ORIGINAL PAPER

Integrating the role of relative density on assessing internal stability of granular filters using existing geometrical methods

Jahanzaib Israr[1,2] · Mubashir Aziz[3]

Abstract

The filtrates under the influence of strong seepage forces accompany finer fractions from broadly graded granular filters, occasionally rendering them ineffective. This phenomenon is termed as internal instability, and its prior assessment in practice is emphasized through some of the well-known geometrical criteria which examine only the shape of the particle size distribution and ignore the level of compaction of soils. In this study, gradient-controlled hydraulic tests conducted over a range of compacted soils were used to analyze some of the geometrical methods, which showed partial success in assessing the stability of test specimens. Further analysis facilitated a useful revision of a well-accepted criterion, whereby the original stability boundaries based on the uniformity coefficient could be set on the basis of relative density to quantify the potential of internal stability with enhanced accuracy. Not surprisingly, all the tests performed in this study along with additional data from published literature validated the revised method. Moreover, the proposed method is endorsed through a real-life design example, thus making it more expedient for practicing engineers.

Keywords Seepage; Internal instability; Hydraulic gradient; Relative density

Introduction

Internal instability is a physical phenomenon whereby the finer particles from a non-uniformly graded soil are eroded through its stable coarser particles and induce marked changes in the original particle size distribution (PSD). There can be a number of factors including physical, hydraulic, hydro-mechanical, and their various combinations that may trigger the internal instability in soils (Israr and Indraratna 2017; Skempton and Brogan 1994). However, the most common is

✉ Jahanzaib Israr
jisrar@uet.edu.pk

1 Department of Civil Engineering, University of Engineering and Technology Lahore, Lahore, Punjab, Pakistan

2 Research Centre for Geomechanics and Railway Engineering (GRE), ARC Industrial Transformation Training Centre for Railroad Technologies (ITTC-Rail), School of Civil, Mining, and Environmental Engineering, University of Wollongong, Wollongong, Australia

3 Department of Civil Engineering, FAST National University of Computer and Emerging Sciences, Lahore, Pakistan

the hydraulic or seepage-induced instability in granular filters and drainage layers that has reportedly contributed up to 46% of all hydraulic structure failures worldwide (Israr and Indraratna 2017). Figure 1 shows that the inception of seepage-induced internal instability can be in the form of segregation piping, suffusion, external or internal erosion, lateral pumping, mud-pumping, and heave (Israr et al. 2016; Richards and Reddy 2007). Given that the filters are required to retain the erodible soils from the protected earth structure, the one suffered from instability may not be fine enough to avoid the erosion. Notably, the problem of instability is not specific to the engineered fills only and can also occur in the more abundant non-uniform natural deposits.

The likelihood of occurrence or the potential of internal instability in soils is the function of their PSD, based on which a number of geometrical criteria have been established thus far (Israr and Israr 2018; Indraratna et al. 2011; Li and Fannin 2008). The instability potential of a soil is governed by the geometrical and physical factors such as its particle gradation and the level of compaction in tandem; hence its constriction size distribution CSD, which is the network of channels connecting two or more adjacent pores together (Indraratna et al. 2015). For example, a nonlinear PSD curve would exhibit a greater risk of suffering from the instability than the
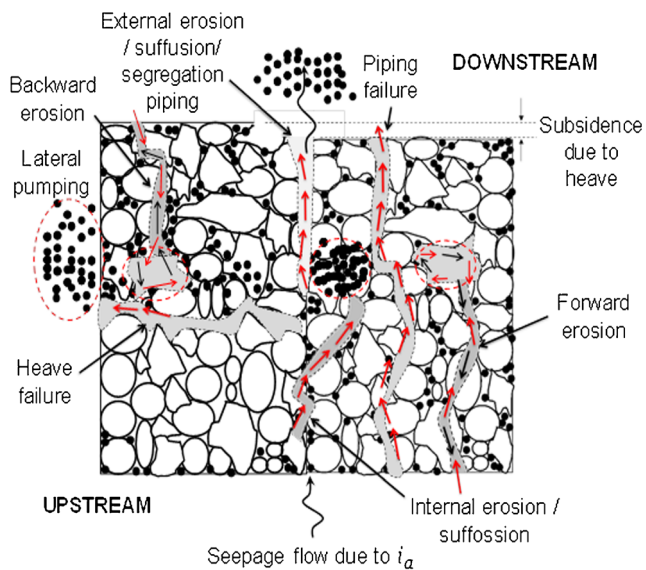
Fig. 1 Types of seepage-induced failures in soils (modified after Israr et al. 2016)

linear (Skempton and Brogan 1994; Kenney and Lau 1985). However, the inception of internal instability is governed by the hydro-mechanical factors, for example, unique combinations of effective stresses in soils and their critical hydraulic gradients (Israr and Indraratna 2017; Li and Fannin 2008). Nevertheless, the current study purports to address and improve upon our understanding of the former aspect of potential of internal instability at varying level of compaction for a select range of cohesion-less soils.

To date, the internal instability potential of soils is evaluated through various PSD-based criteria established through experimental observations of filtration under gravity loading, static loading, and with or without vibrations during tests (Israr and Israr 2018; Indraratna et al. 2015; Kenney and Lau 1985). These criteria would conservatively demarcate a boundary between internally unstable and stable soils such that instability is believed to be occurred only in the former (Skempton and Brogan 1994). Recently, Israr et al. (2016) proposed a CSD-based criterion to examine the internal instability potential of sand-gravel mixtures. In short, the approach would demarcate PSD curve at some arbitrary points to idealize base-filter systems and determine the adequacy of arbitrary filter (coarse fraction) in protecting the base (finer fraction) from erosion to be internally stable. Although this criterion shows promising results for a large body of published experimental data, as an inevitable limitation, it essentially requires computer aid to do complex discretization and analytical calculations to obtain base and filter fractions, PSD of base CSD of filter by surface area techniques. Nonetheless, the recent advancements in computational procedures have sufficiently optimized these complex procedures, the PSD-based methods are still preferred by most practitioners due to their simple operations (Li and Fannin 2008). For instance, a visual

inspection of a PSD curve may evaluate its susceptibility to instability to a sufficient accuracy without the aid of a calculator, as explained in the following section of this paper. Thus, the current study focused on experimentally evaluating and improving upon the well-established existing PSD-based criteria to facilitate practicing engineers.

Critical review of existing particle size distribution-based criteria

USACE (1953), Lubochkov (1969) independently followed by Kezdi (1979), and Sherard (1979) proposed to split PSD curve at arbitrary points to idealize an imaginary base-filter system within the subject soil. The propositions required the determination of maximum value for Terzaghi's retention ratio for granular filter design given by D'_{15}/d'_{85} (Terzaghi 1939). D'_{15} and d'_{85} are the 15th and 85th percentile finer particle sizes for the arbitrary filter and base soils, respectively. According to Kezdi (1979) and Sherard (1979), a soil satisfying the condition $D'_{15}/d'_{85} = 4$ and 5 could be considered internally stable, respectively.

