

Indirect tensile test assessments for rock materials using 3-D disc-type specimens

M. R. M. Aliha

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Abstract Indirect test methods are usually the preferred approach for determining the tensile strength of brittle and quasi-brittle materials such as ceramics, rocks, and concrete. In this paper, the tensile strength of a rock material (Iranian Harsin marble) was obtained indirectly by means of two disc type samples. The test samples were the well-known Brazilian disc and semi-disc specimen under three-point bend loading. Since the existing formula for determining the indirect tensile strength is based on 2-D stress assumption which ignores the effects of specimen thickness, a general 3-D tensile strength formulation was derived for each sample by employing finite element analysis. A series of tensile strength tests were conducted on Harsin white marble using disc and semi-disc specimens with different thicknesses in parallel to finite element analysis. The test results using 3-D formulation showed that the tensile strength of the tested marble depends on the thickness/diameter ratio of the specimen and generally decreases from 11.5 to 7 MPa on increasing the thickness of test samples. The average values of tensile strength were about 8.8 MPa for the Brazilian disc specimen and 9.8 MPa for the semi-disc specimen. The good agreement existing between the results of the two specimens suggests that the semi-disc specimen can also be used for determining the rock tensile strength in addition to using the conventional Brazilian test.

Keywords Rock · Tensile strength · Disc type samples · 3-D numerical analyses · Harsin marble · Experiments

Nomenclature

d Disc diameter
 r Coefficient of correlation

t Thickness
 t/d Thickness to diameter ratio
BD Brazilian disc
 C Loading span correction factor for SDB specimen
 E Modulus of elasticity
FE Finite element
ISRM International Society for Rock Mechanics
 K_t^{BD} Thickness correction factor for BD specimen
 K_t^{SDB} Thickness correction factor for SDB specimen
 P Applied load
 P_f Critical fracture load
 S Loading span in the SDB test
 S/d Loading span to diameter ratio in the SDB specimen
SDB Semi-disc bend

Greek symbols

σ_t Tensile strength
 σ_{xx} Tensile stress component
 ν Poisson's ratio

Introduction

Strength properties of rocks such as tensile, shear, and compressive strengths are important mechanical characteristics which are used as input design parameters in various rock-related projects such as civil and mining fields (e.g., slope stability analysis and rockfall hazard assessments (Verma and Singh 2010a; Singh et al. 2011; KhaloKakaie and Zare Naghadehi 2012; Ansari et al. 2013), hydraulic fracturing, and in situ stress measurements at great depths (Attar et al. 2012)) and also for appropriate selection of rock cutting tools and rock excavation machines (Verma and Singh 2010b; Ataei et al. 2011; Bayram 2012; Torabi et al. 2013). For rocks or rock-like materials, the value of tensile strength is always significantly less than the compressive strength (e.g., Ghazvinian et al. 2012a). Hence, due to the inherent weakness of rock materials against the tensile

M. R. M. Aliha (✉)
Welding and Joining Research Center, School of Industrial Engineering and Fatigue and Fracture Laboratory, School of Mechanical Engineering, Iran University of Science and Technology Narmak, 16846-13114 Tehran, Iran
e-mail: mmm_aliha@iust.ac.ir

stresses, rock structure designers prefer to avoid the tensile loads and tensile stresses in rock mechanics applications such as tunnels and underground mines. Therefore, it is necessary to estimate the value of rock tensile strength (σ_t) by means of appropriate methods. The measurement of σ_t for brittle materials like rocks requires special test procedures and suitable test samples which often differ from common tensile strength experiments used for other materials like metals. For example, despite the direct tensile test which is frequently used for most engineering materials, the indirect tensile test is usually the preferred approach for evaluating the tensile strength of rocks. This is mainly due to the difficulties associated with gripping the rock samples in direct tension loading. Three common methods for indirect tensile tests of rocks and rock-like materials are: (1) the Brazilian test, (2) the point load test, and (3) the three- or four-point flexural beam test. Although in these methods, the loads applied during the tests are compressive, the stresses generated at certain locations in the test specimens become tensile. At some critical level of applied load, the specimen is split due to these tensile stresses. Consequently, the tensile strength of tested rock material can be evaluated by means of the splitting fracture load; the related formulas are available in the literature.

From the practical considerations, because of the cylindrical shape of the rock cores obtained from in situ rock masses, disc type samples are usually the favorite configurations for conducting rock mechanic tests (Sulukcu and Ulusay 2001; Kahraman et al. 2003; Al-Shayea 2005; Aliha et al. 2008, 2010, 2012; Ghazvinian et al. 2012a, b; Erarslan and Williams 2012). For example, the International Society for Rock Mechanics (ISRM) has adopted the Brazilian test as a suggested method for determining σ_t in rock materials (ISRM 2007). However, the tensile strength results obtained using other indirect test methods such as flexural bend beams are not usually consistent with the Brazilian test data and there is a great discrepancy between the different indirect test methods.

