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ORIGINAL PAPER

Locating key stations of a metro network using bi-objective programming: discrete and continuous demand mode

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Abstract

This study proposes two bi-objective optimization problems for locating key stations of a metro network in both discrete and continuous demand modes. Traditionally, designing a metro network based on optimization techniques consists of two approaches. The first approach locates a number of alignments and their stations simultaneously, while the second approach involves locating key stations, designing a core network, and locating secondary stations. In locating key stations processed by a single objective model, the number of produced and attracted trips to the key stations is maximized. This paper considers a second objective for this stage to maximize the coverage of key stations on origin/destination (OD) trips. A fuzzy goal programming model is established to solve the bi-objective model and provide some Pareto-optimal solutions. The previous single objective model and the proposed model with continuous demand mode are applied to a real network. Results show that the proposed model significantly increases the coverage of key stations on OD trips with only a slight reduction in the number of produced and attracted trips.

Keywords Rail rapid transit · Metro network design · Fuzzy goal programming

List of symbols

Sets/Indices

S	The set of proposed stations
D	The set of demand points
C	The set of demand areas
L	The set of catchment levels
s, s'	The index for proposed stations

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i, j The index for demand points/areas
l, l' The index for catchment levels

Input parameters

w_s The coverage of station s of network demand points
$w_{ij}^{ss'}$ The coverage of stations s and s' of demand flow i to j where s and s' are situated around the demand points/areas i and j, respectively
$w^{ss'}$ The coverage of stations s and s' of total demand flows
P The number of key stations in the network
θ_l The cover intensity at the catchment level l
v_i The total number of trip production and attraction in the demand point i
v_{ij} The demand flow from point/area i to point/area j
δ_{isl} A binary input parameter, which is equal to one, if station i belongs to catchment level l of station s, and zero, otherwise
$d_{ss'}$ The Euclidian distance between stations s and s'
d_{min} The minimum permitted Euclidian distance between two key stations
β A constant value based on the gravity model
a_i The total area of demand area i
a_{isl} The common area of demand area i and catchment area l of station s

Decision variables

Z_s A binary decision variable; it is equal to one if station s is selected as key station; otherwise it is equal to zero
$Y^{ss'}$ A binary decision variable; it is equal to one if both stations s and s' are selected as key stations; otherwise it is equal to zero

1 Introduction

Rail rapid transit system is a general term used for transportation systems including metro, light metro, commuter train, monorail, etc. (Laporte and Mesa 2015). Construction of a Rail Rapid Transit Network (RRTN) can help alleviate traffic congestion, air pollution and passengers' travel time. Due to advantages such as high speed, high capacity and independence of the network, metro systems are constructed in many cities throughout the world. According to the World Metro Database (Rhode 2014), 191 cities around the world have metro networks of which 49 inaugurated their metro network in the twenty-first century. Given the high implementation costs and life cycle of metro systems, meticulous attention should be paid to the efficient design of RRTNs (Karlaftis 2004). Finding a ubiquitous layout of a metro network is almost impossible due to the diversity of goals sought by decision makers including traffic engineers, city planners, citizen interest groups, environmentalists, politicians, etc. (Laporte et al. 2005). Hence, rather than an optimal solution, the metro network design problem usually has some Pareto-optimal solutions.

At a strategic level, designing a metro network like other RRTNs includes determining the location of a line and its stations. Traditionally, the process of designing a metro network starts with analyzing the city structure and identifying the major areas that generate or attract trips in a city. Then, a broad corridor is considered based on the mobility pattern of the city by analyzing the origin/destination (OD) matrix. In the next step, to find the best alignment and its stations in the corridor, some scenarios are developed for metro lines. Finally, the best scenario is selected based on various criteria including land-use disruptions (Bay 1985; Wulkan and Henry 1985), environments (Blackledge and Humphreys 1984), traffic and parking (Schabas 1988), and safety (Siegel 1980; Straus 1980). This process, known as scenario-based method, requires a considerable amount of iterations to modify scenarios and finally select the best one (Laporte and Mesa 2015).

Another method of locating metro alignments and their stations is based on optimization. In fact, the optimization helps locate the best alignments and their stations in a city. Other applications of the optimization in metro design include locating the best corridors in a city, before locating metro alignments and their stations (Gutiérrez-Jarpa et al. 2018), and combining proposed alignments to form a metro network, after locating metro alignments and their stations (Laporte and Pascoal 2015). In the literature, two different approaches have been presented for locating alignments and their stations by the optimization technique. In both approaches, forecasting based on the four-step planning model (trip production, trip distribution, mode choice and traffic assignment) is needed.

The first approach involves locating one or more alignments and their stations at the same time (Dufourd et al. 1996; Bruno et al. 1998, 2002). Maximizing the population coverage (Curtin and Biba 2011; Escudero and Muñoz 2009) and trip coverage (Gutiérrez-Jarpa et al. 2013; Laporte et al. 2005) are some objective functions that are widely used in this approach. Considering the population coverage as the objective function has certain advantages and drawbacks. For instance, computing this objective function is inexpensive, as it does not require rich information about OD trips (Bruno et al. 2002). On the other hand, disregarding the station-to-station demand may lead to a sub-efficient solution (Laporte et al. 2005).

The second approach consists of three consecutive stages of locating key stations, designing the core network, and locating secondary stations (Laporte et al. 2002). In the first stage, locating key stations, a number of important and expected crowded stations are selected from many possible sets of stations. These key stations are usually selected based on the number of trip production and attraction in proposed stations. In the next stage, the lines that pass through key nodes are located to maximize the covered station-to-station trips (Laporte et al. 2007). In the last stage, the secondary stations are located on the proposed lines and between the edges of lines, which are supported by two key stations to improve the OD trips attracted to the proposed line (Laporte et al. 2002).

