

# Fractal analysis revisited by the continuous wavelet transform of AVO seismic data

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**Abstract** The main goal of this paper is to establish reservoirs media heterogeneities by the wavelet transform modulus maxima lines. First, we gathered amplitude versus offsets (AVO) amplitudes at the top of the reservoirs, then we calculated the 2D wavelet transform after we calculated its maxima, and we estimated the Hölder exponent at each maxima. Variation of the Hölder exponent can give more information about lithology and fluid nature at any point.

We applied the proposed idea at a 2D synthetic AVO intercept model, obtained results showed that the wavelet transform modulus maxima lines can be used as a seismic image processing tool. We suggest application of the proposed idea on real AVO seismic data and its attributes. It can give more ideas about reservoirs model.

**Keywords** Heterogeneities · WTMM · AVO

## Introduction

The 1D wavelet transform modulus maxima lines WTMM is a multifractal analysis technique based on the summation

of the modulus of the continuous wavelet transform (CWT) on its maxima. The obtained function is used to estimate two spectrums, one is the spectrum of exponents and the other is the spectrum of singularities. The WTMM was used in various domains to resolve many scientific problems (Muller 1994; Ouadfeul 2006, 2007).

A generalized wavelet transform modulus maxima lines WTMM in the 2D domain are used by many researchers to establish physical problems (Kestener 2003; Ouadfeul 2010).

The wavelet transform has been applied in seismic image processing. Miao and Moon (1999) has published a paper on the analysis of seismic data using the wavelet transform. A New sparse representation of seismic data using adaptive easy-path wavelet transform has been developed by Jianwei et al. (2010).

The continuous wavelet transform, has been used for ground roll attenuation (Deighan and Watts 1997). It has been used by Pitas and Kotropoulos (1989) for texture analysis and segmentation of seismic images.

In this paper, we process the intercept attribute (Castagna and Backus 1993) of 3D synthetic seismic AVO data by the 2D WTMM to establish the problem of heterogeneities. It is very complex and need advanced processing tools to get more ideas about morphology of rocks.

## Brownian fractional motion and synthetic model

For  $H \in (0, 1)$ , a Gaussian process  $\{B_H(t)\}_{t \geq 0}$  is a fractional Brownian Motion if for all  $t, s \in \mathcal{R}$  it has (Peitgen and Saupe 1987; Arneodo et al. 2003):

- 1 A mean:  $E[B_H(t)] = 0$
- 2 A covariance:  $E[B_H(t)B_H(s)] = (1/2) \{|t|^{2H} + |s|^{2H} - |t-s|^{2H}\}$

$H$  is the Hurst exponent (Arneodo et al. 2003).

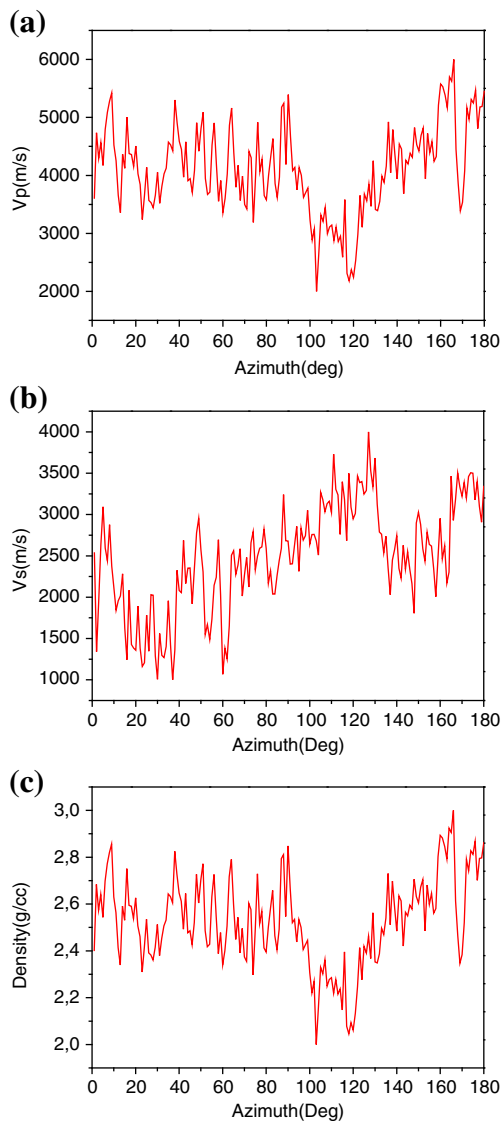
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◀ **Fig. 1** Physical parameters of a synthetic heterogenous layer generated randomly versus azimuth **a** velocity of the P wave; **b** velocity of the S wave; **c** density of the model

