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ORIGINAL PAPER

Global curve-fitting for determining the hydrogeological parameters of leaky confined aquifers by transient flow pumping test

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Abstract Proper management of groundwater resources requires an accurate evaluation of the parameters (hydraulic properties) that control the movement and storage of groundwater. Hydrogeological parameters are the basis of groundwater evaluation, modeling, and management and so on. A global curve-fitting method incorporating pumping test data and water table recovery data was introduced in the present study. The principal and procedures of the method were elucidated in detail. The drawdown and recovery data from two sets of transient flow pumping test conducted in no. 2 water source site of Shizuishan city were used to verify the calculation accuracy of the proposed method. The hydrogeological parameters were also estimated with traditional type curve-fitting method on the basis of formula derived by Hantush and Jacob. The hydrogeological parameters calculated by the two methods were compared and the results show that the parameters obtained by the global curve-fitting method are a little bigger than but very close to those obtained by the traditional type curve-fitting method. The proposed method which possesses three major advantages is feasible and reliable in aquifer parameter identification. A comparative study on various methods for parameter identification is required and expected in future study.

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Introduction

Groundwater, as an important component of water resources, plays an important role in national economic construction and development. In many areas of Asia and the Middle East, intensive aquifer use which has boosted crop production and improved access to relatively clean drinking water has been the single major factor that transformed rural economy in the last 25 years (Van Steenbergen 2006). In China, about one third of the water resource is from fresh groundwater and during the last 20 years, groundwater abstraction has increased at 2 billion cubic meters per year. Over 400 cities in China are exploiting groundwater. Groundwater accounts for 30 % in the urban water supply, especially in North and Northwest China where groundwater accounts for as high as 72 and 66 % of urban water supply, respectively (Liu et al. 2006). However, the irrational use of groundwater has induced many problems such as groundwater table decline, groundwater depletion, groundwater pollution, and soil salinization. Scientific groundwater management and accurate groundwater reevaluation are urgently required.

The determination of accurate hydrogeological parameters is the basis of groundwater evaluation, modeling, and management and so on. It is an essential issue to accurately determine the parameters of an aquifer. Pumping test, a traditional way of determining the aquifer parameters, is still being widely used worldwide nowadays. Many hydrogeologists worldwide have devoted to the research on pumping tests both theoretically and practically all these years (Gernand and Heidtman 1997; Jiang 2002; Van der Schaaf 2004; Singh 2008a, 2010; Miyake et al. 2008; Summa 2010). Cheng and Ni (2009) conducted a series of constant-rate pumping tests in the aquifer between two impermeable layers to understand its hydrogeological characteristics. The generalized radial flow model was used to analyze constant-rate pumping test data and to identify its fractional flow dimensions. Ou and Chen (2010) conducted two single- and group-well pumping tests in gravel formation of the Taipei basin. They developed a simple method to derive the hydraulic parameters from the pumping test results taking into account the site-specific influencing factors. Many other hydrogeologists and engineers also focus on the methods interpreting the pumping test data (Wei et al. 2003; Chenini et al. 2008; Wang and Zhan 2009; Bansal and Das 2009; Çimen 2009; Van Camp and Walraevens 2009; Chang and Yeh 2010; Wen et al. 2010). Chen (1974, 1983) developed an approximate analytical solution to analyze transient groundwater flow to a pumping well in an aquifer which changes from an initially confined system to a system with both unconfined and confined regimes, and Chen and Hu (2008) compared their model with the analytical model developed by Moench and Prickett (1972). Singh (2008b) developed a diagnostic curve of unimodal shape for identifying the confined aquifer parameters from early drawdowns. Malama et al. (2007, 2008) derived semi-analytical solutions to flow in leaky unconfined aquifer–aquitard systems. All these work has accelerated the progress of well flow theory.

For determining the hydrogeological parameters of leaky aquifers, typically there are two kinds of methods. One is analytical method based on analytical solutions of models and the other is numerical method. The traditional type curve-fitting method, water table recovery method, and inflection point method which are widely used are all analytical method for determining leaky aquifer parameters. However, there are some disadvantages using these methods: (1) these methods all depends on a series of curves and the results are usually influenced by subjective judgment and (2) only a part of pumping test data, whether data from the pumping period (Çimen 2009; Van Camp and Walraevens 2009; Chang and Yeh 2010; Wen et al. 2010) or data from water recovery period (Vanden Berg 1975), are used; the parameters determined by different methods (usually with different periods of data) are usually different. Therefore, a method integrating pumping period data with water recovery period data is needed.

