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Performance evaluation of artificial neural networks for planimetric coordinate transformation—a case study, Ghana

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Abstract Two national horizontal geodetic datums, namely,
the Accra and Leigon datum, have been the only available
datum used in Ghana. These two datums are non-geocentric
and were established based on astro-geodetic observations.
Relating these different geodetic datums mostly involves the
use of conformal transformation techniques which could pro-
duce results that are not very often satisfactory for certain geo-
detic, surveying and mapping purposes. This has been ascribed
to the incapability of the conformal models to absorb more of
the heterogeneous and local character of deformations existing
within the local geodetic networks. Presently, application of
new approaches such as artificial neural network (ANN) is
highly recommended. Whereas the ANN has been gaining
much popularity to solving coordinate transformation-related
problems in recent times, the existing researches carried out
in Ghana have shown that only three-dimensional conformal
transformation methods have been utilized. To the best of our
knowledge, plane coordinate transformation between the two
local geodetic datums in Ghana has not been investigated. In
this paper, an attempt has been made to explore the plane
coordinate transformation performance of two different ANN
approaches (backpropagation neural network (BPNN) and ra-
dial basis function neural network (RBFNN)) compared with
two different traditional techniques (six- and four-parameter
models) in the Ghana national geodetic reference network.
The results revealed that transforming plane coordinates from
Leigon to Accra datum, the RBFNN was better than the BPNN
and traditional techniques. Transforming from Accra to Leigon
datum, both the BPNN and RBFNN produced closely related
results and were better than the traditional methods. Therefore,
this study will create the opportunity for Ghana to recognize the
significance and strength of the ANN technology in solving
coordinate transformation problems.

Introduction

Within the last decade, artificial neural network (ANN) has
been widely adopted and applied to different areas of geodesy.
Its suitability as an alternative technique to the classical
methods of solving most geodetic problems has been duly
investigated. Some of these problems are in GPS height con-
version (Fu and Liu 2014; Liu et al. 2011; Lei and Qi 2010;
Tieding et al. 2010; Wu et al. 2012), geodetic deformation
modelling (Bao et al. 2011; Du et al. 2014a, b; Gao et al.
2014; Pantazis and Eleni-Georgia 2013; Yilmaz and Gullu
2012; Yilmaz 2013), earth orientation parameter determina-
tion (Liao et al. 2012; Schuh et al. 2002; Yu et al. 2015),
precise orbital prediction (He-Sheng 2006; Li et al. 2014),
gravity anomaly estimation (Hajian et al. 2011; Hamid and
Mohammad 2013; Tierra and De Frietas 2005), geoid

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determination (Kavzoglu and Saka 2005; Pikridas et al. 2011 ; Stopar et al. 2006; Sorkhabi 2015; Veronez et al. 2006, 2011) and geodetic coordinate transformation (Gullu 2010; Gullu et al. 2011; Lin and Wang 2006; Mihalache 2012; Tierra et al. 2008, 2009; Tierra and Romero 2014; Turgut 2010; Yilmaz and Gullu 2012; Zaletnyik 2004) and to solve many other problems in geodetic applications.

It is well known that integrating geodetic data from different reference systems is a problem of coordinate transformation. In order to solve this problem, different techniques have been applied in literature. Notable among them are the classical methods such as conformal models, affine models and multiple regression to mention but a few. These aforementioned techniques have been applied to solve coordinate transformation problems (Newsome and Harvey 2003; Kinneen and Featherstone 2004; El-Mowafy et al. 2009; Baiocchi et al. 2011; Gledan and Azzeidani 2014). In addition to the traditional coordinate transformation techniques, ANN has been successfully applied in coordinate transformation (Gullu 2010; Gullu et al. 2011; Lin and Wang 2006; Mihalache 2012; Tierra et al. 2008, 2009; Tierra and Romero 2014; Turgut 2010; Yilmaz and Gullu 2012; Zaletnyik 2004).

The general insights gathered from these studies indicate that ANN could produce more accurate transformed coordinate values compared with the classical transformation methods. This is because most traditional coordinate transformation techniques often times do not absorb more of the distortions in the data related to the different geodetic datums (Grgic et al. 2015). Hence, ANN has been recommended as a plausible alternative method for coordinate transformation (Zaletnyik 2004; Tierra et al. 2008, 2009; Turgut 2010; Yilmaz and Gullu 2012; Tierra and Romero 2014).

Although the ANN technique has been applied to solve majorities of geodetic problems, most developing countries, especially in Africa, are yet to adopt, apply and test its efficiency. Ghana is one such post-colonial country where the continual use of two non-geocentric systems, namely, Accra datum and Leigon datum, for national mapping still persists (Mugnier 2000; Ayer 2008).

Moving along with the disheartened political, economic and socio-cultural trend, there are numerous challenges to incorporating data based on the two geodetic datums (Accra and Leigon) in Ghana. The reason is that, currently, only three-dimensional (3D) coordinate transformation using conformal models of Bursa-Wolf, Molodensky-Badekas and Veis as well as 12-parameter linear affine and 3D projective model (Ayer and Tienhah 2008; Dzidefo 2011; Ziggah et al. 2013; Kumi-Boateng and Ziggah 2016) has been applied to transform from global (World Geodetic System 1984) datum to the local datums (Accra and Leigon). Thus, little has been done in integrating the two local reference systems in Ghana. In addition, Ayer and Fosu (2008) showed only the discrepancies between Ghana's local datums with no transformation technique

applied to unify the two geodetic datums. This implies that integrating the data between Accra and Leigon has not been fully investigated. Moreover, to the best of our knowledge, ANN has not been applied in Ghana before for coordinate transformation. It will therefore be interesting to assess the ANN performance for horizontal coordinate transformation between the two local geodetic datums in Ghana's geodetic reference network.

The main objective of this study is to test the predictive capability of ANN in plane coordinate transformation for the first time in Ghana's geodetic reference network against the traditional coordinate transformation methods like four-parameter and six-parameter models. Hence, in this study, to relate the different geodetic datums (Accra and Leigon) in Ghana, two different types of ANN, backpropagation neural network (BPNN) and radial basis function neural network (RBFNN), were applied to the common point coordinates. The ANN results were then compared with the traditional four-parameter and six-parameter transformation models. The findings revealed that the RBFNN was the most efficient technique to be utilized in transforming coordinates from the Leigon to the Accra datum. However, from Accra to Leigon datum, it was observed that both RBFNN and BPNN produced comparable results. Moreover, the overall analysis showed that the ANN was superior to the six- and four-parameter models. This study will create the opportunity for developing countries like Ghana to know the efficiency of applying ANN as a practical alternative technology to the traditional methods for its coordinate transformation. It would also improve the general accuracy involved in cadastral and engineering surveys.

