

# Application of concentration–number (C–N) multifractal modeling for geochemical anomaly separation in Haftcheshmeh porphyry system, NW Iran

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**Abstract** Geochemical anomaly separation using the concentration–number (C–N) method at the Haftcheshmeh porphyry system in NW Iran is the aim of this study. We used litho-geochemical data sets to explore Cu, Mo, Au and Re mineralization in gabbroic, dioritic and monzonitic units at the Haftcheshmeh Cu–Mo porphyry system. The obtained results were interpreted using a rather extensive set of information available for each mineralized area, consisting of detailed geological mapping, structural interpretation and alteration data. Threshold values of elemental anomalies for the mineralized zone were computed and compared with the statistical methods based on the data obtained from chemical analyses of samples for the lithological units. Several anomalies at local scale were identified for Cu (40 ppm), Mo (12 ppm), Au (79 ppb), and Re (0.02 ppm), and the results suggest the existence of local Cu anomalies whose magnitude generally is above 500 ppm. The log–log plots show the existence of three stages of Cu and Mo enrichment, and two enrichment stages for Au and Re. The third and most important mineralization event is responsible for presence of Cu at grades above 159 ppm. The identified anomalies in Haftcheshmeh porphyry system, and distribution

of the rock types, are mainly gabbrodiorite–monzodiorite, granodiorite and monzodiorite–diorite that have special correlation with Cu–Mo and gabbroic and monzonitic rocks, especially the gabbrodiorite–monzodiorite type, which is of considerable importance. The study shows that these elemental anomalous parts have been concentrated dominantly by potassic and phyllic, argillic and propylitic alterations within the gabbroic, monzonitic and dioritic rocks especially in the gabbrodioritic type in certain parts of the area. The results, which were compared with fault distribution patterns, revealed a positive correlation between mineralization in anomalous areas and the faults present in the mineralized system.

**Keywords** Geochemical anomaly separation · Porphyry system · Concentration–number (C–N) method · Multifractal · Haftcheshmeh · NW Iran

## Introduction

Recognition of anomalies and background is the most important aim of geochemical exploration studies. Traditional methods based on classical statistics are used for separation of geochemical anomalies from the background and identification of elemental threshold values (Hawkes and Webb 1979; Li et al. 2003). In these methods, a geochemical threshold value is defined as the sum of mean and 1.5 to 3 standard deviations if the frequency distribution of concentrations is normal. However, the frequency distribution of many elements is not always normal in many cases (Ahrens 1954; Li et al. 2003). Also, spatial distribution of geochemical data is important for separation of geochemical populations, but it is not taken into consideration for statistical methods. Additionally, these methods, which are based on such quantities as mean and standard deviation, cannot identify the anomalies with high

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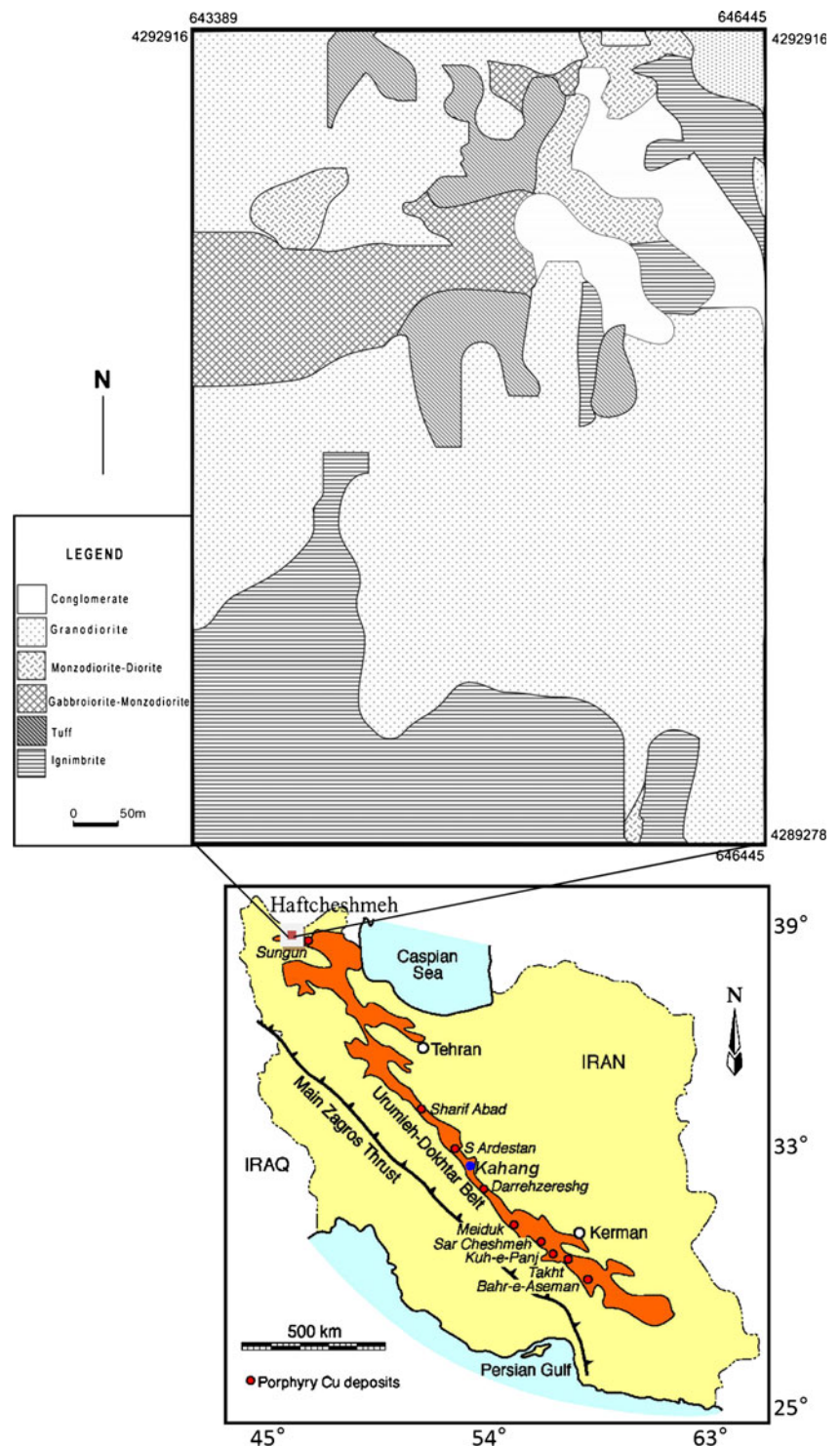
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value of background or even miss weak anomalies in areas with known mineral deposits, because the difference between the background and anomaly in such cases has been investigated in previous studies (Bai et al. 2010).

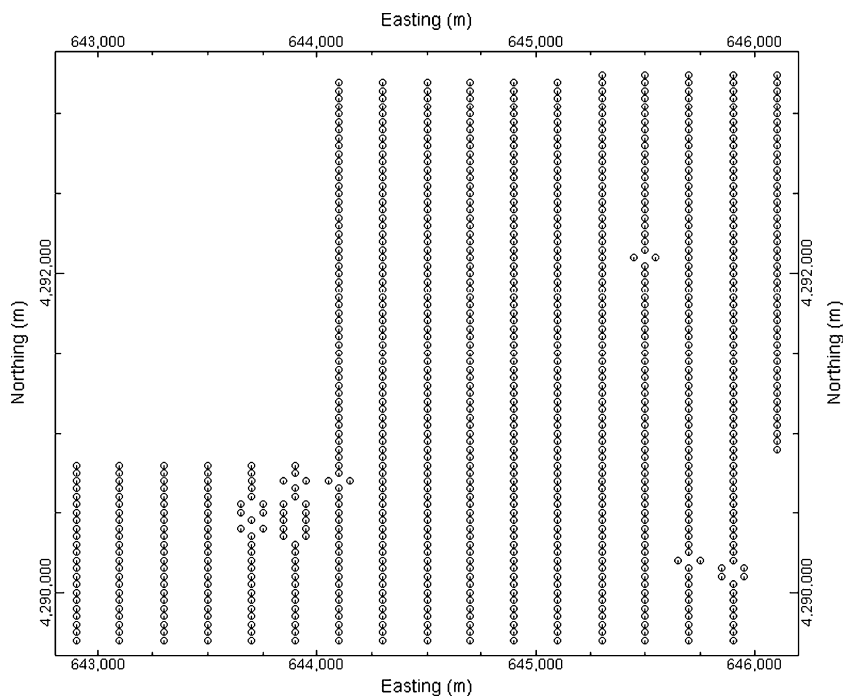
Fractal theory has been introduced by Mandelbrot (1983) as an important non-Euclidean method in geometry. Several

methods have been established based on fractal geometry for application in geosciences since the 1980s (Sanderson et al. 1994; Turcotte 1997, 2002). Problems encountered when using the classical statistics method can be solved by using fractal methods, especially in geochemical exploration. Cheng et al. (1994) and Cheng (1995) proposed concentration–area