The main objectives of the paper are (1) outlining the global curve-fitting method (GCFM) for leaky aquifer parameter determination. Although the method has already been introduced and verified in nonleaky aquifer system by Xiao et al. (2005a, b), it has not been introduced in leaky aquifer and whether it is applicable in leaky aquifer is still not clear. (2) Integrating analytical with numerical methods in pumping test data interpretation and aquifer property identification.

Theories and procedures

Drawdown in leaky confined aquifers

The transient drawdown in and around a fully penetrating well in an infinite, homogeneous, and isotropic leaky aquifer induced by a constant pumping discharge is expressed as (Hantush and Jacob 1955):

$$s(r,t) = \frac{Q}{4\pi T}\int_u^{\infty}\frac{1}{y}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy \tag{1}$$

The integral in the equation is called well function for leaky aquifers expressed by:

$$F(u,\frac{r}{B}) = \int_u^{\infty}\frac{1}{y}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy \tag{2}$$

Then, function (1) can be rewritten as:

$$s(r,t) = \frac{Q}{4\pi T}F(u,\frac{r}{B}) \tag{3}$$

where, $s(r,t)$=drawdown [L], t=time measured since commencement of pumping [T], r=distance between pumping and observation wells [L], F=well function for leaky aquifers [nondimensional], Q=constant pumping discharge [L^3T^{-1}], S=storage coefficient of aquifer [nondimensional], and T=transmissivity of aquifer [L^2T^{-1}], B=leakage factor [L], and u=dummy variable [nondimensional]. The leakage factor and dummy variable are expressed as follows:

$$B = \sqrt{\frac{TM'}{K'}} \tag{4}$$

$$u = \frac{r^2S}{4Tt} \tag{5}$$

where, M' is the thickness of the aquitard and K' is the hydraulic conductivity of the aquitard. The other symbols are the same with the aforementioned symbols.

The residual drawdown in an observation well at a distance r from a pumping well which has been pumped at a constant rate Q over a period t_p is expressed as (Vanden Berg 1975):

$$s' = h_0 - h_t = \frac{Q}{4\pi T}(F(u,\frac{r}{B}) - F'(u',\frac{r}{B})) \tag{6}$$

$$u' = \frac{r^2S}{4Tt'} \tag{7}$$

$$t' = t - t_p \tag{8}$$

where, s' represents the residual drawdown at specific t time [L], h_0 is the initial groundwater level [L], h_t is the groundwater level at t time [L], t_p denotes the pumping time, and t' is the water table recovery time [T]. During the water table recovery period, the pumping well is assumed to continue pumping water from the aquifer and at the same time, an injecting well is assumed to inject water into the same aquifer at the same constant rate Q. $F'(u', \frac{r}{B})$ is the well function for the assumed injecting well and u' is the dummy variable for the assumed injecting well [nondimensional]. The others are the same with the previously mentioned ones.

Nonlinear least squares

The GCFM integrates pumping period data with water table recovery period data. The nonlinear least squares approach is commonly used for the curve-fitting problems (Yeh 1989) and in the present study, it is used for hydrogeological parameters identification.

The discrepancy between observed drawdowns and the drawdowns calculated from (1) and (6) can be calculated, squared, and summed over all the data points to obtain the sum of squares of the deviations which has to be minimized.

$$Z = \begin{cases} \min \sum (s_m - s_c)^2 & \text{pumping period} \\ \min \sum ({s'}_m - {s'}_c)^2 & \text{recovery period} \end{cases} \tag{9}$$

where, Z is the objective function, it is the sum of the square errors; the s_m and s_m' are respectively the observed drawdown in the water pumping period and residual drawdown in the water table recovery period; s_c and s_c' are the predicted drawdown of the water pumping period and residual drawdown in the water table recovery period. By achieving the minimum of the sum of squares of the deviations expressed as Eq. (9), the optimal hydrogeological parameters can be obtained.