We suppose now that we have a geological model of two layers, the first is homogenous with the following physical parameters:

1.  $V_p \approx 3,500 \left(\frac{m}{s}\right)$
2. The density is calculated using the Gardner model (Gardner et al. 1974):  $\rho \approx 1.741 \times VP^{0.25} = 2.38 \left(\frac{g}{cc}\right)$
3. The velocity of the shear wave is estimated using Castagna Mud-rock line (Castagna et al. 1985)

$$V_s = 0.8621XV_p - 1,172.4 = 1,744.95(m/s)$$

The second is a heterogeneous model with the parameters detailed before.

We suppose that the velocity of the P wave, the velocity of the shear wave and the density of the synthetic model are a Brownian fractional motion model versus the azimuth  $\theta$ .  $0 \leq \theta < 180$

A generation of the three geological parameters is represented in Fig. 1a–c

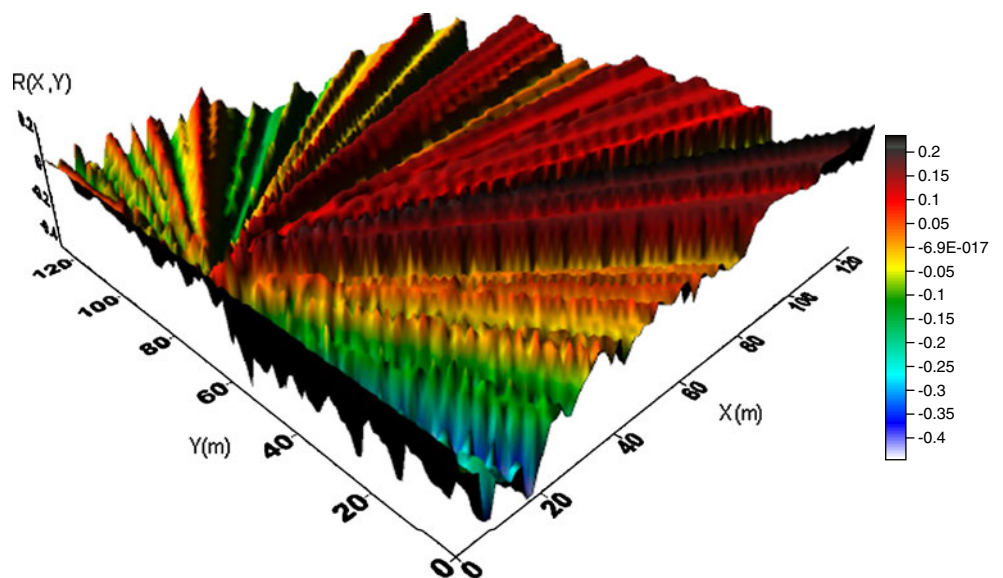
We suppose:

- $2,000 < V_p \leq 6,000$
- $1,000 \leq V_s \leq 4,000$
- $2.1 \leq \rho \leq 3.$

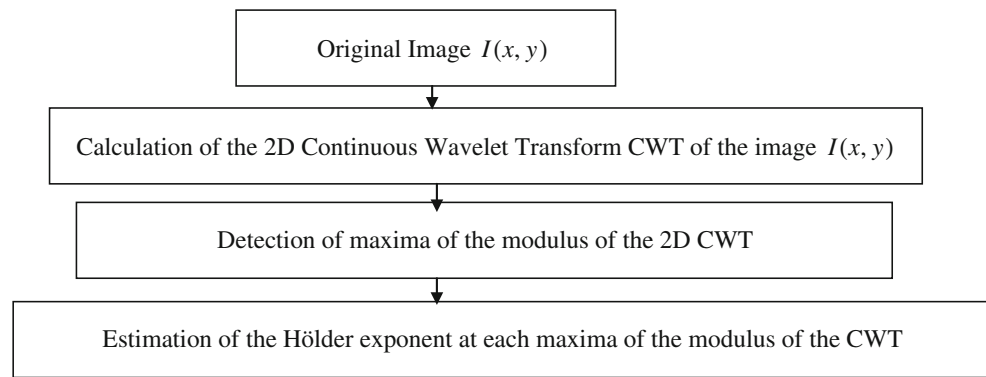
Where  $V_p(m/s)$  is the velocity of the P wave,  $V_s(m/s)$  is the velocity of the shear wave and  $\rho(g/cc)$  is the density.

The second is an heterogeneous model with the parameters detailed above.