Study area and data source

The study area is focused on Ghana's geodetic reference network. Ghana is a country situated in the Western part of Africa sharing borders with Togo to the East, Burkina Faso to the North, Ivory Coast to the West and the Gulf of Guinea to the South. Ghana covers an area of about 239,460 km^2 with the land mass consisting of low plains with a dissected plateau in the South Central area and scattered areas of high relief (Baabereyir 2009). It lies between latitude 4° 30′ and 11° N and longitude 1° E and 3° W (Mugnier 2000). The country has two local geodetic systems, namely, Accra datum and Leigon datum. The reference surface for the Accra datum is the War Office 1926 ellipsoid suggested by the British War Office, with semi-major axis a = 6,378,299.99899832 m; semi-minor axis b = 6,356,751.68824042 m; flattening f = 1/296; and Gold Coast feet to meter conversion factor of 0.304799706846218 (Ayer 2008; Ayer and Fosu 2008). The Leigon datum has the Clark 1880 (modified) ellipsoid as its reference surface, with semi-major axis a = 6,378,249.145 m; semi-minor axis

$b = 6{,}356{,}514.870$ m; flattening $f = 1/293.465006079115$; and British Foot (Sear's) to meter conversion factor of 0.304799470. For planimetric coordinate estimation, the Transverse Mercator projection system is used (Mugnier 2000; Poku-Gyamfi and Hein 2006). Hence, the coordinate system used to indicate positions of features of all survey maps in Ghana is the projected grid coordinates of easting and northing derived from the Transverse Mercator projection.

In this study, secondary data of 27 common control points (Fig. 1) in eastings and northings in both Accra datum and Leigon datum were obtained from the Ghana Survey and Mapping Division of Lands Commission. These datasets provided constitute the national local coordinates of the newly established geodetic reference network referred to as the golden triangle. This golden triangle was established by the Ghana Survey and Mapping Division through the Land Administration Project (LAP) sponsored by the World Bank (Kotzev 2013). The objective for the golden triangle establishment was to enhance the use of Global Navigation Satellite System (GNSS) for land-related positioning undertakings such as geotechnical investigations, traffic and transportation, meteorology, survey and mapping, timing, engineering and many others in Ghana (Wonnacot

Fig. 1 Study area
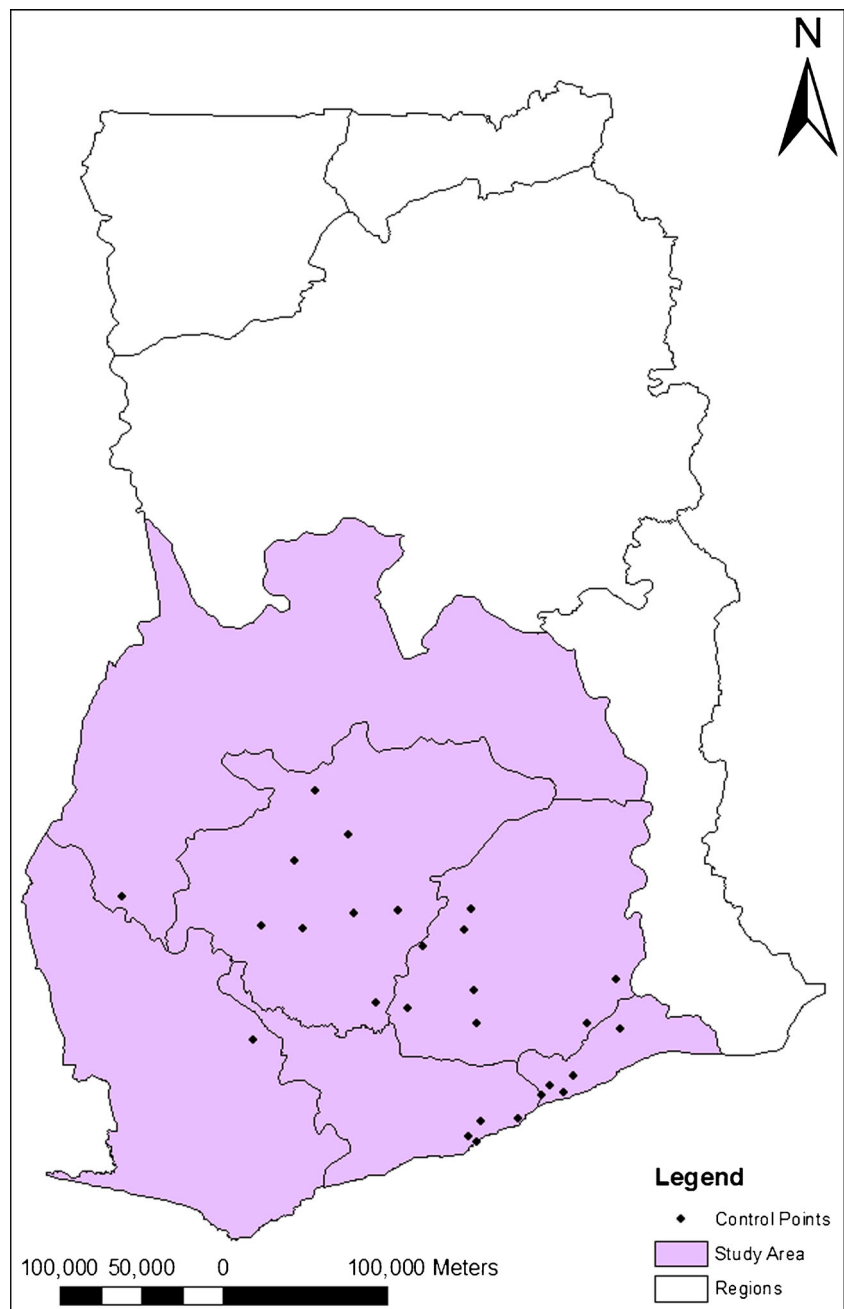
2007; Poku-Gyamfi and Schueler 2008). Also, the unification of all national reference frames for all African countries under the African Reference Frame into a single continental reference system based on the international terrestrial reference system (ITRS) also necessitated the need to renew Ghana's geodetic network.

Artificial neural network methods

Over the past years, artificial neural network (ANN) is a technology that has gained widespread applications for many fields. It is inspired by the structure and behaviour of biological neurons and the nervous system (Kecman 2001). ANN comprises of several neurons connected together with links between variable synaptic weights that process information fed into the network when stimulus is received from the environment. Thus, each neuron unit receives input information weighted by a factor which signifies the strength of the synaptic connection to produce an output. This output is then sent as a new input to another neuron by adapting new weights if the total sum of the weighted inputs is above a certain threshold.

Generally, various ANN types have been proposed based on their architecture, which is structured into different layers. It is worth stating that the application of ANN in geodesy depends on the type of problem to be solved and the training algorithm to be applied. In the present study, the supervised learning algorithm was adopted for the ANN model development and subsequent prediction. This is because in the supervised learning, for each example, the goal is to use the inputs to predict the values of the outputs. The main objective here is to build a forecasting model that can produce accurate transformed coordinate results when independent data are presented to the network. Moreover, the supervised training offers the opportunity to interpret the output results based on the training values. In this study, optimized BPNN and RBFNN models for transforming planimetric coordinates from Accra datum to Leigon datum and vice versa were developed. The choice of these networks was based on their frequent use as universal function approximators (Hartman et al. 1990; Hornik et al. 1989; Park and Sandberg 1991) within the geoscientific disciplines.

In order to develop the BPNN and RBFNN models to achieve the results presented in this paper, the procedural stages adopted are described in the subsequent sections.

Data and selection of input variables

In the present study, 27 common control points in Accra datum and Leigon datum for the Ghana national geodetic reference network were used in the BPNN and RBFNN model formulations. The next issue was to identify the input parameters from the dataset for the ANN training. It is well acknowledged that the input neurons act as control variables with an influence on the desired outputs of the neural network. Hence, the input data

should represent the condition for which training of the neural network is done (Konaté et al. 2015). Consequently, transforming plane coordinates from Leigon to Accra datum, the easting (E) and northing (N) in the Leigon datum denoted as (E_{clark}, N_{clark}) were used as the input layer data, while (E_{war}, N_{war}) in the Accra datum was used as the output layer data. In the transformation from Accra to Leigon datum, (E_{war}, N_{war}) was applied as the input data, while (E_{clark}, N_{clark}) was used as the output layer data, respectively.