The first approach involves proposing candidates as potential stations for alignments. When locating alignments and their stations in a single problem, it should be noted that increasing the number of potential stations will increase the complexity of the problem. In the second approach, the size of the problem, i.e. the number of proposed stations, is decreased in the stage of locating key stations. However, the process of selecting the best alignments and their stations is further divided into three separated problems, implying the difficulty of finding an optimal solution. Therefore, it is crucial to exclude some stations from the set of proposed stations, which requires establishing good measures.

To the best of our knowledge, over the recent years, most studies have adopted the first approach or the second and third stages of the second approach, with few researchers exploring the stage of locating key stations. The key stations located in the network are used as inputs for the next stages of a transit study. Therefore, the final results depend upon the locations of key stations. However, despite the importance of this issue, only one method has been proposed for locating key stations. In view of this gap, the focus of this study is on locating key stations in an efficient manner.

Traditionally, the objective functions for the first and second stages of the second optimization approach are population and trip coverage, respectively (Laporte et al. 2007). Considering these unparalleled objective functions for two consecutive stages to achieve a unique goal may result in a non-optimal solution. Therefore, we believe that locating key stations should be performed by a bi-objective model to maximize population and trip coverage, simultaneously. Figure 1 shows a simple case, suggesting that if the key stations are only located with one objective, a non-optimal solution could be selected. In this figure, three stations (A, B, and C) are proposed as network stations and the goal is to locate two of them as key stations. The number on each dashed line indicates the flow of movements between two demand points (in both directions). The pink circle around each station shows its catchment area. It is assumed that if the demand point i is located in the catchment area of station k, the total trips from/to it will be covered. Moreover, if both origin and destination points of the flow movement i to j are located in the catchment areas of stations k and m, the total trips on i to j will be covered by stations k and m.

Stations A, B and C cover 170, 120 and 120 units of population, respectively. Therefore, if the objective function is to maximize the covered population, station sets {A, B} or {A, C} will be selected as the optimal solution. On the other hand, the number of trips covered by pair stations (A, B), (A, C) and (B, C) is 70,100 and 20, respectively. Hence, if the problem objective is to maximize the covered trips, station set {A, C} will be located as key stations. As can be seen, neglecting trip coverage objective may lead to an inefficient solution by locating {A, B}.

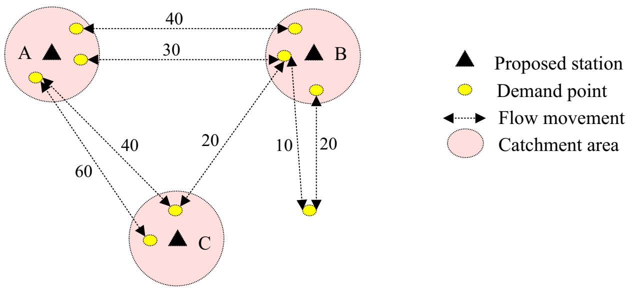

Fig. 1 An example of locating key stations based on maximizing population and trip coverage

Similarly, considering only the trip coverage objective may result in an inefficient solution. Also, since a metro system is a once-in-a-century investment and the forecasted mobility patterns are estimated due to their changes in the long run, disregarding expected crowded stations in the set of key stations may reduce the system utility in the future. Additionally, placing expected crowded stations on metro alignments increases the probability of covering more OD flows in the third stage (secondary stations locating). Hence, we propose two bi-objective problems based on maximizing population coverage as well as trip coverage for locating key stations in discrete and continuous demand modes. A fuzzy goal programming method is utilized to merge the two objectives into a single one. The computed Pareto-optimal solutions can be used as an input for the other next stages. Also, we consider a lower bound for the distance between key stations to avoid concentrating stations in some places and assist the dispersion of key stations throughout the city. In the next stages of the design, it is possible to add the expected crowded stations excluded by this condition to propose alignments by selecting them as secondary stations.

The rest of the paper is organized as follows: In Subsect. 2.2, the method of locating key stations is explained. In Subsect. 2.3, the new bi-objective models are presented for locating key stations based on both population and trip coverage objectives in discrete and continuous demand modes. The fuzzy goal programming method is explained in Sect. 3. Section 4 introduces the metro network of Isfahan, Iran, as the case study of this research. The results of comparing existing and proposed methods of locating key stations in the Isfahan network are examined in Sect. 5. Finally, conclusions are drawn in Sect. 6.

2 Modeling

In this section, first the notations used in this study are described. Then, the method of finding key stations based on the population covering objective is explained for both discrete and continuous network demands. Finally, the proposed bi-objective models of locating the key stations based on both population and trip covering objectives in both discrete and continuous network demands are presented.

2.1 Locating key stations with a population covering objective

Metro stations are points that allow changing the travel mode. This ‘modal shift’ is expected to take place from either cars to metro, in a residential area, or from walking to metro, in a central area (Laporte et al. 2002). One of the important points at this stage is to determine the station coverage on the demand points located in both residential and central areas. The extent of station coverage is defined based on the catchment level around each proposed station. A catchment level is the area with a specific radius served by the proposed station. Stations located within a central business district (CBD), unlike a residential area, have several catchment levels (Laporte et al. 2007). The studies on locating the key stations stage identify the stations with a maximum coverage on the produced and attracted trips (Laporte et al. 2002, 2007). Increasing the coverage of key stations on demand points is not equivalent to increasing the number of passengers attracted to the new metro system (Bruno et al. 2002). It is only a measure to increase the possibility of attracting passengers to the first stage. The anticipated number of passengers attracted to the metro system will be computed in the next stages based on utility functions of all existing modes made by different measures such as level of income, car ownership, travel costs in the modes and accessibility of metro (Dufourd et al. 1996).

Based on the travel cost function between the proposed station and the demand point/area, several different catchment levels are defined. Determining the coverage of proposed stations depends on the nature of the network demand point/area, which is estimated by discrete or continuous catchment levels, respectively. Figure 2 shows an example of continuous and discrete catchment levels.