**Fig. 1** (Above) Simplified geology map of Haftcheshmeh district area within Arasbaran magmatic belt (Hasanpour, 2010) and (below) the known porphyry deposits in Iran (Shahabpour, 1994 and Afzal et al., 2010). Red part indicates the Urmia–Dokhtar magmatic belt that hosts a lot of porphyry copper deposits



**Fig. 2** Lithochemical samples location map of the Haftcheshmeh area



(C–A) and concentration–perimeter (C–P) methods for separation of geochemical anomalies from background and calculation of elemental thresholds values. Additionally, other fractal methods were developed and applied in geochemical exploration such as power spectrum–area (S–A) by Cheng (1999), concentration–distance (C–D) by Li et al. (2003), and concentration–number (C–N) by Mao et al. (2004).

In this study, the C–N fractal method was applied for the separation of geochemical populations (anomalies and background) based on lithochemical data (Cu, Mo, Re and Au) from the Haftcheshmeh Cu–Mo porphyry system, NW Iran. Also, it is debated whether the anomalous threshold values are correlated to the relevant structural, lithological, and alteration data.

**Concentration–number fractal model**

Mao et al. (2004) proposed the elemental concentration–number (C–N) method, as a branch of C–A model, which is

used to define the geochemical background and anomaly threshold values. The model has the general form:

$$N(\geq \rho) \propto \rho^{-\beta} \tag{1}$$

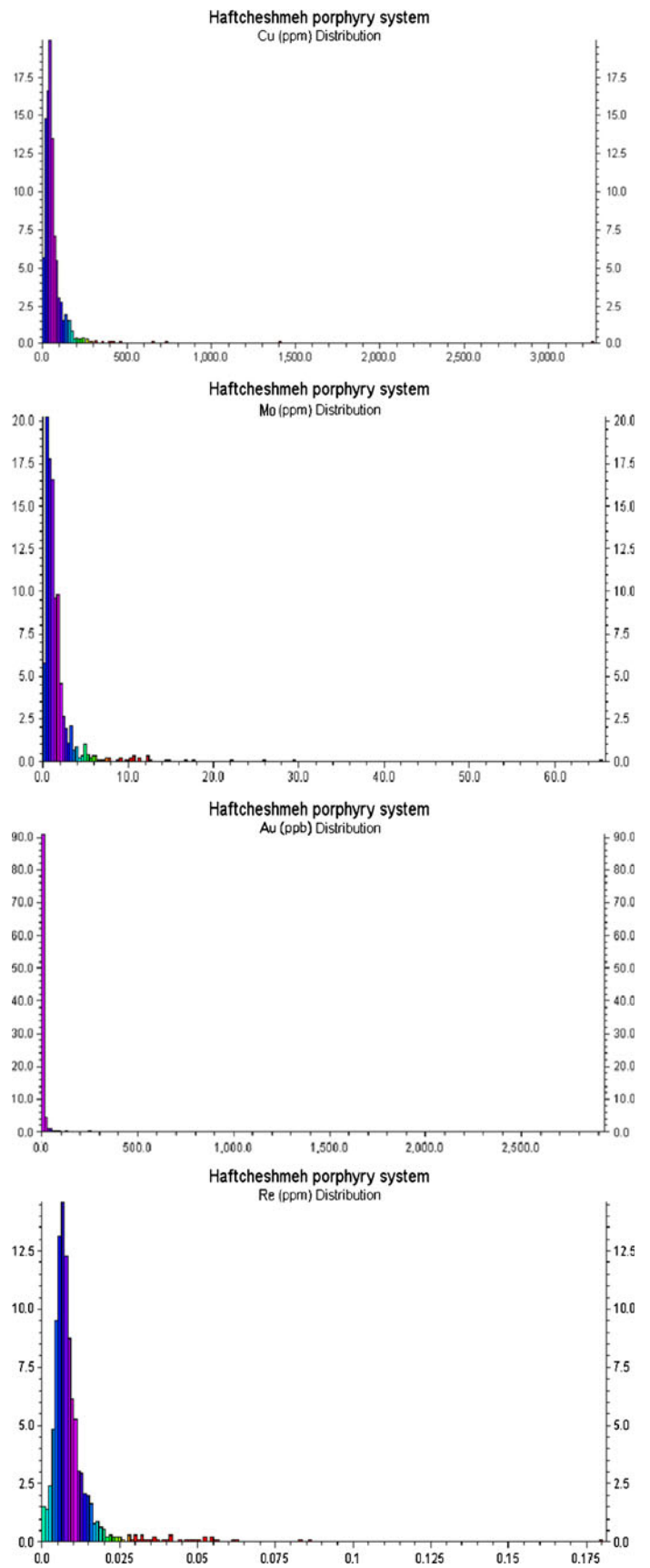
where  $N(\geq \rho)$  denotes the sample number with concentration values greater than the  $\rho$  value.  $\rho$  is concentration of element and  $\beta$  is the fractal dimension. In this method, geochemical data has not undergone pre-treatment and evaluation (Mao et al. 2004).

Fractal methods can illustrate the relationship between the obtained results and the geological, geochemical and mineralogical information (Goncalves et al. 1998; Cheng 1999; Sim et al. 1999; Goncalves et al. 2001; Li et al. 2003; Carranza 2009). Geological differences can be shown in log–log plots resulting from fractal methods, e.g., C–A, C–P and C–N. The breaks between straight-line segments on the plots and the corresponding values of elemental concentration have been used as thresholds to separate geochemical values into different components, representing different causal factors,

**Table 1** Statistical parameters of raw data based on lithochemical sample analysis

Statistical parameter	Cu (ppm)	Mo (ppm)	Au (ppb)	Re (ppm)
Mean	67.00	2.00	15.3	0.01
Median	47.40	1.10	2	0.008
Maximum	3,260.00	65.40	2910	0.18
Minimum	3.90	0.00	0.00	0.00

**Fig. 3** Histogram of Cu, Mo, Au and Re elements



such as lithological differences and geochemical processes (Lima et al. 2003; Wang et al. 2008; Afzal et al. 2010).