Normalization

Usually, the original data to be used for the ANN training and its model formulation are expressed in different units with different physical meanings. Therefore, to ensure constant variation in the ANN model, datasets are frequently normalized to a certain interval such as [−1, 1], [0, 1] or other scaled criteria. This data normalization improves convergence speed and doing so reduces the chances of getting stuck in local minima. In this study, the selected input and output variables were normalized into the interval [−1, 1] using Eq. (1) (Mueller and Hemond 2013)

$$y_i = y_{\min} + \frac{(y_{\max} - y_{\min}) \times (x_i - x_{\min})}{(x_{\max} - x_{\min})}, \tag{1}$$

where y_i represents the normalized data, x_i is the measured coordinate value, while $x_{\min}$ and $x_{\max}$ represent the minimum and maximum values of the measured coordinates with $y_{\max}$ and $y_{\min}$ values set at 1 and −1, respectively.

ANN architecture

In this study, two supervised ANNs, namely, BPNN and RBFNN were utilized due to its frequent application. Both networks have a feedforward topology consisting of input, hidden and output layers that are fully interconnected. A more detailed description of the BPNN and RBFNN structures is given in "Backpropagation neural network" and "Radial basis function neural network" sections, respectively.

Network training

It is a well-known fact that datasets are trained in ANN in order to generate the required desired output for a particular input. Hence, the 27-common point coordinate in Accra datum and Leigon datum for the Ghana national geodetic reference network was divided into reference dataset and testing dataset. Twenty points were selected as the reference points (P_1, P_2, …, P_{20}), while the testing dataset comprised of 7 points (T_1, T_2, …, T_7). In this study, the chosen reference points were used in the BPNN and RBFNN training process. The testing data, on the other

hand, served as an independent check on the performance of the ANN techniques aforementioned.

It is important to note that the training set served as parameterization; thus, it is used in calculating the gradient and for weight adaptation and biases (Yilmaz 2013). The testing data which had no effect on training were applied to the trained models to independently ascertain their performance. For the network training, the Levenberg-Marquardt backpropagation algorithm (Nocedal and Wright 2006) was used to train the BPNN and the gradient descent rule was used to train the RBFNN (Fernandez-Redondo et al. 2006). These networks (BPNN and RBFNN) were allowed to train until no additional effective improvement occurred. Consequently, if there was a significant change in terms of error between the training and testing results, then there was a possibility of overfitting occurring. In such situations, the error on the testing set typically begins to rise although, at the initial phase of training, both the training and testing errors were at a minimum. This implies that the ANN has learned the specific details of the training set instead of the general pattern found in all present and future data (Ziggah et al. 2016).

In determining the optimum BPNN and RBFNN models, the mean squared error (MSE) of all the models was monitored at each stage of training and testing. After several trials, the model with the lowest MSE value was selected as the optimum. Other statistic indicators used for evaluating the BPNN and RBFNN models obtained results are given in "Model performance assessment" section. It is noteworthy that only the results given by the optimum performing BPNN and RBFNN models are presented in this study.

Backpropagation neural network

The backpropagation neural network (BPNN) has gained much popularity over the last decade and has many application areas in geodesy (Lin and Wang 2006; Mihalache 2012; Tierra and Romero 2014; Turgut 2010; Yilmaz and Gullu 2012; Zaletnyik 2004). The BPNN encompasses an input layer, one or more hidden layers and an output layer of processing neurons with each layer feeding input to the next layer in a feedforward fashion through a set of connection weights (Yegnanarayana 2005). Figure 2 shows a typical BPNN architecture of inputs ($X_1, X_2, …, X_N$) and outputs ($Y_1, …, Y_M$).

The input layer is an opening that is responsible for receiving the input data, whereas the output layer gives the final results of the computation. In between these two layers is the hidden layer chamber where the input data are fed to the neurons in the hidden layer for processing. It is important to state that the connections between all the layers of the network are realized through synaptic weights, which are in turn used by the network to solve a specific problem.

It is also well acknowledged that the number of hidden neurons, hidden layers and type of activation functions used
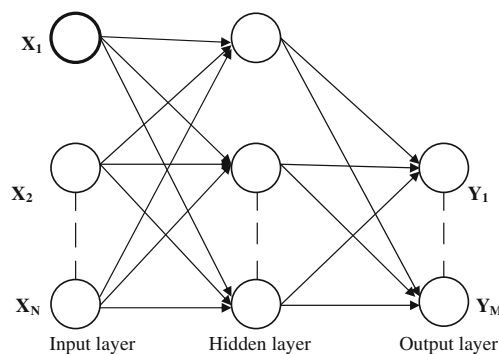
Fig. 2 Schematic BPNN representation

in the BPNN determines its competency. Typically, the number of hidden neurons is obtained through the sequential trial-and-error approach. This is partly due to (i) the type of problem at hand, (ii) the choice of neural network architecture and (iii) the proposed theoretical concepts that are yet to be universally accepted to clarify the number of hidden neurons needed to approximate a given function. In this study, the optimum number of hidden neurons was obtained based on the lowest mean squared error (MSE). The MSE is represented by Eq. (2) as

$$\mathrm{MSE} = \frac{1}{n} \sum_{i=1}^{n} (O_i - P)_i^2, \tag{2}$$

where O and P are the measured and predicted plane coordinates from the BPNN model.

The present study applied one hidden layer in the BPNN. This decision was in line with the conclusion made in Hornik et al. (1989) that the BPNN with one hidden layer could be used as a universal approximator for any discrete and continuous functions. Furthermore, to introduce non-linearity into the network, the hyperbolic tangent activation function was selected for the hidden units, while a linear function was applied for the output units. The hyperbolic tangent function (Yonaba et al. 2010) is defined in Eq. (3) as

$$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1, \tag{3}$$

where x is the sum of the weighted inputs.

It worth stating that the BPNN training can be considered as a non-linear optimization problem, w^* (Konaté et al. 2015), given by Eq. (4)

$$w^* = \mathrm{argmin} E(w), \tag{4}$$

where w is the weight matrix and $E(w)$ is the error function. The purpose of training the network is to find the optimal weight connection (w^*) that minimizes $E(w)$ such that the estimated outputs from the BPNN will be in good agreement

with the target data. This $E(w)$ is evaluated at any point of w shown in Eq. (5) as

$$E(w) = \sum_n E_n(w), \tag{5}$$

where n is the number of training samples and $E_n(w)$ is the output error for each sample n. $E_n(w)$ (Gope et al. 2015) is mathematically defined by Eq. (6)

$$E_n(w) = \frac{1}{2} \sum_j \left(D_{nj} - Y_{nj}(w)\right)^2, \tag{6}$$

where D_{nj} and $Y_{nj}(w)$ are anticipated network outputs and estimated values of the jth output neuron for the nth sample, respectively. Therefore, substituting Eq. (6) into Eq. (5) gives the objective function to be minimized expressed in Eq. (7) as

$$E(w) = \frac{1}{2} \sum_n \sum_j \left(D_{nj} - Y_{nj}(w)\right)^2. \tag{7}$$

Radial basis function neural network

The radial basis function neural network (RBFNN) is based on a feedforward network architecture consisting of a single hidden layer. It has a three-layered topology, thus input, hidden and output layers that are interconnected in a feedforward manner. The input layer accepts information from the external environment, while the output layer provides the final calculated outcomes. The layer that does not have direct access to the external world is known as the hidden layer. Figure 3 shows a typical RBFNN structure of inputs $(X_1, X_2, ..., X_d)$, radial basis functions $(\varphi_1, ..., \varphi_N)$, weights $(W_1, ..., W_N)$ and output (y), respectively.