In Fig. 2, three different catchment levels are drawn around each station s or s'. C_i indicates the demand area i and D_i shows the demand point i. Each catchment level has a specific coverage intensity, θ_i. The value of θ_i is in the range of zero and one, with greater distance from the center of a station yielding lower values (Bruno et al. 2002; Laporte et al. 2002, 2007). The theory adopted to calculate station coverage intensity on the network demand in the discrete mode corresponds to the problems that calculate a partial coverage in the covering location problems. In the partial coverage problems, the coverage intensity is inversely correlated with the travel cost (distance, time, etc.) between the station and the demand point (Berman and Krass 2002; Alexandris and Giannikos 2010; Jones and Simmons 1993). The partial coverage method has been recently applied to other location problems including an implicit covering model (Murray et al. 2010), a gradual coverage decay model (Berman et al. 2003), the complementary edge covering problem (Sadigh et al. 2010) and a hierarchical covering location model (Lee and Lee 2010). Also, a number of studies have defined the station coverage intensity on network demand as a function of the passengers' walking distance from the demand point to the proposed stations (Schabas

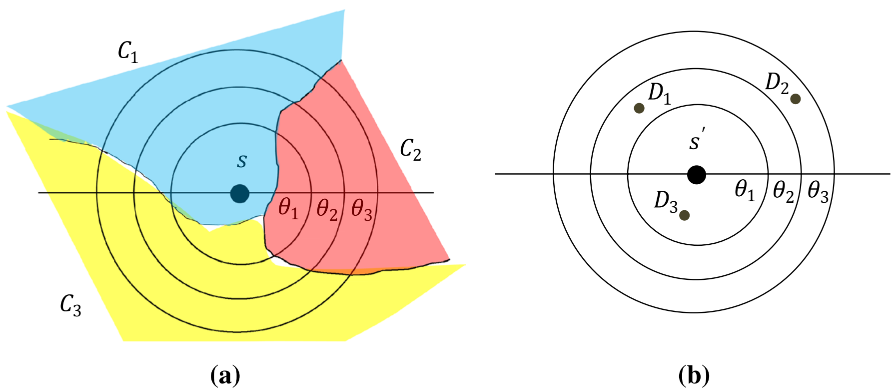

Fig. 2 **a** Continuous, and **b** discrete catchment levels

1988). Maximizing the coverage of network stations in both discrete and continuous modes is based on these equations (Laporte et al. 2002; Dufourd et al. 1996; Bruno et al. 2002; Laporte et al. 2007).

$$\max \sum_{s \in S} w_s Z_s \tag{1}$$

Subject to:

$$\sum_{s \in S} Z_s = P \tag{2}$$

$$Z_s \in \{0, 1\} \quad \forall s \in S \tag{3}$$

Objective (1) maximizes the total production and attraction trips covered by the located key stations. The coverage of station k on network demand points (discrete mode) and demand areas (continuous mode) is obtained from Eqs. (4) and (5), respectively.

$$w_s = \sum_{i \in D} \sum_{l \in L} \theta_l \times v_i \times \delta_{isl} \quad \forall s \in S \tag{4}$$

$$w_s = \sum_{i \in C} \sum_{l \in L} \theta_l \times v_i \times a_{isl}/a_i \quad \forall s \in S \tag{5}$$

In Eq. (4), δ_{isl} is a binary input parameter, which needs to be computed before solving the model based on distance from station s to demand point i. Equation (5) computes the coverage of station s on demand areas. a_{isl}/a_i is the fraction of demand area i that is held in catchment area l in station s. Equation (2) determines the number of key stations in the network. Also, the type of decision variable Z_s is determined by (3).

2.2 Locating key stations by population and OD trip covering objectives

In this section, we propose two bi-objective mathematical models for locating key stations in discrete and continuous modes of the network demand. Unlike the population covering method, in the trip covering method, the coverage of key stations on each OD flow depends on the coverage of key stations in both origin and destination. For clarification, an example of calculating the coverage of stations s and s' on demand flow i to j in both discrete and continuous demand modes is shown in Fig. 3.

Equations (6) and (7) are used to compute the coverage of stations s and s' on demand flow i to j for both discrete and continuous demand modes. The basic form of these equations, which are based on a gravity model, were proposed by Mesa and Ortega (2001). Laporte et al. (2005) utilized this method to design the best alignment by maximizing trip coverage.

$$w_{ij}^{ss'} = \sum_{l,l' \in L, (l \neq l')} \beta^2 \left[\theta_l \left(a_{isl}/a_i\right)\right] \left[\theta_{l'} \left(a_{js'l'}/a_j\right)\right] v_{ij} \quad \forall s, s' \in S \text{ and } i,j \in C \tag{6}$$

