

Cost-of-crowding model for light rail train and platform length

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Abstract With light rail transit (LRT) and other similar rail-based commuter transit systems, train and associated station platform length provides an added dimension of flexibility not available to buses. Train and platform lengths are important factors in the planning and expansion phases of a network. Existing cost models that determine optimal headway by combining passenger and operational costs provide headways that are small and close to a logistical minimum (2–3 min); this type of standard waiting cost model is not sensitive to train and platform length. In this paper, on-board crowding is used as a cost factor and a cost-of-crowding model is developed from supporting psychological research. Two models are proposed and optimized with respect to train length to determine the optimal train and platform length for a many-to-one peak period commuter LRT system. Data from the C-Train network in Calgary, Alberta is used for numerical analysis of the model. The model demonstrated that crowding has an effect on optimal train length. The model produced feasible results when applied to a real-world scenario.

Keywords Crowding · Optimization · Light rail · Train length · Cost minimization

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1 Introduction

Light rail transit (LRT) fills the gap between highly flexible modes such as buses, and high-capacity/high-capital modes such as heavy rail or metro. LRT has the unique ability to provide the reliable, fast, and comfortable service associated with railways while accessing areas under a variety of right-of-way configurations.

In this paper we consider a general LRT system operating mostly on exclusive right-of-way configurations with raised platforms that have a significant construction cost. A high investment in infrastructure is required for such an LRT network. Exclusive right of way and station construction costs are large, and vehicle purchase and maintenance costs are significantly higher than for buses. When considering the logistics of dispatching trains, optimal headways on public transit routes have been modeled and optimized (Newell 1979; Wirasinghe 2003), while platform and train length have been largely ignored as a factor in determining operating policy. Platform and train length are inherently linked, as train length must be less than or equal to platform length. The goal of this paper is to investigate LRT platform and train length as it relates to level of service parameters, particularly on-board passenger crowding.

Determining an appropriate train length for an LRT system is essential in the planning stage of a network. If stations are built with insufficient room for expansion, lengthening trains to accommodate rising demand becomes costly after initial construction. If expansion of an existing network's platform length and fleet size is needed, it is important that the appropriate train length be considered to avoid the excessive cost of constructing an unnecessarily long platform or having to expand again in the near future. By constructing a system with sub-optimal train lengths, the network may experience excessive crowding, or run trains that are empty and costly.

Calgary, Canada has an LRT network known as the C-Train that services the Central Business District (CBD) and extends outwards via four arms to service the rest of the city. Currently the C-Train operates in three-car trains, however expansion of station platforms to allow four-car trains is nearing completion. Data from the C-Train network is used for numerical discussion of the model developed.

This paper proposes a crowding-cost model for an LRT system operating during peak periods at minimal headways. Crowding on-board an LRT train as it progresses towards the CBD is optimally balanced with various operating costs to determine an optimal train and platform length for a given network.

2 Background

2.1 Cost-based approach

There are a number of papers which approach the problem of optimization of urban public transit from the *total cost* perspective. In this method, relevant costs to passengers and the transit operator are combined and weighted to obtain a total

system cost (Byrne and Vuchic 1972; Hurdle 1973; Newell 1979; Strathman and Hopper 1993; Wirasinghe 1990). This combination often results in competing terms which can be analyzed to produce an “optimal result”.

In an unpublished working paper by Beatty (2013), platform construction, fleet, dispatching, and passenger waiting costs were combined in an attempt to find an optimal headway and train length using Newell (1971). Due to the relatively high demand of an LRT system, passenger waiting costs heavily outweighed dispatching costs and headways were reduced to the shortest that was practical for a signaled train system. Because of this extremely short headway, optimal train length was found to be unrealistically short (often less than one car).

Due to the tendency of the model to appease passenger waiting costs by minimizing headways, long run peak-period headways should be chosen as a logistical minimum for the system, and essentially remain fixed. In order to properly utilize the total cost method, train length provides an added degree of flexibility, and passenger costs that are intuitively related to train length must be included. One common level of service parameter that satisfies these requirements is on-board crowding. This paper will investigate two key components of LRT planning using the cost-based approach: train and platform length and its relationship with operations, and how crowding as a passenger cost affects the design of an LRT route. It should be noted that the resulting train lengths arising from the presented model are understood to be subject to safety standards and separation requirements such as the “brick-wall stop” concept (Parkinson and Fisher 1996). This model is investigating scenarios where train separation is minimized but fixed by signals and safety standards, and where train length due to demand is of a reasonable length given the mode of transportation used.