It must be noted that, in RBFNN, the input data are transferred to the hidden layer chamber through unweighted connections. A non-linear activation function is then applied to the hidden layer to transform the received input layer information with each neuron estimating a Euclidean distance between the input to the network and the position of the neuron called the centre. This is then inserted into a radial basis activation function, which calculates and outputs the activation of the neuron (Deyfrus 2005). The present study applied the Gaussian activation function (Gurney 2005) expressed in Eq. (8) as

$$a_j = \phi_j(X) = \exp\left[-\frac{\|X - \mu_i\|^2}{2\sigma_j^2}\right], \tag{8}$$

where X is the input vector, μ_i is the centre of the Gaussian function, σ_j is the spread parameter of the Gaussian function, and $\|X - \mu_i\|$ is the Euclidean norm. The output layer contains the linear activation function and uses the weighted sum of the
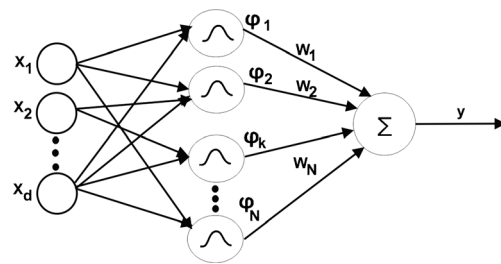

Fig. 3 RBFNN architecture

hidden layer as propagation function expressed in Eq. (9) (Tierra et al. 2008) as

$$Y_k = \sum_{j=1}^{p} W_{jk} \; a_j + W_0. \tag{9}$$

Here, each W_{jk} is the output weight that matches to the association between a hidden node and an output node, while W_0 is the bias and p denotes the number of hidden neurons. It should be noted that through training of the RBFNN, Eq. (9) could be achieved. Therefore, in this study, the supervised learning approach was adopted to establish the input-output mapping relationship. It is well known that the RBFNN training could be regarded as a non-linear optimization problem (Barsi 2001) such that the estimated outputs from the RBFNN will be in good agreement with the existing data. The error e_k in the output of a neuron k is defined as the deviation of the desired value d_k from the computed value Y_k in the first step (Haykin 1999). This is expressed by Eq. (10) as

$$e_k = d_k - Y_k. \tag{10}$$

The RBFNN training process continues until the network error (MSE) reaches an acceptable value.

Traditional coordinate transformation techniques

The traditional coordinate transformation methods applied in this study were the four-parameter and six-parameter models. These transformation models were used to transform

Table 1 Six-parameter model transformation from Leigon datum to Accra datum

Parameters	Values (m)	SD (m)
a	0.99999	3.20E−06
b	−1.13E−05	4.23E−06
c	2.22345	1.417531
d	1.75E−05	3.20E−06
e	1.00001	4.23E−06
f	0.50298	1.417531

Table 2 Four-parameter model transformation from Leigon datum to Accra datum

Parameters	Values (m)	SD (m)
a	1.000001	2.06E−06
b	1.27E−05	2.06E−06
c	2.080065	0.692668
d	2.98E+00	0.693

coordinates from Accra to Leigon datum and vice versa. A description of the methods is given in "Four-parameter similarity model" and "Six-parameter transformation model" sections, respectively.

Four-parameter similarity model

The four-parameter similarity model also known as the two-dimensional conformal model has the characteristics that true shape and angles are retained after transformation, but the length of lines and position of points may change. This model consists of two translations, one scale factor and one rotation parameter. The translations create common origin for the two coordinate systems, while the scale generates equal dimensions in the two coordinate systems. The rotation parameter, on the other hand, makes the reference axes of the two systems parallel (Ghilani 2010).

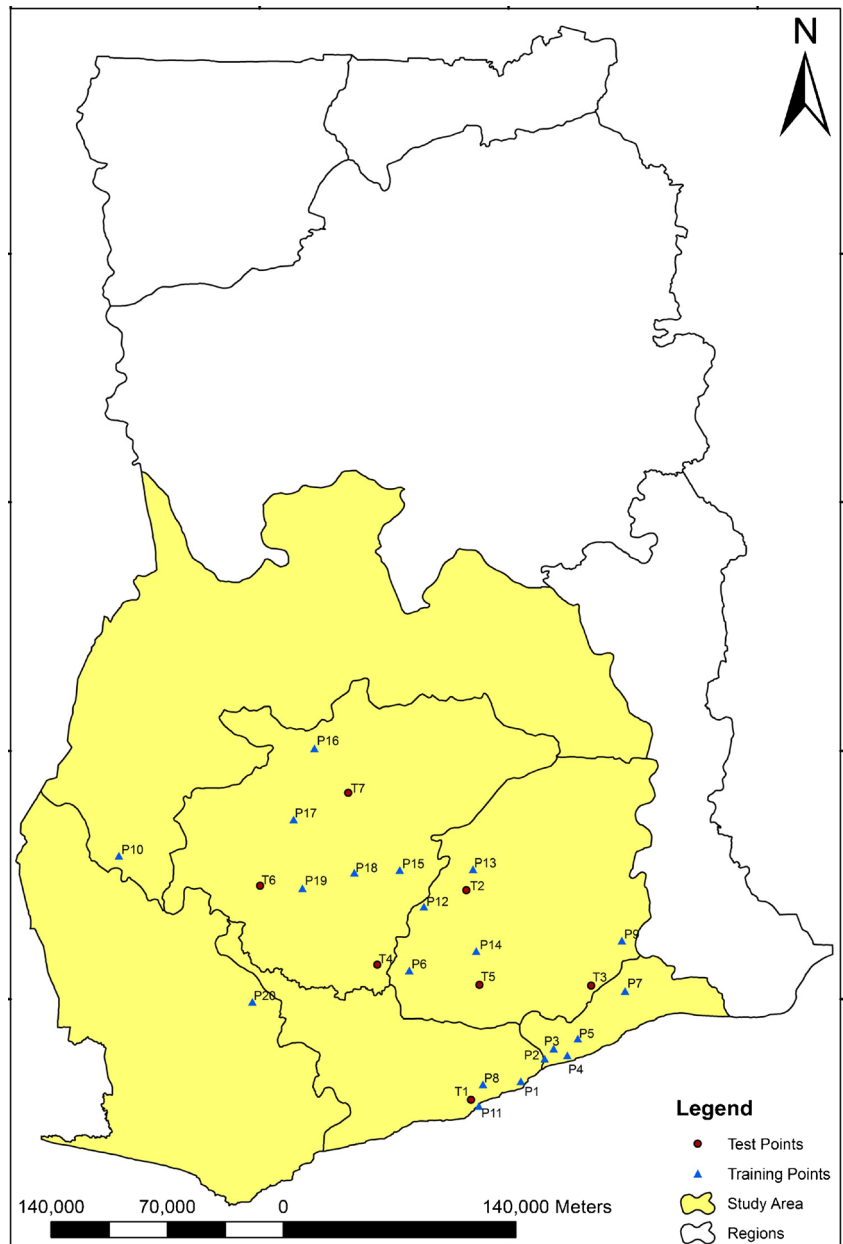**Fig. 4** Training and testing data distribution

