

A comparative study of kriging and simulation-based methods in classifying ore and waste blocks

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Abstract Ore grade is the most important source of uncertainty in a mining operation which plays an important role to classify run-of-mine (ROM) material into ore and waste parcels. As a widely used method, kriging estimator is used to estimate the grade of ore blocks. In conventional mining practices, if the estimated grade of a parcel is above the cut-off grade, this parcel is classified as ore, otherwise, is labelled as a waste parcel. An alternative approach is to simultaneously consider the grade of parcels and the economic consequences of sending parcels to destinations by applying simulation-based methods. In this study, kriging and simulation-based methods including loss and profit functions are applied on a real-world case study to classify ore/waste material based on the initial exploration data. Then, the actual known data, collected from blast holes samples, are compared with the estimated results in order to validate the performance of the presented methods. Outcomes show that simulation-based methods can perform better and show more adjustability with real data.

Keywords Geostatistical simulation · Ore and waste classification · Profit function · Loss function · Kriging

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Introduction

Ore/waste discrimination is a vital stage that should be fulfilled prior to evaluation, investment, design and planning of a mining project. The ore/waste discrimination is performed based on the limited drill holes information obtained at the detailed exploration phase. Due to the high cost of exploration drilling, only a few number of drill holes with large spacing between them are available. In addition, because of the highly complex and variable nature of the mineral deposits, uncertainty is always associated with estimated grade. This uncertainty needs to be addressed in order to prevent underestimation or overestimation. If an ore parcel is sent to a waste dump by mistake, a large amount of money is wasted. Additionally, if a waste parcel is processed at processing, the recovery of process decreases and a huge amount of energy is consumed. Especially, those processing plants, which are very sensitive to the input feed, can face serious problems as a result of the misclassification of ore and waste. Therefore, applying the best ore/waste discrimination technique can lead better planning and decrease the risk associated with the mining operations.

The simplest method to identify ore against waste is to draw ore-waste boundaries manually based on the borehole information (Verly 2005). If a parcel of material is within the ore boundaries, then this parcel is identified as ore and is dispatched to a process circuit. As the structure of orebody is not straightforward, manual classification cannot reach reasonable accuracy and always comes with a high percentage of error. Another ore/waste separation method is to use geological modelling approaches. The conventional geological modelling methods such as triangular prism and polygon use the information of drill holes and generate the shape of the ore body in sections (side-view) and plans (top-view). Then a three dimensional (3D) shape of the orebody is created by combining sections and plans (Sides 1997).

With the emergence of the application of computer in mining industry in the 1960s, the area of interest for mining is represented by a block model. In mining terms, a block is a three-dimensional (3D) prismatic shape spatially represented by the coordinates of its centre. A block model consists of several individual blocks in which different attributes such as density, rock type, and specification of grade are estimated for each individual block. Since the pioneering work by Lerchs and Grossmann (1965), the constructed block model is used for the purpose of mine planning and design. As the whole, planning and design of a mine rely on the block model; it is crucial to heighten the accuracy of the estimated grades.

Statistical and geo-statistical methods are usually applied to estimate the attributes of blocks based on the exploration information (e.g. drill-hole information). Geostatistical methods take priority over the statistical approaches, as they can incorporate data correlation and spatial position and also are able to provide the error of estimation. As a widely used method, kriging estimator is used to estimate block attributes. Kriging is a type of geostatistical method and is known as the best linear unbiased estimator (Cressie 1990). The main disadvantage of kriging is referred to as smoothing which leads to some reduction in variability (Pan 1995). Smoothing causes overestimating of low values and underestimating of the high values. To overcome such smoothing effect, geostatistical simulations have been developed. The proposed simulation techniques such as sequential gaussian simulation, P-field simulation and simulated annealing can produce realizations of block model and address uncertainty in estimation (Vann et al. 2002; Verly 2005). It should be noted that geostatistical simulation only produces realizations of the block model, and each individual realization of the block model is not a good estimation of the block model (Leuangthong et al. 2004). Geostatistical simulation may be conditioned to the original known data such that known data should be retained in each realization. In this case the simulation is named conditional geostatistical simulation (Journel 1974). This study does not discuss the details and applications of kriging and geo-statistical simulation, and readers are referred to relevant references such as (Lantuéjoul 2013; Pyrcz and Deutsch 2014).

Next after grade estimation, the decision should be made about the destinations of mining blocks. This end can be performed based on the grade content of the block. Conventionally, a cut-off grade is defined and the estimated value by kriging (or the average of simulated values) for each block is compared to this cut-off grade. Generally, cut-off grade is defined as a minimum amount of valuable mineral that must be existed in one unit (e.g. one tonne) of material before this material is sent to the processing plant (Rendu 2008). This method is straightforward, but suffers from ignoring the variability of the orebody as mentioned before. An alternative way is to consider the economic consequence of

dispatching a given block to a given destination. In this approach, the loss and profit achieved as a result of sending blocks to destinations are calculated and used to make decisions. This idea of minimizing the loss due to misclassification was first proposed by Isaaks (1991). Glacken (1997) introduced loss and profit functions as two simulation-based methods to determine the destination of run-of-mine material. Cheuiche Godoy et al. (2001) compared kriging and simulation-based methods in performing destination assignment in a case study and compare the results with known data. Verly (2005) discussed the efficiency of simulation, kriging and simulation-based methods in classifying ore and waste by presenting several case studies.