Multifractal theory can be interpreted as a theoretical structure that explains the power-law relationships between number of concentrations below a given value and the actual concentrations itself. To reveal and prove that data distribution

has a multifractal nature requires a rather extensive computation. The C–N method seems to be equally applicable as well to all cases, which is probably rooted in the fact that geochemical distributions mostly satisfy the properties of a multifractal function. There exists some evidence that geochemical distributions are fractal in nature and behavior, at least

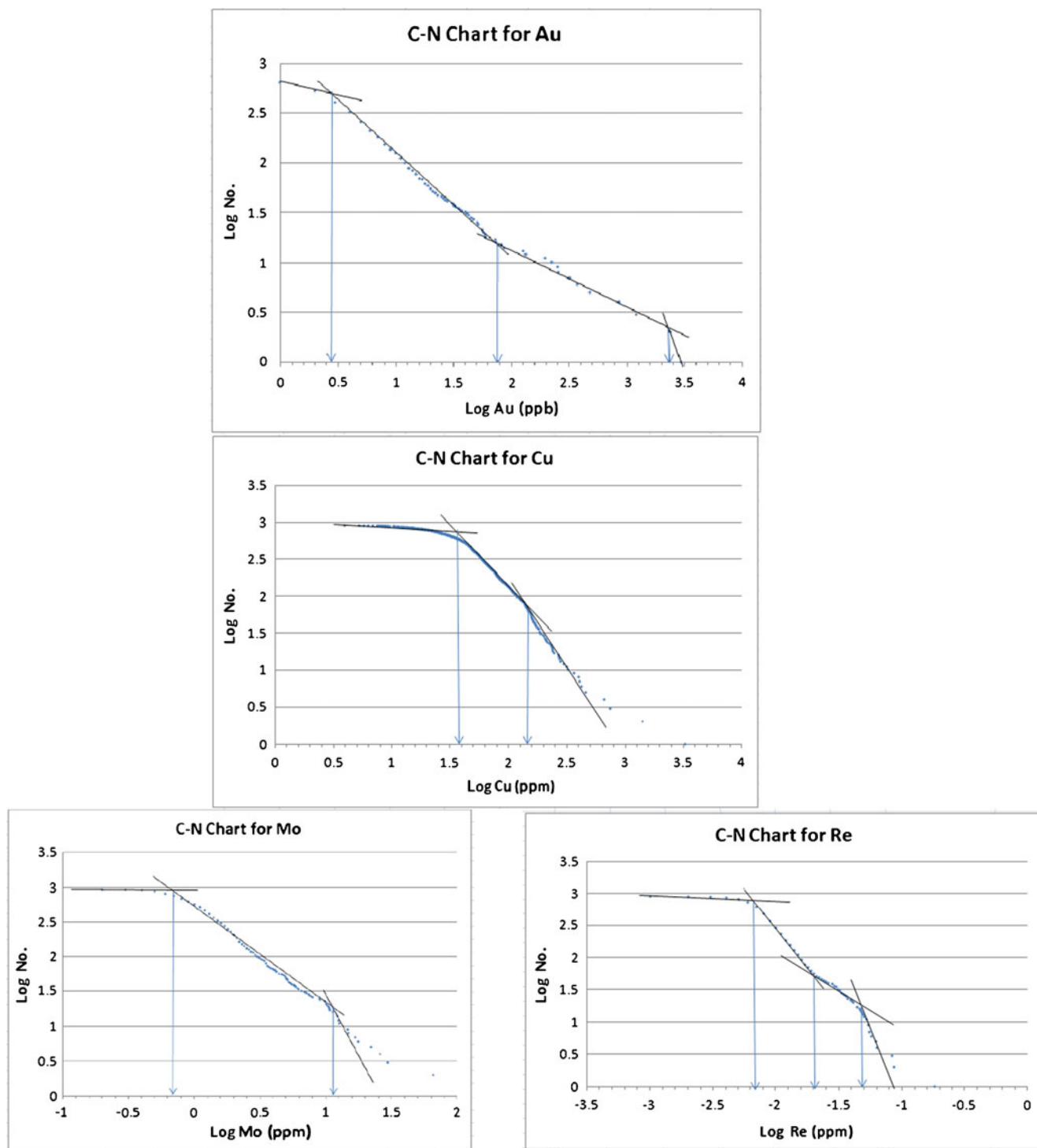
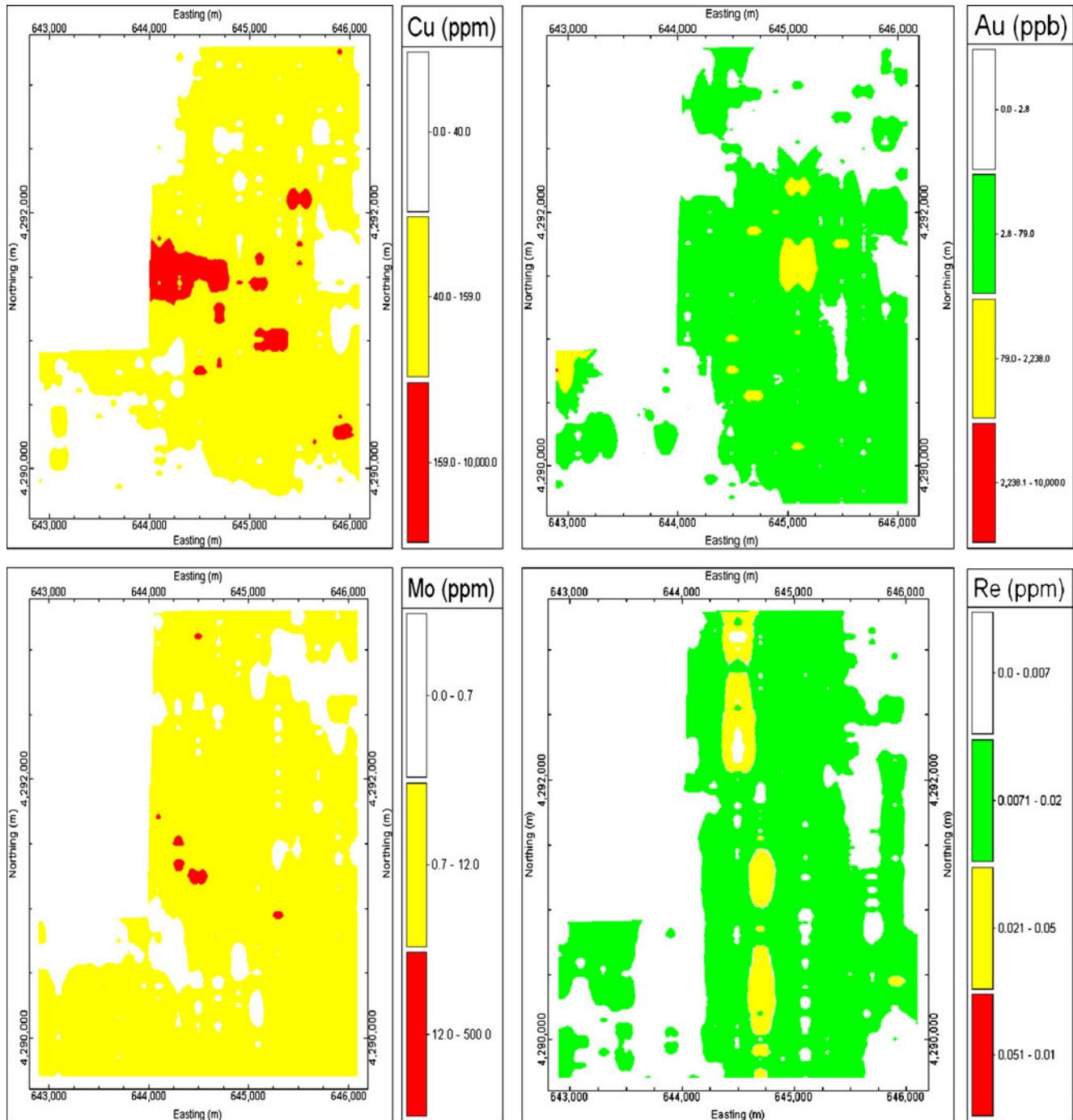


Fig. 4 Log–log plots from C–N method for Cu, Mo, Au and Re

empirically according to Bolviken et al. (1992). Some approaches seem to support the idea that geochemical data distributions are multifractal, although this point is far from being demonstrated (Cheng and Agterberg 1996; Turcotte 1997; Goncalves 2001; Wang et al. 2008; Afzal et al. 2010). This idea may provide and help the development of an alternative interpretation validation and useful methods to be applied to elemental geochemical distributions analysis.