In this study, the Gauss–Newton iteration algorithm was employed to search the solution of the nonlinear least squares minimization Eq. (9). Suppose the initial values of the unknown parameters are T_0, S_0, and K'_0 and the initial drawdown estimated by the initial parameters is s_0. The initial parameter values can be changed by $\triangle T$, $\triangle S$, and $\triangle K'$, namely, $T=T_0+\triangle T$, $S=S_0+\triangle S$, $K'=K'_0+\triangle K'$. Equation (1) is rewritten according to Taylor expansion as (for details, please refer to Appendix):

$$s* \approx s_0(t) + \left(\frac{\partial s}{\partial T}\right)_0 \Delta T + \left(\frac{\partial s}{\partial S}\right)_0 \Delta S + \left(\frac{\partial s}{\partial K'}\right)_0 \Delta K' \tag{10}$$

where, s^* is the calculated drawdown and $s_0(t)$ is the calculated initial drawdown when initial parameters are T_0, S_0, and K'_0. $\left(\frac{\partial s}{\partial T}\right)_0$, $\left(\frac{\partial s}{\partial S}\right)_0$ and $\left(\frac{\partial s}{\partial K'}\right)_0$ are respectively the partial derivatives of s with respect to the aquifer parameters T, S, and K' at (T_0, S_0, K'_0). For the pumping period, the first partial derivatives of s with respect to the aquifer parameters T, S, and K' are (Guo et al. 1994):

$$\begin{cases} \frac{\partial s}{\partial T} = \frac{-Q}{4\pi T^2}\left[F(u, \frac{r}{B}) - \exp(-u - \frac{Tt}{B^2 S}) - \frac{r^2}{4B^2}\int\limits_u^\infty \frac{1}{y^2}\exp(-y - \frac{r^2}{4B^2 y})dy\right] \\ \frac{\partial s}{\partial S} = \frac{-Q}{4\pi TS}\exp(-u - \frac{Tt}{B^2 S}) \\ \frac{\partial s}{\partial K'} = \frac{-r^2 Q}{16\pi T^2 M'}\int\limits_u^\infty \frac{1}{y^2}\exp(-y - \frac{r^2}{4B^2 y})dy \end{cases} \tag{11}$$

And for the recovery period, the first partial derivatives of s with respect to the aquifer parameters T, S, and K' are more complex.

$$\begin{cases} \frac{\partial s}{\partial T} = -\frac{Q}{4\pi T^2}\left[F(u, \frac{r}{B}) - F'(u', \frac{r}{B})\right] + \frac{Q}{4\pi T}\left(\frac{\partial F}{\partial T} - \frac{\partial F'}{\partial T}\right) \\ \frac{\partial s}{\partial S} = \frac{-Q}{4\pi TS}\left[\exp(-u - \frac{Tt}{B^2 S}) - \exp(-u' - \frac{Tt'}{B^2 S})\right] \\ \frac{\partial s}{\partial K'} = \frac{-r^2 Q}{16\pi T^2 M'}\left[\int\limits_u^\infty \frac{1}{y^2}\exp(-y - \frac{r^2}{4B^2 y})dy - \int\limits_{u'}^\infty \frac{1}{y^2}\exp(-y - \frac{r^2}{4B^2 y})dy\right] \end{cases} \tag{12}$$

The formulas (11) and (12) are the general forms of the partial derivatives of s. If their values at (T_0, S_0, K'_0) are needed, all we have to do is replace the parameters T, S, and K' in Eqs. (11) and (12) by T_0, S_0, and K'_0. Further, the integrals in the formulas can be solved by numerical method. When solving the integral, its upper limit has to be assigned because infinitude is not numerically calculable. In our study, the upper limit of the integral was determined to be 50 after many times of tentative calculation since it will shorten the time for calculation, but meanwhile will keep high accuracy required. The integration interval from u (or u') to 50 was further divided into many sub-intervals by geometric progression during the numerical calculation, namely, $\Delta p_{i+1}/\Delta p_i=q$, where, Δp_{i+1} and Δp_i are two adjacent sub-integrals, q is the common ratio of the geometric progression. In the present study, q is 1.00076 which is most proper for rapid calculation but high accuracy.

To get a best fit, the following error function should be minimized.

$$E(\Delta T,\ \ \Delta S,\ \ \Delta K') = \sum_{i=1}^{n} (s_m - s*)^2$$
$$= \sum_{i=1}^{n} \left(s_m - s_0 - \frac{\partial s}{\partial T}\Delta T - \frac{\partial s}{\partial S}\Delta S - \frac{\partial s}{\partial K'}\Delta K'\right)^2 \tag{13}$$