**Fig. 2** Seismic response of a model of two layers the first is homogeneous and the second is heterogeneous



**Fig. 3** Flow chart of the 2D wavelet transform modulus maxima lines WTMM



We calculated the reflection coefficients at the null offset  $R_0$ , which are depending to the azimuth or to  $X$  and  $Y$  coordinates, obtained results are represented in Fig. 2.

**The 2D wavelets transform modulus maxima lines WTMM**

The 2D wavelet transform modulus maxima lines WTMM is a signal processing technique introduced by Kestener and his collaborators in image processing (Kestener 2003; Ouadfeul and Aliouane 2010). It was applied in medical domain as a tool to detect cancer of mammograms and in image processing. The flow chart of this method is detailed in Fig. 3.

**Application on a synthetic model**

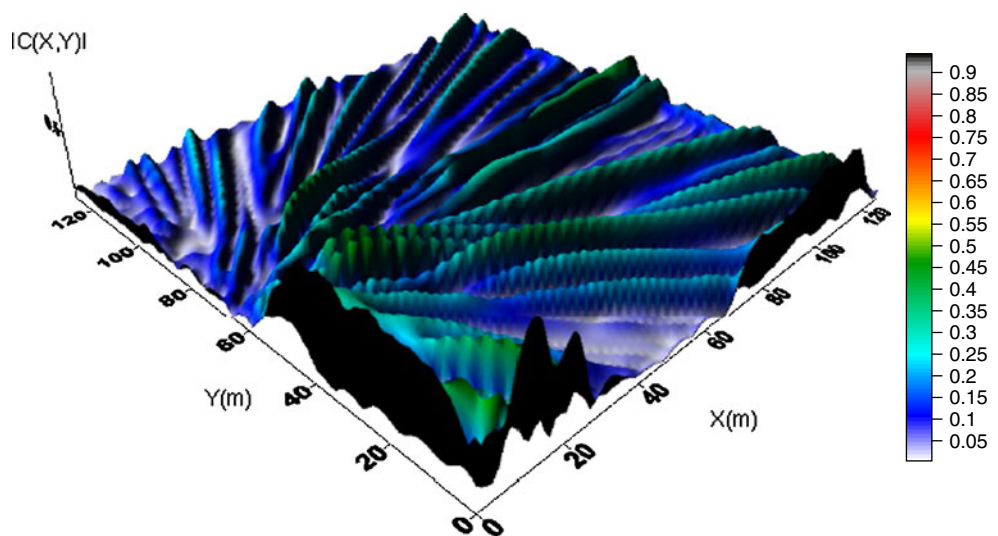
We applied the proposed technique at the synthetic model proposed above. Fig. 4 represents the modulus of the wavelet coefficients at the scale  $a=2.82$  m. Figure 5

represents the phase proposed by Kestener (2003), the skeleton of the modulus of the continuous wavelet transform is represented in Fig. 6, and the local Hölder exponents estimated at each point of maxima is represented in Fig. 7 (see Arneodo et al. 2003). One can remark that the Hölder exponents map can provide information about reflection coefficient behavior. So it can be used as a new seismic attribute for lithology analysis and heterogeneities interpretation.