### Geological setting of the Haftcheshmeh porphyry system

The Haftcheshmeh area is situated about 37 km NE of Varzaghan in NW Iran. This area is located in SE of Arasbaran Cenozoic magmatic belt named the second main Iranian copper belt (after Urmia–Dokhtar magmatic belt) with a NW to SE trend. Northwest of Iran, large porphyry copper deposits such as Sungun, Meiduk and Kighal are



**Fig. 5** Cu, Mo, Au and Re geochemical population distribution maps based on the C–N method

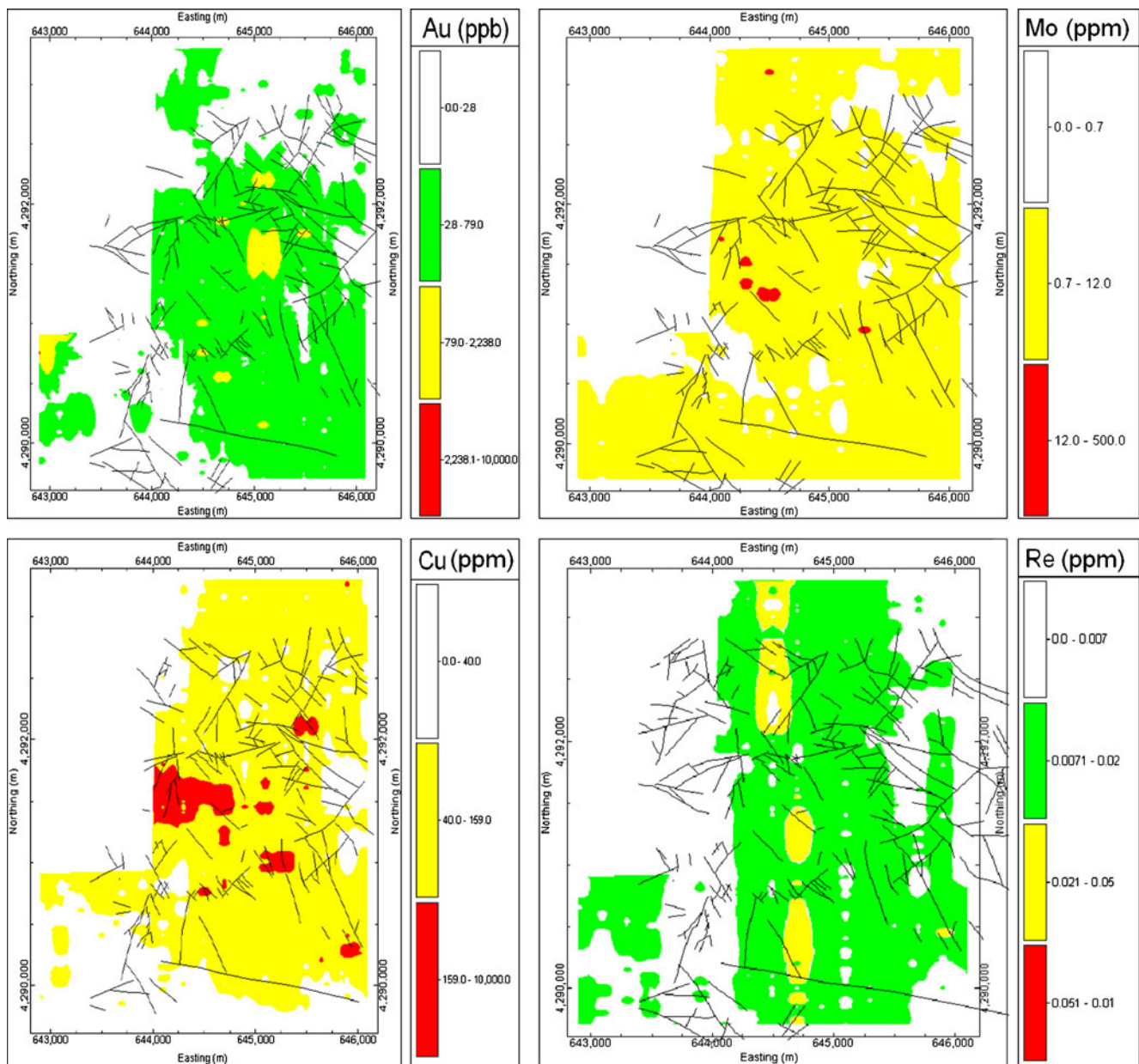
located on this belt, as depicted in Fig. 1. Haftcheshmeh mineralized area is recognized as a Cu–Mo porphyry system (Shahabpour 1994; Hasanpour 2010).

The Haftcheshmeh area is located in a great plutonic unit that has intruded into upper Cretaceous–Paleocene volcanosedimentary–pyroclastic rocks. Then, Miocene dioritic unit intruded the Oligocene gabbroic intrusion, specifically in the western part of the area. Also, there are many NW–SE micro-dioritic and andesitic dykes. The main alteration zones of potassic, phyllic, argillic and propylitic types were established based on the Lowell and Guilbert (1970) model. The ore minerals concluded chalcopyrite, chalcocite, covellite, bornite, pyrite and molybdenite. Also, quartz veins and

veinlets are found to dominantly occur in this area. The main faults systems have NE–SW and NW–SE trends.

### Lithochemical exploration

A total of 914 lithochemical samples were collected from the area and analyzed using the inductively coupled plasma-mass spectrometry (ICP-MS) method for elements which are related to Cu, Mo, Au and Re mineralization. The location map of the samples is shown in Fig. 2. Statistical results illustrate that Cu, Mo, Au and Re mean values are equal to 67 ppm, 2 ppm, 15.3 ppb and 0.01 ppm,