If E is to be the minimum, the first partial derivatives of E with respect to the aquifer parameters T, S, and K' must be zero (Yeh 1987). Thus,

$$\frac{\partial E}{\partial \Delta T} = 0, \frac{\partial E}{\partial \Delta S} = 0, \frac{\partial E}{\partial \Delta K'} = 0 \tag{14}$$

$$\begin{cases} \sum_{i=1}^{n}(s_m - s_0)\frac{\partial s}{\partial T} = \Delta T\sum_{i=1}^{n}(\frac{\partial s}{\partial T})^2 + \Delta S\sum_{i=1}^{n}\frac{\partial s}{\partial S}\frac{\partial s}{\partial T} + \Delta K'\sum_{i=1}^{n}\frac{\partial s}{\partial K'}\frac{\partial s}{\partial T} \\ \sum_{i=1}^{n}(s_m - s_0)\frac{\partial s}{\partial S} = \Delta S\sum_{i=1}^{n}(\frac{\partial s}{\partial S})^2 + \Delta T\sum_{i=1}^{n}\frac{\partial s}{\partial S}\frac{\partial s}{\partial T} + \Delta K'\sum_{i=1}^{n}\frac{\partial s}{\partial S}\frac{\partial s}{\partial K'} \\ \sum_{i=1}^{n}(s_m - s_0)\frac{\partial s}{\partial K'} = \Delta K'\sum_{i=1}^{n}(\frac{\partial s}{\partial K'})^2 + \Delta S\sum_{i=1}^{n}\frac{\partial s}{\partial S}\frac{\partial s}{\partial K'} + \Delta T\sum_{i=1}^{n}\frac{\partial s}{\partial K'}\frac{\partial s}{\partial T} \end{cases} \quad (15)$$

$$\begin{bmatrix} \sum_{i=1}^{n}(\frac{\partial s}{\partial T})^2 & \sum_{i=1}^{n}\frac{\partial s}{\partial S}\frac{\partial s}{\partial T} & \sum_{i=1}^{n}\frac{\partial s}{\partial K'}\frac{\partial s}{\partial T} \\ \sum_{i=1}^{n}\frac{\partial s}{\partial S}\frac{\partial s}{\partial T} & \sum_{i=1}^{n}(\frac{\partial s}{\partial S})^2 & \sum_{i=1}^{n}\frac{\partial s}{\partial K'}\frac{\partial s}{\partial S} \\ \sum_{i=1}^{n}\frac{\partial s}{\partial K'}\frac{\partial s}{\partial T} & \sum_{i=1}^{n}\frac{\partial s}{\partial K'}\frac{\partial s}{\partial S} & \sum_{i=1}^{n}(\frac{\partial s}{\partial K'})^2 \end{bmatrix} \begin{bmatrix} \Delta T \\ \Delta S \\ \Delta K' \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}(s_m - s_0)\frac{\partial s}{\partial T} \\ \sum_{i=1}^{n}(s_m - s_0)\frac{\partial s}{\partial S} \\ \sum_{i=1}^{n}(s_m - s_0)\frac{\partial s}{\partial K'} \end{bmatrix} \quad (16)$$

The $\triangle T$, $\triangle S$, and $\triangle K'$ can be obtained by solving Eq. (14). The real values of T, S, and K' can be calculated by the following iteration equations.

$$\begin{cases} T^{(k+1)} = T^{(k)} + \Delta T^{(k)} \\ S^{(k+1)} = S^{(k)} + \Delta S^{(k)} \\ K'^{(k+1)} = K'^{(k)} + \Delta K'^{(k)} \end{cases} \quad (17)$$

where, k is the iteration step number. Some specified tolerances should be applied to terminate the iterations and they are usually in the following forms:

$$\begin{cases} \left|T^{(k+1)} - T^{(k)}\right| \le \varepsilon_1 \\ \left|S^{(k+1)} - S^{(k)}\right| \le \varepsilon_2 \\ \left|K'^{(k+1)} - K'^{(k)}\right| \le \varepsilon_3 \end{cases} \quad (18)$$

where, the values of ε_1, ε_2, and ε_3 are predefined tolerances depending on the required accuracy. Some scientists have discussed the convergence criteria in detail and 10^{-5} is typically accurate enough for any iteration (Dennis and Schnabel 1996).

Procedures of parameter estimation

There are generally three steps for parameter identification using global curve-fitting method.