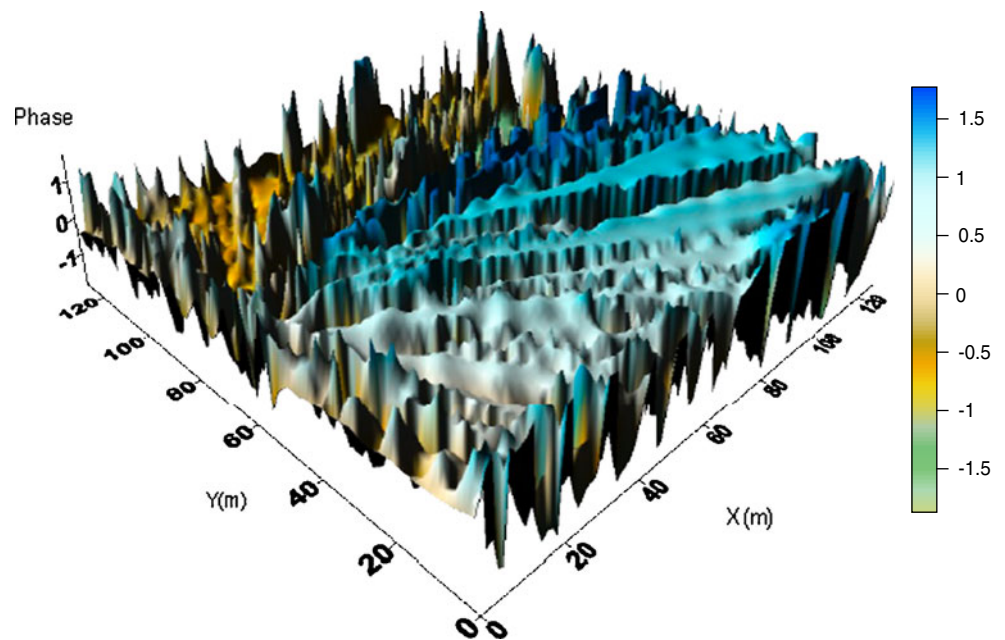
**Application on a 2D fBm process**

We suppose now that heterogeneities pattern is totally random, which is very similar to nature and rock geology. We expect that the Intercept AVO attribute is a 2D fBm process (see Arneodo et al. 2003). Figure 8a is a map of a 2D intercept attribute. This map is generated as a fBm process with an Hurst exponents  $H$  ( $H_x, H_y$ ). In this example we take  $H_x=0.5, H_y=0.4$ (see Arneodo et al. 2003). The WTMM analysis of this seismic response is represented in Fig. 8b–d.

**Fig. 4** Modulus of the wavelet transform plotted at the low dilatation

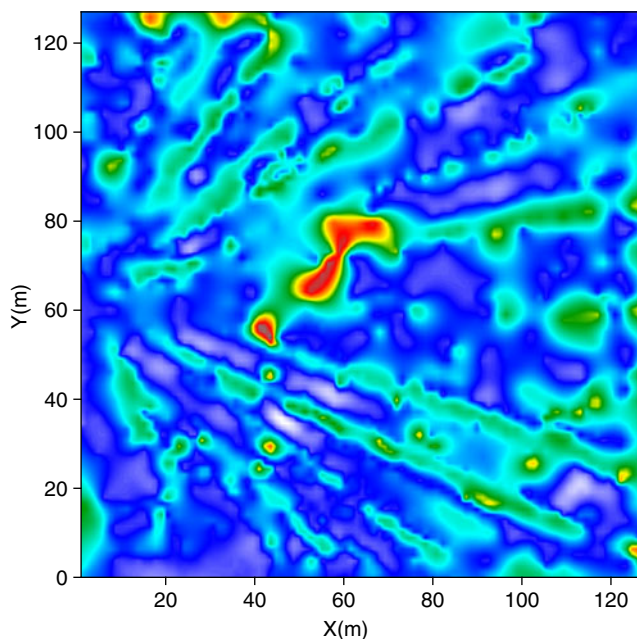


**Fig. 5** Phase of the 2D continuous wavelet transform



### Results interpretation

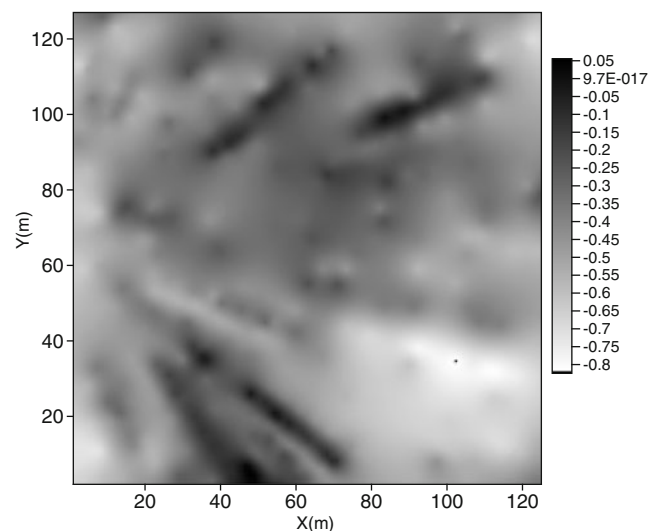
Analysis of the obtained results (Fig. 8b–d) shows that the 2D WTMM analysis can enhance seismic data interpretation. Hölder exponent map (Fig. 8e) is a good candidate for reservoir heterogeneities analysis. This map is an indicator of geological media roughness. For example, points that have the coordinates ( $40 < X < 50$ ;  $70 < Y < 90$ ) in the map of the Hölder exponent (Fig. 8e) have the same geological constitution.