2.2 Impacts of crowding

Emerging research shows that crowding on rail cars has significant effects on passengers’ health, safety, levels of stress and anxiety, perception of time, and overall experience of a transit system. In fact, one study by Cheng (2010) reported that crowding ranks highest (above “delays”) as causing anxiety when using commuter rail systems. It has been shown that crowded commuter trains have an effect on passengers safety and perception of risk (Cox et al. 2006), as well as overall job performance and satisfaction outside of the commute itself (Mahudin et al. 2011). In the literature, the term “crowding” generally refers to the concept of placing people in a confined space. Often, this is manifested as *passenger density*, where the number of passengers per unit area is measured, however in other cases described below it may refer to the proximity of passengers to one another, and their likelihood of contact. This can be measured either in unit area per passenger (the inverse of the former definition), or more specific to the actual layout of the vehicle, such as seats per person. One additional metric used for crowding is *load factor*. A design capacity is determined for the vehicle under an acceptable level of crowding by any of the previous definitions (usually passengers per seat). For example, a load factor of 1.00 may mean that there is one passenger for every seat. In Fig. 1, an

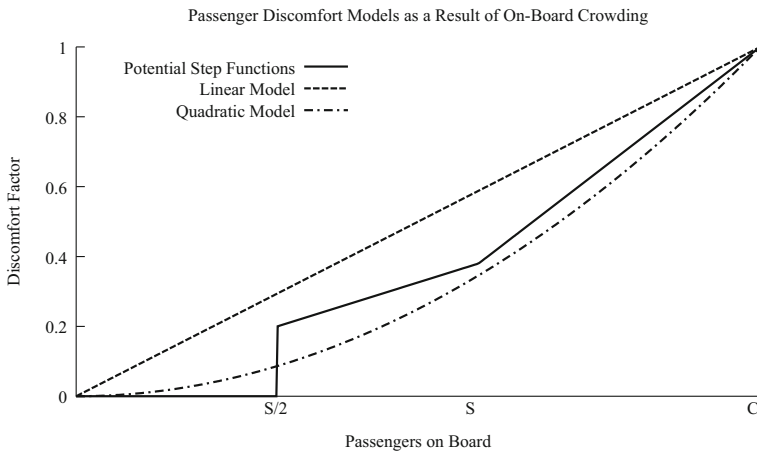


Fig. 1 Step function of discomfort and two continuous models

analog of this definition is used in conjunction with discomfort to produce a discomfort factor. This is mathematically developed in Sect. 3.2.

On a passenger-specific level, stress on board commuter trains was demonstrated to be related to the number of nearby passengers and probability of personal space invasion, as opposed to average passenger density on board the train (Evans and Wener 2007). Passengers reported increased feelings of invasion of privacy as crowding increased (Wardman and Whelan 2011), and are willing to forego an empty seat and stand if they feel their personal space would be unduly encroached upon while seated next to an individual (Hirsch and Thompson 2011). Additionally, there is a significant gap in the level of discomfort between seated and standing passengers (Whelan and Crockett 2009).

The studies presented above imply that a mathematical model describing passenger discomfort should have discomfort vary with the number of passengers on board. If the potential safety issues with a near-empty train are not considered relevant during peak periods, this model of discomfort is likely monotonically increasing with the number of passengers on board. There are key load points at which the function is likely to experience a discontinuous jump: (1) when half the seats are filled, and those who wish to sit must share and those who had a seat of their own are forced to share at the risk of increased personal space invasion, and (2) when all of the seats are filled and one is forced to stand. Additionally, in scenario (1), the change in probability of being seated next to someone from zero to a nonzero value is considered to increase the discomfort of all passengers. Each additional passenger after the jump contributes to an incremental increase in the discomfort of the passengers on board. This effect is potentially reduced at (2) where the first standing passenger is less directly impacting the seated passengers. This lowers the jump in discomfort caused by the S th passenger. If a *discomfort factor* is introduced, with a value of zero for an empty train, and a value of one for a train with a load at design capacity, this piecewise discomfort function is

hypothesized to take on a form similar to the one illustrated in Fig. 1. The exact slopes and heights of the discontinuities in the proposed step function are not known; however, it is likely that the slope between half and full seating capacity is lower than the slope between fully seated and design capacity loads, due to the fact that standing places additional stress on passengers.