- Step 1. Establishing a pumping test database according to the field hydraulic tests. It is quite important to obtain a set of accurate water pumping data and water recovery data since the closer the observed data to the calculated data, the smaller the matching errors. Quality control during the pumping period is required, and the pumping period during which various incidents may easily take place should not be too long. It is okay to stop pumping when the drawdown is relatively stable and there is no need to keep the stable status for a long time.
- Step 2. Programming and calculation. Since a set of accurate test data has been obtained, the hydrogeological workers have to compile a program for best matching based on the theory introduced above. It is alternative to perform the calculation simply in Microsoft Excel if the hydrogeological workers are unable to compile the program. To do the best matching in Microsoft Excel, it is useful to simplify the well function for leaky aquifers expressed as Eq. (2). The simplified well function has been deduced by Swamee and Ojha (1990) and is shown below.

$$F(u, \frac{r}{B}) \approx -\frac{2}{3}\ln\left[(\frac{u}{c_1 + 0.65u})^{\frac{3}{2}} + (\frac{\frac{r}{B}}{1.12 + 0.6(\frac{r}{B})^2})^{\frac{3}{2}}\right] \quad (19)$$

where, c_1 is a constant and in value equals to c_1=exp (−0.577216). However, the simplified well function can only be used when $u<c_1$. If the simplified well function is used to calculate the drawdown data, the calculated data in the first few minutes may not be appropriate to use for the best matching and parameter identification.

- Step 3. Results analysis. The parameters obtained can be used for aquifer property interpretation and groundwater resources evaluation.

Case study

Aquifer structures of the site

Two sets of transient flow pumping tests were carried out during December 2009 to January 2010 in Shizuishan city, Northwest China for the purpose of water supply source investigation. According to sedimentary rhythm and borehole data, in the vertical direction, the aquifers in the study area are divided into five water-bearing groups within the depth of 350 m. From top down, they are phreatic aquifer (the first water-bearing group, represented by I, lower than 30 m), the first confined aquifer (the second water-bearing group, represented by II, within the depth of 30–80 m), the

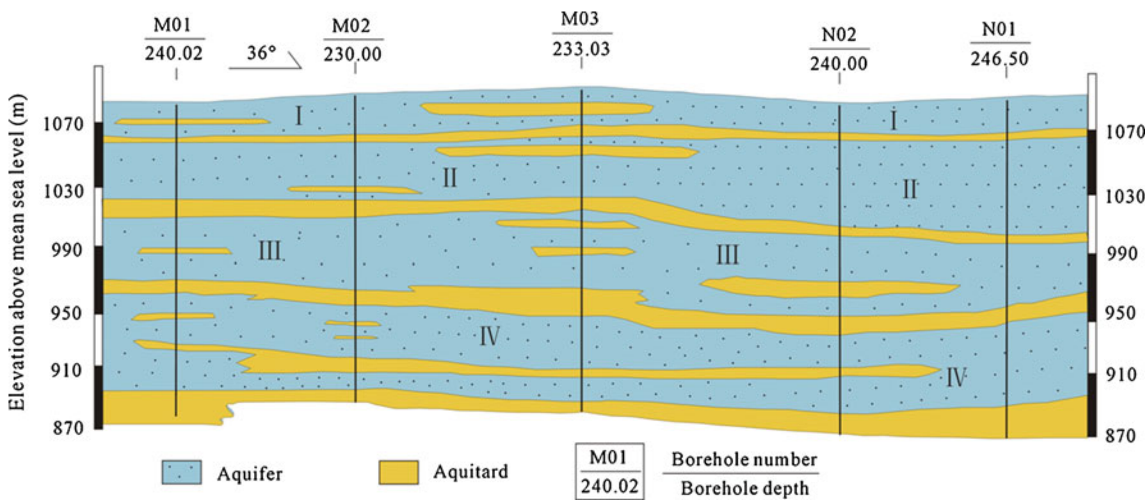

Fig. 1 Hydrogeologic cross-section of the study area (SW–NE direction)

second confined aquifer (the third water-bearing group, represented by III, within the depth of 80–150 m), the third confined aquifer (the fourth water-bearing group, represented by IV, about 150–240 m; Fig. 1), and the fourth confined aquifer (the fifth water-bearing group, represented by V, deeper than 240 m; Zhang et al. 2010). Because of the poor water quality in groups I and II and inconvenience of exploration in group V, only groups III and IV are proper for water supply purpose.

Pumping tests

The two pumping tests were separately conducted in a transient flow field. Each test was carried out with one pumping well and an observation well. The well distributions were shown in Fig. 2. N07 and N08 are pumping wells and O1 and O2 are observation wells, respectively. All four wells are fully penetrating wells, and N07 and O1 are 150 m, and N08 and O2 are both 240 m in depth measured from the ground surface. The first set (set A) of the pumping tests, of which the pumping period lasted 168 h till the drawdown reached stable, and the recovery period lasted 168 h after pump stopping, was designed to determine the parameters of group III, and the second set (set B) was conducted for determining the parameters of group IV. The pumping period of set B continued 168 h, and the recovery period lasted 108 h.