Kenney and Lau (1985) conducted laboratory filtration tests under significant vibrations to examine the role of shape of PSD curve under severe conditions and combined PSD with the CSD of soil. The analysis of test results revealed that an arbitrary particle d on PSD curve can erode through the constriction network of the particles larger than $4d$, which can be protected by the presence of intermediate particle sizes up to $4d$. The ratio $(H/F)_{min}$ was adopted to examine the internal stability, where H is the percentage passing by mass between arbitrary particle sizes d and $4d$. Given that the CSD is a combined function of PSD and R_d, a number of constriction sizes would be formed at different constriction levels such that the fines eroded through one may be retained by another, i.e., local self-filtering. Also, the percentage of erodible fines (F) significantly influences the potential of instability of soil, for which Kenney and Lau (1985) proposed that the erodible particles exist loosely within the pore spaces of soils. They further assumed that the upper limits of F prone to erosion would be 20% and 30% for non-uniform ($C_u > 3$) and uniform ($C_u \leq 3$), respectively.

Lately, Burenkova (1993) and Wan and Fell (2008) proposed similar approaches involving different particle sizes on PSD curve to be examined such as D_{15}, D_{15}, D_{20}, D_{60}, and D_{90}, where subscripts showing the percentages passing by mass. According to each method, the soils would be characterized into stable, unstable, and transition materials based on certain particle size ratios, for instance, D_{20}/D_5 and D_{90}/D_{60} [19] D_{90}/D_{60} and D_{90}/D_{15} (Burenkova 1993). Notably, all of the above methods used for prompt preliminary assessments of risk of internal instability were formulated on the basis of either the shape of PSD curve or specific particle sizes and

none would incorporate the effects of relative density. This may be because the protective filters are expected to be placed at certain minimum relative density levels. For instance, Fell et al. (2018) reported that a relative density of about 70% is desirable. This may be because at times, a higher density may yield an undesirably stiff layer, while lower densities may be susceptible to liquefaction.

This study attempts to experimentally examine the available particle size based criteria and to improve the assessment of internal instability potential of granular soils by compacting the soils at different levels of compaction. In the following sections of this paper, specific details and results are reported from a comprehensive experimental program involving hydraulic tests. Furthermore, the analysis and discussion on the relative performance of existing PSD-based methods and recommendations for most reliable existing criterion are presented. A dataset of 91 experimental results including those from present study is adopted to verify the current proposition, and a real-life filter design example is presented to demonstrate the practical implications of this study.

Laboratory testing program

The granular soils used in this study including a uniform sand A ($C_u = 1.2$) and sand-gravel mixtures B, C, D, E, and F ($C_u = 5, 10, 20, 23,$ and 40, respectively) are shown in Fig. 2. The soil specimens were prepared by compacting at different levels of compaction between $R_d = 5\%$ and 95% to obtain 20 samples, each identified by an alphanumeric identity such as X-#, where X and # represent the soil name and its percentage R_d values, respectively. Varying the level of compaction enabled to determine whether R_d affects the instability potential of granular soils or not. Notably, such low value ($R_d = 5\%$) was selected to examine how a significant variation in relative density would influence the risk of instability.
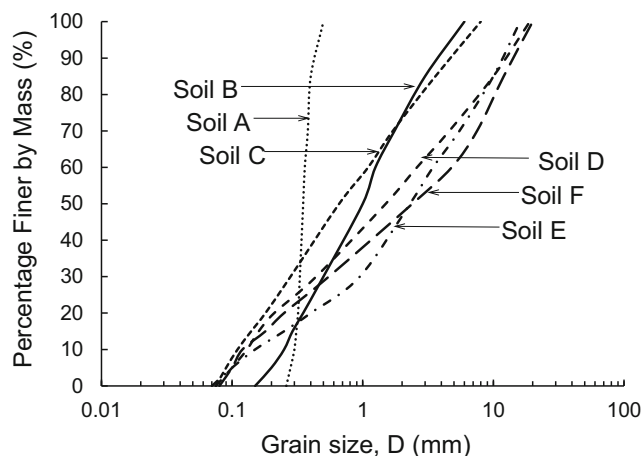

Fig. 2 Particle size distribution curves of the granular soils

For sampling, the soils were dry mixed and compacted in four to five uniform layers in a hydraulic chamber. The target R_d values were then obtained by simultaneously controlling the predetermined dry soil mass for the given specimen volume, optimum moisture content, and compaction energy (E_c) imparted to achieve the required void ratio. Notably, the limiting void ratios (e_{min} and e_{max}), and hence, the actual R_d values were obtained through the standard test procedures (ASTM D4253-16 2016, ASTM D4254-16 2016). For example, to obtain $R_d \approx 5\%$, the soil is placed in layers by hand and allowed to compact under self-weight (Skempton and Brogan 1994). The samples at intermediate compaction levels were prepared in layers using a 0.75-kg steel rod with length of 300-mm diameter of 20 mm ($5\% < R_d < 70\%$), while a sample at $R_d \approx 95\%$ was prepared with a standard compaction hammer (Indraratna et al. 2015). The compaction energies imparted to achieve the desired R_d values of 5%, 30%, 50%, 60%, 70%, and 95% were estimated to be 26, 157, 263, 313, 364, and 564 kJ/m³, respectively. In this study, the specimens' uniformity with respect to compaction as well as particle size distribution was assessed properly. For instance, a number of preliminary tests were conducted by following the procedure of Israr and Israr (2018), whereby the uniformity was evaluated by preparing additional soil specimens using the above techniques. For a given specimen, the local densities of small specimens cored randomly from each specimen were determined and compared to ensure uniform R_d with less than 5% standard deviation. Similarly, the PSD curves of additional and local specimens were also compared to ensure uniformity with respect to particle size distribution. The summary of test results presented in this study (Table 1) shows that the gradation uniformity was also ensured through the comparisons of pre- and post-test PSD analysis results, as indicated by the unchanged C_u values for stable soils. Nevertheless, the percentage erosion would be represented by the loss of fines at D_{10} level that may significantly change the post-test C_u such as soil specimen F-100 showed substantial decrease in C_u from 40 to 12.5 due to erosion and thus proved internally unstable.

