CrossMark

ORIGINAL PAPER

A research of consistencies and progresses of geoid models in Turkey

Nazan Yilmaz[1] · Leyla Cakir[1]

Abstract A number of geoids with various data and names have been determined up to present day in Turkey. In this paper, all Turkish geoids have been reviewed. For this purpose, ten local geoid models have been examined. These are 1976 Turkish Geoid (TG-76), Turkish Geoid-1991 (TG-91), Turkish Doppler Geoid-1992 (TDG-92), Turkish PseudoWGS84Doppler Geoid-1992 (TPDG-92), Turkish Astrogeodetic Geoid-1994 (TAG-94), Turkish GPS/Leveling Geoid, Updated Turkish Geoid-1999 (TG-99A), Turkish Geoid-2003 (TG-03), Turkish Geoid-2009 (TG-09), and Turkish Hybrid Geoid-2009 (THG-09). Within the numerical applications carried out, 30 test benchmarks of the Turkish National Fundamental GPS Network have been selected and geoid heights from each geoid model over these points have been calculated. Later, all of the models have been compared with each other in terms of the criteria based on their developments and geoid height differences have been calculated using local models at the test points. Through these comparisons, the causes of the inconsistencies between the various geoid models are explored while their overall accuracy is determined as well.

✉ Nazan Yilmaz
n_berber@ktu.edu.tr

Leyla Cakir
lcakir@ktu.edu.tr

[1] Department of Geomatics Engineering, Karadeniz Technical University, Trabzon, Turkey

Introduction

The geoid can be broadly defined as the equipotential surface of the Earth's gravity field that corresponds most closely with mean sea level in the open oceans, ignoring oceanographic effects (Heiskanen and Moritz 1967; Grafarend 1994). The geoid forms the reference surface for orthometric heights and can, in practice, be realized as the local vertical datum through geodetic leveling from tide-gauge measurements of mean sea level (Featherstone and Olliver 2001).

Geoid height (N) is the distance between geoid and ellipsoid. Especially in recent years, the widespread use of Global Navigation Satellite System (GNSS) has further increased the need of geoid heights. Ellipsoidal heights (h) obtained by GNSS do not meet practical needs about heights because the ellipsoidal heights do not have any natural meaning in the sense that they represent a mathematical/geometric distance from a mathematical (not realized in the real Earth) surface. But orthometric heights (H) from geoid are more compatible with physical events. Therefore, orthometric heights are successfully used for practical needs about heights. The determination of orthometric heights through the classic combination of spirit leveling and gravity observations is both cumbersome and time consuming. Therefore, we need geoid heights to convert ellipsoidal heights easily obtained by GNSS to orthometric heights. For this reason, an accurate geoid model is needed for the transformation of ellipsoidal heights to orthometric heights.

The accuracy of the geoid depends on several factors, such as the distribution and number of the control points, the interpolation method, the methodology to construct geoid, etc. (Zhan-ji and Yong-qi 1999).

The primary application of a geoid model in land surveying is for the transformation of GPS-derived ellipsoidal heights to elevations on the vertical datum. Obviously, using different

geoid-type models and height systems in the local vertical datum will produce different results (Featherstone and Olliver 2001). Within that frame, geoid modeling studies have been carried out in Turkey over the last decades and they still continue.

Turkish local geoids

1976 Turkish Geoid

1976 Turkey geoid was determined by using deflection of vertical over 106 benchmarks (BMs) of Laplace stations with astrogeodetic method. Reference surface is the 1924 international Hayford ellipsoid, and national datum station is Mesedag Laplace station in Ankara, Turkey. In computation of geoid height, a correction which must be applied due to plumb line has been neglected. The correction has been neglected due to uncertainties in both the topographic structure and the density of the Earth's crust. In addition, the non-linear variation of plumb line has been neglected, too (Ayan 1978).

Turkish Geoid-1991

The geoid, called Turkish Geoid-1991 (TG-91), referenced to the GRS-80 ellipsoid, has been computed by geopotential model, gravity and topographic height data. The geoid has been determined by evaluating least squares collocation of the remove-compute-restore method. In order to determine the geoid by least squares collocation (LSC), the area covering Turkey was divided into 114 blocks of $1 \times 1°$. Moreover, within each $1 \times 1°$ block, a $3' \times 3'$ grid system was established. The geoid was computed at 441 gridding nodes of the 3×3 ft grid system using GEOCOL software (Tscherning et al. 1994). LSC approximation to the geoid based upon the tailored geopotential model GPM2-T1 (Ayhan 1992) was constructed within each block. The model GPM2-T1 up to degree and order 200 was developed by tailoring of the model GPM2 to mean free-air anomalies and mean heights of one degree blocks in Turkey. Within each block was extended up to 0.5°, terrain effect which was reduced to gravity point data at distances of 5×5 ft were used in LSC. The residual terrain model (RTM) reduction of gravity (Forsberg and Tscheming 1981) depending on point heights at 15×20 in. gridding and 5×5 ft and 15×15 ft mean heights was carried out in terrain effect reduction. Indirect effect of RTM on geoid was also taken into account (Ayhan 1992).

Turkish Doppler Geoid-1992

Turkish National Doppler Datum (TUDD-92), consisting of 253 stations was established by Transit Satellite Doppler Positioning surveys between 1983 and 1991. 101 of these stations coincide with first and second order BMs of the Turkish National Vertical Control Network-1992 (TUDKA-92). Turkish Doppler Geoid-1992 (TDG-92) is computed on both TUDD-92 and TUDKA-92 as geometric differences between the ellipsoidal and BM orthometric heights.