Table 3   Statistics of the test data coordinate residuals based on the transformation from Leigon datum to Accra datum

Test point	6 Parameters			4 Parameters			RBFNN			BPNN		
	ΔE (m)	ΔN (m)	HE (m)	ΔE (m)	ΔN (m)	HE (m)	ΔE (m)	ΔN (m)	HE (m)	ΔE (m)	ΔN (m)	HE (m)
T1	−0.148	1.393	1.401	−0.295	0.860	0.909	−0.329	0.012	0.329	−0.359	0.091	0.370
T2	0.638	0.628	0.896	0.675	1.003	1.209	−0.067	0.323	0.330	−0.182	0.347	0.392
T3	0.464	−1.334	1.413	0.320	−1.021	1.070	0.131	−0.137	0.189	−0.233	−0.200	0.307
T4	0.321	0.076	0.330	0.365	−0.132	0.388	0.387	−0.266	0.470	0.424	−0.154	0.451
T5	0.601	0.490	0.775	0.546	0.485	0.731	−0.081	0.349	0.358	−0.122	0.437	0.454
T6	−1.515	0.337	1.552	−1.311	0.140	1.319	−0.407	0.713	0.821	−0.061	0.935	0.937
T7	−0.225	0.813	0.843	−0.010	1.277	1.277	0.599	−0.485	0.771	0.515	−1.576	1.658
ME	0.020	0.343	1.030	0.041	0.373	0.986	0.033	0.073	0.467	−0.003	−0.017	0.653
SD	0.759	0.848	0.441	0.681	0.787	0.337	0.366	0.413	0.240	0.337	0.789	0.490

The four-parameter model was applied to determine transformation parameters suitable to transform coordinates from War Office 1926 ellipsoid (Accra datum) to Clark 1880 (modified) ellipsoid (Leigon datum) and vice versa. It must be known that, to the best of our knowledge, the efficiency of this model in transforming plane coordinates between the two geodetic data in Ghana is yet to be investigated. Hence, there is the need to evaluate the capability of the four-parameter model.

The four-parameter similarity model connecting two datums could be represented by Eq. (11) as (Ghilani (2010))

$$ \begin{aligned} ax - by + c &= X \\ bx + ay + d &= Y \end{aligned} \text{,} \tag{11} $$

where a, b, c and d are the transformation parameters to be determined between the two coordinate systems. Applying the least squares method, Eq. (11) could be represented in matrix form (Eq. (12)) as
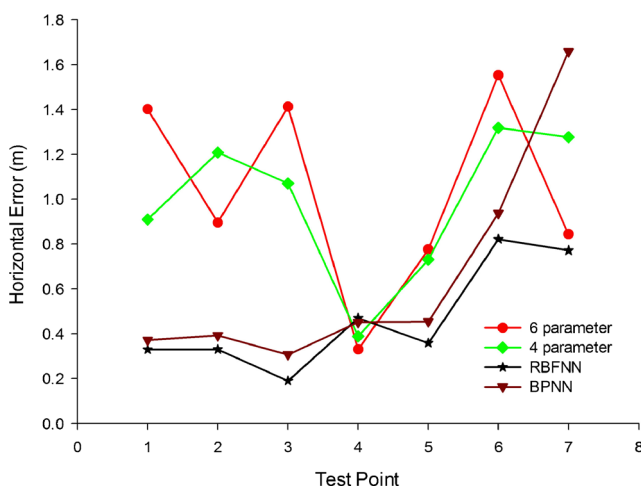
$$ V + BP = L \tag{12} $$

where V is the residual, B is the designed matrix, L is the observation vector matrix and P is the vector of the unknown parameters to be determined. It is worth noting that P was calculated using the relation (Eq. (13))

$$ P = \left(B^T B\right)^{-1} B^T B. \tag{13} $$

Six-parameter transformation model

The six-parameter transformation model is also known as the two-dimensional affine coordinate transformation model. This model involves two translations of the origin, a rotation about the origin and two scale factors: one in the *x*-direction and the other in the *y*-direction, plus a small non-orthogonality correction between the *x*- and *y*-axes resulting in six unknowns (Ghilani 2010). This model was also applied to transform plane coordinates between the two local geodetic data in Ghana. That is, the objective is to estimate parameters suitable to transform coordinates from War Office 1926 ellipsoid
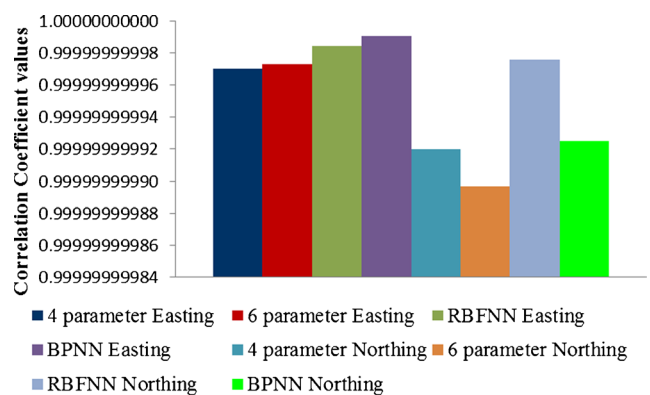

Fig. 5 Horizontal displacement of test coordinates (transformation from Leigon to Accra datum)

Fig. 6 Test data correlation coefficient for the four methods in both coordinate systems (transformation from Leigon to Accra datum)

Table 4 Total horizontal residuals between plane coordinates transformed from Leigon to Accra datum for the test data

PCI	6 Parameters (m)	4 Parameters (m)	RBFNN (m)	BPNN (m)
Max error	1.552	1.319	0.821	1.658
Min error	0.330	0.388	0.189	0.307
MHE	1.030	0.986	0.467	0.653
SD	0.441	0.337	0.240	0.490

(Accra datum) to Clark 1880 (modified) ellipsoid (Leigon datum) and vice versa.

The observation equations for the six-parameter transformation model are given in Eq. (14) (Ghilani 2010) as

$$ax + by + c = X$$
$$dx + ey + f = Y,$$ (14)

where a, b, c, d, e and f are the unknown transformation parameters to be determined between the two coordinate systems. It must be known that Eq. (14) was expressed in Eq. (12) and P was also estimated using Eq. (13).

Model performance assessment

In order to compare the ANN method (BPNN and RBFNN) results with the four-parameter and six-parameter models, the residuals calculated between the desired outputs and the outputs produced by the various techniques were utilized. Hence, to make an objective assessment of the models, performance criteria indices (PCI) of mean error (ME), horizontal position error (HE), mean horizontal position error (MHE), standard deviation (SD) and correlation coefficient (R) were used. Their mathematical expressions are given by Eqs. (15), (16), (17), (18) and (19), respectively.

$$ME = \frac{1}{n}\sum_{i=1}^{n}(O_i - P_i),$$ (15)

$$HE = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2},$$ (16)

$$MHE = \frac{1}{n}\sum_{i=1}^{n}HE_i,$$ (17)

$$SD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e - \bar{e})^2},$$ (18)

$$R = \left(\frac{\sum_{i=1}^{N}(O_i - \overline{O})(P_i - \overline{P})}{\sqrt{\sum_{i=1}^{N}(O_i - \overline{O})^2} \times \sqrt{\sum_{i=1}^{N}(P_i - \overline{P})^2}}\right).$$ (19)

Here, n is the total number of test examples presented to the learning algorithm, O and P are the measured and predicted plane coordinates from the various procedures, while $\overline{P}$ is the mean of the predicted plane coordinates. e denotes the residual between the measured and predicted plane coordinates, and $\bar{e}$ is the average of the residual.