The hydraulic testing apparatus as shown in Fig. 3 consisted of a smooth-wall Perspex glass chamber to obtain test specimens with 200-mm length and 150-mm diameter (Israr 2016). In light of the existing filtration studies, these dimensions were deemed sufficient to avoid potential disturbances to the test specimens including the development of preferential flow paths and boundary friction from the cell walls. For instance, most hydraulic testing in the past was conducted using apparatus with specimen diameter to largest particle size ratio exceeding 8 to successfully avoid the above effects (Indraratna et al. 2015; Indraratna et al. 2018; Zou et al. 2013). Saturation was completed under a back pressure of 120 kPa for a period of 24 to 48 h. To ensure full saturation of the soil, the filtration cell was first filled with de-aired and filtered water to obtain a small constant head of 50 mm, and the saturation was

Table 1   Summary of current test results

Test no.	Sample ID	C_u		$\gamma_{d, max}$ (kN/m³)	$\gamma_{d, min}$ (kN/m³)	R_d (%)	i_{cr}	Assessments of internal stability (S stable)							Failure type
		Pre-test	Post-test					Burenkova 1993	Wan and Fell 2008	Kezdi 1979	Sherard 1979	Kenney and Lau 1979	Indraratna et al. 2015	This Study	
1	A-0	1.2	1.2	14.97	18.71	6.6	1.07	S	S	S	S	S	S	S	Heave
2	A-50	1.2	1.2			52.3	1.18	S	S	S	S	S	S	S	Heave
3	A-100	1.2	1.2			94.3	1.29	S	S	S	S	S	S	S	Heave
7	B-0	5	5	15.31	19.11	6.9	1.0	S	S	S	S	S	S	S	Heave
8	B-50	5	5			51.9	1.05	S	S	S	S	S	S	S	Heave
9	B-100	5	5			92.5	1.10	S	S	S	S	S	S	S	Heave
10	C-0	10	10	15.41	19.26	5.5	0.9	S	S	S	S	S	S	S	Heave
11	C-50	10	10			47.2	1.0	S	S	S	S	S	S	S	Heave
12	C-100	10	10			92.8	1.05	S	S	S	S	S	S	S	Heave
13	D-0	20	23.7	15.50	19.84	6.1	0.45	S[a]	S[a]	S[a]	S[a]	S[a]	U	U	Suffusion
14	D-50	20	22.2			51.4	0.56	S[a]	S[a]	S[a]	S[a]	S[a]	U	U	Suffusion
15	D-70	20	20			71.1	0.90	S	S	S	S	S	S	S	Heave-Piping
16	D-100	20	20			95.6	0.98	S	S	S	S	S	S	S	Heave
17	E-0	23	18.2	15.41	19.26	7.4	0.62	S[a]	S[a]	U	U	S[a]	U	U	Suffusion
18	E-30	23	23			32.1	0.79	S	S	U[b]	U[b]	S	S	S	Heave-Piping
19	E-60	23	23			63.2	0.94	S	S	U[b]	U[b]	S	S	S	Heave
20	E-100	23	23			93.5	1.03	S	S	U[b]	U[b]	S	S	S	Heave
21	F-0	40	10	15.68	19.99	6.3	0.28	S[a]	S[a]	U	S[a]	U	U	U	Suffusion
22	F-50	40	11.2			48.0	0.33	S[a]	S[a]	U	S[a]	U	U	U	Suffusion
23	F-100	40	12.5			92.5	0.40	S[a]	S[a]	U	S[a]	U	U	U	Suffusion

Note: Superscripts a and b show unsafe and conservative predictions, respectively

completed under slow downward flow, while maintaining the head. Notably, the conventional upward saturation was avoided here because of previous experiences, where some loose broadly graded and gap-graded soils experienced some internal disturbance due to particle segregation and premature erosion. Furthermore, a wire mesh with 0.08 mm opening, placed at the bottom, could sufficiently avoid the erosion of fines during saturation.

The test procedure involved subjecting the test specimens to an upward hydraulic flow at controlled pressure until the visual development of failure such as heave, piping, and suffusion (i.e., segregation-piping). In this study, the average applied hydraulic gradient could be deduced from the controlled hydraulic pressure difference across a test specimen. For example, if $p_w^{in}, p_w^{out}, h, and \gamma_w$ represent the inflow pressure, outflow pressure, thickness of soil specimen, and the unit weight of water, respectively, then the average applied hydraulic gradient i_a could be obtained from the ratio of $(p_w^{in} - p_w^{out})$ and $(h \times \gamma_w)$.

For a negligible outflow pressure due to free-draining boundary, the inflow pressure and hence the applied hydraulic gradient i_a could be controlled so that the increments Δi were kept between 0.05 and 0.1 per hour for soils A, B, and C

having linear PSDs and soils D, E, and F with nonlinear PSD curves, respectively. The onset of seepage failures or internal instability was identified by marked increase in effluent turbidity (> 60 NTU) and significant variations in the magnitudes of applied hydraulic gradients i_a (Skempton and Brogan 1994; Indraratna et al. 2018). At such onset of instability, the corresponding i_a values would be considered as critical hydraulic gradients i_{cr}, which may also corroborate with the visual failure tell-tale signs of test specimens such as heave, composite heave-piping, piping, or suffusion. The tested samples were retrieved in 4 to 5 equivalent layers, and their PSDs were determined after the test. Table 1 shows that an internally stable soil could be identified by the unchanged PSD of its middle layer compared with the original soil gradation. Given that such soils would only fail in heave at $i_{cr} \approx \gamma_s/\gamma_w$ (γ_s=unit weight of soil) with no or less than 4% erosion (Israr and Israr 2018).

Results and discussions

The summary of current test results including test series numbers (No.), specimen identity (ID), pre- and post-test
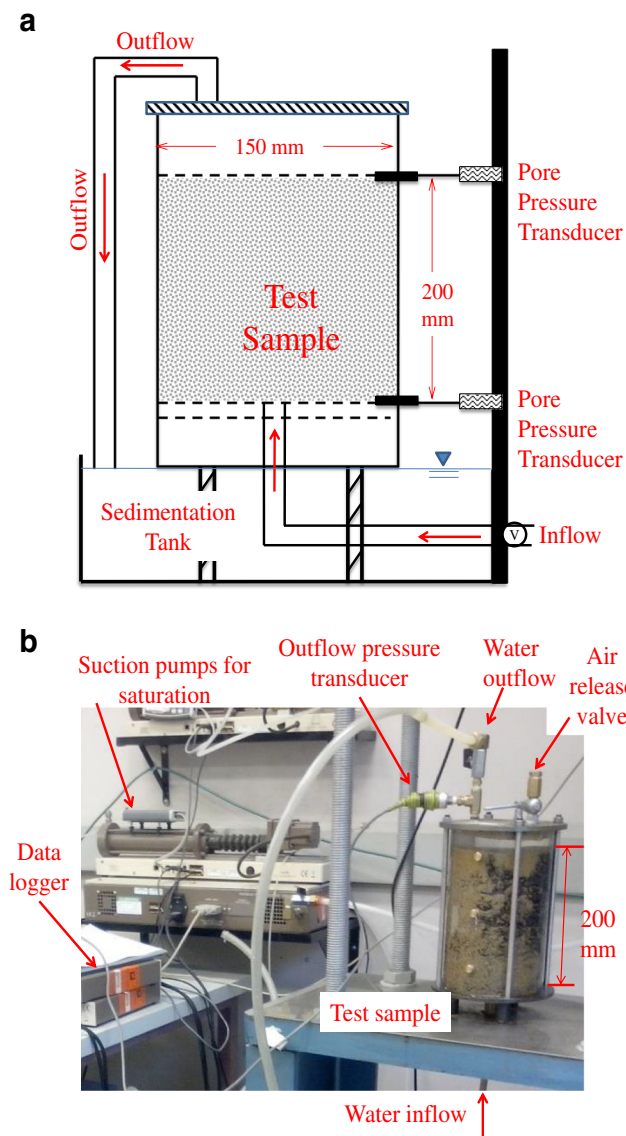
a Schematic illustration showing Outflow, Test Sample (150 mm × 200 mm), Pore Pressure Transducer, Sedimentation Tank, Inflow.