It should be noted that the mentioned simulation-based methods can be applied to construct a long-term resource model with an ore/waste tag for each block. Indeed, the application of simulation-based methods is to use the simulated realizations to classify blocks into ore and waste categories considering a given cut-off grade. As a result, these approaches may not provide much information (e.g. grades of blocks) for mine planning and production scheduling. However, the amount of ore reserve, which is a crucial factor in mine planning and design, can be estimated by simulation-based methods. This paper aims to compare the efficiency of kriging and simulation-based methods (loss and profit functions) in classifying ore and waste blocks. A case study is presented and the proposed methods are validated by comparing the estimated data with actual data. In next section, simulation-based methods are briefly introduced. In Section 3, a case study is presented and the results of grade estimation and comparison are discussed. Finally, Section 4 gives a brief conclusion and a few research directions for future studies.

Simulation-based methods

So far, several geostatistical simulations have been proposed and applied in the literature. A popular and efficient geostatistical simulation method for continuous variables is sequential gaussian simulation (SGS). The SGS algorithm determines random paths and uses a local distribution, created by kriging, at each node to assign a new value to each node. The detail of this algorithm can be found in Isaaks (1991). Conditional version of SGS is termed CSGS and generates a number of realizations of block model such that the statistical and geostatistical parameters of all generated realizations are the same as raw data, and also the real known data stay unchanged in all realizations. After generating realizations, loss function and profit function as two main simulation-based methods are applied to classify material into ore and waste categories.

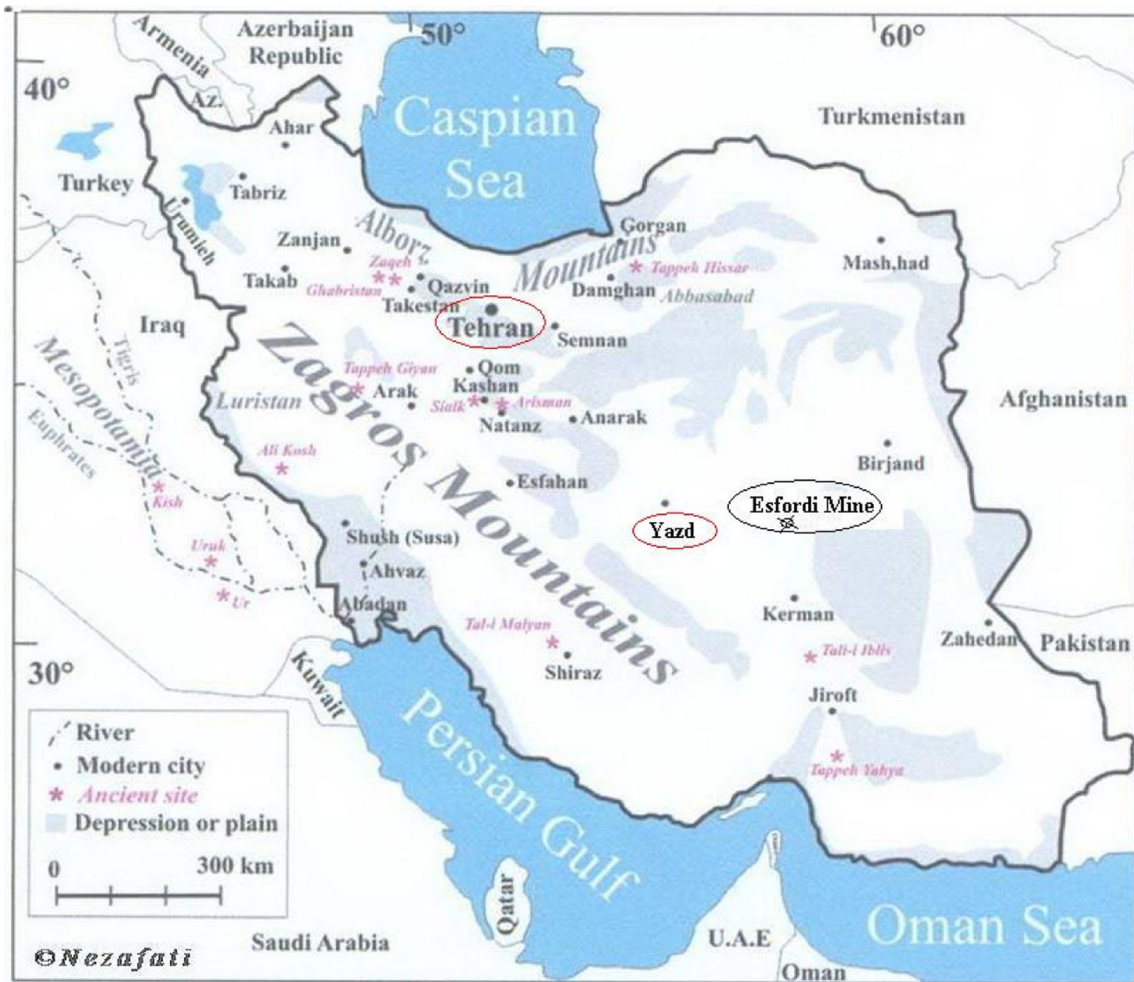


Fig. 1 Location map of Esfordi Mine

Loss function

Loss is defined as amount of money lost as a result of misclassification. In other words, amount of loss for a given block is defined as the potential value of the block minus the recovered value. Let define p as the price of product, r as recovery