Results and discussion

Coordinate transformation from Leigon datum to Accra datum

Tables 1 and 2 present the transformation parameters for the six- and four-parameter models, with their associated SDs, derived from the least square estimation of 20 common control points from the Leigon to Accra datum. Seven
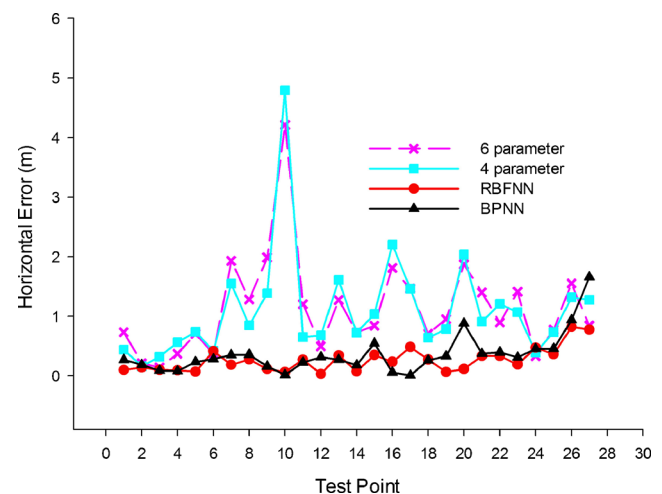

Fig. 7 Horizontal displacement of the whole data (transformation from Leigon to Accra datum)

Table 5 Total horizontal residuals between plane coordinates transformed from Leigon to Accra datum for the whole data

PCI	6 Parameters (m)	4 Parameter (m)	RBFNN (m)	BPNN (m)
Max error	4.22	4.791	0.822	1.658
Min error	0.137	0.158	0.028	0.004
MHE	1.129	1.108	0.260	0.356
SD	0.821	0.896	0.204	0.341

independent common control points which were not involved in the parameter estimation as well as in the ANN trained models were used as the testing data. The computed SD values in Tables 1 and 2 give an indication on the precision of the estimated transformation parameters by showing the extent at which the determined transformation parameters are distant from their most probable value. Figure 4 shows a spatial map of the geographical distribution of training and testing datasets used in this study.

In the ANN (BPNN and RBFNN) training, the plane coordinates of the points in the Leigon datum denoted as (E_{clark}, N_{clark}) were used as the input layer neurons, while (E_{war}, N_{war}) in Accra datum was used as the target data. The MSE (Eq. (2)) was then used as the optimality criterion to aid in determining the optimum ANN structure during the training process. Hence, after several trials, the optimum structures of the BPNN for transforming plane coordinates from Leigon to Accra datum were [2-4-1] and [2-8-1], respectively. Thus, for E_{war} output vector, there are two inputs with four hidden neurons, and for the N_{war} output vector, there are eight hidden neurons with two inputs. The optimum RBFNN architecture selected was two inputs (E_{clark}, N_{clark}) with one hidden layer of 20 neurons for each output vector (E_{war}, N_{war}), that is, [2-20-2].

Table 3 presents the residuals generated when the six-parameter, four-parameter, RBFNN and BPNN test results were subtracted from the measured test plane coordinates. The HE, ME and SD values for each test coordinate are also presented.
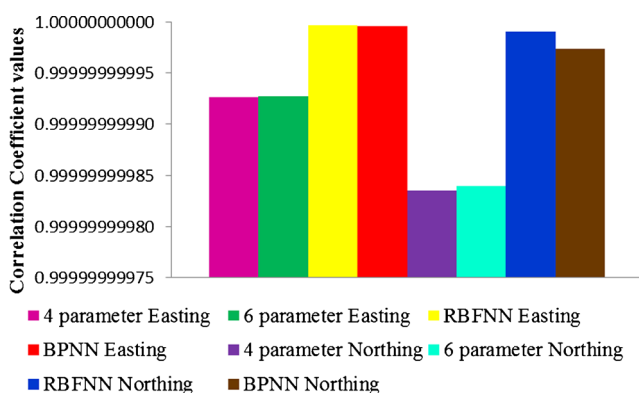
These deviations (ΔE, ΔN) (Table 3) indicate the amount the predicted outcomes produced by the four methods applied depart from the corresponding measured test data. These residuals signify the prediction limitation of the methods utilized in this study.

Analysis of Table 3 shows that the ANN approaches produced results that are better than the traditional techniques. This implies that the RBFNN and BPNN were able to generalize well across the test data than the six-parameter and four-parameter models, respectively. However, comparing the two ANN techniques, the RBFNN showed much improved transformation results compared with the BPNN. This shows that the RBFNN has exhibited greater learning ability and has shown more stability in training and testing than the BPNN. In the light of these, it could be stated that the predicted outcomes rendered by the RBFNN are in better agreement to the measured data than the other methods. These assertions are further confirmed by Fig. 5.

Observation of Table 3 and Fig. 5 demonstrate that the six-parameter, four-parameter, RBFNN and BPNN methods predict the horizontal positional errors with a minimum uncertainty in the order of approximately 0.33, 0.39, 0.19 and 0.31 m, respectively. On the contrary, maximum horizontal positional errors of about 1.55, 1.32, 0.8 and 1.66 m were identified for the six-parameter, four-parameter, RBFNN and BPNN methods, respectively. On the basis of the HEs (Table 3), it could be stated that the RBFNN was capable of absorbing more of the local character deformations existing in both Accra and Leigon data than the six-parameter, four-parameter and BPNN models. Hence, the inference made here is that the effect of the local geodetic network distortions on the final transformed plane coordinates was at a minimum in the case of the RBFNN model, thereby improving the

Fig. 8 Correlation coefficient of the whole data for the four methods in both coordinate systems (transformation from Leigon to Accra datum)

Table 6 Six-parameter model transformation from Accra datum to Leigon datum

Parameters	Values (m)	SD (m)
a	1.00000042	3.20E−06
b	1.13E−05	4.23E−06
c	−2.22343	1.41753
d	−1.75E−05	3.20E−06
e	0.99999	4.23E−06
f	−0.50284	1.41753

Table 7 Four-parameter model transformation from Accra datum to Leigon datum

Parameters	Values (m)	SD (m)
a	0.99999	2.06E−06
b	−1.27E−05	2.06E−06
c	−2.08003	0.69267
d	−2.98E+00	0.69267

Fig. 9 Horizontal displacement of test coordinates (transformation from Accra to Leigon datum)

transformation accuracy. On the account of the results from Table 3, it was noticed that the influence of the local geodetic network distortions on the final transformed coordinates for the traditional techniques and the BPNN was very high. This suggests that the traditional techniques and BPNN could not model out more of the distortions within the two local geodetic networks as compared to the RBFNN.

