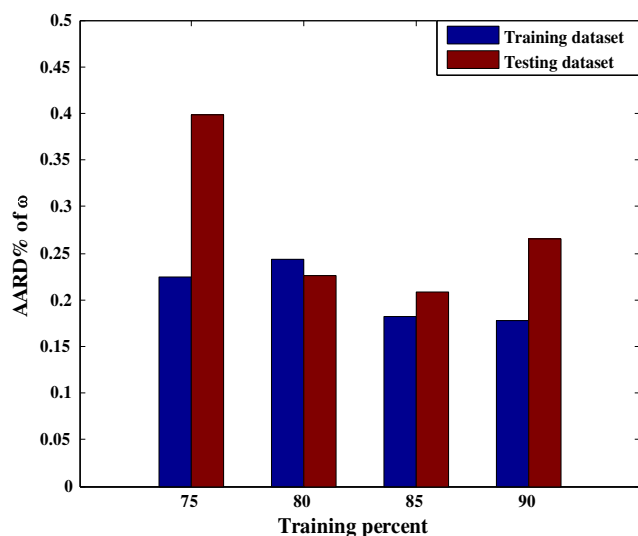
Here,  $n$  is the number of model parameters,  $m$  shows the number of data records. The cutoff value of 3 is typically applied in Eq. (8), representing a range of plus or minus three standard deviations from the mean value of the dataset metric evaluated.

If a model is robust in its predictions, and the dataset does not involve excessive error measurements, most of the data records should be located on a William’s diagram within the following limits:

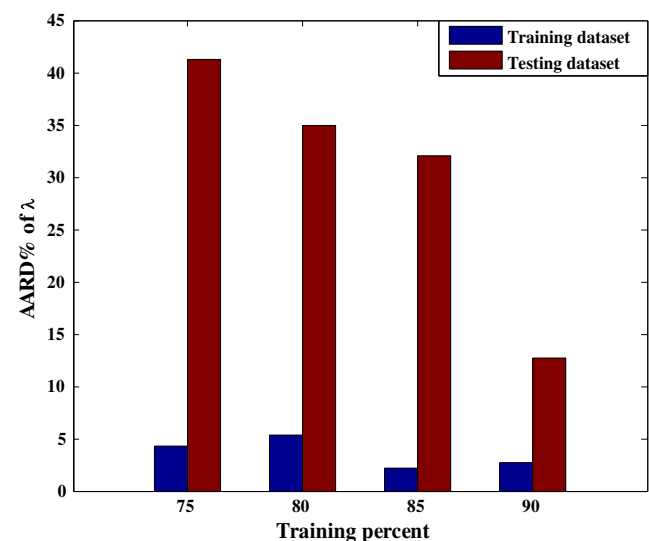
$$0 \leq H \leq 0.1725$$

$$-3 \leq SR \leq 3$$

It is obvious in Figs. 12 and 13 that most of the data points (~94%–95%) for both  $\omega$  and  $\lambda$  fall within the feasible regions.



**Fig. 7** Sensitivity of the MLS-SVR model in prediction of  $\omega$  to the distribution of training and testing subsets



**Fig. 8** Sensitivity of the MLS-SVR model in prediction of  $\lambda$  to the distribution of training and testing subsets

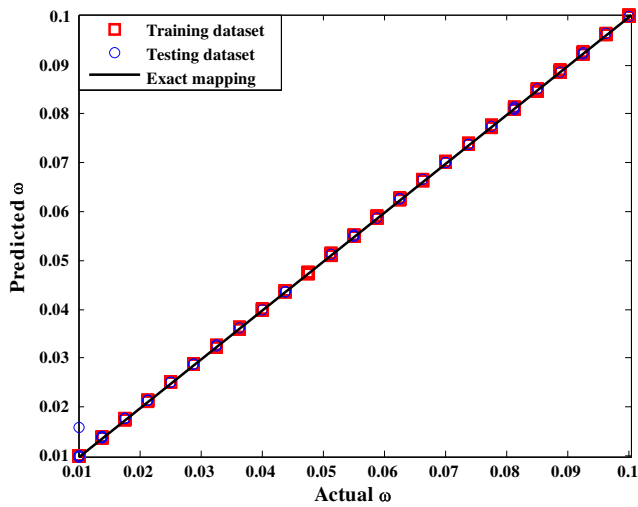


Fig. 9 Comparison of actual values of  $\omega$  versus predicted values by the MLS-SVR model

There are only 28 data records identified as possible outliers for  $\omega$ , and 32 data records identified as possible outliers for  $\lambda$ . This observation indicates high levels of statistical confidence in the predictions generated by the MLS-SVR model.