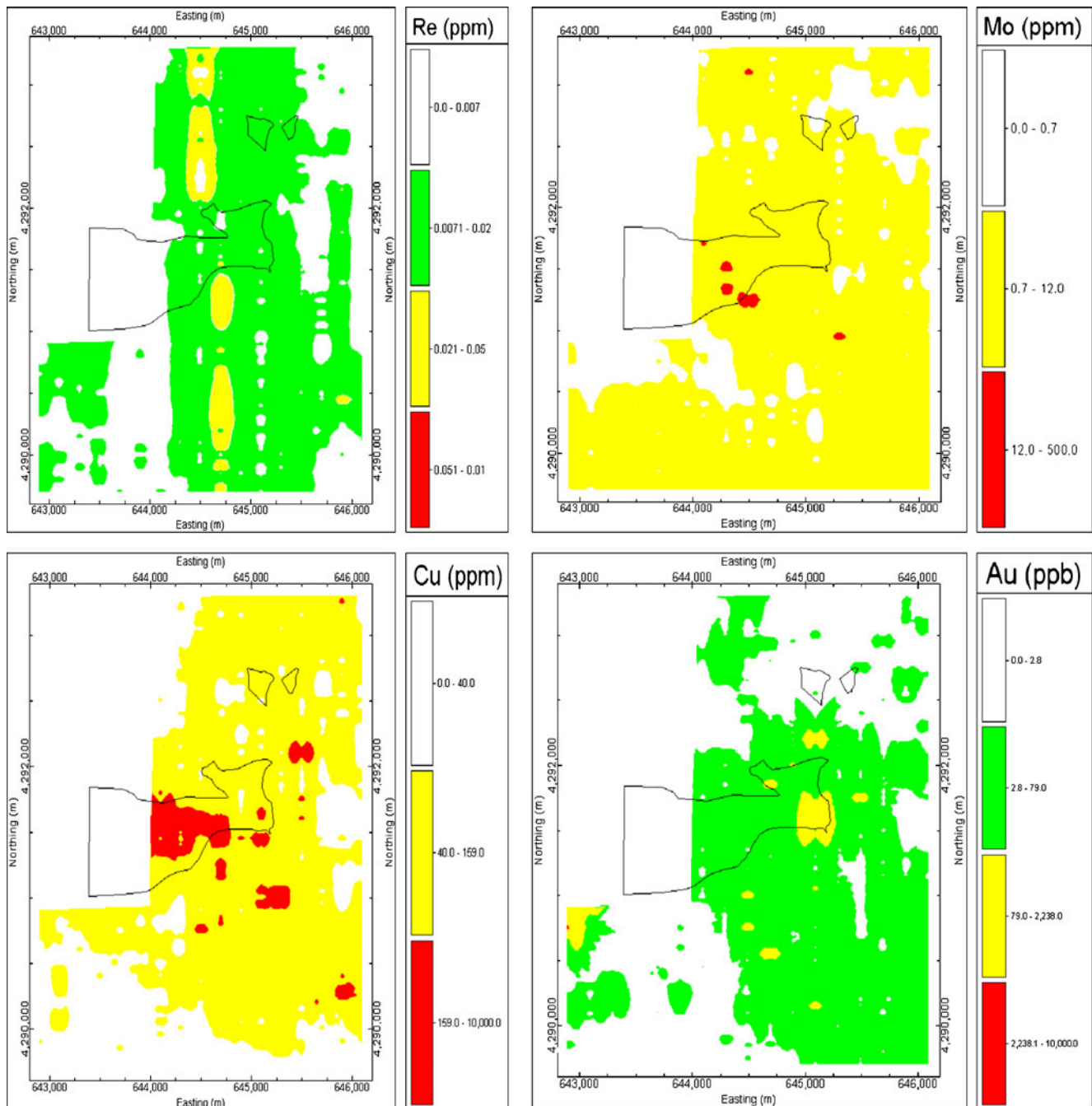


**Fig. 6** Cu, Mo, Au and Re geochemical population distribution maps based on the C–N method imposed on fault location maps (*black lines*)

respectively (see Table 1). The elemental distributions in their histograms are not normal, as shown in Fig. 3. Variations between maximum and minimum for these data reveal a wide range. If the median value is assumed to be equal to threshold values, the existing statistical results are 47.4 ppm for Cu, 1.1 ppm for Mo, 2 ppb for Au and 0.008 ppm for Re.

The analyzed grades in all samples were sorted based on decreasing grades and cumulative numbers were calculated for grades. Finally, log–log plots were generated for Cu, Mo, Au

and Re (Fig. 4). Break points between straight-line segments in those log–log plots show threshold values separating populations of geochemical concentration values representing geological differences due to distinct geochemical processes. Elemental geochemical populations are demonstrated in these log–log plots. On the basis of this process, there are three populations for Cu and Mo and four populations for Au and Re, respectively, as depicted in Fig. 4. The Cu anomalous threshold is 40 ppm and its higher intensity anomaly is 159 ppm. Also, there are three stages of Cu enrichments



**Fig. 7** Relationship between elemental distribution and gabbro-diorite-monzodiorite (*polygons*)



based on the log–log plot (Fig. 4). The first event for Cu C–N variations occurred at grades below 40 ppm. The second event shows up between grades 40 ppm and 159 ppm. The final event included major Cu mineralization which occurred and was interpreted in grades higher than 159 ppm. Mo threshold and higher intensity anomalies are 0.7 and 12 ppm, respectively (Fig. 4). The Mo log–log plot shows that a major Mo enrichment occurred at 12 ppm and higher. The Au anomalous threshold is about 3 ppb. There are two enrichment steps interpreted, as shown in the C–N log–log plot of Au (Fig. 4). Major Au enrichment started from 79 ppb, and 2,238 ppb concentration is the beginning of the high-intensity Au anomaly. The C–N log–log plot for Re

illustrates that two enrichment steps can exist in this area. Re threshold is equal to 0.007 ppm, and major Re enrichment begins from 0.02 ppm. Additionally, 0.05 ppm concentration marks the start of the high-intensity Re anomaly.

Each geochemical population in this study was assumed to have various kinds of distributions, and its various components, such as individual chemical elements and their concentrations, could be fitted into a straight line on log–log plot. Obviously, due to the non-uniform behavior of the elements, if plotted on log–log coordinates, the plot will have different slopes and various straight-line segments which connect their points at an angle or with breaks on the plot.