TUDD-92 is orthogonal coordinate system (X, Y, Z). The origin of the system is close to the geocenter of the earth. z-axis almost coincides with the rotation axis of the earth, and x-axis is close to the Greenwich astronomical meridian. But the system is a special satellite coordinate system created in conditions of Turkey (Ayhan and Kılıçoğlu 1993).

Turkish PseudoWGS84 Doppler Geoid-1992

To obtain both PseudoWGS84 coordinates and Turkish PseudoWGS84 Doppler Geoid (TPDG), TUDD-92 coordinates of 101 BMs were transformed into ITRF89 by using 16 collocated points, the ITRF89 coordinates of which are computed by GPS. These coordinates are called PseudoWGS84. Turkish PseudoWGS84Doppler Geoid-1992 (TPDG-92) is computed on both TUDD-92 (in ITRF89) and TUDKA-92 as geometric differences between the ellipsoidal and BM orthometric heights (Ayhan and Kılıçoğlu 1993).

Turkish Astrogeodetic Geoid-1994

Observed deflections of the vertical in ED-50 at 200 astronomical points scattered throughout Turkey were utilized to determine an accurate astrogeodetic geoid, which is required for the reduction of some geodetic observations and datum transformations. However, the deflections of the vertical belong to points located on the earth's surface and have horizontal non-linear variations due to rough topography of Turkey and the large distance between the points (Heiskanen and Moritz 1967). Astrogravimetric leveling method was applied to recover these effects. Firstly, uncorrected geoidal heights between points were computed by astronomical leveling. Secondly, the gravimetric and orthometric corrections corresponding to the non-linearity effect and reduction to the geoid due to the curvature of plumb line, respectively, were determined. In order to compute the gravimetric correction, TG-91 was used. Gravimetric deflections of the vertical and geoidal heights at the points were interpolated by bicubic splines from a grid file of TG-91 geoidal heights at 3×3 ft nodes. Finally, after obtaining corrected geoidal heights, Turkish Astrogeodetic Geoid-1994 (TAG-94) was computed by least squares (Ayhan and Alp 1994).

Turkish GPS/Leveling Geoid

Of Turkish National Fundamental GPS Network (TUTGA-99A), 197 points were selected to determine GPS/

Leveling Geoid heights. TUTGA-99A is a network which is defined by GRS-80 (Moritz 1984) in ITRF96 International Terrestrial Reference Frame. The selected points of TUTGA-99A were connected to the Turkish National Vertical Control Network-1999 (TUDKA-99) by geometric leveling. Orthometric heights of points were calculated. Geoid heights of points (N) were computed by $N = h - H$ equation. The difference between round-trip measurements in geometric leveling measurements was expected to be small from $6\sqrt{S}$ mm (here, S is km units). Actual gravity values must be known to calculate orthometric heights. These gravity values were interpolated with ±3 mgal accuracy from National Gravity Catalog (Ayhan et al. 2002).

Updated Turkish Geoid (TG-99A)

TG-91 was computed by LSC using point gravity observations, a digital terrain model and an earth geopotential model in a remove-restore procedure. The ellipsoidal heights of the points referring to ITRF96 were made available by the establishment of TUTGA-99A. Precise and homogeneous Helmert orthometric heights all over Turkey can be computed after the re-adjustment of TUDKA-99. Helmert orthometric and ellipsoidal heights (TUTGA-99A) were determined at 197 points scattered over Turkey. TG-91 geoid heights at those points were interpolated. The differences between GPS/Leveling and TG-91 geoid heights, TG-91 being the reference surface, were computed at 197 points. In the modeling of the differences, the deterministic part (trend) was first determined by a 6-parameter polynomial surface, and it was subtracted from the differences in order to obtain the residual differences. The residual differences have stochastic properties, and were modeled and gridded by using adjustable tension continuous curvature surface gridding algorithm. The Updated Turkish Geoid-1999A (TG-99A) for Turkey was computed by summing up known TG-91 geoid heights, computed trend values and gridded residual differences at 3 × 3 ft grid nodes (Kılıçoğlu 2002). TG-99A geoid heights values were computed as follows:

$$N_{\text{TG99A}} = N_{\text{TG91}} - t - \mathrm{d}N \tag{1}$$

where
  N_{TG99A} = Geoid heights in TG99A geoid
  N_{TG91} = Geoid heights in TG91 geoid
  t = trend
  $\mathrm{d}N$ = residual differences.
  Six-parameter polynomial surface was used for trend surface (t). $\mathrm{d}N$ was coumpted as follows:

$$\mathrm{d}N_i = \delta N_i - t_i \tag{2}$$

  δN_i = Difference between TG91 and GPS/Leveling Geoid heights (Ayhan et al. 2002).

Turkish Geoid-2003

Turkish Geoid-2003 (TG-03) was computed as new and more data became available. Heterogeneous data (gravity, topography and geoid heights) were used by LSC in a remove-restore procedure. EGM96 (Lemoine et al. 1998) was used as the reference model of the Earth's geopotential. The data used consist of surface gravity anomalies, gravity anomalies derived from ERS1, ERS2, and TOPEX/POSEIDON altimetry data, GPS/Leveling Geoid heights, and topographic heights. Surface gravity values are in Modified Potsdam Datum, and the free-air anomalies were computed in GRS80. No surface gravity data were used outside the Turkish border while topographic heights were obtained from GTOPO30 (US Geological Survey 1996) global topography. The RTM effect of the topography was computed using a high-resolution Digital Terrain Model (450 × 450 m). The DTM used consists of high-resolution topographic heights within the borders, and dense bathymetry near the shoreline. Evenly distributed GPS/Leveling Geoid heights were introduced so as to compute the final geoid in agreement with GPS ellipsoidal heights. Ellipsoidal heights of the GPS/Leveling points refer to well-established Turkish National GPS Network (aligned to ITRF96), while orthometric heights refer to Turkish National Vertical Datum (fixed to mean sea level) (Kılıçoğlu et al. 2005).