Fig. 4 Effects of compaction on critical hydraulic gradient

b Test apparatus showing Suction pumps for saturation, Outflow pressure transducer, Water outflow, Air release valve, Data logger, Test sample (200 mm), Water inflow.

Fig. 3 Experimental setup. **a** Schematic illustration. **b** Test apparatus (after Israr 2016)

uniformity coefficients (C_u), relative density (R_d), observed critical hydraulic gradients for instability (i_{cr}), geometrical assessments from six different existing criterions, and the type of seepage failure observed in this study have been outlined in Table 1. The variations of i_{cr} at different R_d values are presented in Fig. 4, where as a general trend, any increase in R_d of soil markedly increased its i_{cr}, regardless of internal instability potential of soil. As two limiting cases, both soils A and F showed significant increase in their i_{cr} from 1.07 to 1.29 and 0.28 to 0.40, respectively, when their R_d increased from 5 to 95%. Nevertheless, with a similar increase in R_d, samples D and E exhibited significant increase in their i_{cr} values from 0.45 to 0.98 and from 0.62 to 1.03, respectively. Interestingly, this increase in a relative density could markedly alter the mode of seepage failure of soils D and E. For instance, suffusion in soil-E at $R_d \approx 5\%$ transformed to heave
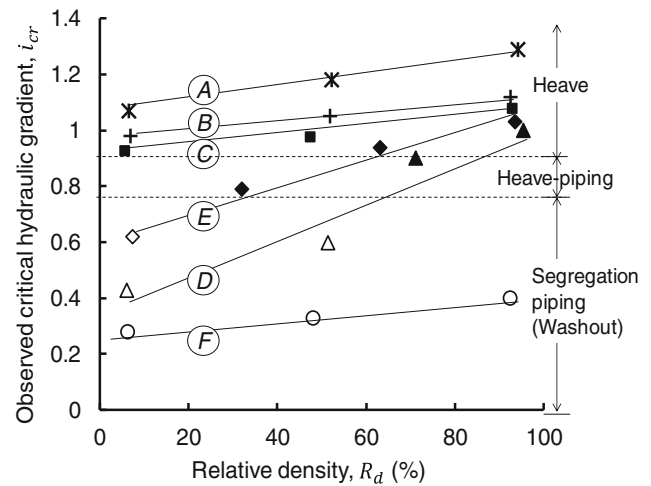
at $R_d \geq 30\%$. Similarly, suffusion in soil-D at $R_d \approx 5\%$ transformed to a composite heave-piping failure with reduced erosion in D-70 ($i_{cr} \geq 0.90$) and eventually to heave failure at $R_d \approx 95\%$ (i.e., $i_{cr} \approx 1.0$). This could clearly show that the increase in relative density would markedly reduce the constriction sizes, thereby minimizing the erosion of fines initiating localized self-filtering. Effects of compaction on eroded fines have been represented in Fig. 5 showing that soils A, B, and C have negligible erosion of fines (i.e., 0, 0.20%, and 0.23% at $R_d = 5\%$, respectively) which reduced further with the increase in their levels of compaction (e.g., 0.03% and 0.08% in soils B and C at $R_d = 100\%$, respectively). Nonetheless, the erosion in soils D and E reduced significantly from nearly 8 to 1% when their R_d increased from 5 to 95%. This is mainly because the compaction minimizes erosion by bringing some of the loose fines together to become an active part of the primary fabric which is responsible for sustainable load transfer and volume changes in granular soils (Skempton and Brogan 1994; Indraratna et al. 2018). These observations also establish that the assumption of loose mobile fines inside the pore spaces
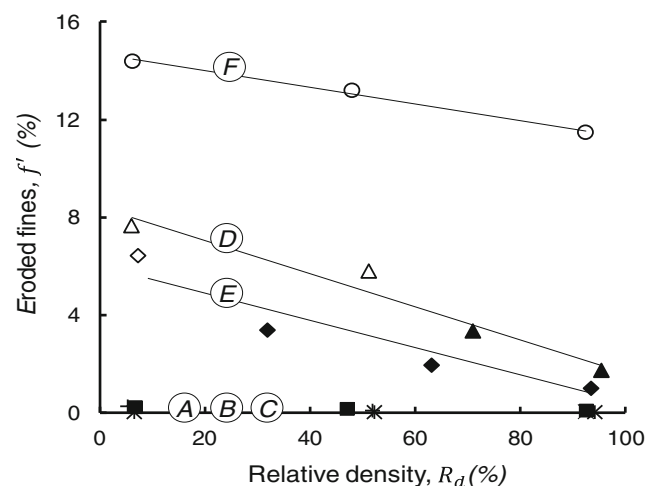

Fig. 5 Effects of compaction on seepage-induced erosion of fines

may not always be correct. This implies that the limiting values of fines (F) proposed by Kenney and Lau (1985) would actually depend upon R_d of soils than their C_u values. For example, soil F showed up to 3% reduction in erosion when its R_d increased from 5 to 95%; although it was still characterized as internally unstable due to significant changes in its original PSD curve as a result of up to 11.5% erosion.

Figure 6 illustrates the initiation and development of seepage failures in selected soil specimens A-50, D-50, and F-50. It can be observed in Fig. 6a that the heave in A-50 initiated in the form of small hairlines at the bottom of specimen at $i_a \approx 1.06$ and became matured at $i_a \approx 1.18$ in the form large visible channel at the base with negligible erosion (Fig. 6b). Similarly, slight rearrangements of fines in D-50 began at $i_a \approx 0.5$ and transformed to a composite heave-piping at $i_a \approx 0.56$ (Fig. 6c, d). Nevertheless,
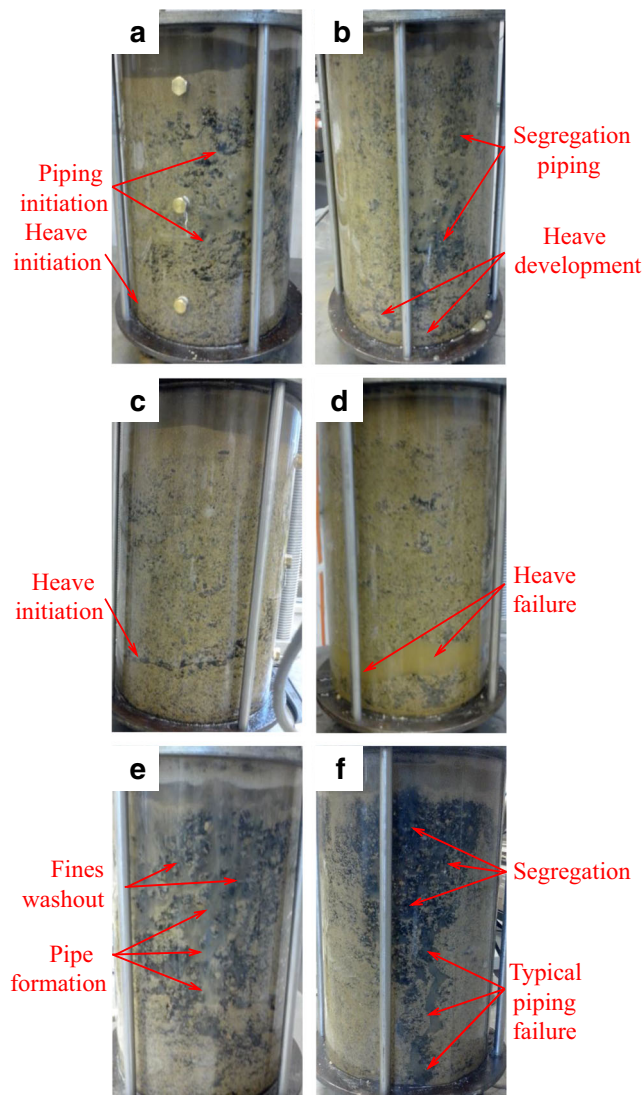
a relatively small erosion in F-50 at $i_a \approx 0.29$ converted into mature suffusion at $i_a \approx 0.33$, as shown in Fig. 6e, f.