In this research, the tensile strength of a rock material (an Iranian marble) is obtained experimentally by means of two disc type test configurations namely; the Brazilian disc (BD) and the semi-disc bend (SDB) specimens. Figure 1 shows the schematic geometry and loading conditions of these test specimens. The BD specimen is a circular disc of diameter d and thickness t which is loaded by two opposite diametral compressive force P . The SDB specimen is a semi-disc bend specimen of diameter d and thickness t which is subjected to three-point bend loading. The distance between the bottom supports in the SDB specimen is shown as S . Simple geometry, easy preparation from rock cores with less machining efforts, and the ability of conducting the tests by using common test machines are some of the advantages for these two specimens. Hence, the circular disc has been used frequently in the past for tensile strength experiments (e.g., Timoshenko

and Goodier 1970; Mellor and Hawkes 1971; Singh et al. 2001; Sulukcu and Ulusay 2001; Claessona and Bohloli 2002; Kahraman et al. 2003; Al-Shayea 2005; Yu et al. 2006; Kang et al. 2006; Mikaeil et al. 2012; Afifipour and Moarefvand 2013). However, unlike the BD specimen, the semi-disc has been used only by a few researchers working in the field of rocks and especially asphalt concrete materials (Huang et al. 2005; Arabani and Ferdowsi 2009; Dai et al. 2010) for evaluating the static and dynamic tensile strength but for very limited loading conditions (i.e., a fixed span/diameter ratio (S/d) value). Thus, in this research, more general tensile strength equation is proposed for the SDB specimen by considering S/d as variable. Moreover, the practical ability of semicircular bend specimen for the tensile strength experiments of rocks is examined experimentally and the obtained results for a typical marble rock are compared with the Brazilian disc test data.

The determination of tensile strength using these specimens requires accurate relations in terms of the applied load and the specimen dimensions. Hence, the first aim of this paper is to derive the required equations for calculating the tensile strength of the BD and SDB specimens. A review of literature shows that the σ_t of the BD specimen is often determined from a 2-D elasticity solution as Mellor and Hawkes (1971):

$$\sigma_t = -\frac{2P}{\pi t d} \quad (1)$$

where P is the diametral compressive load and t and d are the thickness and diameter of disc specimen, respectively. However in practice, the test samples have a 3-D state of stress and hence the effect of thickness to diameter ratio (t/d) needs to be considered for more accurate tensile strength evaluation. Thus, the existing 2-D relation is modified to general 3-D relations by considering a thickness correction factor (K). In order to obtain a 3-D model for the tensile strength, it is suggested in this paper to use:

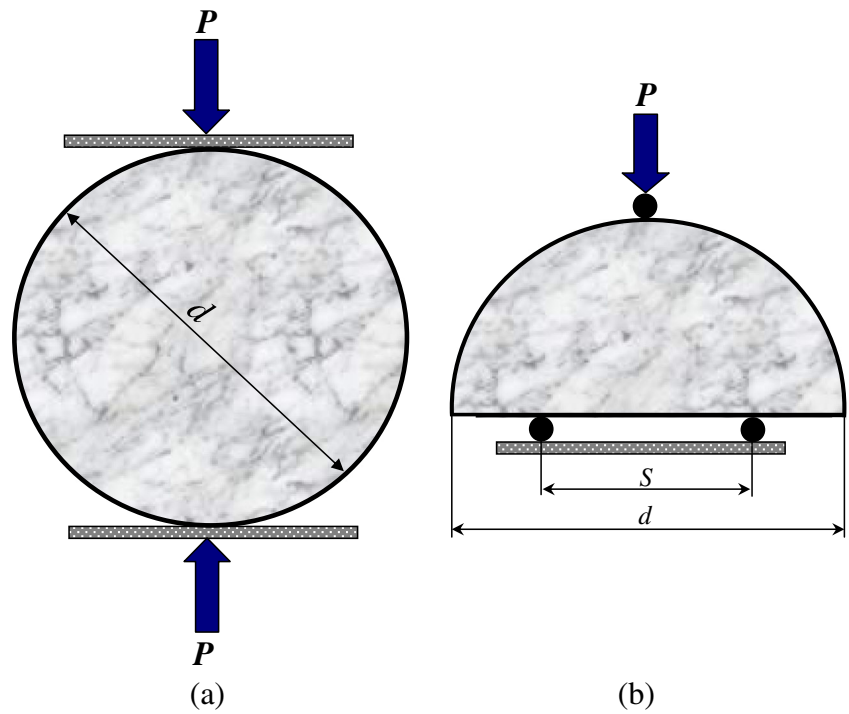
$$\sigma_t(\text{BD}) = -K_t^{\text{BD}} \frac{2P}{\pi t d} \quad (2)$$

for the BD specimen and the relation:

$$\sigma_t(\text{SDB}) = -K_t^{\text{SDB}} \frac{2P}{\pi t d} f(S) \quad (3)$$

for the SDB specimen, where σ_t (BD) and σ_t (SDB) are the tensile strengths obtained from BD and SDB specimens, respectively. K_t^{BD} and K_t^{SDB} in Eqs. (2) and (3) are the thickness correction factors for the BD and SDB specimens, respectively, and $f(S)$ is function of the loading span (S) for the SDB specimen. In the next section, finite element method is used

Fig. 1 **a** Brazilian disc specimen subjected to diametral compression and **b** the semi-disc specimen under three-point bend loading



to present general equations for determining the tensile stress of these two test samples. The generalized equations consider the

effects of three-dimensional stress state (i.e., the specimen thickness) and also the loading span S .