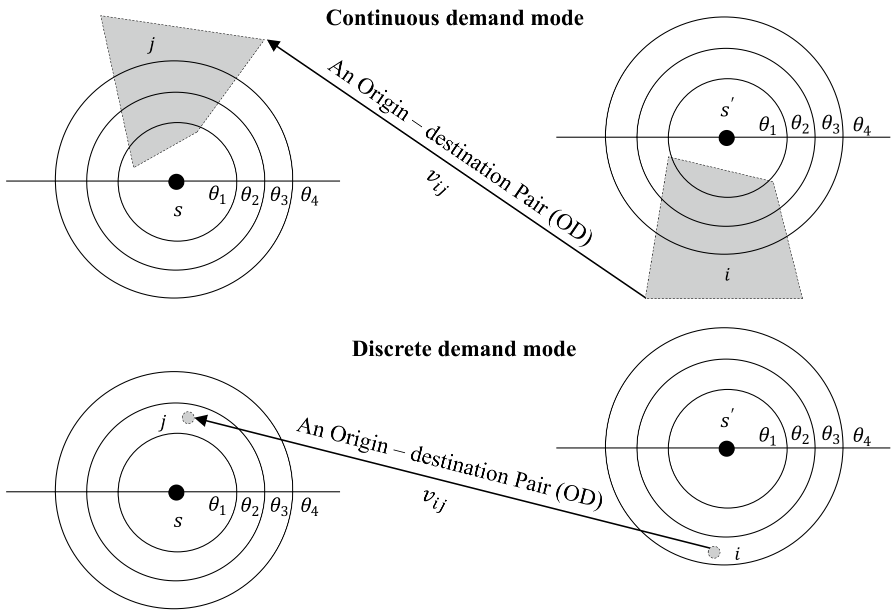

Fig. 3 An example of computing trip coverage

$$w_{ij}^{ss'} = \sum_{l,l' \in L,(l \neq l')} \beta^2 \left(\theta_l \times \delta_{isl}\right)\left(\theta_{l'} \times \delta_{js'l'}\right) v_{ij} \quad \forall s, s' \in S \ and \ i, j \in D \tag{7}$$

$$w^{ss'} = \sum_{i \in C} \sum_{j \in C} w_{ij}^{ss'} \tag{8}$$

$$w^{ss'} = \sum_{i \in D} \sum_{j \in D} w_{ij}^{ss'} \tag{9}$$

In Eq. (6), $\theta_l\left(a_{isl}/a_i\right)$ is the coverage of station s on trip origin i. Also, $\theta_{l'}\left(a_{js'l'}/a_j\right)$ is the coverage of station s' on trip destination j. Hence, the product of these two specifies the coverage of stations s and s' on demand flow i to j. Equation (7) is similar to Eq. (6), but it represents a discrete form of $w_{ij}^{ss'}$. Equations (8) and (9) calculate the coverage of stations s and s' on total network demand by aggregating $w_{ij}^{ss'}$ on demand nodes.

By computing $w^{ss'}$, the mathematical form of the multi-objective key station locating problem for both discrete and continuous modes can be presented as follows:

$$Max \ O_1 = \sum_{s \in S} \sum_{s' \in S} w^{ss'} \times Y^{ss'} \tag{10}$$

$$\max \ O_2 = \sum_{s \in S} w_s Z_s \tag{11}$$

Subject to:

$$\sum_{s \in S} Z_s = P \tag{12}$$

$$Y^{ss'} \le Z_s \quad \forall s, s' \in S \tag{13}$$

$$Y^{ss'} \le Z_{s'} \quad \forall s, s' \in S \tag{14}$$

$$Y^{ss'} \ge Z_s + Z_{s'} - 1 \quad \forall s, s' \in S \tag{15}$$

$$Y^{ss'} = Y^{s's} \tag{16}$$

$$d_{ss'} \ge d_{min} \times Y^{ss'} \quad \forall s, s' \in S \tag{17}$$

$$Y^{ss'}, Z_s \in \{0, 1\} \quad \forall s, s' \in S \tag{18}$$

In (10), the first objective (O_1) is to maximize the coverage of located key stations on the total demand flows for both continuous and discrete demand modes. Moreover, in (11), the second objective (O_2) is to maximize the coverage of located key stations on produced and attracted trips in the network. In Eq. (12), it is assumed that the number of key stations is equal to P. In Constraints (13) and (14), if the value of $Y^{ss'}$ is equal to one, then the value of both Z_s and $Z_{s'}$ must be equal to one, meaning these stations are counted as located key stations. Accordingly, if both Z_s and $Z_{s'}$ are equal to one, Constraint (15) ensures that the value of $Y^{ss'}$ is equal to one. Equation (16) decreases the number of $Y^{ss'}$, as decision variables, by half and contracts the size of the problem. According to Constraint (17), if two stations s and s' are located as key stations ($Y^{ss'} = 1$), their distance must be greater or equal to d_{min} (the minimum limited distance between key stations). (18) shows the nature of the decision variables (binary variables).

3 Fuzzy goal programming

Unlike a single objective optimization (SOO) model, a multi objective optimization (MOO) model usually does not have an optimal solution due to conflicting objectives. For MOO models, identifying the non-dominated solutions (Pareto-solutions or efficient solutions) is of great importance. A non-dominated solution is a solution in which the value of no objective can be improved without deteriorating the values of other objectives. There are several methods for solving MOO models, but none can be claimed to be generally superior to all the others. In fact, the specific features of the problem and the decision maker's judgements are important factors in selecting an appropriate method (Miettinen 2012). The solving approaches of MOO models can be categorized in various ways. One popular classification was presented by Hwang and Masud (1979), who categorized MOO solving approaches into four groups based on the level of utilizing the preference information:

- No-preference methods (e.g. global criterion).
- A priori methods (e.g. goal programming or fuzzy goal programming).
- A posteriori methods (e.g. ϵ-constraint methods).
- Interactive methods (e.g. NIMBUS method).

In previous methods, the preferences of a decision maker are incorporated in the initial formulation of an appropriate SOO. This category includes methods such as value function (e.g. weighted global criterion method, weighted sum method, weighted min–max method, exponential weighted criterion and weighted product method), lexicographic ordering and goal programming. Goal Programming (GP) is a useful method that helps decision makers model the real-world MOO problems and find a set of non-dominated solutions. However, this method requires precise determination of the goals for objective functions. It is difficult to define precise goals for decision makers, as they only contain partial information. In this regard, Fuzzy Goal Programming (FGP) offers a useful approach for importing imprecision and uncertainty. For instance, in a MOO model, one objective is maximizing profits. To convert the model into a single problem with GP, a precise goal like 500 must be considered for the profit. By setting the goal, deviations are minimized. However, in FGP, the goal defined as the profit should be a value around 500. As a result, the goal itself is imprecise and fuzzy in nature.