Turkish Geoid-2009

Turkish Geoid-2009 (TG-09) was computed as new and more data were gathered. Data concerning gravity, topography and geoid heights, which came from different sources, were used by LSC in a remove-restore procedure. Earth Gravitational Model 2008 (EGM08) (Pavlis et al. 2012) was used as the reference model of the earth's geopotential. Data and assessments used in computation of this geoid are the same as TG-03 geoid (Kılıçoğlu et al. 2009a).

Turkish Hybrid Geoid Model-2009

Turkish Hybrid Geoid Model-2009 (THG-09) was calculated with Fast Fourier Transformation (FFT) technique using EGM08 (Pavlis et al. 2012) model, 1 × 1 ft DNSC08 sea gravity anomalies derived from satellite altimetry, surface gravity observations in 262212 points, 3 × 3 in. digital terrain model and 2714 GPS/Leveling points (Kılıçoğlu et al. 2009b). The release of the EGM08, the collection of new surface gravity observations, the advanced satellite altimetry-derived gravity over the sea, and the availability of the high-resolution digital terrain model encouraged us to compute a new geoid model for Turkey. A new Earth Gravitational Model (EGM08) to degree and order 2160

was released to the geodetic community. EGM08 incorporates 5 × 5 ft gravity anomalies and benefits from the latest GRACE solutions. Improved altimetry-derived gravity anomalies and its implied Dynamic Ocean Topography model were also used in the computations (Pavlis et al. 2008).

The computation of THG-09 was based on the remove-restore procedure and on EGM08 and RTM was applied in reducing the surface gravity data. FFT technique was then used to obtain the residual quasi-geoid from the reduced gravity. The individual contributions of EGM08 and RTM to the whole Quasi-Geoid Height (TQG-09) were restored. Since the Helmert orthometric height system is adopted in Turkey, the Quasi-Geoid Model (TQG-09) was then converted to the Geoid Model (TG-09) by making use of Bouguer gravity anomalies and digital terrain model:

$$N_{TG-09} = \varsigma_{TQG-09} + \frac{g-\gamma}{\overline{\gamma}} H \qquad (3)$$

where, $\overline{g}$ is the mean gravity along the plumb line between geoid and the ground, and $\overline{\gamma}$ is the mean normal gravity along the plumb line between ellipsoid and the telluroid.

Finally, in order to obtain a hybrid geoid model (THG-09) to be used in GPS positioning applications, gravimetric geoid model and GPS/Leveling Geoid heights were combined (Kılıçoğlu et al. 2011).

Application area and methodology for geoid evaluation

In this paper, 30 points of Turkish National Fundamental GPS Network were used. Those 30 test points, carefully distributed as homogenously as possible, are shown in Fig. 1. (Yılmaz and Karaali 2011).

Calculation of geoid heights of test points in different Turkish local geoids

Geoid heights of 30 test points in 1976 Turkish Geoid (TG-76) geoid models have been computed by multiquadratic interpolation. The most general representation of multiquadratic surface is as follows:

$$\Delta N = \sum_{i=1}^{m} C_i [Q(x, y, x_i, y_i)] \qquad (4)$$

Here, C_i is the coefficient of the unknown calculated with the ΔN residual height values, and $Q(x, y, x_i, y_i)$ is the kernel function. Other kinds of multiquadratic surfaces are also

available in addition to Eq. (4). For example, they can be expressed as:

$$\Delta N = \sum_{i=1}^{m} C_i \left[(x-x_i)^2 + (y-y_i)^2 + \delta^2 \right]^{1/2} \qquad (5)$$

As the sum of bicuspid circular hyperboloid series, or as:

$$\Delta N = \sum_{i=1}^{m} C_i \left[(x-x_i)^2 + (y-y_i)^2 + \delta^2 \right] \qquad (6)$$

as the sum of the circular paraboloid series. The δ in the relations is an optional coefficient (Zhan-Ji 1998). In this study, δ has been taken zero.

A program has been designed in MATLAB software to create multiquadratic surface and a trend surface has been created by using orthogonal polynomial in the program. A quadratic surface has been selected as trend surface. Significance testing of parameters has been done and all parameters have been found to be statistically significant. In this method, latitude, longitude, and geoid height of 106 Laplace stations regarding the TG-76 have been used as control points. Geoid heights of test points have been calculated by taking the geoid height of Mesedag station—the national datum station—as zero.

Geoid heights of test points in TAG-94 have been calculated in geoip.exe program with GRAVSOFT software package (Tscherning et al. 1994) using the grid file of TAG-94 geoid model.

Geoid heights of test points in TDG-92 and TPDG-92 models have been calculated in NETCAD software package (Netcad Yazılım 1989). Equal geoid height maps of these models have been transferred to NETCAD, and affine transformation has been made. Later, geoid height maps have been digitized and geoid heights of curves have been entered. The selected 30 test points have been placed on the map based on latitudes and longitudes. Also, with this program, geoid heights of test points have been calculated according to the method of Delaunay triangulation.

Geoid heights of test points in TG-91, TG-99A, TG-03, and THG-09 have been used from the data by (Yılmaz and Karaali 2011).