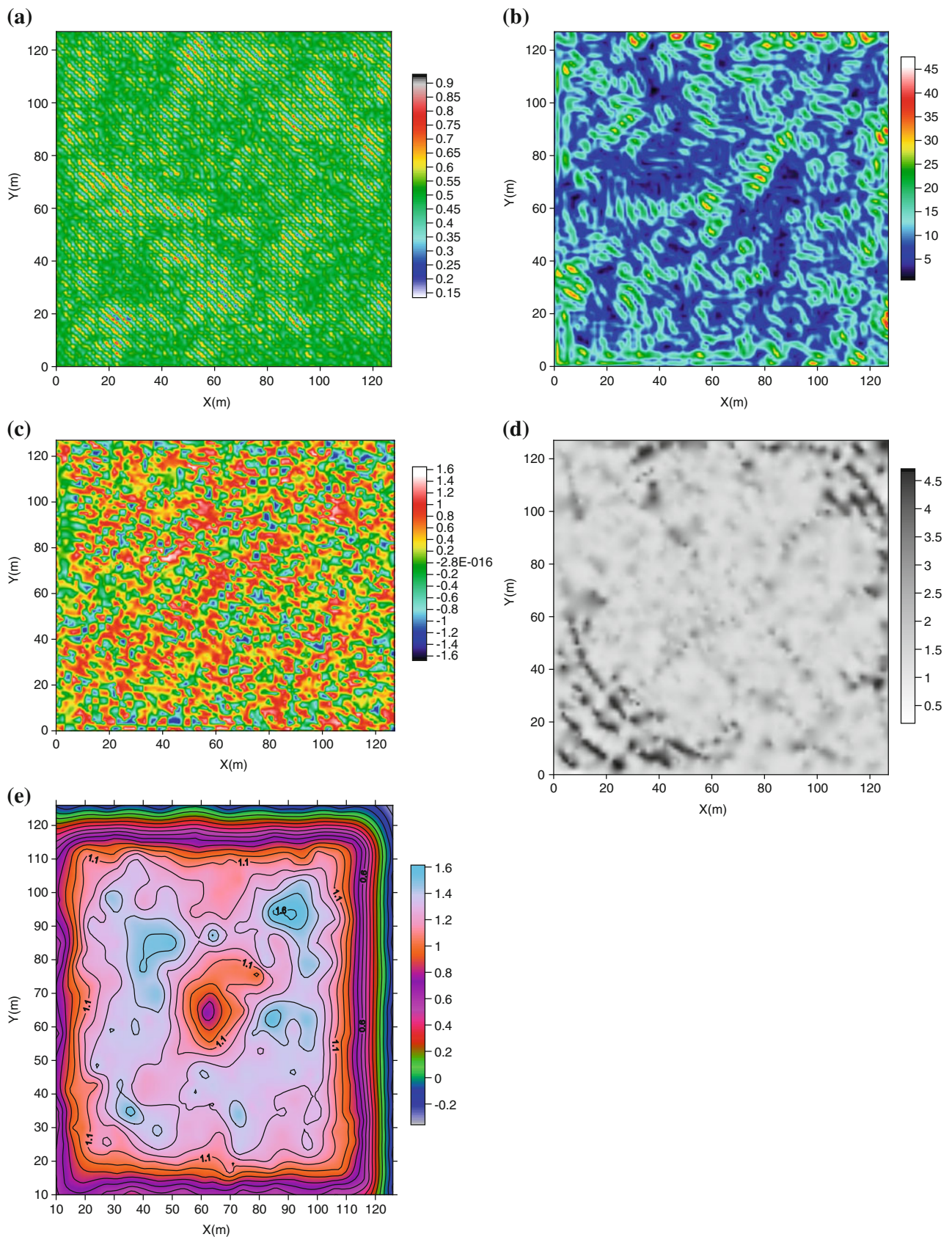
**Fig. 6** Skeleton of the module of the 2D wavelet transform at the scale ( $\alpha=2.82$  m)

### Conclusion

Application of the proposed technique at an AVO synthetic heterogeneous model shows that the 2D WTMM is able to give more information about reservoir rock heterogeneities. The local Hölder exponent can be used as a supplementary seismic attribute to analyze reservoir heterogeneities. The proposed analysis can help the hydrocarbon trapping research and analysis, fracture detection and fractured reservoirs analysis. We suggest application of the proposed philosophy at real seismic AVO data and its attribute, for example the intercept, the gradient, the fluid factor proposed by Smith and Gildow and the attribute product intercept  $\times$  gradient.



**Fig. 7** Map of local Hölder exponents



**Fig. 8** WTMM analysis of the seismic response: **a** AVO seismic response; **b** modulus of the 2D CWT at the low dilatation; **c** phase of the modulus of the 2D CWT; **d** skeleton of the modulus of the wavelet coefficients; **e** local Hölder exponents estimated at the maxima

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