Figure 7 shows the pre-test and post-test PSD analysis for the middle layers of select specimens B-50, D-70, and F-50. The negligible erosion of fines from B-50 did not induce significant changes in the original PSD and therefore proved B-50 as internally stable (Fig. 7a). Similarly, D-70 showed limited erosion, which could not cause significant changes to its PSD and thus proved it internally stable (Fig. 7b). In contrast, F-50 suffered from excessive suffusion, which resulted into marked changes in its original PSD and therefore characterized it as internally unstable (Fig. 7c). In summary, soil specimens A-0, A-50, A-100, B-0, B-50, B-100, C-0, C-50, C-100, D-70, D-100, E-30, E-60, and E-100 were characterized as stable, while the rest as internally unstable soils.

Geometrical assessments and proposed revision

A number of well-known existing PSD and CSD-based criteria (Table 1) were adopted to examine the potential of internal instability for the currently tested soils. Notably, the methods of Burenkova (1993) and Wan and Fell (2008) made

Fig. 6 Seepage failures in soils: **a** A-50 at $i_a \approx 1.06$, **b** A-50 at $i_a \approx 1.18$, **c** D-50 at $i_a \approx 0.5$, **d** D-50 at $i_a \approx 0.56$, **e** F-50 at $i_a \approx 0.29$, and **f** F-50 at $i_a \approx 0.33$
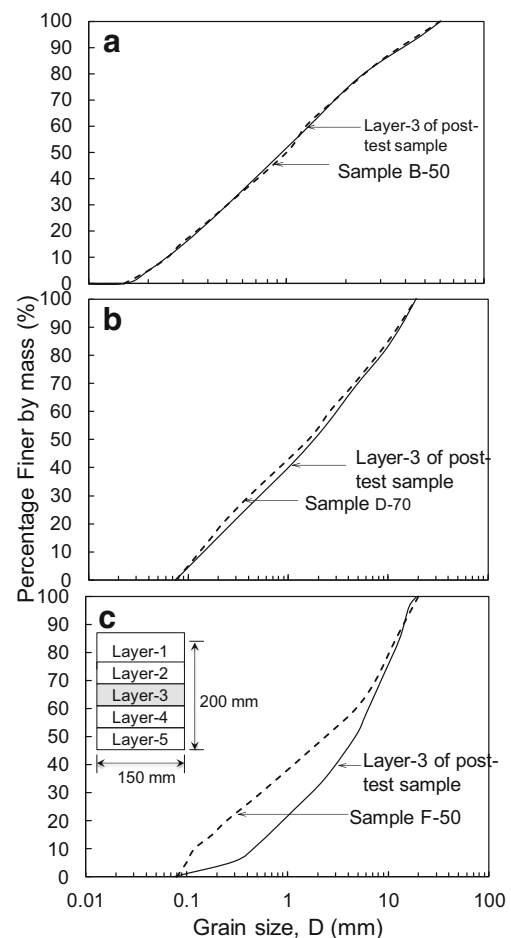
Fig. 7 Post-test PSD analyses for selected soil specimens: **a** B-50, **b** D-70, and **c** F-50

six inconsistent non-conservative assessments each, Kezdi (1979) showed five inconsistent predictions (two unsafe and three conservative), and Sherard (1979) obtained eleven inconsistent assessments (eight unsafe and three conservative). Notably, the criterion of Kenney and Lau (1985) obtained only three inconsistent predictions. Nevertheless, CSD-based criterion of Indraratna et al. (2015) showed 100% success in correctly evaluating the internal stability of tested soils. In essence, the criterion of Kenney and Lau (1985) proved to be the most effective criterion among the available PSD-based criteria.

Figure 8 demonstrates the variations in percentage erosion plotted against the (H/F) values for the tested soils. According to the original criterion of Kenney and Lau (1985), for all well-graded soils with $Cu > 3$, the limiting value of $F = 20\%$ was considered and soils B, C, D, and E were characterized as internally stable except soil F. Interestingly when analyzed for $F > 20\%$, the $(H/F)_{min}$ was obtained at $F = 30\%$ for soils B, C, D, and F (Fig. 8). Here, soils D, F, G, and H were characterized as internally unstable. A probable explanation of this discrepancy would be the role of relative density, whereby a compacted soil mass having smaller constrictions would exhibit a lesser risk of internal instability compared to loose.

In this study, this role of compaction level could be integrated into existing criterion of Kenney and Lau (1985), wherein it is proposed that the evaluation limit for $(H/F)_{min}$ be extended from $F \leq 20$ to $F \leq 30\%$ for soils with R_d up to 70%, while keeping the original limit of $F \leq 20\%$ for soils with $R_d \geq 70\%$. Given that the increasing R_d would significantly reduce the constriction sizes, thus reducing the risk of internal erosion of finer fraction from the soil mass. For instance, the controlling constriction sizes D_{c35} of coarser fraction for the select soil D decreases markedly from 0.205 to 0.13 mm when the relative density increased from 5 to 95%, respectively (Fig. 9). The reduced constriction size could then sufficiently retain the erodible particles from the finer fraction, thereby making the soil more stable (Israr and Israr 2018; Indraratna et al. 2015). For a quick comparison, Figure 10a, b presents the assessments from the criteria of Kenney and Lau (1985),
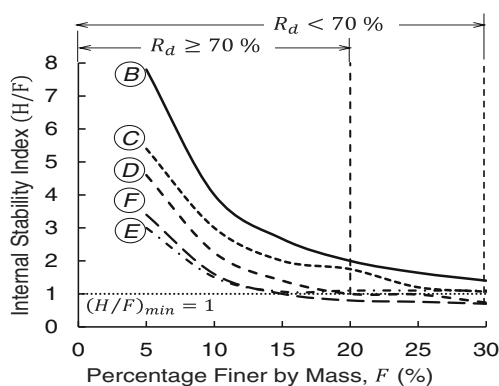
Fig. 8 Variations of internal stability index for original and revised criteria
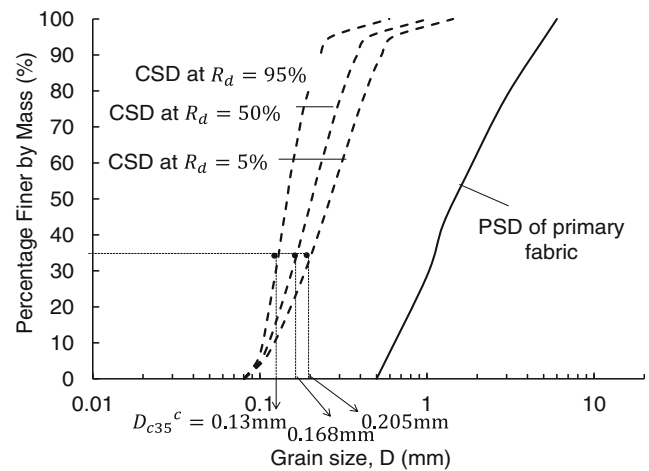
Fig. 9 Compaction-induced variations in constriction sizes of soil D and retention analysis