Fig. 2 3-D finite element mesh patterns and line-type load and boundary conditions along the thickness direction used for **a** BD and **b** SDB specimens

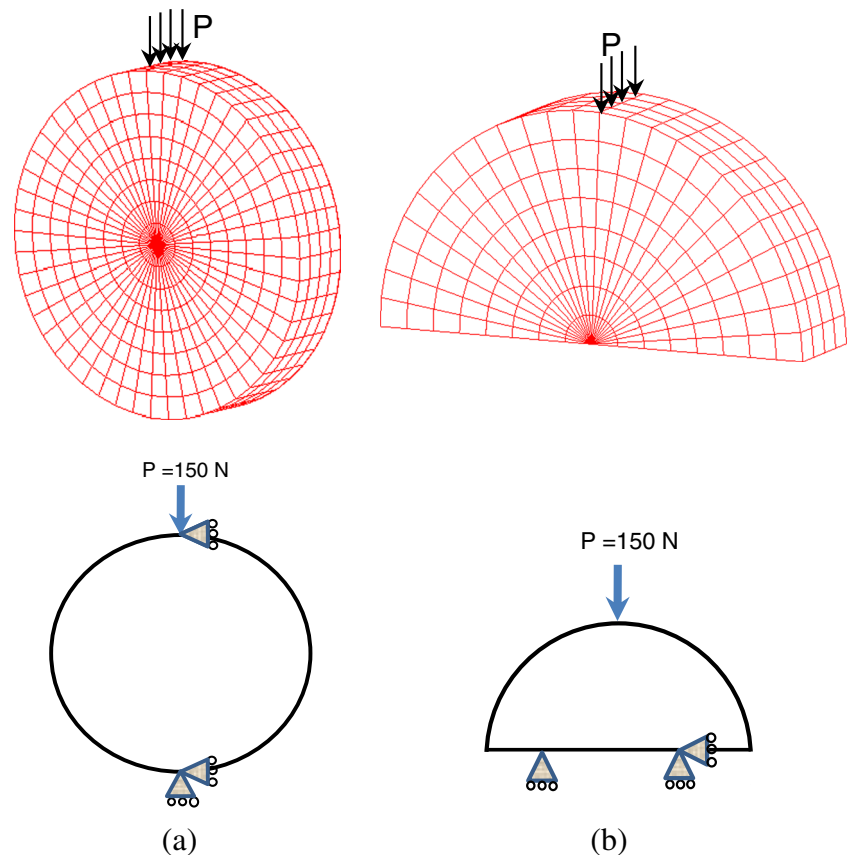
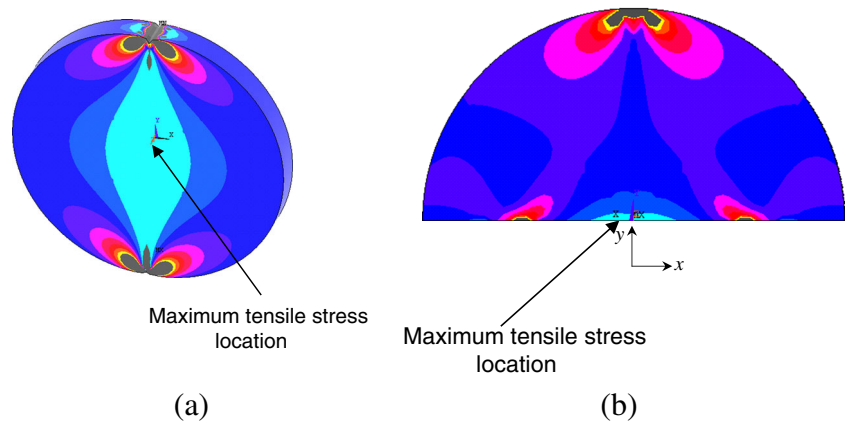


Fig. 3 Tensile stress (σ_{xx}) contour for 3-D models of **a** BD and **b** SDB specimens



Finite element simulation

For deriving general formulas to obtain the σ_i in both BD and SDB specimens, a large number of 3-D finite element analyses were performed in the ABAQUS code for various dimension ratios of t/d and S/d . This code is a powerful tool for computer-aided engineering and finite element analyses in a wide range of industrial applications such as mechanical, aerospace and automotive engineering, civil and mining engineering, etc. For example, the ABAQUS finite element code has been used recently by Sazid and Singh (2012) for simulating the rock blasting process. The ABAQUS code provides easily all the required modules for modeling and analysis of components and assemblies. These modules include: (1) preprocessing (creating the finite element model, meshing, application of loads and boundary conditions, and definition of material properties); (2) analyzing (e.g., different linear and nonlinear (standard and explicit) analyses for both solid and fluid materials and coupled solid–fluid media), and finally (3) visualizing and postprocessing the finite element results (such as stress, strain, energy, and deformation fields).