On the basis of the computed correlation coefficient (R) values using Eq. (19), it was noticed that the changes in the R values obtained for all the methods applied were only exhibited as the number of decimal places increased. Consequently, a graphical illustration of the R values for the four-parameter, six-parameter, BPNN and RBFNN models in easting and northing coordinates was the preferred way of presenting the efficiency of the four methods. Figure 6 is an R distribution of eastings and northings for the transformation techniques. This distribution shows the efficiency of the various transformation methods based on the testing data. In addition, it determines the strength of the relationship existing between the estimated test coordinate results and the measured data. In the case of this study, a stronger correlation was exhibited by all the methods (Fig. 6). This indicates that the transformation methods could produce satisfactory results. However, comparatively, it is obvious from Fig. 6 that the RBFNN had the highest R value and thus is superior to four-parameter, six-parameter and BPNN models.
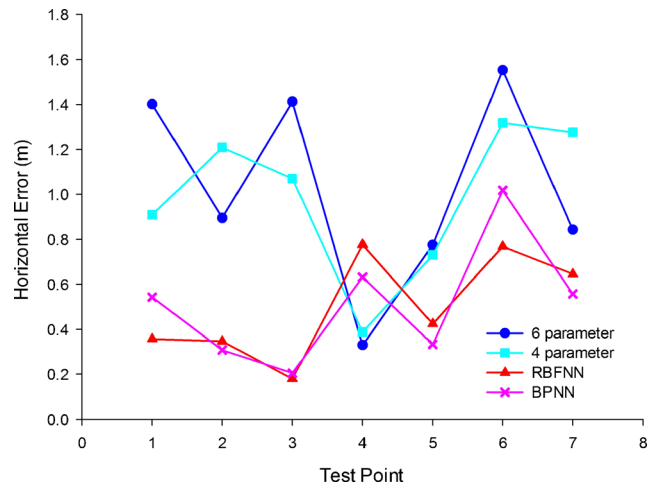
A summary of the total error attained when the four methods were used for transforming coordinates from Leigon to Accra datum is presented in Table 4. Analysis of Table 4 shows that the RBFNN-obtained results were significantly better than the other methods. The interpretation made in line with the maximum and minimum error values (Table 4) is that the RBFNN model-forecasted values varied by not more than 0.821 m, whereas 1.552, 1.319 and 1.658 m were gotten by the six-parameter, four-parameter and BPNN models, respectively. The SD values (Tables 3 and 4) calculated show a practical expression for the accuracy of the transformed test coordinates. A check from Tables 3 and 4 indicate that the RBFNN had the least SD values, which further show the limit of the error bound by which every value within the RBFNN transformed test dataset varies from its most probable value.

In order to further access the generalization capability of the optimum ANNs formulated and the determined transformation

Table 8 Statistics of the test data coordinate residuals based on the transformation from Accra datum to Leigon datum

| Test point	6 Parameters			4 Parameters			RBFNN			BPNN		
	ΔE (m)	ΔN (m)	HS (m)	ΔE (m)	ΔN (m)	HS (m)	ΔE (m)	ΔN (m)	HS (m)	ΔE (m)	ΔN (m)	HS (m)
T1	0.148	−1.393	1.401	0.295	−0.860	0.909	0.334	0.125	0.356	0.405	0.363	0.543
T2	−0.638	−0.628	0.896	−0.675	−1.003	1.209	0.072	−0.339	0.346	0.112	−0.287	0.308
T3	−0.464	1.334	1.413	−0.320	1.021	1.070	−0.150	−0.098	0.180	−0.130	−0.158	0.204
T4	−0.321	−0.076	0.330	−0.365	0.132	0.388	−0.376	0.680	0.777	−0.629	0.060	0.632
T5	−0.601	−0.490	0.775	−0.546	−0.485	0.731	0.081	−0.416	0.424	−0.006	−0.333	0.333
T6	1.515	−0.337	1.552	1.311	−0.140	1.319	0.417	−0.645	0.768	0.303	−0.972	1.018
T7	0.225	−0.813	0.843	0.010	−1.277	1.277	−0.604	0.230	0.647	−0.482	0.280	0.558
ME	−0.020	−0.343	1.030	−0.041	−0.373	0.986	−0.033	−0.066	0.500	−0.061	−0.150	0.514
SD	0.759	0.848	0.441	0.681	0.787	0.337	0.369	0.450	0.232	0.384	0.451	0.271
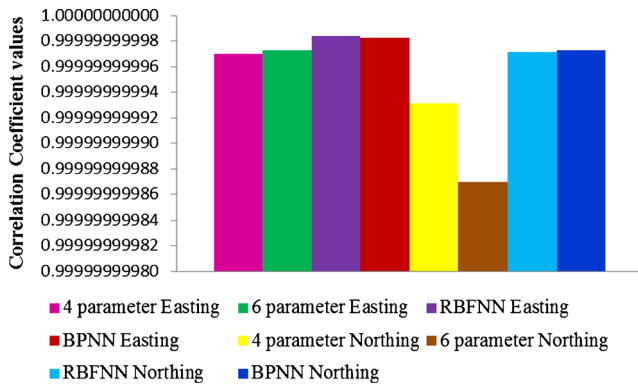
Fig. 10 Test data correlation coefficient for the four methods in both coordinate systems (transformation from Accra to Leigon datum)

Fig. 11 Horizontal displacement of the whole data (transformation from Accra to Leigon datum)

parameters of the traditional techniques when the dataset is increased, the whole data (27 common points) were used as the testing data. Figure 7 displays the horizontal errors when the whole data were used to test the already determined six-parameter, four-parameter, optimum RBFNN and BPNN models.

The results from Fig. 7 clearly show that within the Ghana national geodetic reference network, both the artificial intelligence techniques of RBFNN and BPNN could serve as a credible alternate technology to be applied for plane coordinate transformation from Leigon datum to Accra datum. This assertion is further confirmed by the total error attained by each method as shown in Table 5. However, on the basis of the performance criteria indicators (PCI) used, the RBFNN had a little edge over the BPNN. Given the values of R (Fig. 8), an intuitive interpretation depicts the strength of the ANN methods over the classical transformation techniques. Moreover, analysis of Fig. 8 duly demonstrates that the RBFNN is the most appropriate procedure for transforming coordinates from Leigon datum to Accra datum.

Coordinate transformation from Accra datum to Leigon datum

The resulting parameters obtained from least square estimation of coordinate transformation from the Accra datum to Leigon datum with their related SD values based on the six-parameter and four-parameter models are presented in Tables 6 and 7, respectively.

In the case of the RBFNN and BPNN training, plane coordinates of the points in Accra datum represented as (E_{war}, N_{war}) were used as the input layer neurons, while (E_{clark}, N_{clark}) in Leigon datum was used as the output layer neurons. In order to establish the ANN architecture, the MSE (Eq. (2)) of all trained models was examined at each phase of training and testing. The model that gave the smallest MSE in the testing dataset was selected as the best ANN structure. In this study, the optimum BPNN for carrying out the transformation was [2-3-1] and [2-10-1], respectively. Thus, for E_{clark} output vector, there are two inputs with three hidden neurons, while for the N_{clark} output vector, there are ten hidden neurons with two inputs.

Table 8 shows the coordinate differences (ΔE, ΔN) obtained compared with the coordinates known from Leigon datum (E_{clark}, N_{clark}).