In order to consider crowding as a viable passenger cost in a total cost model, an understanding of passengers' cost of discomfort is required. In terms of passenger cost per unit time, this value has been investigated directly by Douglas and Karpouzis (2006) using stated preference surveys. A review of this and several other studies regarding passenger willingness to pay with regards to crowding is provided by Li and Hensher (2011).

While there have been some efforts to incorporate on-board crowding into demand model estimations (Li and Hensher 2011), there are no peer-reviewed studies investigating the link between passenger crowding and train configuration. The potential for variation in train lengths to compensate for changes in crowding means it is important that these effects be linked and investigated. This will provide insight into how transit companies may want to construct right-of-ways and station platforms, and provide additional information for developing proper fleet-size models.

The hypothesized form of crowding discomfort is one that may only reflect passenger attitudes in certain locations. The studies discussed above were done in cultural milieus that are quite averse to crowding, and other areas of the world with different attitudes towards crowding may experience a different discomfort curve. The discomfort curve proposed is therefore assumed to be reflective of passenger discomfort in countries culturally similar to where the previous research was performed. Additionally, it is understood that the model can be extended beyond the typical design capacity, when the passenger load on board reaches so-called "crush" levels. These levels may be the norm in certain countries, however this paper assumes that a certain design capacity is used to determine outcomes of the model developed, and crush load amounts will only occur in rare and unplanned circumstances.

3 Development of an analytical model

3.1 Description of system

To derive an appropriate cost function for crowding, it is first necessary to develop an expression for the number of people on board a train at a given time along the route, t . The route has n stations, with an equal travel time between each station of τ . Assume n is large enough (or τ small enough) to perform a continuum approximation (Newell 1973), and let the number of people on board the train as it progresses along its route be given as a linear function $x(t) = at + b$. The system has well-defined initial and final conditions; at $t = 0$, the first set of passengers have boarded, and at $t = T$ all passengers must be on board. If trains arrive in time intervals of h , and passengers arrive at a rate of p/n at each station for a total route

demand of p passengers per unit time, the train will depart the first station with ph/n passengers on board. As such, $x(0) = ph/n$. At the end of the route, when the train has travelled its total travel time $t = T = (n - 1)\tau$ the train has visited all boarding stations and there must be ph passengers on board, so $x([n - 1]\tau) = ph$. Thus

$$x(t) = \frac{ph}{n\tau}t + \frac{ph}{n} \quad (1)$$

3.2 Crowding costs

Based on the earlier review of the literature, a discomfort factor $\alpha(t)$ is introduced, which is dependent on the number of passengers on board, $x(t)$. This discomfort factor is not a density, which would require dividing $x(t)$ by an area, but is instead a form of load factor. In this case, however, two models are proposed when considering how passengers' discomfort factor varies with x (see Fig. 1). The first model is a linear relationship, given as

$$\alpha_1(t) = \frac{1}{C}x(t) \quad (2a)$$

for a total train design capacity of C passenger spaces. This approximation is better suited for trains with few seats, where discomfort is more directly related to the raw number of passengers on board, without major differences in seating and standing comfort. This model is referred to as the "linear" model throughout the rest of the paper. Note that the requirements outlined in Sect. 2.2 are met: $\alpha_1(x = 0) = 0$ and $\alpha_1(x = C) = 1$.

In the second model, the discomfort factor grows at an increasing rate with the number of passengers on board. For this analysis, the discomfort factor is proportional to the square of the number of passengers on board. This quadratic relationship with x is given as

$$\alpha_2(t) = \frac{1}{C^2}x(t)^2 \quad (2b)$$

This model is referred to as the "quadratic" model throughout the rest of this paper.