(mining, processing and melting), c_m as the cost of mining, c_p as the cost of mineral processing and z as the grade of block. For an actual ore block estimated as waste:

Potential value $prz - c_m - c_p$

Recovered value $-c_m$

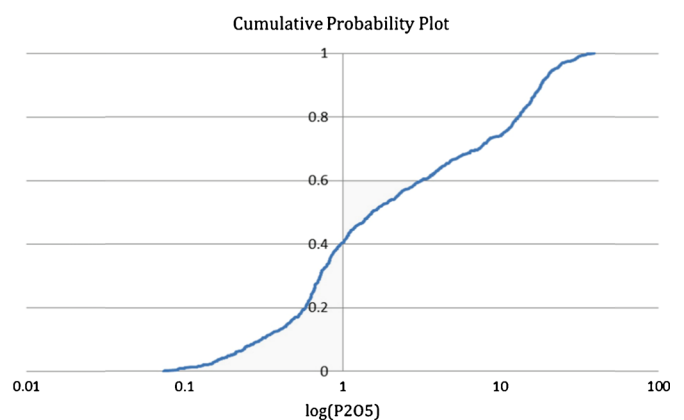
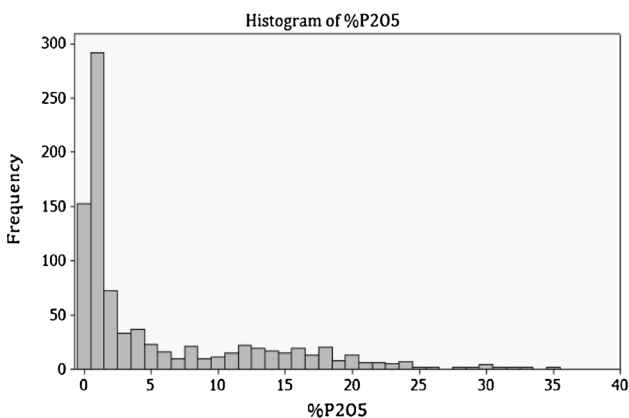