Judging from the results in Table 8, it was realized that the ANN methods were superior to the traditional techniques. This could be ascribed to the ANN methods' ability to better learn and generalize well when unseen data are introduced to the network. In addition, the superiority of the ANN approaches could be that the distortions in data related to the Accra and Leigon geodetic datums which could not be more absorbed by the traditional techniques were captured better by the ANN models. Therefore, in conformance with the results (Table 8), it can fairly be stated that in our case of transforming plane

	PCI	6 Parameters (m)	4 Parameters (m)	RBFNN (m)	BPNN (m)
Table 9 Total horizontal residuals between plane coordinates transformed from Accra to Leigon datum for the test points	Max error	1.552	1.319	0.777	1.018
	Min error	0.330	0.388	0.180	0.204
	MHE	1.030	0.986	0.500	0.514
	SD	0.441	0.337	0.232	0.271

Table 10 Total horizontal residuals between plane coordinates transformed from Accra datum to Leigon datum for the whole data

PCI	6 Parameters (m)	4 Parameters (m)	RBFNN (m)	BPNN (m)
Max error	4.216	4.791	0.798	1.018
Min error	0.137	0.158	0.051	0.018
MHE	1.129	1.108	0.339	0.299
SD	0.821	0.896	0.236	0.233

coordinates from Accra to Leigon datum, the ANN approaches produced the most practicable transformation results.

However, comparing the two ANN methods, it was detected that the BPNN had the least HE values for about 57.14 % of the test points, while 42.86 % was achieved by the RBFNN. This is intuitively confirmed by Fig. 9 where the HEs for the test points are displayed. Here, the BPNN attained slightly better horizontal positional accuracies of the transformed test coordinates compared to the RBFNN. This means that the BPNN was able to demonstrate better generalization capability than the RBFNN. A critical look at Tables 3 and 8 show the same obtained results but different arithmetic signs for the four-parameter and six-parameter models. This phenomenon as stated by Bašić (2006) cited in the study by Grgic et al. (2015) is due to the least square estimation process, in which the system of equation contains the actual values of the source and target coordinates, depending on the direction of the transformation. Hence, the computed transformation parameters' absolute values differ in both directions of the transformation. The BPNN and RBFNN, on the other hand, did not show such occurrences and might possibly be attributed to their non-parametric nature whereby results are dependent on the input and output data supplied to the network and the number of hidden neurons that could learn and generalize well on the training and testing data, respectively.

By virtue of the R estimated results (Fig. 10), it was realized that closely identical values were produced by the BPNN and RBFNN. In comparison, it was noticed that the BPNN and RBFNN attained higher R values than the four-parameter and six-parameter models. This means that the outputs rendered by the BPNN and RBFNN are more satisfactory and are in better agreement with the measured test coordinates than those produced by the four-parameter and six-parameter models, respectively.

The statistics of total horizontal residuals attained using the traditional techniques and ANN methods of transforming plane coordinates from Accra to Leigon datum is summarized in Table 9.

From Table 9, it could be seen that the magnitude of distortions was significantly reduced in the case of the ANN methods compared with the six- and four-parameter models. This suggests that the ANN methods could absorb most of the uncertainties in both the training and testing datasets, thereby achieving good generalization. On the account of the maximum and minimum horizontal positional

errors (Table 9), it was observed that a positional accuracy of approximately 0.2 m was gotten by the ANN methods, while approximately 0.4 m was attained by the six- and four-parameter models, respectively. These positional errors further point out that the transformed test points generated by the ANN methods are in closer agreement to the measured test coordinates than the traditional methods. Moreover, considering Table 9, employing the ANN methods identified an improvement of about 90 % in MHE when compared with the six- and four-parameter model results. The SD was then used as a criterion to further evaluate the accuracy of the techniques applied. The results (Table 9) indicate differences in SD values, starting with a maximum SD of 0.441 m for the six-parameter model and ending up with 0.232 m for the RBFNN. The determined SD values (Table 9) for the RBFNN and BPNN show that they are the most suitable for carrying out coordinate transformation from Accra to Leigon datum.

Furthermore, the whole dataset was applied as the testing data onto the already developed six-parameter models, four-parameter models and the optimum ANN models. The objective here is to ascertain how well the developed models could generalize when more dataset is introduced. With reference to Fig. 11, the horizontal residuals denote the range that the transformed coordinates produced by the RBFNN, BPNN, six-parameter and four-parameter models differ from the measured plane coordinates. It also shows the positional accuracy of the transformed data in horizontal terms to the measured data. In comparison, the RBFNN and the BPNN yielded
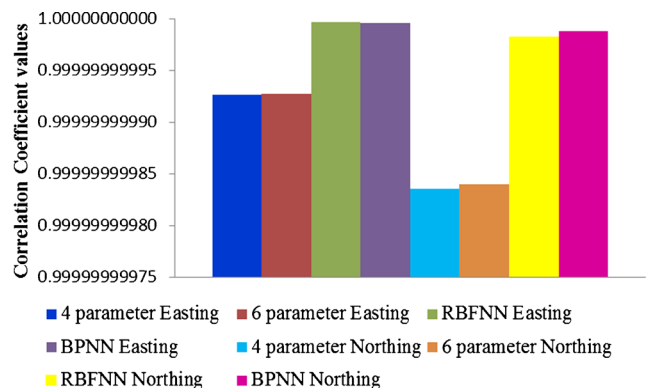

Fig. 12 Correlation coefficient of the whole data for the four methods in both coordinate systems (transformation from Accra to Leigon datum)

closely related horizontal positional accuracy than the six-parameter and four-parameter models as illustrated in Fig. 11.

Table 10 shows that transforming plane coordinates from Accra to Leigon datum, the RBFNN and BPNN are both practicable and could produce better transformation results. Besides, it can be stated based on graphical evidence (Fig. 12) of the *R* values that among the four methods, BPNN and RBFNN methods have demonstrated better capabilities of producing satisfactory results from the coordinate transformation processes. It can obviously be concluded from Table 10 and Fig. 12 that the ANN methods are better than six-parameter and four-parameter transformation models.

Conclusion

Coordinate transformation is essential in developing countries like Ghana where the local geodetic networks applied for surveying and mapping purposes are non-geocentric and highly heterogeneous in nature. Many methods for coordinate transformation have been developed and applied over the years. One of such method is the artificial intelligence technique of artificial neural network (ANN). Although ANN has been tested for its efficacy in coordinate transformation research, its suitability for plane coordinate transformation has not been accessed in Ghana's geodetic reference networks. Hence, the main contributions of this study are to evaluate, compare and discuss the capability of ANNs as a realistic alternative technology to transform coordinates between the two classical geodetic reference networks, namely, Accra datum and Leigon datum.

To this end, backpropagation neural network (BPNN) and radial basis function neural network (RBFNN) based on the supervised learning technique as well as the six-parameter and four-parameter models have been presented. The findings revealed that the BPNN and RBFNN gave more satisfactory transformation results compared with the six-parameter and four-parameter models. It is important to know that the outcomes rendered by the six-parameter and four-parameter models further confirm the presence of heterogeneity in the classical geodetic networks of Ghana. As a result, the transformation parameters determined could not model and absorb more of the distortions in the coordinates relating the Accra and Leigon data.

On the other hand, the RBFNN and BPNN were able to model out and compensate the local character deformation existing in the local geodetic networks in a more effective way than the traditional techniques. However, transforming coordinates from Leigon datum to Accra datum, the RBFNN compared to BPNN showed superior stability and more accurate transformation results. In the case of transforming coordinates from Accra datum to Leigon datum, both the BPNN and RBFNN achieved closely related results.

To conclude, it can reasonably be stated that the ANN models based on the results achieved could be used for practical survey works such as cadastral, topographic mapping surveys and engineering surveys. In the future, other artificial intelligence techniques such as generalized regression neural network should be tested when the newly established GNSS geodetic reference network is expanded to cover the rest of Ghana.

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