and this study for the soils tested in this study, respectively. Not surprisingly, the current proposition yields 97% success with only one incorrect prediction (i.e., E-5) in comparison to the original KL criterion with three inconsistent results (i.e., D-5, D-50, and E-5).

Fig. 10 Internal instability potential of test samples: **a** Kenney and Lau (1985) method, **b** revised method (*Note: solid symbol = stable; hollow symbol = unstable*)

Validation and observations

Table 2 presents the laboratory dataset of 91 results including this study and those adopted from 11 published works to verif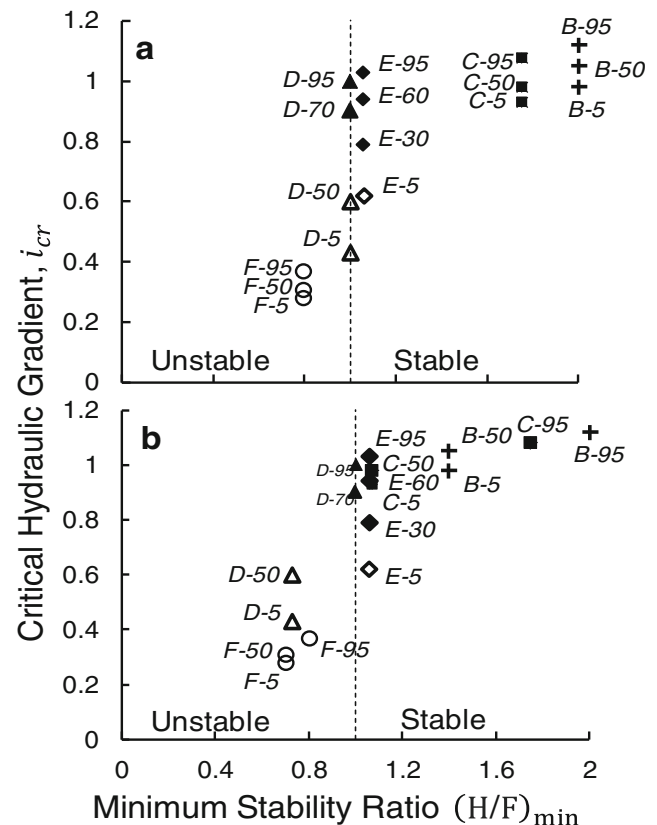y the current proposition, i.e., the revised method of Kenney and Lau (1985). The data table outlines the test numbers (No.), sample identities (ID), R_d values (%), $(H/F)_{min}$ values, and the internal stability assessment results from both methods and experiments. For convenience, original and revised criteria of Kenney and Lau (1985) are abbreviated as KL and RKL, respectively. Figure 11a shows that the KL method yielded eight inconsistent predictions including two conservative or safe (e.g., A_s and G1-c) and six non-conservative or unsafe assessments (e.g., A, B(S), D-5, D-50, E-5, and G3-c). On the contrary, the revised method (RKL) gave only three inconsistent assessments (i.e., one conservative, A and two non-conservative, A and G3-c), as shown in Fig. 11b. It is clear that for those samples, RKL method is more conservative than the original KL method, which is insensitive to the effects of level of compaction and occasionally tends to be unsafe. Interestingly, both the KL and RKL criterion could not correctly assess the internal instability potential for specimen A and proved to be unsafe. A probable explanation of this discrepancy would be the application of severe vibrations applied during the hydraulic test on soil A (Kenney and Lau 1985), and both the methods are also insensitive to the effects of physical disturbance. Furthermore, the well-graded soils B and C ($C_u = 5$ and 10, respectively) were found stable regardless of their level of compaction and none of the soils with $C_u < 10$ reportedly showed instability during hydraulic tests as shown in Tables 1 and 2. The analysis also showed that the uniform soils ($C_u \leq 3$) do not suffer from seepage-induced internal instability. Furthermore, the level of compaction was found to affect markedly the potential of instability, increasing which could transform some of the marginally unstable specimens into internally stable. For instance, B-5, B-50, and C-5 transformed from internally unstable to stable when their R_d respectively increased from 50 to 70% and 5 to 30%. It can be established that the criterion based on R_d (proposed in this study) proved to be more practical than those based on C_u to correctly assess internal instability potential of granular soils.

Implications and scope

Internal stability of selected nine granular soils compacted at different R_d values between 0 and 100% and subjected to upward seepage were experimentally evaluated in this study. The soils tested herein conformed to current industry practices of subballast filter selection for railway industry in New South Wales (NSW) and downstream protective filters in embankment dams worldwide (Israr et al. 2016; Kenney and Lau 1985; Trani and Indraratna 2010; Moffat and Fannin 2006). Notably, the revised criterion proposed here requires only

PSD of soil to assess its internal stability and therefore can easily be coupled with existing filter design methods such as Raut and Indraratna (2008), NRCS (1994), and/or ICOLD (1994). For instance, as a pre-requisite to filter design, the internal stability of base soil, filter soil, and also the anticipated self-filtering layers must be evaluated in order to assure both effectiveness and longevity of granular filters and drainage layers. The natural broadly graded and gap-graded soils would exhibit a higher risk of occurrence of internal instability and most self-filtering layers are also broad- and gap-graded. A timely assessment of their internal stability would allow for precautionary measures to be taken such as compaction, multiple filter placements, re-grading, and replacement.

Notably, the hydraulic equilibrium and the tortuous flow paths in various full-scale problems may not be comparable with those considered here for simplified experimental simulations. Similarly, as a common limitation for most experiments, the scale of the laboratory simulations is not a true scale representative of the actual field problems. Nevertheless, the applicability of the proposed revision in practical problems must be validated further, although it can still be recommended as a means of prompt and preliminary assessments prior to a comprehensive analysis.