Fig. 2 Histogram and cumulative probability plot of the data

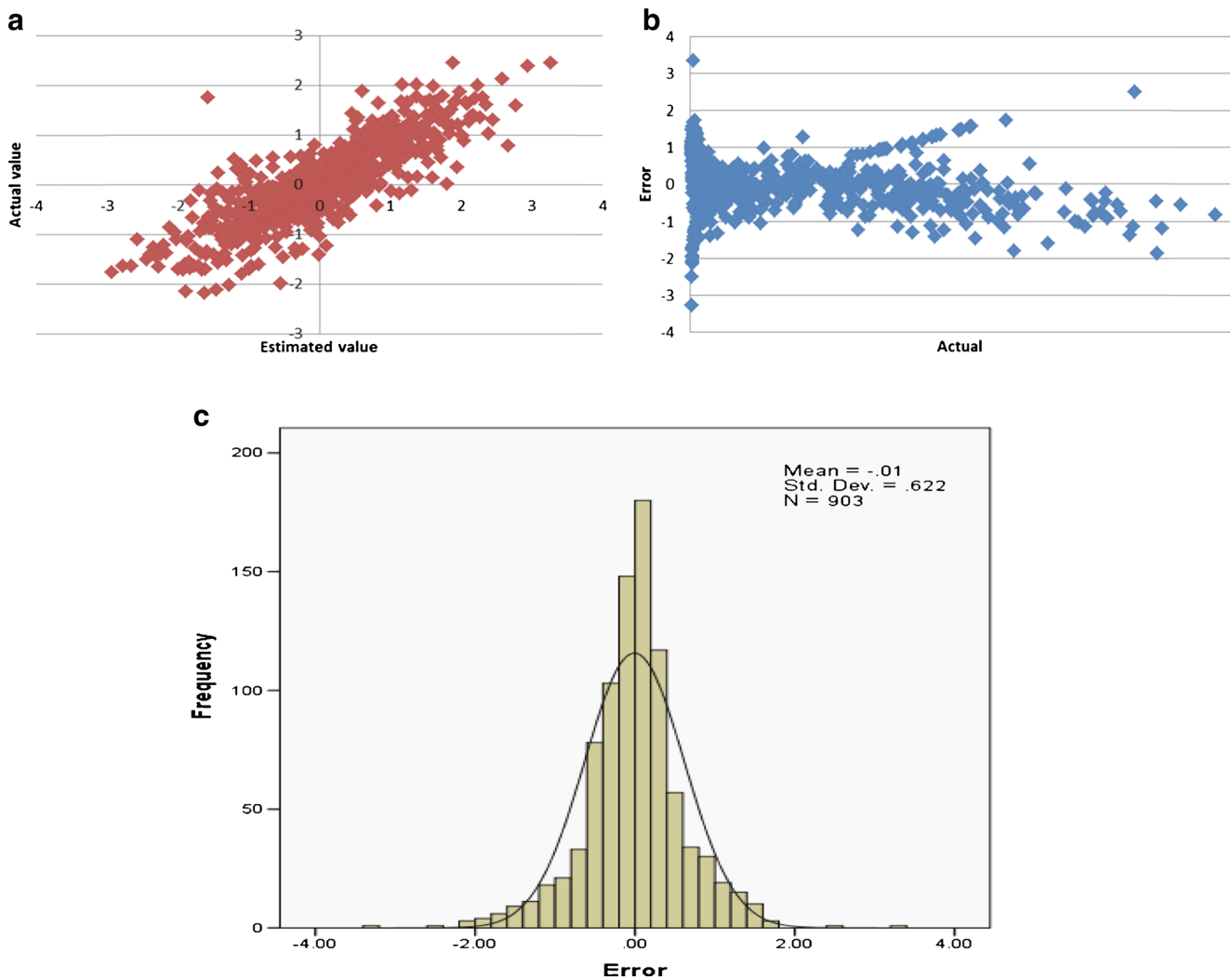


Fig. 3 a Scatter plot of estimated values versus actual values. b Estimation error versus the actual data. c Error distribution

For an actual waste block estimated as ore:

$$\begin{aligned} \text{Potential value} &= -c_m \\ \text{Recovered value} &= prz - c_m - c_p \end{aligned}$$

Thus, loss function for a block estimated as waste (L_w) is as follows:

$$L_w = \begin{cases} 0 & z \leq z_c \\ (1-P_o) \times [(prz^+ - c_p)] & z > z_c \end{cases} \quad (1)$$

Table 1 Properties of variogram models for the main anisotropy directions

Direction	Maximum	Minimum
Model type	Spherical	Spherical
Dip	5	88
Strike	85	108
Range (m)	145	43

Here P_o is the probability of having a grade greater than cut-off grade and z_c is the cut-off grade. z^+ represents the average of the simulated values above the cut-off grade.

Loss function for a block estimated as ore (L_o) is computed as follows:

$$L_o = \begin{cases} (P_o) \times [-(prz^- - c_p)] & z \leq z_c \\ 0 & z > z_c \end{cases} \quad (2)$$

Here z^- denotes the average of the simulated values below the cut-off grade. The values of L_o and L_w are calculated for each individual block. If $L_o < L_w$ for a given block, then this block is classified as ore, otherwise, is assigned to waste category. For a given block and a category (ore or waste), the predicted grade (z) is the average of simulated realizations.

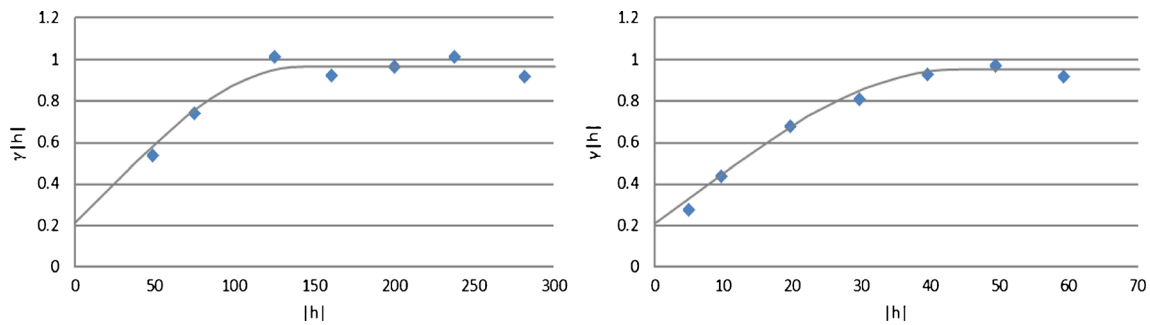


Fig. 4 Variograms in maximum and minimum direction of continuity (computed on the normalized data)

Profit function

Glacken (1997) explained a so called profit function which aims to maximize the potential profit obtained through ore/waste classification. In profit function approach, the expected profit associated with each class (ore or waste) is computed, and the classification that yields the maximum expected profit is selected.

Profit function for a block estimated as ore (Pr_o) is (Glacken 1997; Godoy et al. 2001):

$$Pr_o = \begin{cases} 0 & z > z_c \\ (P_o) \times (prz^+ - c_p) + (1 - P_o) \times (prz^- - c_p) & z \leq z_c \end{cases} \quad (3)$$

Profit function for a block estimated as waste (Pr_w) is (Glacken 1997; Godoy et al. 2001):

$$Pr_w = \begin{cases} 0 & z \leq z_c \\ -(P_o) \times (prz^+ - c_p) & z > z_c \end{cases} \quad (4)$$

If $Pr_o > Pr_w$ then block is classified as ore, otherwise, is assigned to waste category. For example, if the average of simulated value for a given block is above the cut-off grade, profit function classifies this block as ore, if $-(P_o) \times (prz^+ - c_p) < 0$.