**Performance of the developed MLS-SVR model applied to a published signal**

Figure 14 presents a pressure derivative graph for a drawdown test performed on a horizontal well in a dual-porosity reservoir (Cheng 2004). The values of storativity ratio and interporosity flow coefficient for this signal are reported in that work as 0.1 and  $1 \times 10^{-6}$ , respectively.

The developed MLS-SVR approach is applied to this signal, and predicts storativity ratio and interporosity flow

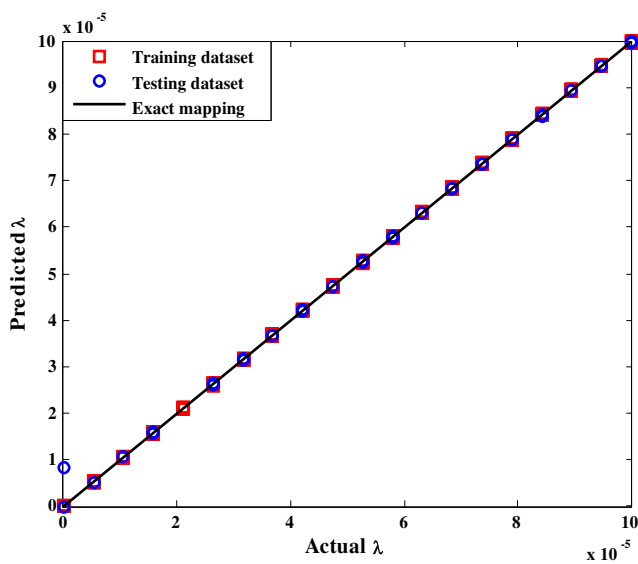


Fig. 10 Comparison of actual values of  $\lambda$  versus predicted values by the MLS-SVR model

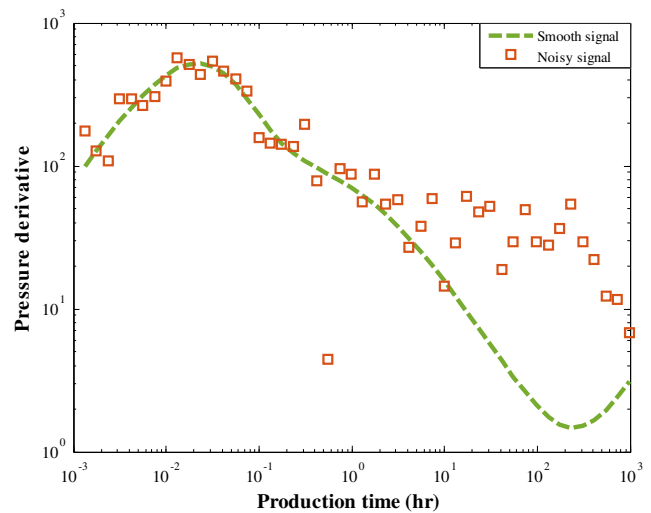


Fig. 11 Pressure derivative curve for a noisy record data from the dual-porosity NFR

coefficient as 0.08419 and  $1.16 \times 10^{-6}$ . The MLS-SVR model is clearly able to identify and predict that signal accurately.

**Conclusion**

In this study, a multi-output least squares support vector regression model developed and successfully applied to predict two key parameters of naturally fractured reservoirs, i.e., interporosity flow coefficient and storativity ratio. A huge databank having 500 PTRs is generated using an analytical solution of partial differential equations defining fluid flow in a fractured reservoir. These PRT then converted to the pressure derivative curves to facilitate the MLS-SVR for discrimination among different values of  $\omega$  and  $\lambda$ . AARD of 0.177% and 2.755% were obtained for trained  $\omega$  and  $\lambda$  predictions by the MLS-SVR, respectively. The leverage method identified that up to about 6% of the predictions made by the method were potential outliers. These results suggest that the MLS-SVR method developed and applied in this study to a dataset of pressure derivative curves provides a useful alternative to conventional PTA methods, mainly when applied to more complex reservoirs dual-porosity natural fractured reservoir flow system evaluated.