The GPS/Leveling and TG-09 geoid heights of these 30 test points have been used from the data by (Yılmaz and Karaali 2010).

Comparisons of geoid models

Turkish local geoid models have been compared with each other in two ways: through the criteria based on developments of Turkish local geoid models and by taking the geoid height differences in test points.
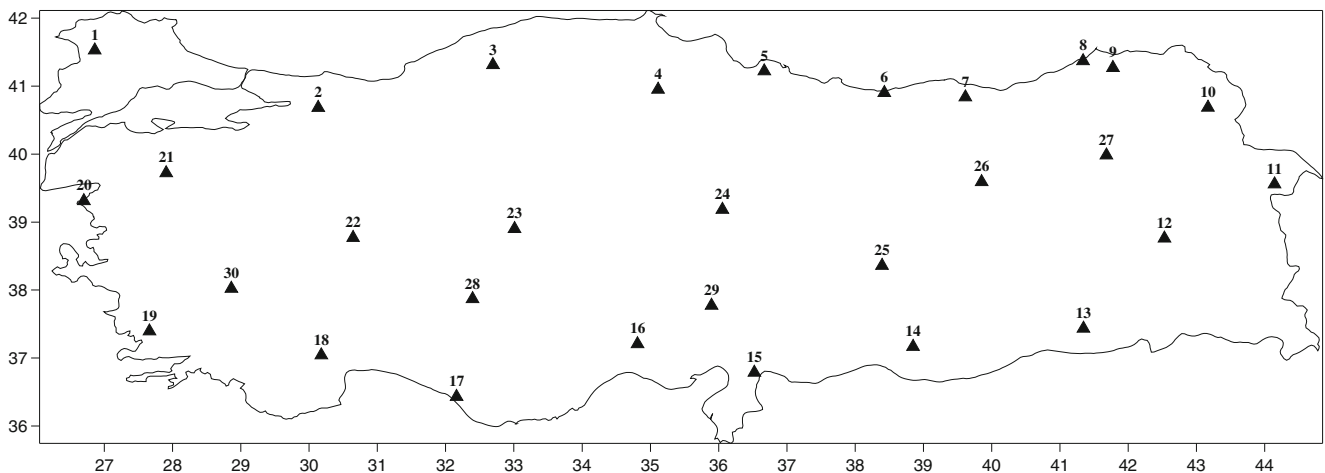
Fig. 1 Positions of test points

Comparisons in terms of the criteria based on developments of Turkish local geoid models

Turkish geoid models are different from each other in terms of datum, ellipsoid, data, methods, and benchmark number. These differences are shown in Table 1.

Results

In this section, geoid height differences obtained from various geoid models have been evaluated statistically and some findings have been obtained. These findings and the criteria based on developments of Turkish Local Geoid Models (Table 1) have been taken into account, and the causes of inconsistencies of different Turkish local geoid models have been examined.

Analysis of and findings from geoid height differences calculated from different Turkish local geoid models

Geoid height () ($i = 1, 2, …, 10$) differences calculated from 10 local geoid models have been evaluated. Evaluations have been made by taking the minimum, maximum, mean, standard deviation, and root mean square values of deviations from each other in those different geoid models. Geoid height differences have been created in the form of $\left(\Delta N_{ij} = N_{m_i}^{LM} - N_{m_j}^{LM} \right)$ and root mean square (RMS) values of the differences have been calculated by RMS = $\left[\left(\sum\limits_{i=1}^{k} \Delta N_{ij}^2 \right)/k \right]^{1/2}$ equation, with k being 30. The minimum, maximum, mean, standard deviation, and RMS values of deviations from each other in those different geoid models and signs of geoid height differences have been shown in Table 2. In this table, if the signs of geoid height differences are always

positively (+) or always negatively (−) oriented, the signs have been shown as being one-way and if the signs of geoid height differences are sometimes positively (+) or sometimes negatively (−) oriented, the signs have been shown as being two-way.

In order to investigate the causes of the inconsistencies among the various local geoid models, plane trend surfaces have been applied by taking the mean of geoid height differences (ΔN_{ij}) separately. The value (mean value) of each trend surface has been subtracted from geoid height differences and the remaining differences have been evaluated. After applying trend surfaces, the minimum, maximum, and RMS values of deviations from each other in those different geoid models have been shown in Table 3. Trend surfaces have been applied to find datum inconsistency between geoid models.

Examining the differences between Turkish local geoid heights

The differences in geoid heights calculated from various Turkish local geoid models in test points have been examined by taking into account both statistical data (Tables 2 and 3) and the criteria based on developments of models (Table 1) and these examinations have been shown in Table 4. The column showing the inconsistency degrees between geoid models in Table 4 has been filled by taking into account the root mean square (RMS) of deviations from each other (Table 2). If the RMS value is higher than 2 m, these geoid models are expressed as highly inconsistent. If the RMS value varies by 1 to 2 m, these geoid models are expressed as inconsistent. If, however, the RMS value is at centimeter level, these geoid models are expressed as consistent, all of which are displayed in Table 4. To determine the causes of inconsistency, the following criteria have been taken into account: If the signs of geoid height differences (Table 2) are one-way, there is definitely a datum inconsistency between two models. To find