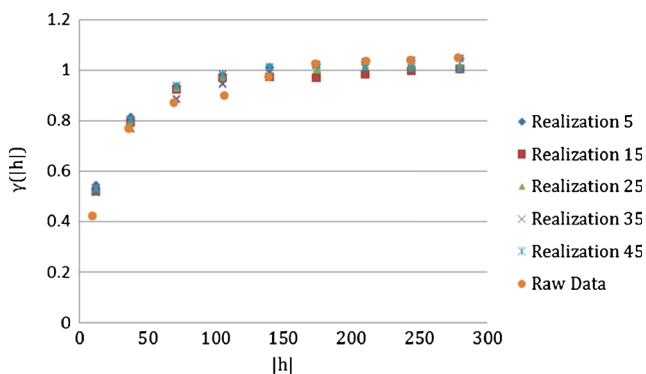


Fig. 5 Omnidirectional computed variograms for five realizations and raw data

Similarly, if the average of simulated value for a given block is below the cut-off grade then this block is sent to a waste dump, if $(P_o) \times (prz^+ - c_p) + (1 - P_o) \times (prz^- - c_p) \leq 0$. Note that in both loss and profit approaches, it is assumed that all blocks must be mined and ore and waste mining costs are equal. Thus, mining costs are not considered into the expressions.

Case study: Esfordi Phosphate Mine

To illustrate the efficiency of kriging and simulation-based methods in a real case study, these techniques have been implemented on Esfordi Phosphate Mine. This mine is located in Bafgh district in Yazd province, Iran (Fig. 1). The main product of mine is phosphate while there are iron and chlorine as by products and ore impurity, respectively.

Jami et al. (2007) studied the geology of the Esfordi deposit and explained that Esfordi deposit is an apatite-magnetite deposit. As Jami et al. (2007) explained “Esfordi deposit which is recognized as the most P rich deposits in Bafgh district is hosted by a sequence of intercalated shallow-water sediments and early Cambrian rhyolitic volcanic rocks”. Three different geological formations in the Esfordi region have been recognized: (a) igneous and volcanoclastic including alkaline rhyolite, lamprophyre, diabase, gabbro, tuffs and breccia volcanic, (b) metamorphic rocks, including amphibolites, quartzite, schist and metarhyolite and (c) sedimentary rocks including shale, conglomerate, dolomite and limestone. Stratigraphy in the esfordi deposit shows that there are three main ore zones including apatite-iron ore in the lower section covered by green strongly altered volcanic rocks and apatite dykes (Rajabzadeh et al. 2015).

Esfordi mine is the only phosphate mine in Iran and its mineral processing plant is very sensitive to ore grade variation (Sayadi et al. 2011). Therefore, it is necessary to control the grade of ore material dispatched to the process plant. In this study, ordinary kriging (OK) and simulation-based methods including loss function (LF) and profit function (PF) have been used to classify dispatched run-of-mine materials (ore or waste). The 3D block model of the orebody has been created