To the best of our knowledge, Narasimhan (1980) was the first to propose an FGP method. This FGP method was further improved in a number of studies to enrich computational efficiency (Hannan 1981a, 1981b). Among the presented FGP methods, those with an additive function that consider the priority of objectives are suitable for this study. These methods are classified into weighted FGP methods, FGP methods with preemptive priority and FGP methods with different achievement degrees. In this study, a weighted FGP method is used for our model (Tiwari et al. 1987; Amid et al. 2011). The weighted FGP method is selected not only for its simplicity, but also its ability to find a set of Pareto-optimal solutions by changing the weights of fuzzy goals. For further information about the methodology of the other two methods, interested readers can see Chen and Tsai (2001).

3.1 Weighted additive fuzzy goal programming

Narasimhan (1980) proposed an FGP method using membership functions to specify the aspiration levels of goals in a fuzzy environment. The membership functions used in Narasimhan (1980) are actually inspired by a fuzzy programming approach presented by Zimmermann (1978). The linear membership functions for maximization goals $\mu_{O_a}(x)$ $(a = 1, 2, \dots, q)$ and minimization goals $\mu_{O_b}(x)$ $(b = 1, 2, \dots, q')$ are similar to Eqs. (19) and (20). Also, Fig. 4 shows the linear membership functions,

$$\mu_{O_a}(x) = \begin{cases} 1 & if\ O_a(x) \geq O_a^{max} \\ \frac{O_a(x) - O_a^{min}}{O_a^{max} - O_a^{min}} & if\ O_a^{min} \leq O_a(x) \leq O_a^{max} a = 1, 2, \dots, q \ , \\ 0 & if\ O_a(x) \leq O_a^{min} \end{cases} \tag{19}$$

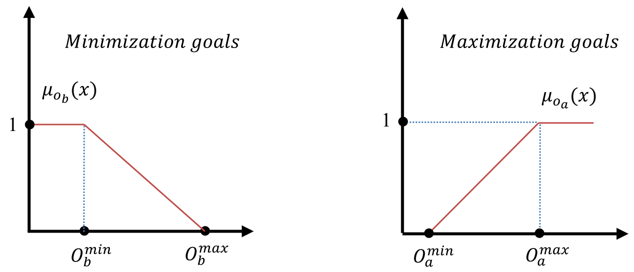

Fig. 4 Objective function as fuzzy number

$$\mu_{O_b}(x) = \begin{cases} 1 & if\ O_b(x) \le O_b^{min} \\ \frac{O_b^{max} - O_b(x)}{O_b^{max} - O_b^{min}} & if\ O_b^{min} \le O_b(x) \le O_b^{max} b = 1, 2, \dots, q' \\ 0 & if\ O_b(x) \ge O_b^{max} \end{cases}, \tag{20}$$

where $O_a(x)$ and $O_b(x)$ are the values of maximization and minimization goals. O_a^{max} and O_a^{min} are the ideal and nadir solutions for maximization goals. Also, O_b^{min} and O_b^{max} are the ideal and nadir solutions for minimization goals. In multi-objective programs, the ideal solution of each objective is calculated by its optimization irrespective of other objectives. It is difficult to compute the nadir solution for each objective especially for problems with more than two objectives. It can be estimated with the payoff table of objective values but this estimation may not be reliable (Miettinen 2012). However, for bi-objective problems, the value of the first/second objective when optimized individually will be a nadir solution for the first/second objective.

Based on the linear membership functions (Eqs. (19) and (20)), Tiwari et al. (1987) proposed an additive fuzzy goal programming that takes into account the weights of objectives. The method is formulated as follows:

$$Max \sum_{j=1}^{q} \alpha_{o_j} \mu_{o_j}(x) \tag{21}$$

Subject to:

$$\mu_{O_a}(x) = \frac{O_a(x) - O_a^{min}}{O_a^{max} - O_a^{min}} \quad for\ maximization\ goals\ a = 1, 2, \dots, q \tag{22}$$

$$\mu_{O_b}(x) = \frac{O_b^{max} - O_b(x)}{O_b^{max} - O_b^{min}} \quad for\ minimization\ goals\ b = 1, 2, \dots, q' \tag{23}$$

$$\sum_{j=1}^{q} \alpha_{o_j} = 1, \alpha_{o_j} \ge 0 \tag{24}$$

$$Ax \leq b \ \& \ x \geq 0 \tag{25}$$

$$0 \leq \mu_{O_a}(x), \mu_{O_b}(x) \leq 1 a = 1, 2, \ldots, q \ \& \ \ b = 1, 2, \ldots, q' \tag{26}$$

where α_{o_j} is the weight of objective j, and q is the number of objectives. Also, a and b are the vectors of coefficients and constants of non-fuzzy constraints, respectively. Objective (21) maximizes the weighted additive fuzzy goals. Equation (24) ensures that the sum of objectives' weights is equal to one. The system constraints in vector notation are provided by (25). The remaining constraints provide the linear membership functions, as explained above.

4 Data presentation

The presented model was applied to Isfahan, which is one of the Iranian metropolises with a population of 2.25 million and a total area of 551 km^2. Its metro system has been under construction since 2001, but it has not been completed yet. The studies that led to the construction of this network can be traced back to 2000. The city center is made up of 178 traffic zones, and the OD matrix of travel demand between each pair of these zones was obtained based on the results of a home survey in 2016, which was conducted as a part of the Isfahan's transportation comprehensive study (Isfahan University of Technology, 2014). Figure 5-a shows the city road network map and its 178 traffic zones. Figure 5-b shows the metro network (stations and lines) of this city. It consists of 47 stations in three different lines (Isfahan University of Technology 2014).