Practical design example

The Shoalhaven City Council in consultations with the University of Wollongong installed permeable barriers at Bombaderry in 2007 for an economic and effective treatment of acid-sulphate-contaminated ground water (Raut 2006). Figure 12a shows the PSD of protected base soil B mixed with the ground water that was characterized as a highly dispersive clayey-silt with sand fraction. Initially, a non-reactive permeable barrier F1 was installed as a protective filter to avoid erosion of soil B, while allowing the contaminated ground water to seep through freely. Figure 12b shows the estimated self-filtering layer B-F1, which was obtained by using the following Eq. (1) from Indraratna and Raut (2006). For brevity, given the porosity of soils B (n_B) and F1 (n_{F1}), the percentage finer by mass for the self-filtering layer B-F1 (P_{B-F1}) could be obtained using the following equation:

$$P_{B-F1} = \left[\frac{1-n_{F1}}{1-n_B}\right] \times \frac{P_B}{n_{F1}} \qquad (1)$$

Notably, the resulting self-filtering layer B-F1 was characterized as internally unstable by most existing methods including the current approach. Therefore, an additional permeable reactive barrier F2 was installed on the downstream of F1 with a twofold objective of protecting B-F1 and treating the contaminated ground water prior to its disposal into the river. Not surprisingly, this technique could effectively prevent F2 from clogging, thereby assuring enhanced longevity of the filtering

Table 2 Published data for the validation of revised Kenney and Lau method

Test no.	Sample ID	Reference	R_d(%)	C_u	$(H/F)_{min}$	Assessments of internal stability (S stable)		
						Kenney and Lau (1985)	Revised (Kenney and Lau 1985)	Experimental
1	A	Kenney and Lau (1985)	90	30	1	S	S	U
2	A_s		90	24	0.9	U	U	S
3	D		90	30	0.5	U	U	U
4	D_s		90	3.67	3	S	S	S
5	1		90	20	1.2	S	S	S
6	2		80	6	1.4	S	S	S
7	3		90	17.5	1.9	S	S	S
8	20		90	8	2	S	S	S
9	21		90	6.2	1.2	S	S	S
10	23		90	20	3.4	S	S	S
11	K		90	4	3.4	S	S	S
12	X		90	31.8	0.5	U	U	U
13	Y		90	50	0.7	U	U	U
14	Y_s		90	40	0.9	U	U	U
15	F-1 (I)	Indraratna and Vafai (1997)	90	2.87	1.67	S	S	S
16	F-2 (I)		90	2.87	2.33	S	S	S
17	Fine	Indraratna et al. (1996)	50	1.28	--##	S	S	S
18	Medium		50	1.45	--##	S	S	S
19	Coarse		50	1.47	--##	S	S	S
20	F-1 (I)	Indraratna et al. (2007)	70	3	2.33	S	S	S
21	F-2 (I)		70	3	1.76	S	S	S
22	F-4 (I)		70	1.2	--##	S	S	S
23	F-5 (I)		70	1.2	--##	S	S	S
24	F-6 (I)		70	1.2	--##	S	S	S
25	LF2	Lafleur et al. (1989)	70	11.6	1.11	S	S	S
26	LF3		70	5.9	1.33	S	S	S
27	LF4		70	4	2	S	S	S
28	LF5		70	3	9	S	S	S
29	L1	Locke et al. (2001)	70	4	2.9	S	S	S
30	L2		70	3.7	2.72	S	S	S
31	A(S)	Skempton and Brogan (1994)	5	24	0.1	U	U	U
32	B(S)		5	10	1	S	S	U
33	C(S)		5	7	1.5	S	S	S
34	D(S)		5	4.5	1.67	S	S	S
35	NG1	Nguyen et al. (2013)	70	4	--##	S	S	S
36	NG2		70	3.33	--##	S	S	S
37	NG3		70	2.67	2.07	S	S	S
38	NG4		70	2	1.87	S	S	S
39	NG5		70	1.67	1.77	S	S	S
40	NG6		70	1.33	1.67	S	S	S
41	NF1	Indraratna et al. (2012)	70	2.1	--##	S	S	S
42	NF2		70	1.9	--##	S	S	S
43	NF3		70	1.8	--##	S	S	S
44	NF4		70	5.1	1.35	S	S	S
45	NF5		70	4.5	1.75	S	S	S
46	NF6		70	3.5	2.2	S	S	S

Table 2 (continued)

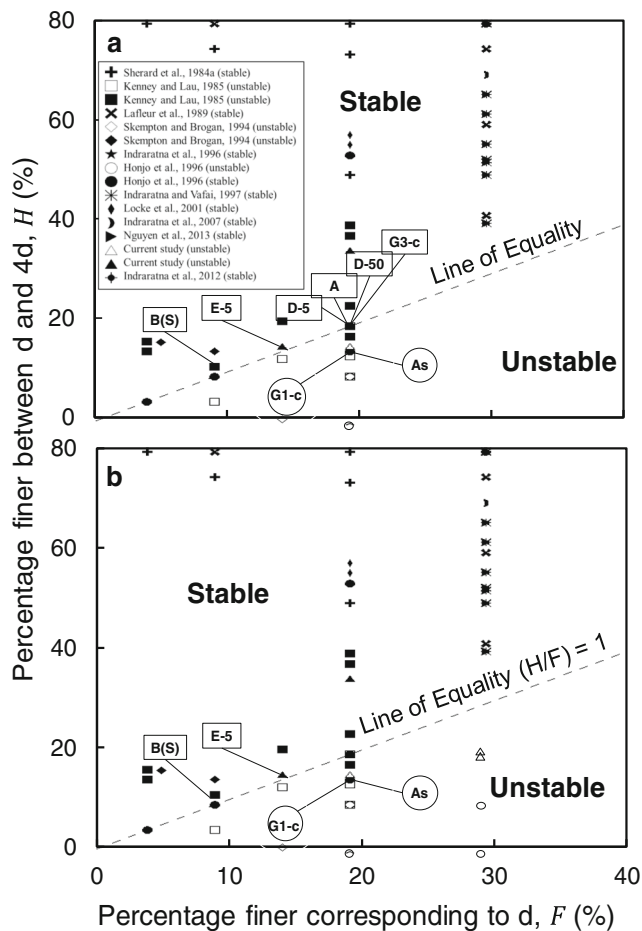| Test no.	Sample ID	Reference	R_d (%)	C_u	(H/F)$_{min}$	Assessments of internal stability (S stable)		
						Kenney and Lau (1985)	Revised (Kenney and Lau 1985)	Experimental
47	G1-a	Honjo et al. (1996)	70	GG	1	S	S	S
48	G1-b		70	GG	1	S	S	S
49	G1-c		70	GG	0.75	U	U	S
50	G2-a		70	GG	2.7	S	S	S
51	G2-b		70	GG	1.6	S	S	S
52	G2-c		70	GG	1.3	S	S	S
53	G3-a		70	GG	0	U	U	U
54	G3-b		70	GG	0	U	U	U
55	G3-c		70	GG	0.3	S	U	U
56	G4-a		70	GG	0	U	U	U
57	G4-b		70	GG	0	U	U	U
58	G4-c		70	GG	0.3	U	U	U
59	SF1	Sherard et al. (1984)	70	6.3	1.83	S	S	S
60	SF2		70	2.9	7.5	S	S	S
61	SF3		70	2.2	18	S	S	S
62	SF4		70	1.9	--##	S	S	S
63	SF5		70	1.8	--##	S	S	S
64	SF7		70	1.1	--##	S	S	S
65	SF8		70	2.7	4	S	S	S
66	SF9		70	2.8	3.7	S	S	S
67	SF10		70	2.2	--##	S	S	S
68	SF11		70	1.5	--##	S	S	S
69	SF12		70	1.6	--##	S	S	S
70	SF13		70	1.1	--##	S	S	S
71	SF14		70	1.2	--##	S	S	S
72	A-0	This study	7	1.2	--##	S	S	S
73	A-50		52	1.2	--##	S	S	S
74	A-100		94	1.2	--##	S	S	S
75	B-0		7	5	1.4	S	S	S
76	B-50		52	5	1.4	S	S	S
77	B-100		93	5	2	S	S	S
78	C-0		6	10	1.07	S	S	S
79	C-50		47	10	1.07	S	S	S
80	C-100		93	10	1.8	S	S	S
81	D-0		6	20	0.73	S	U	U
82	D-50		52	20	0.73	S	U	U
83	D-70		71	20	1	S	S	S
84	D-100		96	20	1	S	S	S
85	E-0		7	23	1.02	S	S	U
86	E-30		32	23	1.02	S	S	S
87	E-60		63	23	1.02	S	S	S
88	E-100		94	23	1.06	S	S	S
89	F-0		6	40	0.75	U	U	U
90	F-50		48	40	0.75	U	U	U
91	F-100		93	40	0.7	U	U	U