Set A was carried out in December 2009. Groundwater was pumped from N07 at a rate of 3,447.86 m^3/day and the maximum drawdown observed at O1 is 6.32 m. The mean aquifer thickness observed at O1 and N07 is 50.13 m. Set B was performed in January 2010 and the pumping rate at N08 was 3,059.57 m^3/day. The stable drawdown observed at O2 is 3.65 m. The mean aquifer thickness for group IV observed at N08 and O2 is 68.61 m. The thicknesses of aquifers are used to estimate the hydraulic conductivities (*K*). The geometry of the pumping and observation wells is shown in Fig. 3.

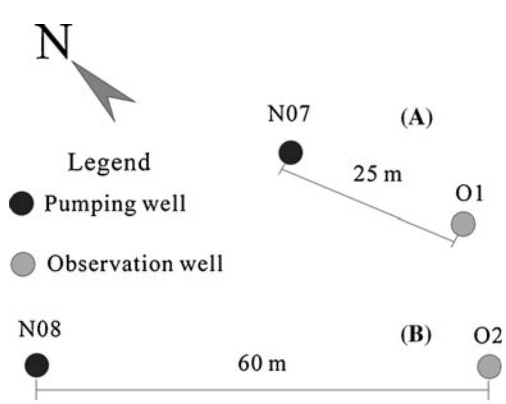

Fig. 2 Sketch maps of locations of the two sets of wells; **a** set A, **b** set B

Results and discussion

Under natural conditions, the groundwater levels of these aquifers are assumed to be the same and no leakage will occur. When pumping is conducted in group III, downward leakage will occur from groups II to III and upward leakage will also occur from groups IV to III. When pumping is carried out in group IV, similar situation will occur, that is downward leakage from groups III to IV and upward leakage from groups V to IV. The calculation will be rather complex if the hydraulic conductivities of the upper and lower aquitards are different. Therefore, in the present study,

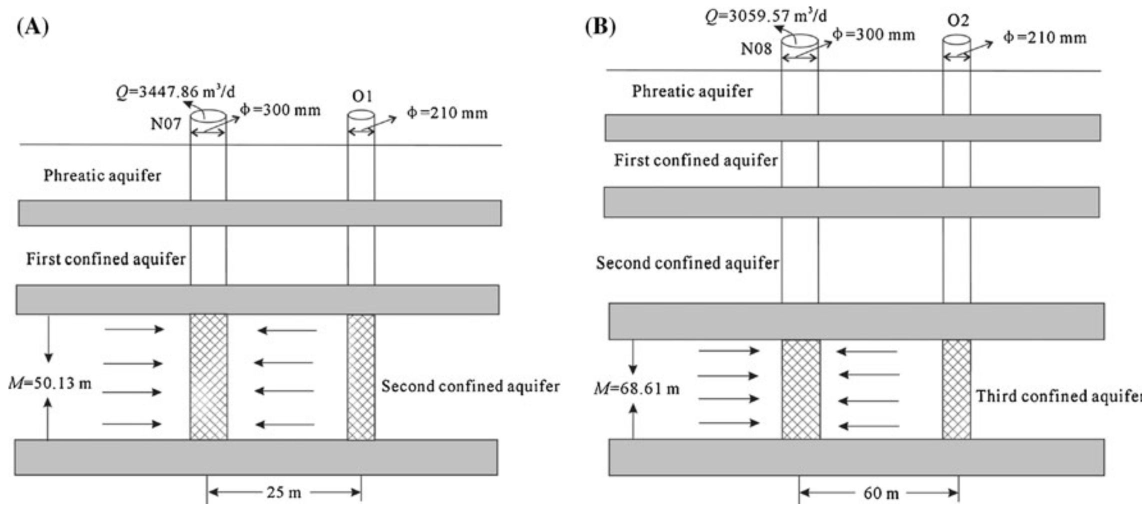

Fig. 3 Geometry of the pumping and observation wells; **a** set A, **b** set B

the hydraulic conductivities of the aquitards were considered to be equal to each other to reduce the complexity.

According to the hydrogeological properties of the aquifers in the study area, the groundwater systems are conceptualized as leaky aquifers and the parameters can be estimated using above introduced GCFM. The calculated parameters were listed in Table 1. In order to verify the accuracy of the parameters estimated by GCFM, they were also estimated using traditional type curve-fitting method (TCFM) based on Eq. (1) performed with Aquifer Test 3.0 designed by Waterloo Hydrogeologic Inc. (incorporated into Schlumberger Water Services since 2005). The results were also listed in Table 1 for comparison. The curve-matching curves for sets A and B using GCFM were shown in Figs. 4 and 5.