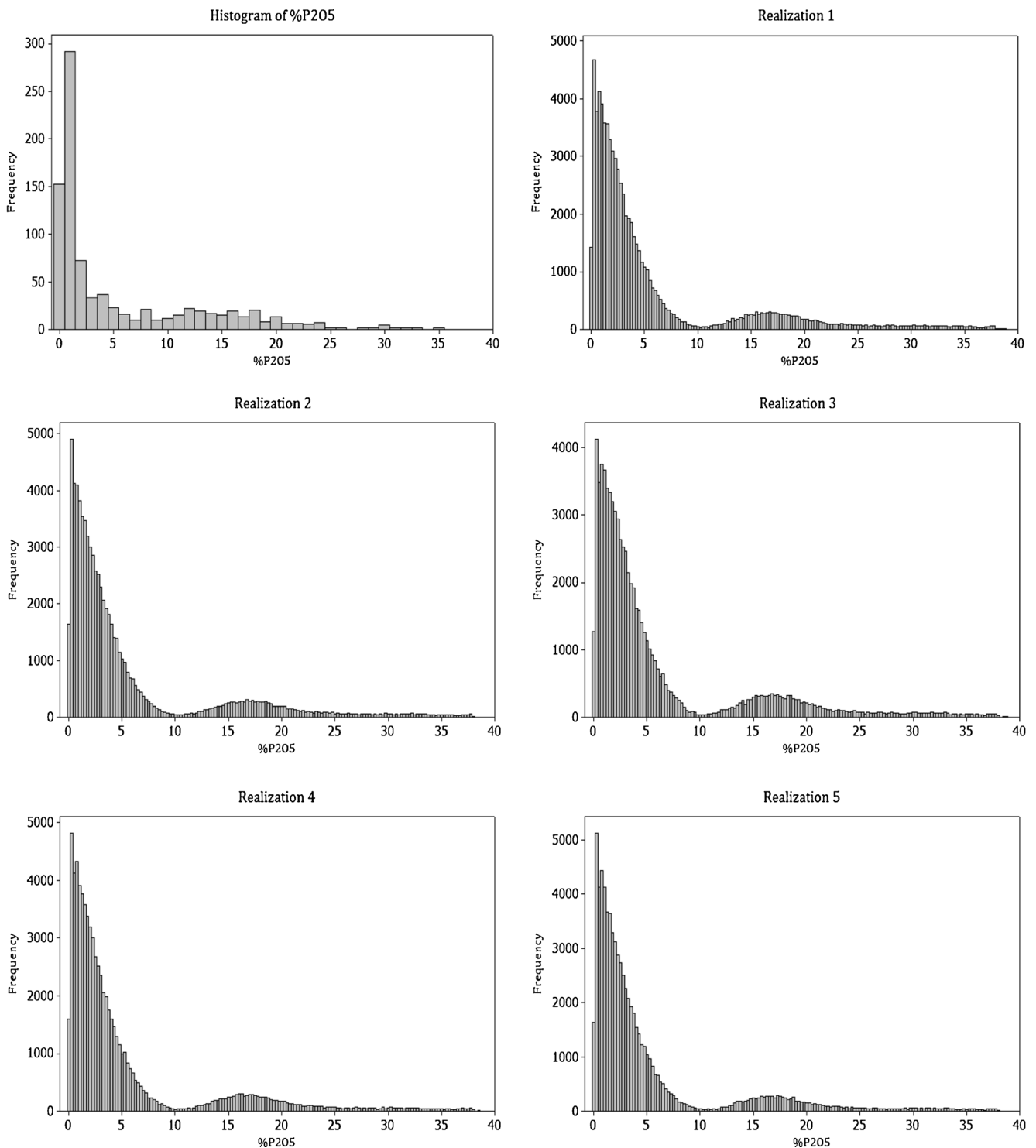


Fig. 6 Histograms of five realizations and raw data

based on the drill holes information. The OK has been used to estimate the phosphate grade for each individual block and conditional sequential gaussian simulation (CSGS) has been applied to create 50 realizations of the block model. For the estimation purpose, the provided exploration drill holes data by the end of 2008 has been used. In addition, the real grades of

179 blocks have been collected. Note that the real grade of each block is calculated based on the sampling results performed at the time of blasting and extraction. The sampling had been performed on the blast hole particles. As blast holes are dug very close (about 2-m distance), the average value of samples can be a good approximation of the real grade.

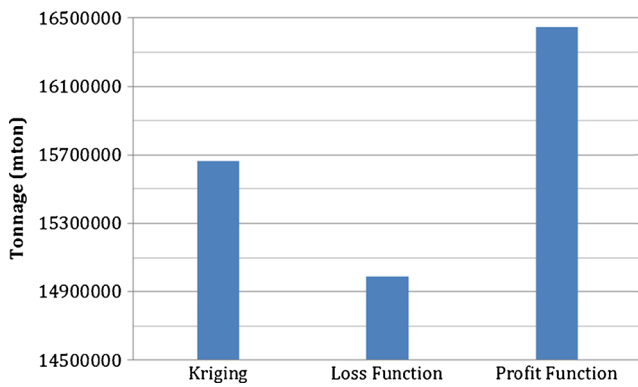


Fig. 7 Ore tonnage obtained by each method

Grade estimation

In the detailed exploration phase of Esfordi mine, a number of 59 boreholes have been drilled. A total number of 903 composited sample dataset, constructed from these boreholes, have been used to estimate the grade of blocks. Rock core drilling method was used to dig drill holes in an irregular pattern. The average distance between two adjacent drill holes is about 40 m. The average length of original assay intervals is 0.85 m. For the sake of estimation, assay intervals have been converted to five-meter composites. The compositing method was down-the-hole method in which the compositing starts from the bottom of each drill hole and the original sample grades are weighted by their length.

Histogram and the cumulative probability plot of the raw data are shown in Fig. 2. As can be seen from this figure, the distribution of data is not normal and should be transferred to a normal distribution before applying CSGS method. In addition, from the histogram of %P2O5, the grade distribution is bimodal with a gap around the 6% of P2O5. However, we have performed this study considering all data as a single population.