Figure 2 shows general mesh patterns used for the 3-D finite element modeling of the BD and SDB specimens. Typically 1,400 and 700 twenty-noded cubic solid elements were used for simulating the BD and SDB specimens. The diameter of discs or semi-discs was 100 mm and their thicknesses varied from 10 to 50 mm. In the finite element models, the elastic modulus $E=70$ GPa and Poisson's ratio $\nu=0.28$ (corresponding to typical marble rocks; Whittaker et al. 1992; Saghafi et al. 2010) were considered. The load and boundary conditions applied to each three-dimensional model are shown schematically in Fig. 2, where the nodes located along the support lines in the 3-D models were fixed in horizontal and/or vertical directions. A constant and reference vertical uniformly distributed load of $P=150$ N was applied to the finite element models along the thickness direction. The finite element analysis of the BD specimen under diametral

compression and the SDB specimen under three-point bend loading showed that for both specimens the maximum tensile stress occurred at the center of disc or at the middle point of flat edge in the semi-disc and along the direction of applied load P . This can be seen in Fig. 3, which shows the contours of tensile stress (σ_{xx}) for both BD and SDB specimens. Therefore, by performing a linear static analysis for each model and using the given load P , the corresponding σ_{xx} were obtained from the finite element analyses for each specimen. Since the applied load P and its corresponding generated maximum tensile stress at the center of disc or at the middle of flat edge in the semi-disc specimens are proportional, the corresponding values of thickness correction factors K_t^{BD} and K_t^{SDB} was extracted for each thickness from Eqs. (2) and (3) by applying a fixed and reference load of $P=150$ N for all the finite element models and extracting the maximum tensile stress value from the finite element analyses at the center of BD or at the middle of flat edge in the SDB specimen.

Figure 4 shows the variations of the determined correction factor with the t/d in the BD specimen. By fitting a linear

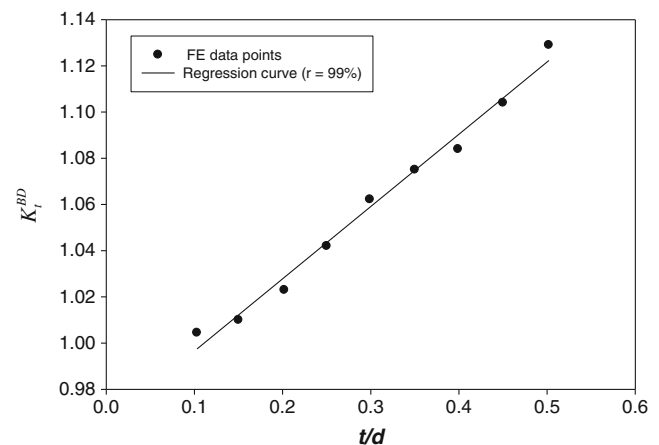
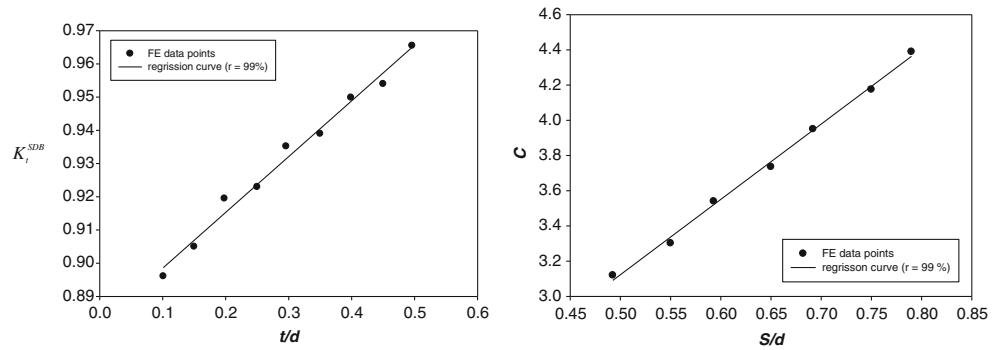


Fig. 4 Thickness correction factor K_t^{BD} for the BD specimen

Fig. 5 Thickness correction factor (K_t^{SDB}) and loading span factor (C) for the SDB specimen



curve to the finite element results, the following equation was extracted for K_t^{BD} in the BD specimen.

$$K_t^{BD} = 0.312\left(\frac{t}{d}\right) + 0.964 \tag{4}$$

Therefore, the generalized 3-D formulation for determining σ_t in the BD specimen can be written as:

$$\sigma_t(BD) = -\left(0.312\left(\frac{t}{d}\right) + 0.964\right) \frac{2P_f}{\pi t d} \tag{5}$$

where P_f is the critical fracture (splitting load of the BD specimen) which is obtained experimentally.