Table 1 Criteria based on developments of Turkish local geoid models

Model	Datum	Ellipsoid	Data	Method	Benchmark number
1976 Turkish Geoid (TG-76)	Turkish National Datum-1954 (TUD-54)	Hayford ellipsoid: $a = 6,378,388$ m; $f = 1/297$	Astrogeodetic deflections of vertical	Astrogeodetic method	106
Turkish Geoid-1991 (TG-91)	Approximate geocentric	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	GPM2-T1 earth potential model, gravity and topographic heights	Remove-compute-restore Technique (RCR) and least squares collocation method (LSC)	441
Turkish Doppler Geoid-1992 (TDG-92)	Turkish National Doppler Datum	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	Ellipsoidal heights obtained from Doppler measurements and Helmert orthometric heights of TUDKA-92	GPS/leveling method $(h = H + N)$	83
Turkish PseudoWGS84 Doppler Geoid-1992 (TPDG-92)	ITRF89	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	Ellipsoidal heights obtained from Doppler measurements and Helmert orthometric heights of TUDKA-92	GPS/Leveling method $(h = H + N)$	83
Turkish Astrogeodetic Geoid-1994 (TAG-94)	Turkish National Datum-1954 (TUD-54)	Hayford ellipsoid: $a = 6,378,388$ m; $f = 1/297$	Astrogeodetic deflections of vertical, gravimetric deflections of vertical and geoid heights obtained from TG-91 geoid models by interpolation	Astrogravimetric leveling and least squares method	200
Turkish GPS/Leveling Geoid	ITRF 96	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	Ellipsoidal heights regarding Turkish National Fundamental GPS Network-1999 and orthometric heights regarding TUDGA-99	GPS/Leveling method $(h = H + N)$	197
Updated Turkish Geoid-1999 (TG-99A)	ITRF 96	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	TG-91 geoid heights, trend value in common points between TG-91 and GPS/Leveling Geoid (t), residual measurements (dN)	$N_{TG99A} = N_{TG91} - t \cdot dN$	197
Turkish Geoid-2003 (TG-03)	ITRF 96	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	EGM96 earth potential model, land gravity anomalies, sea gravity anomalies, digital terrestrial model, GPS/Leveling geoid heights	RCR technique and LSC method	197
Turkish Geoid-2009 (TG-09)	ITRF 96	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	EGM08 geoid model, land gravity anomalies, gravity anomalies from ERS1, ERS2 and TOPEX/POSEIDON altimetry data, topographic heights, GPS/Leveling geoid heights	RCR technique, LSC method and fast Fourier transform (FFT)	197
Turkish Hybrid Geoid Model-2009 (THG-09)	ITRF 96	GRS-80 ellipsoid: $a = 6,378,137$ m; $f = 1/298.257222101$	EGM08 geoid model, DNSC08 gravity anomalies from sea, land gravity measurements, digital terrestrial model and GPS/Leveling geoid heights	RCR technique and FFT method	197

Table 2 Statistical findings regarding the geoid height differences calculated from different Turkish local geoid models (TG76: Astrogeodetic geoid; TG-91: Gravimetric geoid; TDG-92, TPDG-92 and GPS/Lev: GPS/Leveling geoid; TAG-94: Astrogravimetric geoid; TG-99A, TG-03 and THG-09: combined geoid)

Compared geoid models	Minimum value of deviations from each other (m)	Maximum value of deviations from each other (m)	Mean (m)	Standard deviation (m)	RMS (m)	Signs of geoid height differences
TG91-TG76	29.810	49.735	37.918	5.303	38.274	One-way
TDG92-TG76	35.278	54.454	43.475	5.604	43.823	One-way
TPDG92-TG76	29.708	49.633	38.223	5.671	38.628	One-way
TAG94-TG76	−2.176	10.749	2.964	3.053	4.219	Two-way
TG99A-TG76	30.543	48.949	37.839	4.647	38.114	One-way
TG03-TG76	30.684	48.928	37.744	4.647	38.019	One-way
TG09-TG76	30.785	48.867	37.742	4.614	38.014	One-way
THG09-TG76	30.751	48.859	37.815	4.576	38.081	One-way
TG99A-TG91	−1.750	2.752	−0.078	1.082	1.067	Two-way
TG03-TG91	−2.238	2.546	−0.174	1.076	1.072	Two-way
TG09-TG91	−1.718	2.728	−0.175	1.080	1.076	Two-way
THG09-TG91	−1.871	2.936	−0.103	1.141	1.126	Two-way
TG91-TDG92	−2.542	−12.007	−5.558	2.476	6.067	One-way
TAG94-TDG92	−30.210	−49.322	−40.511	5.152	40.826	One-way
TG99A-TDG92	−2.655	−11.188	−5.636	2.141	6.016	One-way
TG03-TDG92	−2.522	−11.257	−5.731	2.145	6.107	One-way
TG09-TDG92	−2.496	−11.376	−5.733	2.162	6.114	One-way
THG09-TDG92	−2.430	−11.303	−5.661	2.162	6.047	One-way
TG91-TPDG92	2.606	−6.674	−0.306	2.408	2.387	Two-way
TDG92-TPDG92	4.546	5.942	5.252	0.326	5.262	One-way
TAG94-TPDG92	−24.764	−43.379	−35.259	5.234	35.633	One-way
TG99A-TPDG92	2.915	−5.997	−0.384	2.132	2.131	Two-way
TG03-TPDG92	3.048	−6.066	−0.480	2.135	2.154	Two-way
TG09-TPDG92	3.074	−6.185	−0.481	2.150	2.168	Two-way
THG09-TPDG92	3.140	−6.112	−0.409	2.157	2.160	Two-way
TG91-TAG94	24.433	41.945	34.953	5.203	35.325	One-way
TG99A-TAG94	25.166	41.119	34.875	4.643	35.172	One-way
TG03-TAG94	25.307	40.935	34.779	4.622	35.075	One-way
TG09-TAG94	25.408	40.976	34.778	4.603	35.071	One-way
THG09-TAG94	25.374	41.008	34.850	4.568	35.138	One-way
TG09-TG99A	0.242	−0.381	−0.097	0.137	0.166	Two-way
THG09-TG99A	−0.185	0.225	−0.025	0.111	0.112	Two-way
TG99A-TG03	−0.141	0.488	0.095	0.156	0.180	Two-way
TG09-TG03	−0.181	0.520	−0.002	0.130	0.128	Two-way
THG09-TG03	−0.145	0.475	0.071	0.173	0.184	Two-way
THG09-TG09	−0.153	0.448	0.072	0.125	0.143	Two-way
GPS/Lev-TG76	30.671	48.928	37.856	4.606	38.125	One-way
GPS/ Lev-TG91	−1.689	2.767	−0.062	1.099	1.083	Two-way
GPS/ Lev-TDG92	−2.462	−11.199	−5.620	2.168	6.010	One-way
GPS/ Lev-TPDG92	3.108	−6.009	−0.368	2.160	2.155	Two-way
GPS/ Lev-TAG94	25.294	41.121	34.891	4.600	35.183	One-way
GPS/ Lev-TG99A	−0.103	0.193	0.016	0.065	0.066	Two-way
GPS/ Lev-TG03	−0.079	0.549	0.112	0.144	0.181	Two-way
GPS/ Lev-TG09	−0.114	0.417	0.113	0.107	0.154	Two-way
GPS/ Lev-THG09	−0.182	0.189	0.041	0.082	0.090	Two-way