In geostatistical approach, the so-called variogram is used to show the spatial variability of ore grade. A variogram plot shows the average squared differences of pairs of samples for

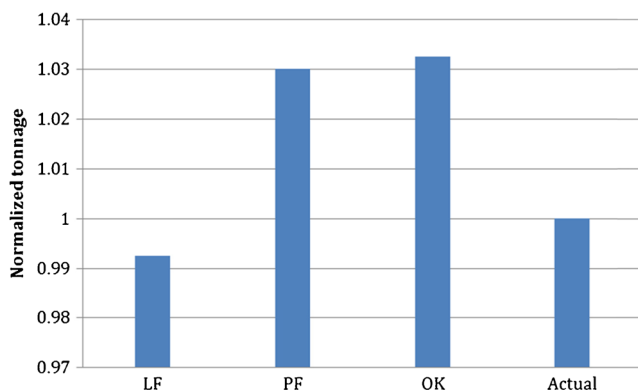


Fig. 8 Normalized ore tonnage obtained by each method and actual amount

Table 2 Misclassification percentage of each method

	Ore estimated as waste (%)	Waste estimated as ore (%)
OK	5.0	8.4
LF	6.1	5.6
PF	5.6	7.8

each class of distance. From a variogram, the distance at which data are uncorrelated can be estimated. This distance is called range. If the values of ranges are not same in different directions, then a geometric anisotropy exists in the orebody. Directional variography is used to plot variograms in several different directions. Direction variography for Esfordi phosphate mine shows that there is geometric anisotropy. We have used principal component method (Wold et al. 1987) and directional variography to obtain the main directions of anisotropy. To apply PCA, a covariance matrix for the three dimensional components of each lag (x, y and z for each h) is calculated. Given the covariance matrix, by finding the eigenvalues and eigenvectors of the covariance matrix, the main directions of anisotropy can be obtained. The largest eigenvalue represents the direction of the strongest correlation in the dataset. In addition, the orientations of maximum, minimum and medium correlation can be measured by using eigenvectors. To fit variograms in main directions, a cross validation approach is used. In cross validation method, each actual datum is dropped once and re-estimated again. Note that to estimate a composite (sample) in a drill hole, other composites in the same hole are dropped in order to avoid having an artificial high-density of samples in the neighbour of the dropped composite. The main criteria suggested to analyse the re-estimated samples are error distribution, estimation error versus estimated values plot and the scatter plot of actual value and the estimated value (Davis 1987; Deutsch and Journel 1992).

These criteria have been analysed for Esfordi mine as shown in Fig. 3. Figure 3a shows the scatter plot of actual and estimated values with a 0.84 correlation coefficient. Figure 3b represents error versus estimated values and as expected most of the points are close to zero. Finally, the error distribution shows a distribution with mean of 0.01 and variance of 0.6 (Fig. 3c).

The main parameters of the fitted variograms in maximum and minimum directions (computed on the normalized data) are listed in Table 1, and the corresponding variograms are graphically shown in Fig. 4.

It should be noted that composite data are declustered prior to estimation. Variograms have been computed for raw data and normalized data to be used in kriging and CSGS, respectively. Declustering, normalization/back transformation, variograms computation as well as upcoming kriging and simulation are performed by component of the WinGslib software (Deutsch and Journel 1992).