For the SDB specimen, in addition to the three parameters P , t , and d , the state of stress is affected by the distance between the bottom supports S . The available formulas in the published papers are limited to a fixed S/d ratio. For example, Molenaar et al. (2002) and Huang et al. (2005) proposed the following equation ($\sigma_t = \frac{3.56P_f}{d}$) for obtaining the tensile strength of a two-dimensional SDB specimen using finite element analysis. However, there is no general formulation for σ_t in the SDB specimen in the literature for taking into account the effects of t/d and S/d ratios. Therefore, similar to the BD specimen, the following general form of σ_t is considered in this study for the SDB specimen:

$$\sigma_t = -K_t^{SDB} \cdot C \frac{2P}{\pi t d} \tag{6}$$

where K_t^{SDB} and C represent the effects of t/d and S/d , respectively. In order to obtain the necessary relations for K_t^{SDB} and C , several 3-D finite element analyses were performed for different t/d and S/d , for $t/d=0.1$ to 0.5 and $S/d=0.5$ to 0.8 . Similar to the procedure described for the BD specimen, the results shown in Fig. 5 were obtained for the correction factors K_t^{SDB} and C for the SDB specimen. Indeed, the SDB specimen were subjected to a uniform distributed load of $P=150$ N and the value of maximum tensile stress at the middle of bottom edge of the SDB specimen where extracted from the linear

static numerical analysis. Also, for obtaining each correction factor (for example K_t^{SDB}), only the thickness was assumed as variable and the other parameter (i.e., S) was kept constant and vice versa. Thus, using the finite element analysis results, two separate linear relations were extracted for K_t^{SDB} and C as:

$$K_t^{SDB} = 0.146\left(\frac{t}{d}\right) + 0.8896 \tag{7}$$

and

$$C = 4.02\left(\frac{S}{d}\right) + 1.052$$

Thus, the tensile strength σ_t for the SDB specimen at the onset of tensile rupture where the applied load P reaches its critical value P_f can be determined from the following equation for various values of t/d and S/d .

$$\sigma_t(SDB) = -\left(0.146\left(\frac{t}{d}\right) + 0.8896\right) \left(4.02\left(\frac{S}{d}\right) + 1.052\right) \frac{2P_f}{\pi t d} \tag{8}$$

Indirect tensile strength experiments

In order to investigate the validity of Eqs. (5) and (8) and to evaluate the effect of specimen thickness on the tensile strength, a series of indirect tensile strength experiments were conducted on an Iranian white marble rock (called Harsin

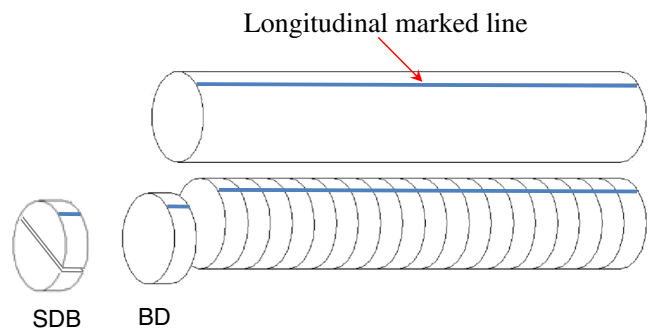
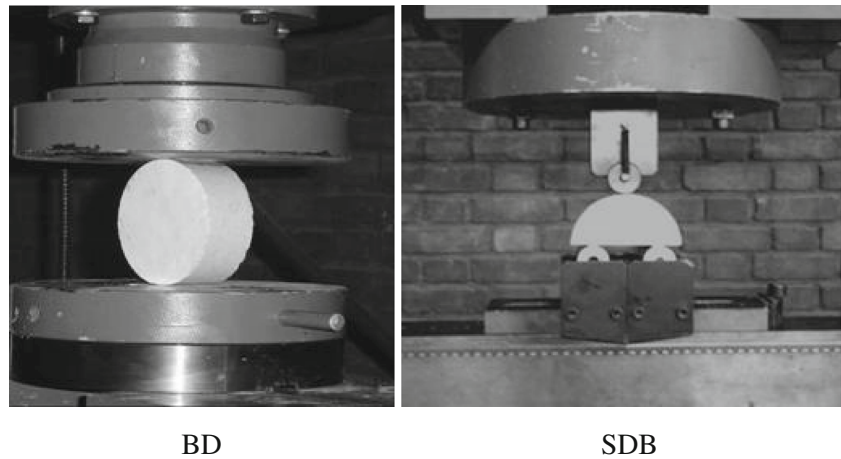


Fig. 6 Schematic picture representing the marking and slicing of the cylindrical marble samples to create the disc-shape samples

Fig. 7 Test set up for conducting the indirect tensile strength test using the BD and SDB specimens



marble), using both BD and SDB specimens. The Harsin marble is a homogenous and isotropic rock (excavated from Kermaanshah province minis in west of Iran) which can be prepared and cut easily for manufacturing the test specimens. This marble is widely used in many applications, particularly in Iran and thus its mechanical characteristics such as the tensile strength should be known for rock structure engineers and designers. For manufacturing the BD and SDB specimens, three rods of marble with diameters of approximately 100 mm and length of 900 mm were prepared from a Harsin marble rock mass. In order to reduce the possible effects of rock anisotropy, the specimens were all loaded in a similar direction and along a same diameter of each rod. Hence, as shown schematically in Fig. 6, the marble cylinders were marked before cutting with a narrow line along the longitudinal direction of the cylinders. Then, the rods were sliced with a diamond saw blade to obtain disc specimens with different thicknesses of 10, 20, 30, 40, and 50 mm. For creating the SDB specimens, each disc was split into two halves along the direction shown in Fig. 6 by a very thin saw blade of thickness 0.25 mm. Therefore, several BD and SDB specimens with different thickness to diameter ratios were prepared for indirect tensile strength tests.