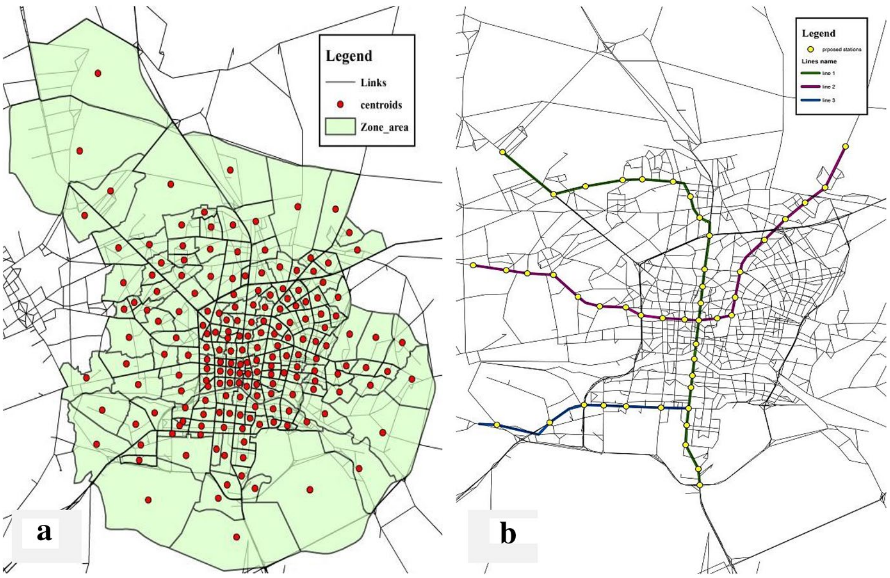

Fig. 5 **a** Isfahan's traffic zones. **b** The metro network

To apply the proposed model, a set of 165 possible stations were considered. The set of proposed stations also included the existing metro stations (47 stations). Typically, the people living within a certain distance from stations are attracted to the system. This distance is limited to 400 m for populated regions and could be extended to 1 km for less populated areas (Laporte and Mesa 2015; Gutiérrez-Jarpa et al. 2018). There are various measures for depicting the walking distance such as Manhattan, Block, Euclidean norm, etc. (Laporte et al. 2005). In this study, we assume a maximum walking distance of 400 m and use a revised Euclidean norm based on the pattern of Isfahan's streets. Since the structure of the streets in cities is typically based on a rectangular grid, the average walking distance in each catchment level (d_i) is more than the radius (r_i) considered in the Euclidean norm. Accordingly, we decreased the radius of the catchment levels (10 percent) to provide the expected walking distance. The characteristics of catchment levels are as follows:

- Catchment level 1: $0 \leq d_1 \leq 130$ & $0 \leq r_1 \leq 120$ & $\theta_1 = 1$.
- Catchment level 2: $130 < d_2 \leq 270$ & $120 < r_2 \leq 240$ & $\theta_2 = 0.5$.
- Catchment level 3: $270 \leq d_3 \leq 400$ & $240 < r_3 \leq 360$ & $\theta_3 = 0.25$.
- Catchment level 4: $400 < d_4$ & $360 < r_4$ & $\theta_4 = 0$.

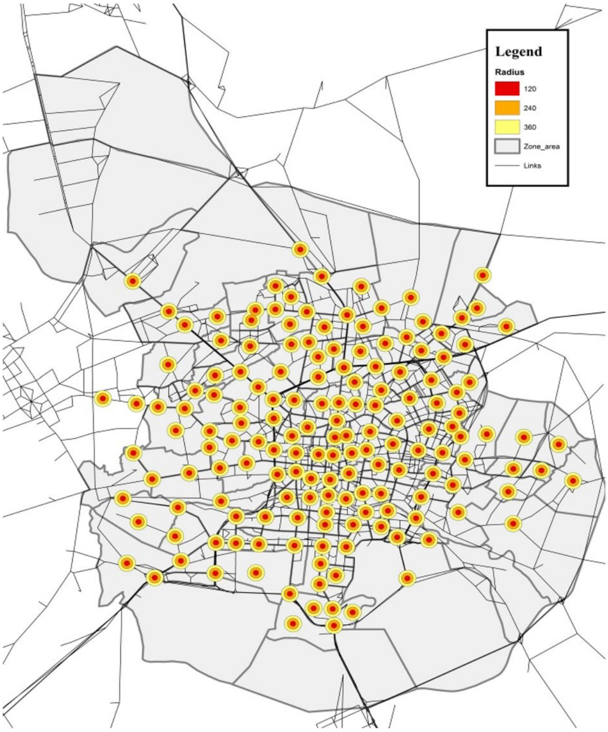

Fig. 6 Proposed stations for the metro network

Figure 6 shows the proposed stations and their catchment levels.

5 Results and discussion

After determining the catchment levels of each proposed station, first parameters $w_{ij}^{ss'}$ and w_s were calculated using MATLAB software. Then, IBM ILOG CPLEX 12.6.1 software was used to solve the previous and presented models for locating key stations. A computer (Intel(R) Core(TM) i7 CPU @2.13 GHz with 8G RAM under the 64-bit Windows 7 OS) was used to solve the models. According to previous studies, the number of key stations generally varies between 5 and 22 (Laporte et al. 2007). Thus, this study presents the results of three scenarios with 10, 15, and 20 key stations. In this section, first a comparison is drawn between the results of (a) the previous model and (b) the proposed model based on the continuous demand mode. Then, the results of the proposed model are compared with the metro system of Isfahan to evaluate the efficiency of the considered design plan.

In order to apply the proposed model to three scenarios, the ideal and nadir solutions for all scenarios were needed. Table 1 shows the value of the ideal and nadir solutions for the two objectives of the proposed models and all scenarios. The solutions are computed assuming that the values of a and d_{min} are 1 and 500 m, respectively.

The Pareto-optimal solutions for the bi-objective proposed model were computed by changing the weight of the first fuzzy goal α_{O_1} (trip coverage objective) from zero to one in increments of 0.1. The solutions of the previous method are shown in the fifth column of Table 1 (the ideal solution for population covering objective O_2). Figure 7 shows the Pareto-optimal solutions for all scenarios and the solutions achieved from the previous method.