Table 3 Statistical findings regarding the geoid height differences calculated from different Turkish local geoid models after applying trend surfaces (TG76: Astrogeodetic geoid; TG-91: Gravimetric geoid; TDG-92, TPDG-92 and GPS/Lev: GPS/Leveling geoid; TAG-94: Astrogravimetric geoid; TG-99A, TG-03 and THG-09: Combined geoid)

Compared geoid models	Minimum value of deviations from each other (m)	Maximum value of deviations from each other (m)	RMS (m)
TG91-TG76	−8.108	11.818	5.214
TDG92-TG76	−8.197	10.979	5.510
TPDG92-TG76	−8.515	11.409	5.576
TAG94-TG76	−5.141	7.785	3.002
TG99A-TG76	−7.296	11.110	4.569
TG03-TG76	−7.060	11.184	4.569
TG09-TG76	−6.957	11.125	4.536
THG09-TG76	−7.064	11.045	4.499
TG99A-TG91	−1.672	2.830	1.064
TG03-TG91	−2.064	2.720	1.041
TG09-TG91	−1.543	2.903	1.045
THG09-TG91	−1.768	3.039	1.108
TG91-TDG92	3.015	−6.449	2.434
TAG94-TDG92	−8.811	10.301	5.065
TG99A-TDG92	2.981	−5.552	2.105
TG03-TDG92	3.210	−5.525	2.109
TG09-TDG92	3.237	−5.643	2.125
THG09-TDG92	3.231	−5.642	2.125
TG91-TPDG92	2.911	−6.368	2.367
TDG92-TPDG92	0.690	−0.706	0.320
TAG94-TPDG92	−8.121	10.495	5.146
TG99A-TPDG92	3.300	−5.613	2.096
TG03-TPDG92	3.528	−5.586	2.100
TG09-TPDG92	3.555	−5.704	2.114
THG09-TPDG92	3.549	−5.703	2.121
TG91-TAG94	6.992	−10.520	5.115
TG99A-TAG94	6.244	−9.709	4.565
TG03-TAG94	6.156	−9.472	4.544
TG09-TAG94	6.198	−9.370	4.526
THG09-TAG94	6.158	−9.476	4.491
TG09-TG99A	−0.284	0.339	0.135
THG09-TG99A	−0.160	0.250	0.109
TG99A-TG03	−0.236	0.393	0.153
TG09-TG03	−0.180	0.521	0.129
THG09-TG03	−0.216	0.404	0.172
THG09-TG09	−0.225	0.376	0.123
GPS/Lev-TG76	−7.184	11.073	4.529
GPS/Lev-TG91	−1.627	2.829	1.060
GPS/Lev-TDG92	3.158	−5.580	2.131
GPS/Lev-TPDG92	3.476	−5.641	2.123
GPS/ Lev-TAG94	6.230	−9.597	4.523
GPS/ Lev-TG99A	−0.119	0.176	0.064
GPS/ Lev-TG03	−0.190	0.437	0.142
GPS/ Lev-TG09	−0.227	0.304	0.105
GPS/ Lev-THG09	0.141	−0.230	0.081

Table 4 Examining the geoid height differences calculated from different Turkish local geoid models other than GPS/Leveling geoid (TG76: Astrogeodetic geoid; TG-91: Gravimetric geoid; TDG-92, TPDG-92 and GPS/Lev: GPS/Leveling geoid; TAG-94: Astrogravimetric geoid; TG-99A, TG-03 and THG-09: combined geoid)