**Appendix 1. Multi-output least squares support vector regression**

Support vector regression maps nonlinear patterns into higher dimensional feature space that can approach infinite dimensions (Khandelwal and Kankar 2011; Tikhamarine et al. 2019; Quan et al. 2020). It then applies linear regression to the mapped feature space (Chao et al. 2018; Xu and Chen



**Table 3** The prediction performance of the MLS-SVR model applied to example noisy data records of well-test pressure derivative curves for a dual-porosity NFR

Real value of $\omega$	Predicted by MLS-SVR	Real value of $\lambda$	Predicted by MLS-SVR
0.0100	0.0101	$1 \times 10^{-8}$	$1.09 \times 10^{-8}$
0.0138	0.013	$5.273 \times 10^{-6}$	$5.32 \times 10^{-6}$
0.0175	0.018	$1.053 \times 10^{-5}$	$1.13 \times 10^{-5}$
0.0213	0.021	$1.580 \times 10^{-5}$	$1.6 \times 10^{-5}$
0.0250	0.024	$2.106 \times 10^{-5}$	$2 \times 10^{-5}$
0.0288	0.028	$2.632 \times 10^{-5}$	$2.32 \times 10^{-5}$
0.0325	0.033	$3.159 \times 10^{-5}$	$3.01 \times 10^{-5}$
0.0363	0.036	$3.685 \times 10^{-5}$	$3.89 \times 10^{-5}$
0.0400	0.038	$4.211 \times 10^{-5}$	$4.01 \times 10^{-5}$
0.0438	0.042	$4.737 \times 10^{-5}$	$4.35 \times 10^{-5}$

2019). Considering a dataset of  $N$  records existing in multi-dimensional feature space, its  $i$ th data record or element can be expressed as:

$$[(x_i, y_i), i = 1, 2, \dots, N]$$

Here,  $x_i$  and  $y_i$  represent an actual and predicted value of the  $i$ th data record. For such a data set, the support vector regression can be expressed as follows (Xu et al. 2013):

$$F(x) = \langle w, x \rangle + b \tag{9}$$

where  $\langle, \rangle$  denotes the dot product of the matrix elements involving all the  $x$  data records,  $w$  is the weight vector of the SVR regression function, and  $b$  is an intercept of the SVR regression function.

Accepting a certain level of error ( $\epsilon$ ), the objective of SVR is to establish a function  $F(x)$  that estimates the values of  $y$  from  $x$  data for a training dataset that maintains deviations at or less than the value specified for  $\epsilon$ . The risk function ( $R$ ) can then be solved using appropriate optimization techniques

(Chen et al. 2017; Deng et al. 2019; Liu et al. 2019a; Cao et al. 2020a, b, c; Qu et al. 2020).

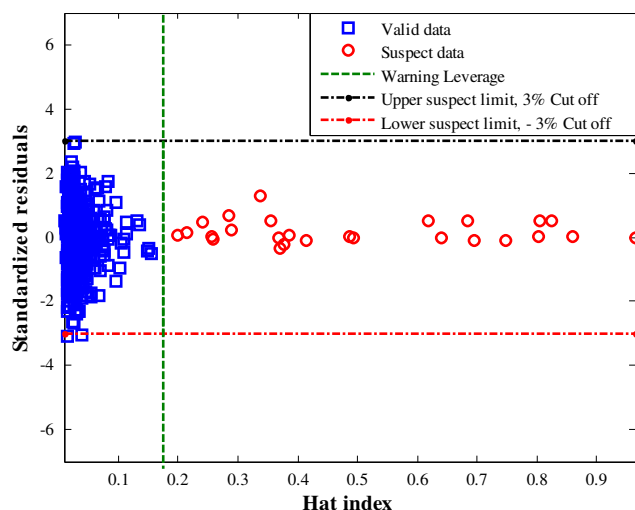
$$R(f) = \frac{1}{N} \sum_{i=1}^N L(f(x_i) - y_i) + \frac{1}{2} \|w\|^2 \tag{10}$$

where:

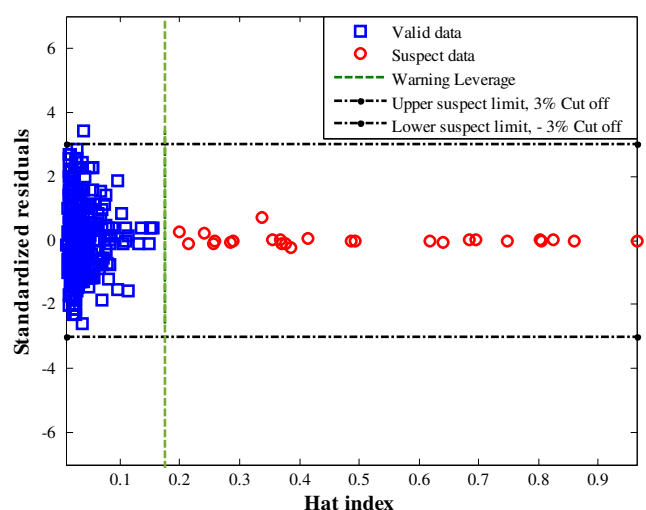
$$L(f(x) - y) = \begin{cases} \|L(f(x) - y)\| - \epsilon & \text{if } |f(x) - y| > \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Equation (4) expresses an error insensitive loss function.  $\epsilon$  determines the regression’s precision by essentially defining the radius of a cylinder surrounding the regression function,  $f(x)$ , within which acceptable values may exist. By substitution of Eq. (3) into Eq. (2), it is possible to determine functions that fit the records of the SVR training subset with deviations of no more than  $\epsilon$ .

The acceptable error components associated with SVR minimization can be further defined as follows (Smola and Schölkopf 2004; Chen et al. 2019):



**Fig. 12** William’s plot for detection of suspected outlier predictions of  $\omega$  by the MLS-SVR model. The dashed vertical line represents the  $H^*$  value calculated using Eq. (8)



**Fig. 13** William’s plot for detection of suspected outlier predictions of  $\lambda$  by the MLS-SVR model for the dual-porosity NFR dataset analyzed. The dashed vertical line represents the  $H^*$  value calculated using Eq. (8)

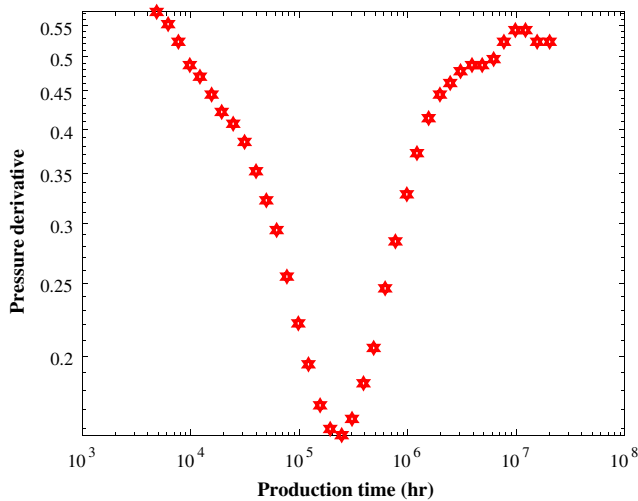


Fig. 14 Pressure derivative graph from literature (Cheng 2004)

$$\text{Minimize } \frac{1}{2} \|w\|^2 + C \sum_{i=1}^N (\zeta_i + \zeta_i^*) \quad (12)$$

This minimization involves the constraints listed in Eq. (5):

$$\text{s.t. } \begin{cases} y_i - \langle w, x_i \rangle + b < \varepsilon + \zeta_i \\ \langle w, x_i \rangle + b - y_i < \varepsilon + \zeta_i^* \\ \zeta_i, \zeta_i^* > 0 \\ i = 1, 2, \dots, N \end{cases} \quad (13)$$

where  $C$  is a positive regularization constant,  $\zeta_i$  and  $\zeta_i^*$  are positive slack variables.

$C$  is a metric that establishes a trade-off between a solution’s ability to be generalized across all the elements of a data subset (e.g., the SVR training subset) versus achieving acceptable levels of accuracy (expressed in terms of error tolerance by  $\varepsilon$ ).  $\zeta_i$  and  $\zeta_i^*$  quantify the distance from the values to the boundaries values of the error cylinder defined by  $\varepsilon$ .