In Fig. 7, the solutions in yellow circles demonstrate that the bi-objective model is more effective than the single objective model with the population coverage objective. These Pareto-optimal solutions are slightly different in the population coverage objective, but they are considerably different in the trip coverage objective. This is especially evident in case $p = 15$. In this case, the solution of the single objective model based on population coverage is point A, which covers 41,337 people and 931 trips. Also, the closest solution of the bi-objective model to point A is B. The population and trip coverage objectives of point B are 41,178 and 1011, respectively. A comparison of these points shows that if the population objective distances is just

Table 1 The ideal and nadir solutions for all scenarios

Scenario ID	No. key stations (P)	OD trip covering objective (O_1)		Population covering objective (O_2)	
		Ideal	Nadir	Ideal	Nadir
1	10	868	651	31,367	27,282
2	15	1226	931	41,337	37,196
3	20	1585	1257	49,908	46,443

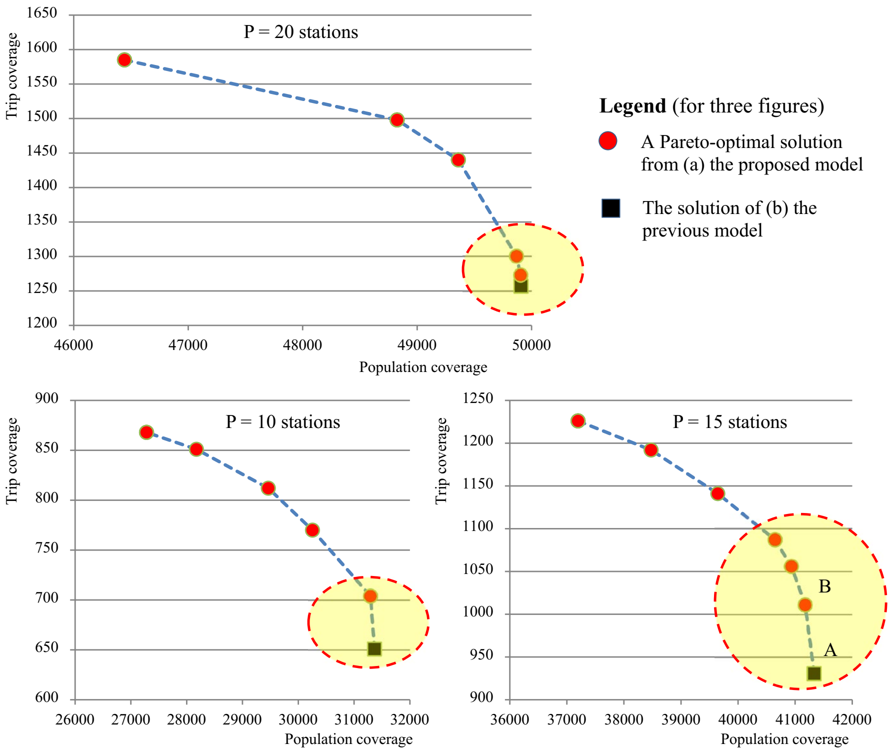

Fig. 7 Pareto-optimal solutions for all scenarios

less than 4 percent from its optimal value, the trip coverage objective can approach its optimal value by about 28 percent. Therefore, the bi-objective model, by introducing some Pareto-optimal solutions, can help making the best decision.

The results of the bi-objective model were compared with the existing metro system in Isfahan for the following reasons:

(1) Evaluating the efficiency of the existing system based on a comparison of the position of the proposed key stations in the bi-objective model and the existing stations.
(2) Recognizing the most important key stations not covered in the existing metro lines and proposing them as key stations for development plans.
(3) Prioritizing the existing lines based on their coverage of selected key stations.

Table 2 shows located key stations in all Pareto-optimal solutions. In this table, the located stations are signified with number 1 in cells. The last row of the table shows the repetition of each station in all Pareto-optimal solutions. Therefore, stations with higher repetition take priority over other stations. Figure 8 shows the

Table 2 The located key stations for all Pareto-optimal solutions

P	α_{o_1}	Located stations ID (1 = located)																										
		9	10	11	13	14	15	16	31	34	42	52	57	81	95	96	97	98	99	100	101	107	116	132	142	148	161	162
10	0.0	1	1						1	1		1	1				1			1				1			1	
10	0.1	1	1							1		1	1				1		1	1				1			1	
10	0.2	1	1							1		1	1				1		1	1				1			1	
10	0.3	1	1							1		1	1				1		1	1				1			1	
10	0.4	1	1							1		1	1				1		1	1				1			1	
10	0.5	1	1									1	1		1		1		1	1				1			1	
10	0.6	1	1									1	1		1				1	1				1		1	1	
10	0.7	1	1										1	1	1				1	1				1		1	1	
10	0.8	1											1	1	1				1	1				1		1	1	1
10	0.9	1											1	1	1				1	1				1		1	1	1
10	1.0	1											1	1	1				1	1				1		1	1	1
15	0.0	1	1						1	1	1	1	1			1	1		1	1				1	1	1	1	
15	0.1	1	1						1	1	1	1	1			1	1		1	1				1	1	1	1	
15	0.2	1	1						1	1	1	1	1		1	1	1		1	1				1		1	1	
15	0.3	1	1						1	1	1	1	1		1	1	1		1	1				1		1	1	
15	0.4	1	1	1			1		1	1		1	1		1		1		1	1				1		1	1	
15	0.5	1	1	1			1		1	1		1	1		1		1		1	1				1		1	1	
15	0.6	1	1	1			1			1		1	1	1	1		1		1	1				1		1	1	
15	0.7	1	1	1			1			1		1	1	1	1		1		1	1				1		1	1	
15	0.8	1	1	1		1	1	1				1	1	1	1				1	1				1		1	1	
15	0.9	1	1	1		1	1	1				1	1	1	1				1	1				1		1	1	
15	1.0	1	1	1		1	1	1				1	1	1	1				1	1				1		1	1	
20	0.0	1	1				1		1	1	1	1	1		1	1	1		1	1	1	1	1	1	1	1	1	
20	0.1	1	1		1		1		1	1	1	1	1		1	1	1		1	1		1	1	1	1	1	1	