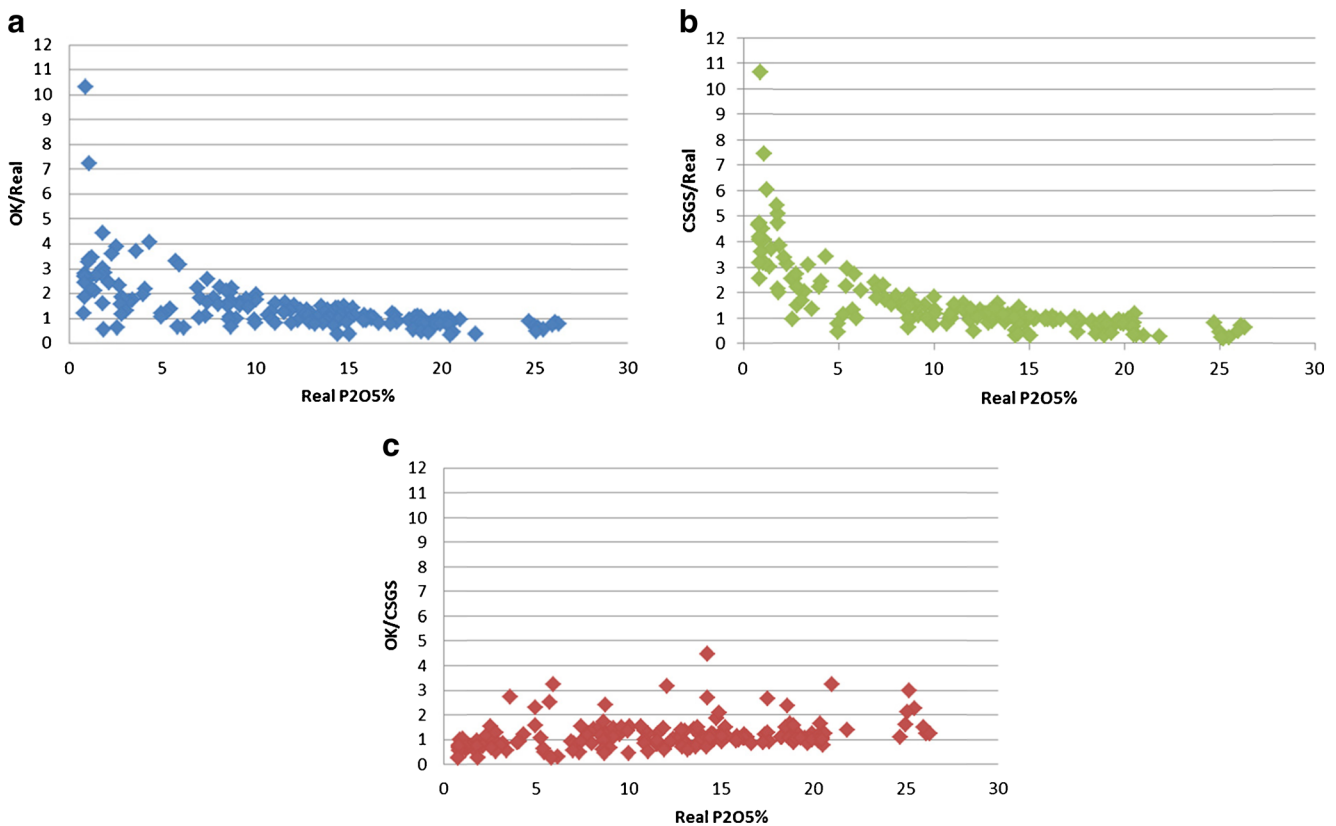


Fig. 9 The ratios of OK, Average CSGS and actual data

Ordinary Kriging (OK) has been applied to estimate the grades of blocks on the basis of sample raw data, the variogram models for raw data and the estimation parameters. Block size is $15 \times 15 \times 5$ m. Additionally, 50 realizations of the block model have been created by CSGS. A multiple grid simulation has been performed. The number of multiple grid refinements is three. Descriptive statistics and computed variograms have been analysed to validate the produced simulation results. Computed variograms and histograms for raw data and five realizations are shown in Fig. 5 and Fig. 6.

Regarding the estimated grade and the simulation results and also considering a given 6% phosphate cut-off grade, the ore tonnages (for whole block model) have been calculated by ordinary kriging (OK), loss function (LF) and profit function (PF) and shown in Fig. 7. Outcomes show that LF and PF provide minimum and maximum ore tonnage, respectively.

Comparing OK, LF and PF with real data

For the Esfordi mine, the actual grade of 179 blocks have been calculated using blast hole samples. The calculated actual grade for each block is an average of about 16 samples. These blocks have been extracted in 2009 and most of them were ore blocks processed at mineral processing plants.

Given a 6% cut-off grade, each technique has been applied to determine the destination of these 179 blocks, and the

amount of ore material has been calculated. The results are given in Fig. 8.

Results show that LF has the best performance while OK and PF overestimate the ore amount. The amounts of deviations are 3.26, -0.75 and 2.9% for OK, LF and PF, respectively, which are not significant. The ore tonnage may not be a good indicator, because some waste blocks may be considered as ore or some ore blocks may be estimated as ore. Hence, the misclassification percentage have been calculated and given in Table 2.

Table 2 shows that LF has the least error in assigning waste blocks to process. This happen because LF is a conservative method which tries to minimize potential loss as much as possible. The smoothing effect of OK leads to the classification of some low grade blocks as ore, consequently the percentage of wrong waste classification increases. PF works on the basis of maximizing potential profit, therefore blocks has more chance to be assigned as ore to process.

More investigations have been performed on the estimated values by OK and simulated results by CSGS. The ratios of OK/Actual, average of CSGS/Actual and OK/average of CSGS have been calculated and graphically shown in Fig. 9. The horizontal axis of these plots is actual values of P2O5% for 179 blocks. The desirable ratios for Ok/Actual and CSGS/Actual at each data point are one (or close to one). For P2O5% > 8 good correlations for these ratios are observed

in Fig. 9a, b. However, for P2O5% around 5%, the ratios are not promising. This happens because the average of raw data (used for the purpose of estimation and simulation is about 6%). This is the point that smoothing effect appears. Another interesting clue is that the ratio of OK/CSGS is poor for the range 0–7%, as can be seen in Fig. 9c. The reason of such miscorrelation may arise from the fact that the distribution of data is not normal and transformation has been performed for CSGS. Therefore, two main findings can be concluded. First, the estimated grades which are close to the average value of raw data may come with higher risk. Second, where the ratios of OK/Simulation are not promising, the chance of misclassification increases.

Conclusions

Ore grade estimation is a vital step in designing mine and mineral processing plant. The ore/waste classification is performed based on the estimated grade, and all planning and designs are conducted according to the results of ore/waste discrimination. In this study, ordinary kriging and simulation-based methods including loss and profit functions as the two main approaches used to determine the destination of run-of-mine material have been presented and compared for a case study. The efficiency of proposed techniques has been evaluated by comparing estimated results with actual data for 179 blocks. The outcomes show that loss function can marginally perform better than kriging and profit function. Loss function provides conservative tool for destination determination and can be considered suitable to have reliable estimation, if the exploration data is not enough. Additionally, comparing kriging and simulation results demonstrates that if the estimated grade for a block by kriging is different from the average value of simulation, then more attention must be taken into account for this block. For the future work, comparison of a larger number of blocks is suggested to verify this study. In addition, it is suggested to apply these techniques for other types of deposits to investigate the efficiency of the proposed techniques.

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