SVR can then be expressed as a dual problem in the form of Eq. (6) to be maximized (Borges 1998; Bian et al. 2016):

$$\begin{aligned} &\text{maximize } \frac{1}{2} \\ &\times \sum_{i,j=1}^N (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) (x_i x_j) - \varepsilon \sum_{i=1}^N (\alpha_i + \alpha_i^*) \\ &+ \sum_{i=1}^N y_i (\alpha_i + \alpha_i^*) \end{aligned} \quad (14)$$

$$\text{s.t. } \begin{cases} \sum_{i=1}^N (\alpha_i - \alpha_i^*) = 0 \\ \alpha_i, \alpha_i^* > 0 \\ i = 1, 2, \dots, N \end{cases} \quad (15)$$

Here,  $\alpha_i$  and  $\alpha_i^*$  are Lagrange multipliers derived from quadratic equation solutions.

The SVR single space function can then be mathematically expressed by Eq. (8):

$$f(x) = (\alpha_i - \alpha_i^*) \langle x_i, x \rangle + b \quad (16)$$

The SVR dual space regression function is expressed by Eq. (9):

$$f(x) = (\alpha_i - \alpha_i^*) k(x_i, x) + b \quad (17)$$

where  $k(x_i, x)$  indicates the kernel function that satisfies Mercer’s conditions.

Those data records in the dataset determined by Eq. (9) to have non-zero coefficients are the support vectors. A kernel function commonly applied in Eq. (9) is the radial basis function (RBF) expressed as Eq. (10):

$$k(x_i, x_j) = \exp(-\gamma \|x_i, x_j\|), (\gamma > 0) \quad (18)$$

where  $\gamma$  shows the width of the RBF.

The critical variables in establishing acceptable SVR optimization solutions (regression functions) and the complexity of those solutions are  $\varepsilon$  (Eq. 3),  $C$  (Eq. 4), and  $\gamma$  (Eq. 10). SVR optimization, therefore, focuses upon optimizing the variables  $\varepsilon$ ,  $C$ , and  $\gamma$ .

In MLS-SVR, each data record in the dataset has multiple independent-dependent variables (Yan et al. 2020). For the case of our study, the digitized pressure derivative curve with 28 points is the independent variable, while  $\omega$  and  $\lambda$  constitute the target vector.

**Nomenclature**  $V$ , bulk volume;  $c$ , compressibility;  $k$ , permeability;  $L$ , half-length of the horizontal well;  $p$ , pressure;  $t$ , time;  $R$ , risk function;  $w$ , weight vector;  $b$ , intercept of the SVR regression function;  $k(x_i, x)$ , kernel function;  $N$ , number of data records;  $H$ , Hat matrix;  $X$ , matrix of standardized residuals;  $H^*$ , warning leverage value;  $n$ , number of model parameters;  $m$ , number of data records

**Greek symbols**  $\lambda$ , interporosity flow coefficient;  $\omega$ , storativity ratio;  $\phi$ , porosity;  $\alpha$ , shape factor;  $\Delta$ , difference;  $\varepsilon$ , error;  $\zeta_i$ , slack variable;  $\alpha_b$ , Lagrange multiplier;  $\gamma$ , width of the RBF

**Abbreviations**  $NFR$ , naturally fractured reservoirs;  $PTR$ , pressure transient response;  $MLS-SVR$ , multi-output least squares support vector regression;  $PD$ , pressure derivative;  $AARD\%$ , absolute average relative deviations;  $R^2$ , regression coefficients;  $PTA$ , pressure transient analyses;  $ANN$ , artificial neural networks;  $RBF$ , radial basis function;  $MSE$ , mean square error;  $PV$ , parameter value;  $SR$ , cutoff value for standardized residuals;  $hr$ , hour

**Superscripts/subscripts**  $f$ , fracture;  $m$ , matrix;  $t$ , total;  $act.$ , actual value;  $cal.$ , calculated value;  $T$ , transpose;  $-I$ , Inverse;  $D$ , dimensionless

**Declarations**

**Conflict of interest** The author (s) declare that they have no competing interests.

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