It can be seen from Table 1 that most of the parameters calculated by GCFM are a little bigger than but very close to those obtained by TCFM, which indicates the GCFM is logical for hydrogeological parameter identification. The results are accurate enough for aquifer structure interpretation and groundwater resources evaluation. As a matter of fact, the parameters obtained by GCFM may be more accurate than those calculated by TCFM because GCFM incorporates the pumping period data and water recovery period data and ensures the uniqueness of the parameter calculation. GCFM may be more reliable than TCFM because it does not require manual curve fitting but TCFM does. It can also be seen from Figs. 4 and 5 that the fitting is generally good between the testing curve and the theoretical curve, and the errors at most points are less than 0.1 and 0.05 m for tests A and B, respectively. One more advantage of GCFM is that some anomalous points caused by any incident during the pumping period can be omitted during the matching, which ensures the minimization of the errors.

There are generally three advantages of GCFM that can be summarized according to the above analysis:

- GCFM can calculate the desired hydrogeological parameters rapidly and accurately by computer and can avoid human impact on calculation.
- GCFM uses all the available data including the pumping and water recovery period data, which ensures the uniqueness of the parameter calculation.
- The anomalous points can be omitted, which ensures the square errors are minimal and the parameters are optimal.

However, more discussions are needed on the accuracy of different calculation methods, and this issue will be implemented in a separate paper.

Table 1 Parameter estimation results using GCFM and TCFM

Well no.	Method	T (m^2/day)	M (m)	K (m/day)	S	K' (m/day)
O1	GCFM	430.09	50.13	8.58	4.90×10^{-4}	7.05×10^{-3}
O1	TCFM	406	50.13	8.10	1.36×10^{-4}	4×10^{-3}
O2	GCFM	530.16	68.61	7.73	8.26×10^{-4}	1.79×10^{-3}
O2	TCFM	453	68.61	6.60	1.37×10^{-3}	2×10^{-4}

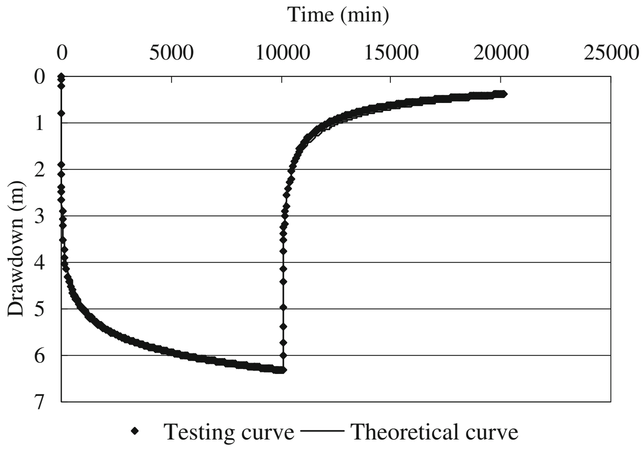

Fig. 4 Curve matching of set A using GCFM

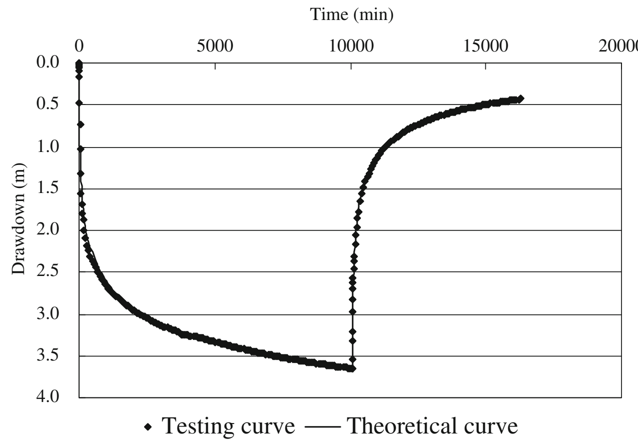

Fig. 5 Curve matching of set B using GCFM

Conclusions

A global curve-fitting method incorporating pumping test data with water recovery data was introduced in the study. The method is based on the analytical solution of transient flow to a well in leaky aquifer deduced by Hantush and Jacob, and the principal and procedures of the method were elucidated in detail. A case was studied to show the feasibility and reliability of the method in parameter identification and the results were compared with those obtained by the traditional type curve-fitting method. The following conclusions were reached:

- The method proposed in the paper is feasible and reliable in aquifer parameter identification. The results obtained by the proposed method are a little bigger than but very close to the values calculated by the traditional type curve-fitting method.
- Three advantages of the global curve-fitting method in parameter identification were summarized and these advantages make it a more promising and worthy of promoting method for aquifer parameter calculation than other traditional methods.
- A further comparative analysis on the accuracy of parameters calculated by different methods is expected. The comparative study will determine which method is optimal for parameter calculation. This issue will be discussed in future study.