For conducting the tests, a servo hydraulic tension–compression test machine (Zwick/Roell) with a loading capacity of 500 kN was used. For Brazilian disc tests, the BD samples were located between two flat plate fixtures and then the specimens were loaded with diametral compression along the marked lines. For conducting the semi-disc bend tests, the SDB specimens were located inside a three-point bend fixture. The loading span in the SDB experiments was kept constant at 60 mm. Thus, the span to diameter ratio S/d was 0.6 for the tested SDB specimens. Figure 7 shows the loading set up for the BD and SDB specimens.

The test samples were loaded with a constant rate of 0.5 mm/min. The load–displacement data for each specimen were recorded during the test up to the final fracture and sudden

splitting of the test sample. A total number of 25 BD specimens and 33 SDB specimens with different thicknesses were tested successfully. The σ_t of each specimen (with given t/d and S/d ratios) was then determined by substituting the critical peak load obtained from the test into Eqs. (5) or (8). The dimensions of each specimen, the P_f and the corresponding σ_t values for the tested BD and SDB samples are presented in Tables 1 and 2, respectively.

Table 1 Dimensions (t and d), critical peak load (P_f) and the tensile strength (σ_t) values obtained for the BD specimens

t/d	d (cm)	t (cm)	K_t^{BD}	P_f (kN)	σ_t (MPa) [from Eq. 5]	Average value (MPa)
0.107	10.3	1.1	0.9974	22.557	12.642	10.623
0.097	10.3	1	0.9943	14.963	9.279	
0.097	10.3	1	0.9943	15.332	9.62	
0.107	10.3	1.1	0.9974	21.082	11.815	9.1135
0.107	10.3	1.1	0.9974	18.159	10.651	
0.214	10.3	2.2	1.0308	39.553	11.43	
0.204	10.3	2.1	1.0276	33.396	9.642	9.464
0.194	10.3	2	1.0245	22.650	7.172	
0.206	10.2	2.1	1.0283	26.870	8.21	
0.307	10.1	3.1	1.0598	33.963	7.32	10.304
0.294	10.2	3	1.0557	41.714	9.642	
0.294	10.2	3	1.0557	53.170	11.68	
0.294	10.2	3	1.0557	40.908	9	9.39
0.304	10.2	3.1	1.0588	47.464	10.157	
0.41	10	4.1	1.0919	53.271	9.258	
0.4	10	4	1.0888	54.247	9.4	10.304
0.41	10	4.1	1.0919	56.650	9.59	
0.4	10	4	1.0888	68.504	11.871	
0.4	10	4	1.0888	65.813	11.404	9.39
0.505	10.1	5.1	1.1216	66.765	9.255	
0.505	10.1	5.1	1.1216	72.355	10.029	
0.495	10.1	5	1.1184	63.295	8.92	9.39
0.5	10.2	5.1	1.1200	68.263	9.365	

Table 2 Dimensions (t and d), critical peak load (P_f) and the tensile strength (σ_t) for the SDB specimens

t/d	d (cm)	t (cm)	K_t^{SDB}	C	P_f (kN)	σ_t (MPa) [from Eq. 8]	Average value (MPa)
0.108	10.2	1.1	0.9054	3.464	6.44	11.466	10.753
0.108	10.2	1.1	0.9054	3.464	5.89	10.503	
0.098	10.2	1	0.9039	3.464	6.536	12.78	10.019
0.12	10.2	1.2	0.9071	3.464	5.054	8.262	
0.22	10.2	2.2	0.9217	3.464	12.995	11.768	
0.196	10.2	2	0.9182	3.464	10.432	10.36	
0.22	10.2	2.2	0.9217	3.464	12.995	11.768	
0.203	10.3	2.1	0.9192	3.464	9.239	8.663	
0.196	10.2	2	0.9182	3.464	10.433	10.36	
0.22	10.35	2.25	0.9217	3.464	8.346	7.32	
0.21	10.2	2.1	0.9203	3.464	10.690	10.126	
0.21	10.2	2.1	0.9203	3.464	7.907	7.49	
0.196	10.2	2	0.9182	3.464	12.406	12.319	8.315
0.3	10	3	0.9334	3.464	11.084	7.608	
0.304	10.2	3.1	0.9340	3.464	10.654	6.942	
0.3	10	3	0.9334	3.464	15.909	10.919	
0.3	10	3	0.9334	3.464	13.399	9.196	
0.3	10	3	0.9334	3.464	16.667	11.439	
0.304	10.2	3.1	0.9340	3.464	11.083	7.222	
0.304	10.2	3.1	0.9340	3.464	11.631	7.578	
0.307	10.1	3.1	0.9344	3.464	10.392	6.842	
0.294	10.2	3	0.9325	3.464	10.737	7.218	
0.304	10.2	3.1	0.9340	3.464	14.818	9.656	7.256
0.28	10.2	2.9	0.9305	3.464	10.505	7.295	
0.41	10	4.1	0.9495	3.464	13.393	6.839	
0.396	10.1	4	0.9474	3.464	14.097	6.585	
0.396	10.1	4	0.9474	3.464	14.097	7.292	
0.382	10.2	3.9	0.9454	3.464	15.826	8.297	
0.374	10.3	3.85	0.9442	3.464	13.934	7.318	
0.49	10.2	4.95	0.9611	3.464	18.607	7.454	
0.5	10	5	0.9626	3.464	14.121	9.208	
0.5	10	5	0.9626	3.464	13.630	5.788	
0.51	10	5.1	0.9641	3.464	21.685	5.887	