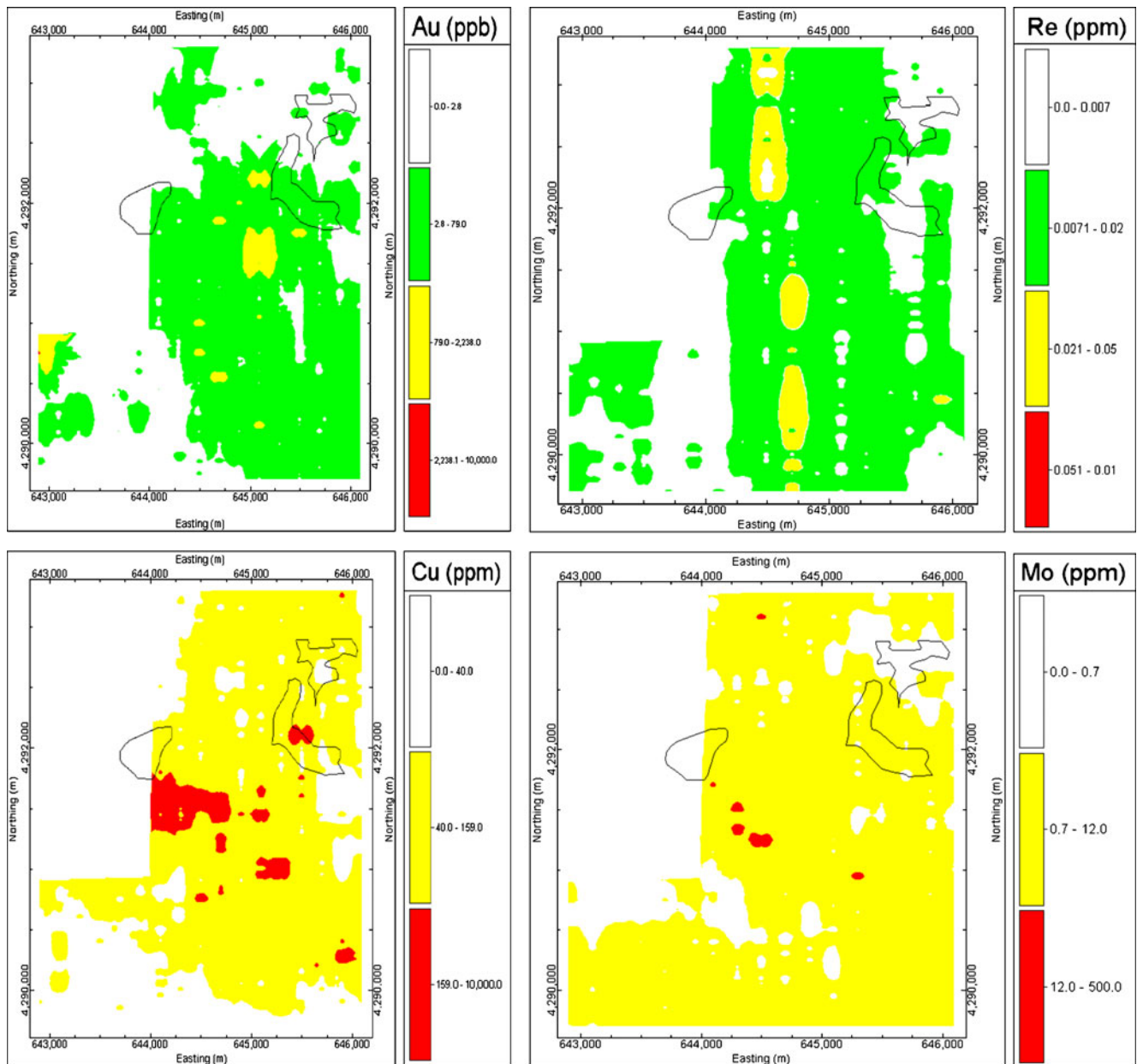


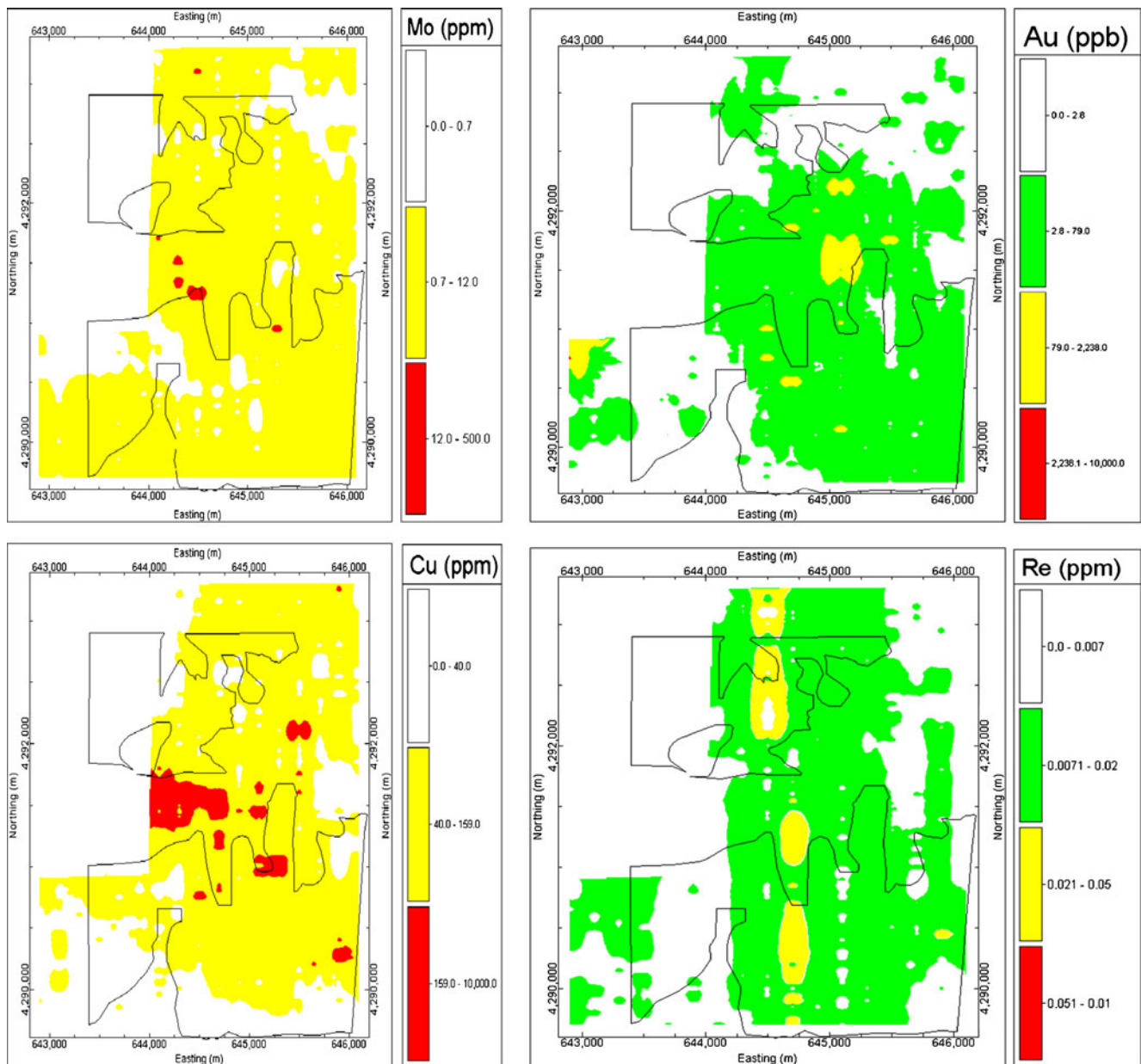
Fig. 8 Relationship between elemental distribution and monzodiorite–diorite (*polygons*)

Geochemical maps of the elements were generated using the RockWorks™ v. 15 software package. Based on the results of the C–N method, elemental grade distribution maps were drawn (Fig. 5). Most of the Cu anomalies are located in the central parts of the area, especially high-intensity Cu anomalies (over 500 ppm). Few parts of these anomalies were also located in the SE, NE and eastern parts of the studied area. High-intensity Mo anomalies (More 12 ppm) were situated in the central and eastern parts, but they are small anomalies. Au anomalies are located in the SW and central parts of the area, and very small high-intensity anomalies are located in the western part in the Haftcheshmeh area. Low intensive anomalies of Re (between 0.02 and

0.05 ppm) are located in the central, northern and southern parts of the area. No high-intensity Re anomalies are present (over 0.05 ppm). Based on these maps, the potential presence of these elements are located in the central and eastern parts as presented in Fig. 5. Also, a small but high-intensity Au anomaly is determined on the SW part of the area (see Fig. 5).

#### Comparison of C–N method results with geological information

The resulting threshold values obtained from the C–N method are compared and correlated to specific geological

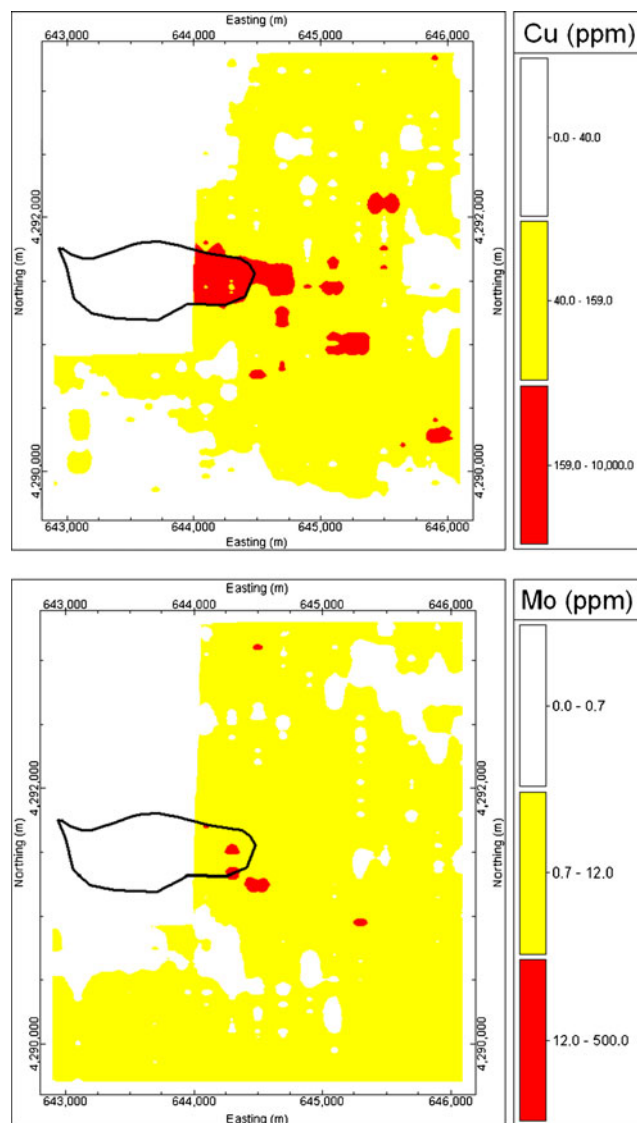


**Fig. 9** Relationship between elemental distribution and granodioritic units (*polygons*)

characteristics of the region including considering the nature of lithological units, faults and alterations. Distributions of Cu, Mo, Au and Re elements in the Haftcheshmeh area, and the faults map are presented in Fig. 6. The anomalous parts clearly indicates the main identified faults especially in the central and eastern parts of the area, which fit well with existing structural settings and controls as indicated in Fig. 6. Comparison between positions of faults and Cu, Mo, Au and Re anomalies, reveals that faults intersect the anomalies situated near those structures as depicted in Fig. 6. On the other hand, faults and elemental anomalies have a proportional relationship. High-grade elemental anomalies occur inside and within the fault zones or located on faults intersection areas (Fig. 6). It can be deduced that fault density has a direct positive correlation with mineralization especially in the western parts of the studied area.