Table 2 (continued)

P	α_{o_1}	Located stations ID (1 = located)																										
		9	10	11	13	14	15	16	31	34	42	52	57	81	95	96	97	98	99	100	101	107	116	132	142	148	161	162
20	0.2	1	1	1			1		1	1	1	1	1		1	1	1		1	1		1	1	1	1	1	1	
20	0.3	1	1	1	1		1	1	1	1	1	1	1		1	1	1		1	1	1			1		1	1	
20	0.4	1	1	1	1		1	1	1	1	1	1	1		1	1	1		1	1	1			1		1	1	
20	0.5	1	1	1	1		1	1	1	1		1	1	1	1	1	1		1	1	1			1		1	1	
20	0.6	1	1	1	1		1	1	1	1		1	1	1	1	1	1		1	1	1			1		1	1	
20	0.7	1	1	1	1		1	1	1	1		1	1	1	1	1	1		1	1	1			1		1	1	
20	0.8	1	1	1	1	1	1	1				1	1	1	1	1	1	1	1	1	1			1		1	1	
20	0.9	1	1	1	1	1	1	1				1	1	1	1	1	1	1	1	1	1			1		1	1	
20	1.0	1	1	1	1	1	1	1				1	1	1	1	1	1	1	1	1	1			1		1	1	
Grand total		33	30	16	9	6	18	11	15	21	9	29	33	15	26	15	25	3	32	33	9	3	3	33	5	27	33	3

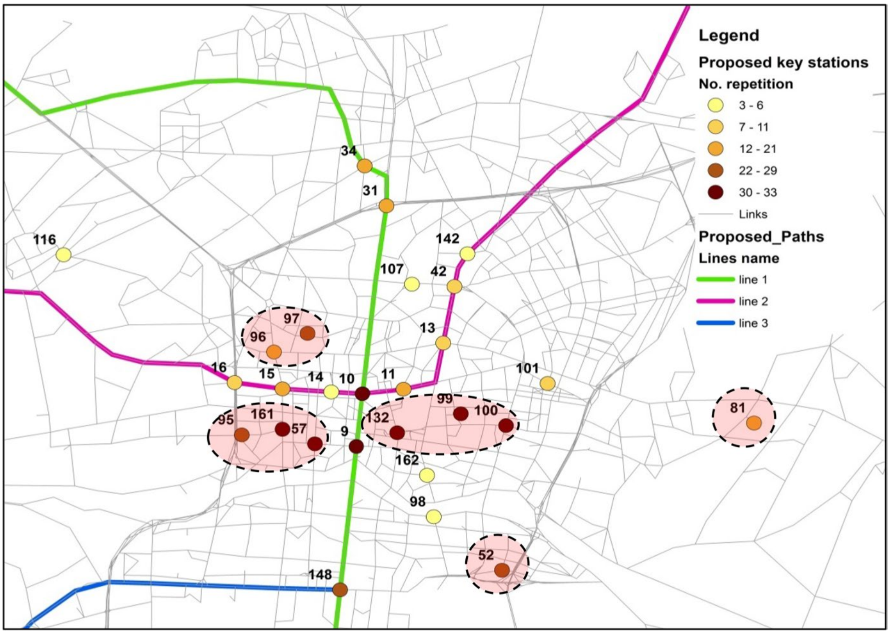

Fig. 8 The key stations based on the bi-objective model and their repetitions in all scenarios

spatial location of key stations. As can be seen, the located key stations are classified into five groups based on the frequency of their repetition in all solutions.

The results show that the existing metro lines covered 12 of 27 stations selected in Pareto-solutions. Based on the objectives of this study, the current Line 3 is unable to cover important key stations and is therefore an inefficient line. Moreover, some important key stations (with high repetitions) are far from all proposed paths. These stations are indicated in Fig. 8 with a red circle. The existing Line 2 covers eight key stations and seems to be an efficient line although some key stations with high repetition are around this line (e.g. points 95, 161, 57, 132, 92, and 96). The existing Line 1 is an efficient south-to-north line that covers two key stations with high repetition in the center of the city (i.e. points 9 and 10). Also, some important key stations are located at the beginning and end of this line.

The key stations not covered by existing lines have a greater potential to be covered in the development of the metro system in the future. Among these stations, 57, 100, 132, and 161 take priority. The results also suggest that Lines 1 and 2 take higher priority for opening than Line 3. To determine the priority of Lines 1 and 2, the two objectives considered in this study were computed based on the locating of the key stations belonging to these lines. The population coverage objective for Lines 1 and 2 were 1424 and 1531, respectively. Moreover, the trip coverage objective for Lines 1 and 2 were 98.6 and 248.4, respectively. Therefore, Line 2 takes priority over Line 1.

6 Conclusion

This study presented two bi-objective optimization problems for locating key stations in the design of a metro network in both discrete and continuous demand modes. The first objective maximizes the coverage of key stations for total produced and attracted trips in the city. The second objective maximizes the coverage of key stations on OD trips. The results showed that simultaneous consideration of the two objectives could provide more effective solutions than a single objective problem. To solve the bi-objective problem, a fuzzy goal programming technique was applied to convert it into a single objective problem. The proposed model was applied to a real case study in the city of Isfahan. The results suggested that the proposed model could expand the coverage of key stations on OD trips with only a small reduction in the coverage of produced and attracted trips. Furthermore, the comparison of results with the metro network of Isfahan revealed that some important areas of the city do not have access to any of the metro lines.

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