Results and discussion

Since the existing formulations for obtaining the value of indirect tensile strength of BD and SDB specimens are valid only for plane stress (2-D) conditions, modified formulations were presented in this paper using several finite element analyses which consider the 3-D stress state and hence the specimen thickness. Based on the finite element analysis results for the BD specimen (see Fig. 4), the commonly used Eq. 1 (that is based on 2-D assumption), is only valid for thin BD specimens (i.e., for $t/d < 0.1$ for which K_t^{BD} is nearly equal to 1). Thus for higher values of t/d , Eq. (1) requires a modification to consider

the effect of specimen thickness. The thickness correction factor K_t^{BD} was determined numerically for the BD specimen in the present research for $t/d < 0.5$ and previously in Yu et al. (2006) for $t/d > 0.5$.

The equation derived for the SDB specimen in this paper (Eq. 8), provides also a general relation for determining σ_t for any desired ratios of t/d and S/d . According to results of this research (presented in Figs. 4 and 5) and also Yu et al. (2006), the thickness correction factors K_t^{BD} and K_t^{SDB} increase generally for the thicker BD and SDB specimens and hence it is expected that the difference between 2-D and 3-D tensile strength determinations become more pronounced when the thickness of the test specimen is increased. Consequently, it can be concluded that the ISRM suggested method which uses the BD samples with $1 > t/d > 0.5$ (ISRM 2007) provides conservative estimates for the tensile strength of rock materials.

According to the finite element analysis results, the maximum tensile stresses generated in both BD and SDB specimens

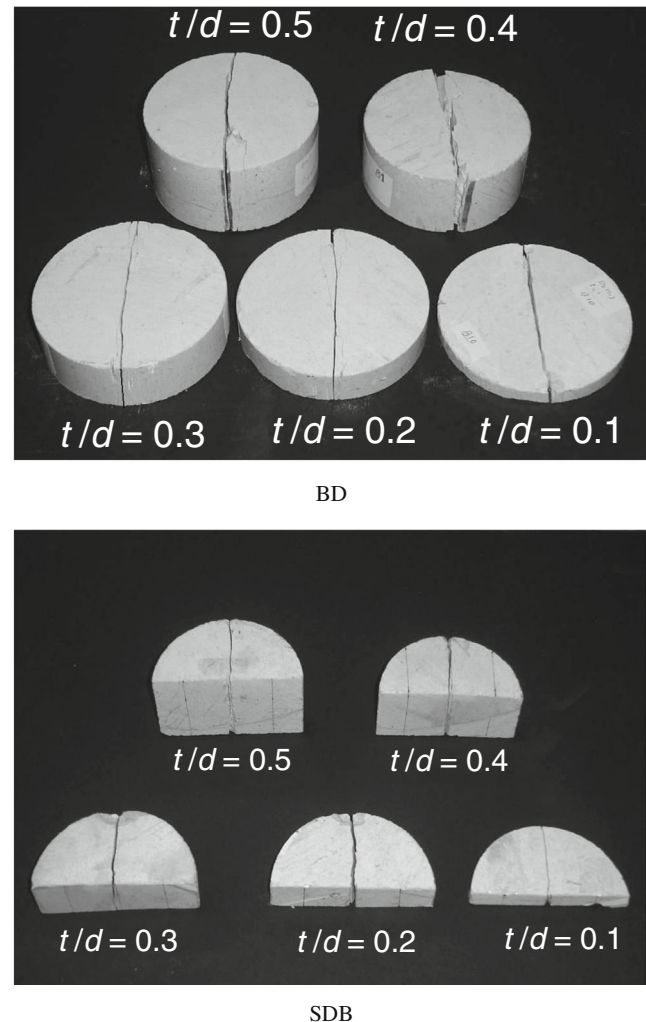
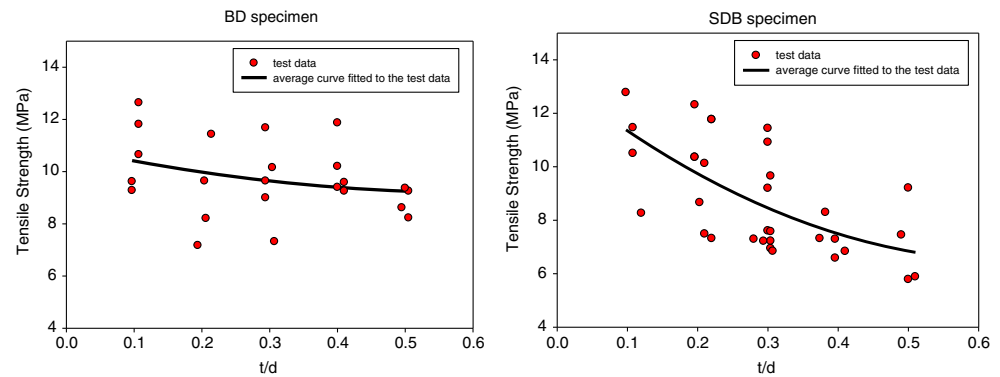