Compared models	Examination	
	Degree of inconsistency	Causes of inconsistency
TG91-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TDG92-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TPDG92-TG76	Highly inconsistent	Datum, size and location of ellipsoids, data, method
TAG94-TG76	Highly inconsistent	Data, method
TG99A-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG03-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG09-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
THG09-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG99A-TG91	Inconsistent	Data, method
TG03-TG91	Inconsistent	Data
TG09-TG91	Inconsistent	Data
THG09-TG91	Inconsistent	Data
TG91-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TAG94-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG99A-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG03-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG09-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
THG09-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG91-TPDG92	Inconsistent	Data, method
TDG92-TPDG92	Highly inconsistent	Datum
TAG94-TPDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG99A-TPDG92	Inconsistent	Data, method
TG03-TPDG92	Inconsistent	Data, method
TG09-TPDG92	Inconsistent	Data, method
THG09-TPDG92	Inconsistent	Data, method
TG91-TAG94	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG99A-TAG94	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG03-TAG94	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG09-TAG94	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
THG09-TAG94	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
TG09-TG99A	Consistent	–
THG09-TG99A	Consistent	–
TG99A-TG03	Consistent	–
TG09-TG03	Consistent	–
THG09-TG03	Consistent	–
THG09-TG09	Consistent	–

datum inconsistency again, if the inconsistency between two models has not been recovered very well after applying trend surfaces (Table 3), it indicates that there is an inconsistency resulting from the sizes and the locations of ellipsoids besides datum problem between two geoid models. If ellipsoids in criteria based on developments of models (Table 1) are the same, it is considered that there is a problem of ellipsoid orientation. If the deviations of models

from each other do not change after applying trend surface, it is considered that there is an inconsistency resulting from data and methods. If the inconsistency between two models disappears completely after applying trend surface, it is considered that there is only datum problem between two geoid models. Because GPS/Leveling Geoid gives geoid undulation value directly ($N = h - H$) and because there is a relationship between the geoid and the Vertical Control

Table 5 Examining the geoid height differences calculated between different Turkish local geoid models and GPS/Leveling geoid

Compared models	Examination	
	Degree of inconsistency	Causes of inconsistency
GPS/Lev-TG76	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
GPS/Lev-TG91	Inconsistent	Data, method
GPS/Lev-TDG92	Highly inconsistent	Datum, sizes and locations of ellipsoids, data
GPS/Lev-TPDG92	Inconsistent	Data
GPS/Lev-TAG94	Highly inconsistent	Datum, sizes and locations of ellipsoids, data, method
GPS/Lev-TG99A	Consistent	–
GPS/Lev-TG03	Consistent	–
GPS/Lev-TG09	Consistent	–
GPS/Lev-THG09	Consistent	–

Network, these examinations have been shown in two separate tables (Tables 4 and 5).

Discussion/conclusions

Conclusions with respect to different geoid models can be summarized as follows:

- TG-76 and TAG-94 geoids are highly inconsistent with all local geoids. The deviation of TG-76 from the other geoid models is nearly 38 m, and the deviation of TAG-94 from the other geoid models is nearly 35 m. The basic cause of that inconsistency is a problem of constant systematic datum and ellipsoid. In addition, the data and the method used for determining these geoids are quite different from the other geoids.
- TG-91 geoid is inconsistent with other geoids except Doppler, TG-76, and TAG94 geoids. The deviations of geoid models from each other are nearly 1 m. The basic cause of this inconsistency is a different data.
- TDG-92 is highly inconsistent with all local geoids. The main reason for that inconsistency is a problem of constant systematic datum. Deviations of geoid models from each other, except TG-76 and TAG-94, are nearly 6 m. It has been observed that TDG-92 has both datum problems and ellipsoid orientation problems with other geoids, except TG-76 and TAG-94 geoids, judging by the examinations after applying trend surface. It has been concluded that TDG-92 may have datum problems and ellipsoid problems with TG-76 and TAG-94 geoids.
- TPDG-92 is inconsistent with other geoids except TDG-92, TG-76, and TAG-94 geoids. The deviations of geoid models from each other are nearly 2 m. The basic cause of the inconsistency is a different data and method.
- It has been seen that GPS/Leveling, TG-99A, TG-03, TG-09, and THG-09 geoids are consistent with each other. The deviations of geoid models from each other are nearly

between 7 and 12 cm. The main cause of the consistency is the same datum and reference ellipsoids that they have. In addition to this, there are similarities in data and methods used in calculations.
- The geoids which are mostly compatible with each other are THG-09 and TG-99A. The deviations of geoid models from each other are nearly 11 cm. The basic cause of the consistency is the same datum and reference ellipsoids that they have. In addition to this, the data and the methods used in calculations are quite similar.
- The geoids which are mostly incompatible with each other are TDG-92 and TG-76. The deviations of geoid models from each other are nearly 44 m. It can be argued that the basic cause of that inconsistency is the fact that the selected datum and reference ellipsoids, the data and the methods based on developments of these geoids are quite different from each other.
- The geoid model which has the highest deviation from Turkish local GPS/Leveling is TG-76. The deviations of geoid models from each other are nearly 38 m. It can be suggested that the reason why there exists such inconsistency is because the selected datum and reference ellipsoids, the data and the methods based on developments of these geoids are quite different from each other.
- The geoid model which is mostly compatible with Turkish local GPS/Leveling is TG-99A. The deviations of geoid models from each other are nearly 7 cm. It can be argued that the reason why there exists such consistency is because the selected datum and reference ellipsoids are the same, and that there are some similarities in their data.
- Transition to orthometric heights can be made at decimeter level with geoids determined after 1999. It has been observed that orthometric height is nearly 1.8, 1.5, and 1 dm precisely using TG-03, TG-09, and THG-09 geoids, respectively.
- Geoid models have appeared to be more compatible with each other in recent years. This situation is thought to be

caused by rapidly evolving and increasingly sensitive data collection techniques, the new positioning systems, evolving data analysis and modeling methods. Therefore, geoid models are continuously improving.

- Orthometric heights are directly determined in cm level using traditional measurement methods. Thus, geoid and ellipsoidal heights which are used to indirectly determine orthometric heights are expected to approach the same level accuracy. Today, it is not exactly the case.