Using (1), the two discomfort models can be written as a function of time, and a cost of discomfort parameter γ_z (with units of money per unit time) is included to convert this discomfort into a passenger cost. Additionally, if passengers are assumed to occupy all train cars with equal passenger load, the total train capacity C can be written as $C = Lc$ for L train cars with a design capacity of c passenger spaces each.

So far, the passenger cost presented has been for one passenger only. To account for all on-board passengers, another factor of $x(t)$ is required, and the cost of on-board crowding becomes

$$z_1(t) = \frac{\gamma_z}{Lc}x(t)^2 \quad (3a)$$

for the linear model, and

$$z_2(t) = \frac{\gamma_\alpha}{(Lc)^2} x(t)^3 \tag{3b}$$

for the quadratic model.

Since the train length cannot vary as it progresses along a route, the optimal train length, once found, remains constant. With this in mind, the average cost of on-board crowding over the entire period T is considered, and multiplied by a factor of T/h to obtain an instantaneous cost of crowding on the entire inbound route during peak periods. These costs are expressed for the linear model as

$$\bar{z}_1 = \frac{T}{hT} \int_0^T z_1(t) dt \tag{4a}$$

and for the quadratic model as

$$\bar{z}_2 = \frac{T}{hT} \int_0^T z_2(t) dt \tag{4b}$$

Both integrals can be solved with the substitution of (1) into (3a) or (3b) for the linear and quadratic model, respectively, and then into (4a) or (4b). The integrals are evaluated to produce

$$\bar{z}_1 = \frac{\gamma_\alpha p^2 h T}{3Lc} \beta_1 \tag{5a}$$

for the linear model, and

$$\bar{z}_2 = \frac{\gamma_\alpha p^3 h^2 T}{(2Lc)^2} \beta_2 \tag{5b}$$

for the quadratic model, where

$$\beta_1 = 1 + \frac{1}{n} + \frac{1}{n^2} \tag{6a}$$

and

$$\beta_2 = 1 + \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} \tag{6b}$$

Note that for large n , β_1 and β_2 rapidly approach 1.

3.3 Dispatching costs

There is a cost of dispatching trains, separated into two terms. Head end costs account for costs such as driver wages and do not vary with train length, and are therefore not included. Car-specific costs, λ_c , account for electricity consumption and maintenance. Since this cost is generally expressed per unit time for a single

train, it is necessary to multiply this term by the factor T/h in order to account for all trains in the system at a given time. The dispatching cost per unit time is therefore

$$z_d = \frac{TL\lambda_c}{h} \quad (7)$$

3.4 Capital costs

Fleet size and platform construction costs play an important role in the determination of future infrastructure developments. These costs are considered as a discounted cash flow when incorporated with a day-to-day cost model. The capital costs z_c are a combination of car purchase costs A_c , and platform construction costs A_p expressed as

$$z_c = NA_c + LA_p \quad (8)$$

The fleet size is related to train length and headway by $N = TL/h$ and thus (8) becomes

$$z_c = \frac{TL}{h}A_c + LA_p \quad (9)$$

The units of A_p are in dollars per unit time per car length. It should be noted that for a route with n platforms, this value must be multiplied by n to obtain the cost along the entire route. The numerical discussion in Sect. 5 applies that reasoning to obtain the values presented in Table 2.

3.5 Objective function

Combining (5a) or (5b), (7), and (9) produces the total cost per unit time, and the objective function

$$Z_1 = \frac{\gamma_\alpha p^2 h T}{3Lc} \beta_1 + \frac{TL\lambda_c}{h} + \frac{TL}{h}A_c + LA_p \quad (10a)$$

for the linear model, and

$$Z_2 = \frac{\gamma_\alpha p^3 h^2 T}{(2Lc)^2} \beta_2 + \frac{TL\lambda_c}{h} + \frac{TL}{h}A_c + LA_p \quad (10b)$$

for the quadratic model.