Fig. 8 Some of the broken BD and SDB samples made of Harsin marble with different thicknesses

Fig. 9 Variation in tensile strength of the Harsin marble with t/d for BD and SDB specimens tested



are along the direction of applied loads. Therefore, the finite element analysis results suggest that cracking in both specimens should be started from the center of specimens (where the tensile stress is greatest) and then grow along the direction of applied load such that the specimen is split into two equal halves at a critical level of P_f . A series of tensile strength tests were conducted on a rock sample (Iranian white marble) using both BD and SDB specimens and with various thicknesses. Some of the broken BD and SDB samples are shown in Fig. 8. It can be seen that the cracking and fracturing pattern in the tested BD and SDB specimens are in very good agreement with the finite element predictions. Figure 9 shows the variations of tensile strength results with t/d for the tested Harsin marble. As seen from this figure, the tensile strength of the tested rock is dependent on t/d ratio. The tensile strength σ_t for both specimens decreased typically from 11.5 to 7 MPa on increasing the t/d ratio from 0.1 to 0.5. However, the sensitivity of σ_t to specimen thickness is more pronounced for the SDB specimen. The decrease in the tensile strength value with the specimen thickness has also been reported by Yu et al. (2006) for $t/d > 0.5$. The average values of σ_t obtained from the complete experiments are about 8.8 MPa for the Brazilian disc and 9.8 MPa for the semi-disc specimens which are in the typical range reported for different types of marble rocks (Xeidakis et al. 1996; Ersoy and Atıcı 2004; Yavuz and Topal 2007; Mikaeil et al. 2012; Sengun and Altindag 2013). While the tensile strength results obtained from other flexural bend methods such as rectangular beam subjected to three- or four-point bending are two to three times greater than the Brazilian test results (Jaeger and Cook 1976; Brook 1993; Kaklis et al. 2009; Sengun and Altindag 2013), the good consistency that exists between the test data obtained from ISRM suggested Brazilian test method and the SDB specimen, suggests that the SDB specimen can also be used as an alternative specimen for estimating the indirect tensile strength of rock materials in addition to the well-known Brazilian test. Indeed, in comparison with the flexural rectangular beam specimens, the SDB method can provide

more accurate evaluations for the tensile strength of brittle materials because of its more consistency with the ISRM-suggested Brazilian test data.

Furthermore, a comparison between the BD and SDB test procedures showed that the SDB specimens need smaller load for fracture. Based on the fracture loads presented in Tables 1 and 2, it can be seen that for any given t/d , the required fracture load for the SDB specimen is about three times less than the corresponding fracture load for the BD specimen. Therefore, the SDB experiment can also be conducted using low-capacity testing machines. Also, for situations where the rock cores are limited or core preparation is difficult and expensive, the use of semi-disc specimen rather than the full circle samples may be more cost effective. In such conditions, testing on sub-sized specimens ($t/d < 0.5$) is probably more favorite than the commonly used specimens (with $0.5 < t/d < 1$) for extracting a typical value of σ_t for a rock sample. Therefore, considering advantages such as smaller fracture load and smaller amount of rock material required for specimen preparation, the SDB specimen can be considered as an alternative indirect test specimen for estimating the tensile strength of rocks in addition to the conventional methods and specimens such as the indirect Brazilian test method.

Conclusions

The following conclusions are drawn from the present study.

1. Modified 3-D formulations were derived for obtaining the indirect tensile strength of the BD and SDB specimens using finite element analysis method.
2. The presented equations take into account the effects of the t/d and the loading S/d for evaluating a more accurate tensile strength value of rocks using the actual 3-D laboratory test samples.

3. The thickness correction factors for both BD and SDB specimens increase by increasing the thickness of specimen. Therefore, the existing 2-D based formulations would provide conservative estimates for the tensile strength of practical three-dimensional test specimens.
4. A series of tensile stress tests were conducted on an Iranian white marble using both BD and SDB specimens with different t/d ratios ranging from 0.1 to 0.5. Based on the results obtained, the tensile strength of the tested rock (Harsin marble) was seen to decrease by increasing the specimen thickness.
5. Average value of σ_t obtained from BD and SDB specimens made of Harsin marble were about 8.8 and 9.8 MPa, respectively. The consistency and good agreement that existed between the BD and SDB results for the tested rock material demonstrates the practical ability of the SDB specimen to use as a new alternative test specimen for conducting tensile strength test of similar rocks in addition to the conventional ISRM suggested BD specimen.

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