Three main different rock types — gabbrodiorite–monzodiorite, monzodiorite–diorite and granodiorite — are present in the area. Correspondence between rock types and elemental distribution from C–N method shows that gabbrodiorite–monzodiorite rocks have a relationship with Cu and Au anomalies, especially in the eastern and central parts of the Haftcheshmeh area (Fig. 7). Monzodiorite–diorite units are situated in NE part of the area and correlated with low-intensity elemental anomalous parts, but a small part of this rock type is correlated within the high-intensity Cu anomaly (over 159 ppm), as depicted in Fig. 8. Granodioritic units correlated with low intensive elemental anomalous in the central part of the area, e.g., between 0.7 and 12 ppm for Mo (Fig. 9). These show a good relationship between element distributions with intrusion units.

Alteration zones have a strong positive relationship with Cu, Mo, Au and Re anomalies. All of the anomalous parts are covered by phyllic, argillic and specifically potassic alterations. Potassic alteration is associated with high intensive Cu and Mo anomalies, especially Cu anomalies as shown in Fig. 10. Cu concentrations higher than 500 ppm and Mo concentrations higher than 50 ppm have anomalies in the central part of the area with covering by potassic alteration zones. Low-intensity anomalous parts of Au (between 2.8 and 79 ppb) and Re (0.0071–0.02 ppm) are correlated within the potassic alteration zone. Also, phyllic alterations correlate with Cu (40–160 ppm) Mo (0.7–12 ppm) and Re (0.0071–0.02 ppm) low intensive anomalies in the central parts as shown in Fig. 11. A small part of high-intensity Cu anomaly, higher than 159 ppm, in the eastern part of the area is covered by phyllic alteration. However, the alteration zone is correlated with the higher intensity of Au anomalies (over 1,000 ppb) in the central part of the area. Argillic alteration zone covers small parts of eastern and southern Cu anomalies, where Cu grade is 500 ppm and more. This alteration is related to the higher intensity Re anomalies in the southern portion of the area, higher than 0.021 ppm and Mo low intensive anomalies

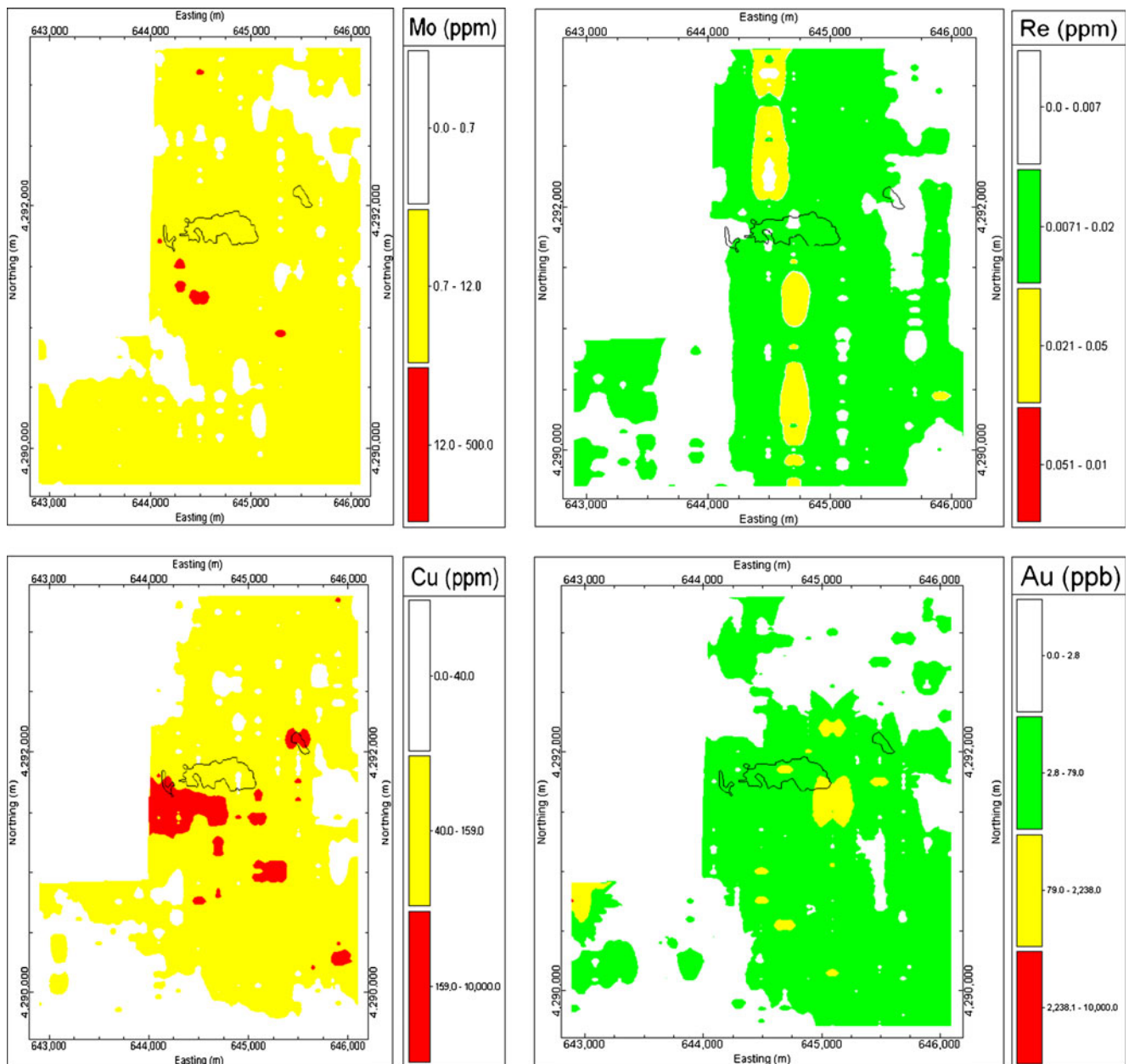


**Fig. 10** Relationship between Cu and Mo anomalies and potassic alteration zone (*black polygons*)

between 0.7 and 12 ppm, as shown in Fig. 12. Au anomalies in the central part, between 79 and 2,338 ppb, of the area have a rather good relationship with argillic alteration zones (Fig. 12). Propylitic alterations zones are situated in the eastern, northern and southern parts of the area and correlated with Cu content of higher than 500 ppm, Mo concentration higher than 10 ppm, Au higher than 2,000 ppb and Re higher than 0.05 ppm — constituting the high intensive anomalies in the eastern and southern parts of Haftcheshmeh area, as depicted in Fig. 13.