- As vertical crustal movements have reached significant amounts in Turkey, GPS/Leveling Geoid has a dynamic property and the geoid model must be regenerated. GPS/Leveling Geoid is based on Turkish National Vertical Control Network, and there is vertical deformation in vertical control points.

- Geoid determination works should be continued to determine orthometric height more accurately in our country too. When we obtain more accurate data or when new methods are developed, geoid determination works should be repeated.

References

Ayan T (1978) 1976 Turkish Geoid. Map Journal 85:5–17, in Turkish

Ayhan ME (1992) Turkey Geoid-1991 (TG-91). Map Journal 108:1–17, in Turkish

Ayhan ME, Alp O (1994) Turkish Astrogeodetic Geoid-1994. Internal Report, Jeof 94–2, HGK, Ankara (in Turkish)

Ayhan ME, Kılıçoğlu A (1993) Turkish Doppler Geoid (TDJ). Prof. Dr. H. Wolf Geodesy Symposium. Yildiz Technical University, Proceedings, Istanbul, pp 409–435, in Turkish

Ayhan ME, Demir C, Lenk O, Kılıçoğlu A, Aktuğ B, Açıkgöz M, Açıkgöz M, Fırat O, Cingöz A, Gürdal MA, Ocak M, Türkezer A, Yıldız H, Bayazıt N, Ata M, Çağlar Y, Özerkan A (2002) Turkish national fundamental GPS network-1999A (TFGN-99A). Map Journal, Special Edition 16:1–69, in Turkish

Featherstone WE, Olliver JG (2001) A review of geoid models over the British Isles: progress and proposals. Surv Rev 36(280):78–100

Forsberg R, Tscheming CC (1981) The use of height data in gravity field approximation by collocation. JGR 86(89):7843–7854

Grafarend EW (1994) In: Vanicek P, Christou NP (eds) What is geoid? In geoid and its geophysical interpretations. CRC press, Boca Raton, pp 3–32

Heiskanen WA, Moritz H (1967) Physical geodesy. W.H. Freeman and Company, San Francisco and London

Kılıçoğlu A (2002) Updated Turkish Geoid (TG-99A). Scientific Meeting of Turkish National Geodesy Commission for 2002, Tectonics and Geodetic Networks Workshop, (in Turkish).

Kılıçoğlu A., Fırat O., Demir C. (2005) Observations and methods used in the computation of new Turkish Geoid (TG-03). Scientific Meeting of Turkish National Geodesy Commission for 2005, Proceedings of the Geoid and Vertical Datum Workshop, Karadeniz Technical University, Trabzon, pp. 53–65

Kılıçoğlu A, Yıldız H, Direnç A, Lenk O, Bağcı H, Simav M (2009a) Measurements and methods used in computation of Turkey Geoid (TG-09). The First Symposium of Interministerial Mapping Coordination and Planning Committee, METU Cultural and Convention Center, Ankara, in Turkish

Kılıçoğlu A, Direnç A, Yıldız H, Bölme M, Aktuğ B, Simav M, Lenk O (2009b) Turkish Hybrid Geoid Model-2009 (THG-09). Internal Report of General Command of Mapping, Ankara, in Turkish

Kılıçoğlu A, Direnç Y, Yıldız H, Bölme M, Aktuğ B, Simav M, Lenk O (2011) Regional gravimetric quasi-geoid model and transformation surface to national height system for Turkey (THG-09). Stud Geophys Geod 55(2011):557–578

Lemoine FG, Kenyon SC, Factor JK, Trimmer RG, Pavlis NK, Chinn DS, Cox CM, Klosko SM, Luthcke SB, Torrence MH, Wang YM, Williamson RG, Pavlis EC, Rapp RH, Olson TR (1998) The development of the joint NASA GSFC and the national imagery and mapping agency (NIMA) geopotential model EGM96. NASA technical paper NASA/TP1998206861. Goddard Space Flight Center, Greenbelt, USA

Moritz H (1984) Geodetic reference system 1980. Bulletin géodésique 58 (3):388 398

Netcad Yazılım AŞ (1989) NETCAD 5.0 GIS for WINDOWS, http://www.netcad.com

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2008) An Earth Gravitational Model to Degree 2160: EGM2008. Geophysical Research Abstracts, 10, 2-2-2008. Full version released by National Geospatial-Intelligence Agency, Bethesda, MD (http://www.dgfi.badw.de/typo3_mt/fileadmin/2kolloquium_muc/2008-10 08/Bosch/EGM2008.pdf)

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2012) The development and evaluation of the Earth gravitational model 2008 (EGM2008). J Geophysıcal Res 117:B04406. doi:10.1029/2011jb008916

Tscherning CC, Knudsen P, Forsberg R (1994) Description of the GRAVSOFT package. University of Copenhagen, Geophysical Institute, Copenhagen, Denmark

US Geological Survey (1996) GTPO30 global digital elevation model. EROS Data Center, https://lta.cr.usgs.gov/GTOPO30

Yılmaz N, Karaali C (2010) Comparison of global and local gravimetric geoid models in Turkey. Sci Res Essays 5(14):1829–1839

Yılmaz N, Karaali C (2011) Investigation of discrepancies of some geoids determined using various methods for Turkey. Int J Phys Sci 6(19): 4656–4666

Zhan-Ji Y (1998) Precise determination of local geoid and its geophysical interpretation. Hong Kong Polytechnic University, Hong Kong, Doctorate These

Zhan-ji Y, Yong-qi C (1999) Determination of local geoid with geometric method: case study. J Surv Eng 136