GG gap-graded soil

Fig. 11 Assessment predictions for published data and current test results: **a** original (Kenney and Lau 1985) criterion, **b** proposed criterion *(Note: inconsistent predictions are marked in accordance with specimen IDs in Tables 1 and 2 (circle = unsafe; rectangle = safe)*

Fig. 12 Particle size distribution curves for **a** erosible acid sulphate soil-B to be protected and non-reactive filter barrier-F1 and **b** self-filtering layer B-F1 obtained by the method of Indraratna and Raut (2006) and reactive protective filter barrier-F2

facility. Given that the above facility is still fully operational today in 2018, the internal stability of this dual layer filtering system is re-examined using the approach proposed in this study (see also Table 3).

At 50% relative density, the criterion of Keney and Lau (Kenney and Lau 1985) assessed all soils B, F1, and F2 as internally stable except self-filtering layer B-F1, whereas the proposed criterion would conservatively characterize both B and B-F1 as unstable. At $R_d = 90\%$, both criteria would assess the gradations B, F1, and F2 as internally stable, the self-filtering layer B-F1 as unstable. This clearly justifies the need of a second protective filter F2 in order to protect the self-filtering layer B-F1. As discussed earlier, the original criterion of Kenney and Lau (1985) tends to be unsafe, while the current criterion proved to be more accurate and conservative. For example, the original criterion does not recommend the second protective filter F2 as it could not assess the correct potential of internal instability of soil B. Notably, the second protective filter F2 was agreed
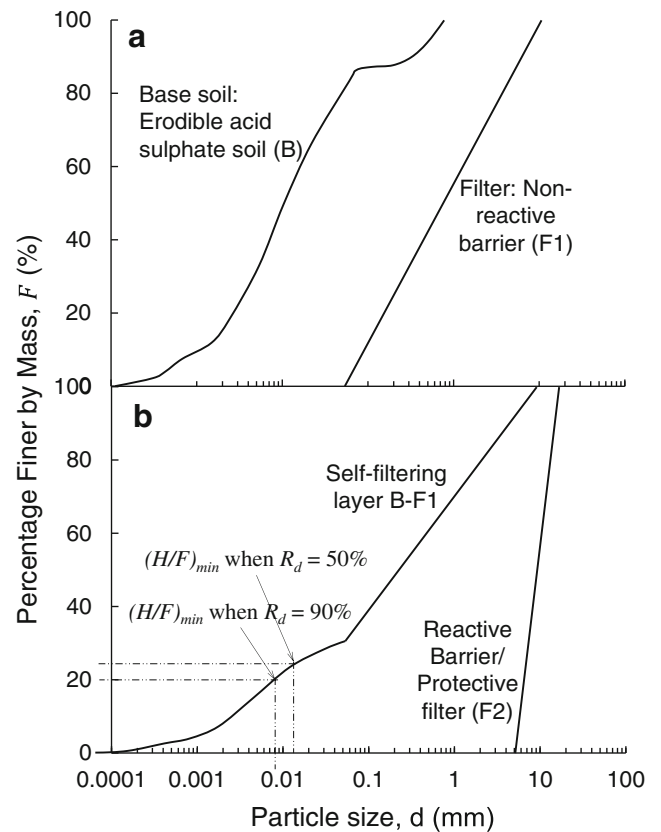
upon by the Shoalhaven City Council as an environmental protection measure and not on the basis of instability of both F1 and B-F1. Nonetheless, the current study scientifically justifies the provision of the second filter F2.

Table 3   Re-evaluation of an existing filtering system for water treatment

Soil	R_d (%)	C_u	Kenney and Lau (1985)		This study	
			$(H/F)_{min}$	Result	$(H/F)_{min}$	Result
B	50	125	1.45	S	0.97	U
	90		1.45	S	1.45	S
F1*	50	11	1.5	S	1.13	S
	90		1.5	S	1.5	S
F2**	50	1.5	--#	S	--#	S
	90		--#	S	--#	S
B-F1	30	135	0.5	U	0.33	U
	75		0.5	U	0.5	U

*Permeable non-reactive barrier

**Permeable reactive barrier

#Infinite value from a highly stable uniform soil

Conclusions

The results are reported from a series of hydraulic gradient-controlled filtration tests on different granular soils at various levels of compaction and the following conclusions are drawn:

• The risk of occurrence of internal instability of granular soils in the form of heave, suffusion, and piping failure depends significantly on both the particle size distribution and the degree of compaction. Nevertheless, the soil would exhibit instability at unique magnitude of hydraulic gradient, which may be lower for a soil with greater potential of internal instability such as broadly graded and gap-graded soils.

• The test results showed that the risk of instability in some soils could markedly be reduced by increasing the level of compaction. For example, the broadly graded soils D and E showed marked reduction in the erosion and thus proved stable at $R_d > 70\%$. Nonetheless, the soil F showed up to 3% reduction in the erosion at $R_d > 95\%$, although the pre- and post-test PSD analysis results proved it was still internally unstable.

• A revision of a widely accepted particle size-based criterion of Kenney and Lau (1985) considering the relative densities of filters has been presented. Although, the original criterion showed acceptable degree of success in evaluating the instability potential of soils, there is an obvious need to make it more precise. In essence, the original boundaries based on the uniformity coefficients of soils could be more appropriately established on the basis of level of compaction. For instance, it was observed that the soils compacted below 70% relative density may be examined up to 30% finer by mass (i.e., $F \leq 30\%$) when evaluating $(H/F)_{min}$ values. Nevertheless, the limit of $F \leq 20\%$ finer by mass would still be conservative for granular soils at relative density above 70%.

• Although a large set of published data satisfactorily validated the findings of this study, it is recommended that further studies should be carried out to investigate additional practical problems such as cohesive soils, and artificial mixtures of broadly and gap-graded powders and chemicals.

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