4 Analysis

4.1 Optimization

Setting the derivatives of (10a) and (10b) to zero with respect to L produces

$$\frac{\partial z}{\partial L} = 0 = -\frac{\gamma_z p^2 h T}{3cL^2} \beta_1 + \frac{T\lambda_c}{h} + \frac{TA_c}{h} + A_p \tag{11a}$$

$$\frac{\partial z}{\partial L} = 0 = -\frac{2\gamma_z p^3 h^2 T}{(2c)^2 L^3} \beta_2 + \frac{T\lambda_c}{h} + \frac{TA_c}{h} + A_p \tag{11b}$$

Note that $\partial^2 z / \partial L^2 > 0$ for both models, confirming that the extreme point is a minimum. Solving (11a) and (11b) for L yields optimal train lengths of

$$L_1^* = ph \sqrt{\beta_1 \frac{\gamma_z}{3c(\lambda_c + A_c + (h/T)A_p)}} \tag{12a}$$

for the linear model, and

$$L_2^* = ph^3 \sqrt{\beta_2 \frac{\gamma_z}{(2c)^2(\lambda_c + A_c + (h/T)A_p)}} \tag{12b}$$

for the quadratic model.

4.2 Discussion

Intuitive and logical analysis of both (12a) and (12b) agrees with expectations. Train length is directly proportional to demand. As the per-car capacity increases, the number of cars required to accommodate the crowding decreases. L does not scale with n since in this scenario the many-to-one demand is “spread out” over all stations. An increase in overall route length is represented by an decrease in the ratio h/T for a fixed headway, and results in longer optimal train and platform length due to the extended time passengers must experience crowding at any level. Since the headway h is understood to be a static, minimized value for this model, the effect of the ratio h/T on the platform cost A_p can be thought of as a ‘dilution’ of the effect of A_p by the increased route length. As the route becomes longer for the same number of stations, the platform cost becomes decreasingly important. High capital costs in A_c and A_p will force train lengths to be shorter.

Aside from cost considerations, there are a number of logistical changes required when using longer trains. Dwell times at stations increase due to longer station clearing times and adjusted acceleration and deceleration rates, however boarding times per door will decrease, countering the above increase in dwell times. Longer trains may not fit on sections of track that shorter ones could occupy, and would have to be stored further away for switching operations. When headways are small during peak periods this can cause additional congestion and increase total travel time, which in turn increases optimal train length.

This model is concerned with optimization of train length when headways are already minimized, and trains are spaced as close together as safety will allow. Train control systems such as signals will regulate the spacing between trains and serve to mitigate the variation in the number of boarding passengers that comes

from variation in headways. With that in mind, however, the model does include the simplifying assumption that no bunching of vehicles occurs.

4.3 Sensitivities

4.3.1 Sensitivity to Headway

Figure 2 describes the response of the optimal train length to headways using data from one of Calgary's C-Train arms. When $h \ll T$, optimal train length approximates a linear function of h . As headways increase, the sensitivity becomes increasingly sub-linear; however, this model is developed with the understanding that headways are kept short, so in both the linear and quadratic model the sensitivity to headways remain relatively constant.

4.3.2 Sensitivity to crowding cost

The cost of crowding parameter γ_α may vary from system to system, and have inaccuracies in its determination. The optimal train length varies by its square root for the linear model, and the cube root for the quadratic model. This allows for some inaccuracy in the determination of γ_α . An explanation of the value chosen for γ_α is given in Sect. 5.1.1, and the optimal train length's response to varied values of γ_α for both models is given in Fig. 3.

4.3.3 Sensitivity to number of stations

The factors β_1 and β_2 both converge rapidly to one as the number of stations n increases. For the smallest practical n of two stations, $\beta_1 = 1.75$ and $\beta_2 = 1.86$, which will increase the optimal train length by a factor of $\sqrt{1.75} = 1.32$ and

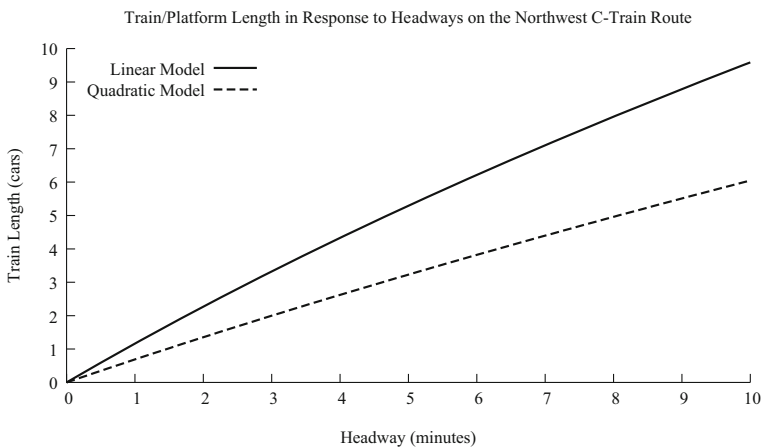


Fig. 2 Two models of optimal train and platform length in response to varied headways