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Appendix

Mathematically, a function of two variables expressing as $f(x', y')$ can be expanded at (x, y) using Taylor expansion:

$$\begin{aligned}
f(x',y') &= f(x,y) + \Delta x \frac{\partial f(x,y)}{\partial x} + \Delta y \frac{\partial f(x,y)}{\partial y} \\
&+ \frac{1}{2!}\left[(\Delta x)^2 \frac{\partial^2 f(x,y)}{\partial x^2} + 2\Delta x \Delta y \frac{\partial^2 f(x,y)}{\partial x \partial y} + (\Delta y)^2 \frac{\partial^2 f(x,y)}{\partial y^2}\right] \\
&+ \frac{1}{3!}\left[(\Delta x)^3 \frac{\partial^3 f(x,y)}{\partial x^3} + 3(\Delta x)^2 \Delta y \frac{\partial^3 f(x,y)}{\partial x^2 \partial y} + 3\Delta x (\Delta y)^2 \frac{\partial^3 f(x,y)}{\partial x \partial y^2} + (\Delta y)^3 \frac{\partial^3 f(x,y)}{\partial y^3}\right] + \cdots \\
&= f(x,y) + \frac{1}{1!}\left(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}\right) f(x,y) \\
&+ \frac{1}{2!}\left(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}\right)^2 f(x,y) + \frac{1}{3!}\left(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}\right)^3 f(x,y) + \cdots \\
&= \sum_{i=0}^{\infty} \frac{1}{i!}\left(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y}\right)^i f(x,y)
\end{aligned} \tag{A1}$$

Where, $x' = x + \Delta x, y' = y + \Delta y$

Usually, the higher order terms of the expansion are omitted for approximate uses. Thus,

$$f(x', y') \approx f(x, y) + \Delta x \frac{\partial f(x, y)}{\partial x} + \Delta y \frac{\partial f(x, y)}{\partial y} \tag{A2}$$

Similarly, a function of three variables can be approximately expressed with Taylor expansion as

$$f(x', y', z') \approx f(x, y, z) + \Delta x \frac{\partial f(x, y, z)}{\partial x} + \Delta y \frac{\partial f(x, y, z)}{\partial y} + \Delta z \frac{\partial f(x, y, z)}{\partial z} \tag{A3}$$

Where, $x' = x + \Delta x, y' = y + \Delta y, z' = z + \Delta z$

In the present study, $T = T_0 + \triangle T, S = S_0 + \triangle S, K' = K'_0 + \triangle K'$. Drawdown in a specific time is determined by T, S, and K'. Thus,

$$s(T, S, K') \approx s(T_0, S_0, K_0') + (\frac{\partial s}{\partial T})_0 \Delta T + (\frac{\partial s}{\partial S})_0 \Delta S + (\frac{\partial s}{\partial K'})_0 \Delta K' \tag{A4}$$

Where, $s(T_0, S_0, K_0')$ is the initial drawdown at (T_0, S_0, K'_0), $(\frac{\partial s}{\partial T})_0$, $(\frac{\partial s}{\partial S})_0$ and $(\frac{\partial s}{\partial K'})_0$ are respectively the partial derivatives of s with respect to the aquifer parameters T, S and K' at (T_0, S_0, K'_0). Then, the $s(T, S, K')$ is expressed by $s*$, and $s(T_0, S_0, K_0')$ is represented by s_0 thus,

$$s* \approx s_0 + (\frac{\partial s}{\partial T})_0 \Delta T + (\frac{\partial s}{\partial S})_0 \Delta S + (\frac{\partial s}{\partial K'})_0 \Delta K' \tag{A5}$$

Drawdown is also varies with time (t), and it is also a function of time (t), thus, drawdown at different time is:

$$s* \approx s_0(t) + (\frac{\partial s}{\partial T})_0 \Delta T + (\frac{\partial s}{\partial S})_0 \Delta S + (\frac{\partial s}{\partial K'})_0 \Delta K' \tag{A6}$$

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