## Conclusions and future challenges

The study conducted on the Haftcheshmeh Cu–Mo porphyry system presents the potential use of the C–N method for



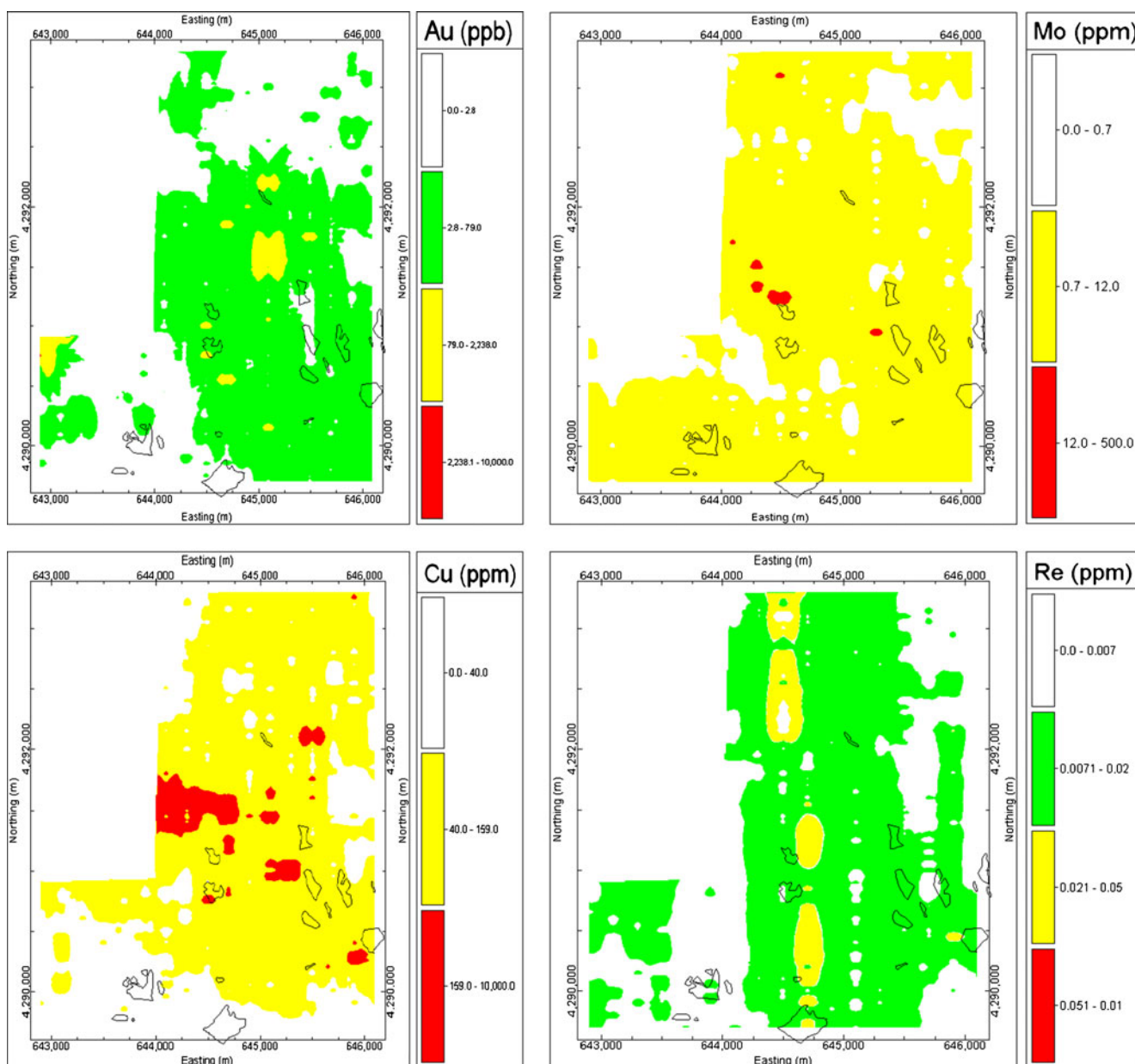
**Fig. 11** Relationship between elemental anomalies and phyllic alteration zone (*black polygons*)

geochemical anomaly separation as a useful tool for geochemical exploration. The advantages of this method rely essentially on its simplicity, and easy computational implementation, as well as the possibility of computing a numerical value of concentrations, i.e., the anomalous threshold, which is the most useful criteria for cross-examination of information with numerical data from different sources, commonly used in litho-geochemistry studies.

Interpretation of C–N log–log plots of Cu, Mo, Au and Re shows that Cu and Mo concentrations in the area may be a result of the three steps of enrichment, i.e., mineralization and later dispersions. Also, there are may be four steps of

enrichment, i.e., mineralization and later dispersions for Au and Re. Major Cu and Mo mineralization occurred by the intrusion of Oligocene gabbrodiorite–monzodiorite bodies in this area.

The resulting elemental anomalies have direct relationships with the faults in the Haftcheshmeh area. High-intensity element anomalies are mostly situated at fault intersections, especially in the central parts of the area. Also, there is a good correlation between potassic, phyllic, argillic and propylitic alterations and anomalous concentration of Cu, Mo, Au and Re. Potassic alteration zones have a good relationships with high grade anomalous elemental enrichment parts in the central part of the area, specifically for Cu and Mo elements.

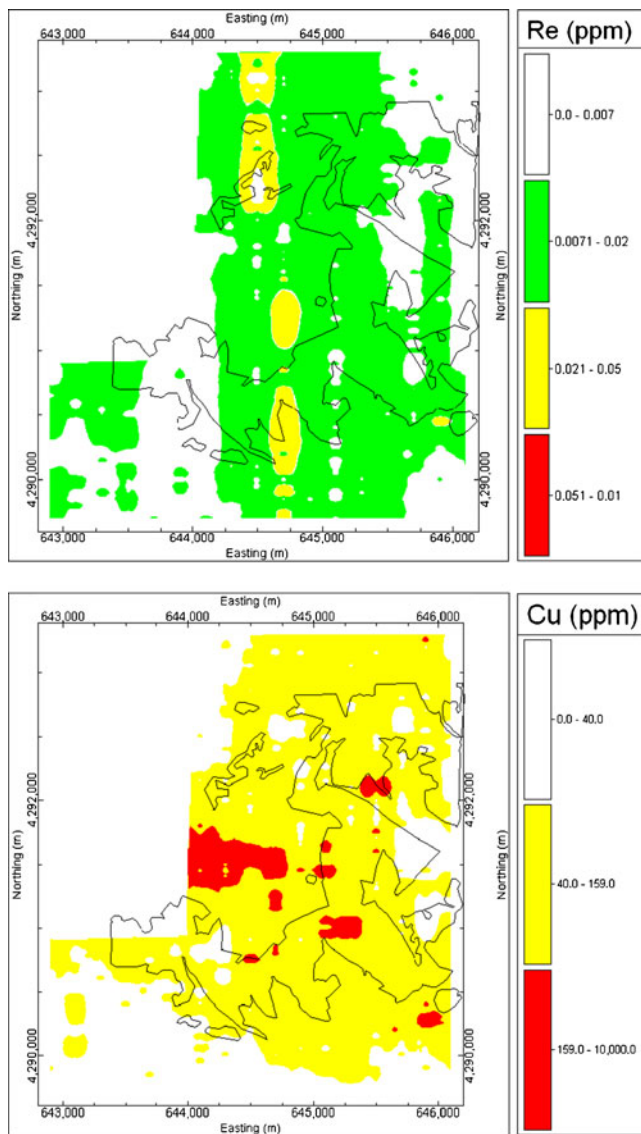


**Fig. 12** Relationship between elemental anomalies and argillic alteration zone (*black polygons*)

Furthermore, geological evidence including lithological units, alteration zones and position of faults proved that accurate results can be obtained using the C–N method. The richest parts of these elements correlated direction to tectonics, and many of other particulars of these rocks are situated at fault intersections. The enrichment of Cu, Mo, Au and Re element processes is correlated with the lithological units and intrusion emplacement. Gabbroic, dioritic and monzonitic rock types especially gabbrodiorite–monzodiorites were the last intrusion emplacements in this area, and these units host high-grade Cu and Mo anomalies. Re anomalies in the southern and northern parts of the area are located in gabbrodiorite–monzodiorites units. Monzodiorite–dioritic units host low-grade of Cu–Mo,

Au and Re mineralization in the NE part of the area. Granodioritic units exist in the central parts of the area and host low-intensity elemental anomalies.

Although it may be easier to study the geochemical anomalies with the C–N method, the multifractal nature of C–N log–log curves could be of essential help to geoscientists for interpreting the stages at which an element is enriched. The developments in multifractal theory and their usage could provide a favorable ground for the stochastic simulation of geochemical distributions, and their understanding and interpretations. We propose to continue our litho-geochemical studies in the west and northwestern of the area to estimate the anomaly trends especially with regard to Cu, Mo and Au.



**Fig. 13** Relationship between Cu and Re anomalies and propylitic alteration zone (black polygons)

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