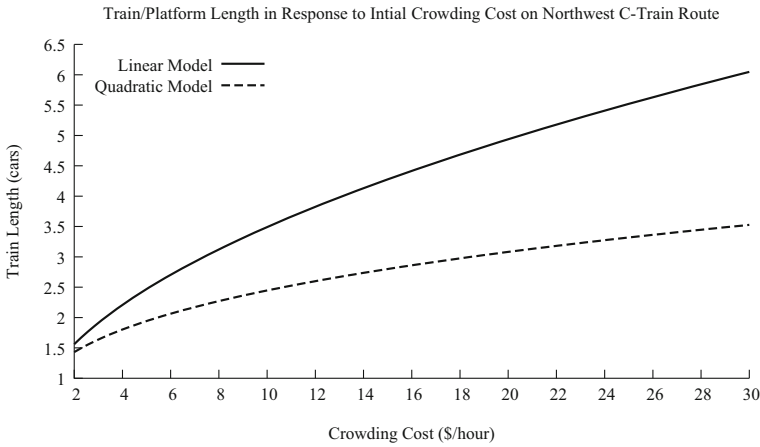


Fig. 3 Two models of optimal train and platform length in response to varied values of time for crowding, γ_x

$\sqrt{1.86} = 1.36$ for the linear and quadratic models, respectively. For $n > 10$, $\beta_1, \beta_2 \in (1, 1.11]$ and so the factor under the square root closely approximates unity, and has little effect on the optimal train length.

4.4 Variability in demand

In both models, the optimal train length given by (12a) and (12b) is a linear function of the demand per hour p , that is

$$L_1^* = Ap \tag{13a}$$

and

$$L_2^* = Bp \tag{13b}$$

where

$$A = h\sqrt{\beta_1 \frac{\gamma_x}{3c(\lambda_c + A_c + (h/T)A_p)}} \tag{14a}$$

and

$$B = h^3\sqrt{\beta_2 \frac{\gamma_x}{(2c)^2(\lambda_c + A_c + (h/T)A_p)}} \tag{14b}$$

If p is allowed to be distributed with a mean of \bar{p} and a variance of σ^2 , L_1^* will be distributed with a mean $A\bar{p}$ and a variance $A^2\sigma^2$. Similarly, L_2^* will be distributed with a mean $B\bar{p}$ and a variance $B^2\sigma^2$. The least presumptive probability distribution function for L_1^* and L_2^* , given that only the mean and variance is known, is the

normal distribution (Jordaan 2005). Using this, L_1^* and L_2^* can be estimated to any required “reliability”. In this instance, setting a reliability would be choosing a probability that the train length chosen is *long enough*. This serves a practical purpose of providing a certain confidence of not overcrowding trains, however given the concave nature of (10a) and (10b) the chosen length will also be sub-optimally long. If there is an understanding that \bar{p} will continue to rise over time, the chosen probability can be seen as the confidence with which a transit system will be able to accommodate an increase in demand.

5 Numerical example

Calgary’s C-Train network is divided into four arms that emanate radially from the CBD, as shown in Fig. 4. These four branches can be analyzed individually for optimal train length to provide an overview of the appropriate train length suggested for this network. Branch-specific values are shown in Table 2, and any general values used are the typical values shown in Table 1 which are introduced in Sect. 5.1

5.1 Sources of data

5.1.1 Crowding value of time

Since γ_c represents the cost of crowding at design capacity, the value of crowding for the highest load factor reported (200 %) was used from Douglas and Karpouzis (2006). The value of \$16.97 in 2003 AUD was converted to 2003 CAD using historical exchange rates (Onada 2015) and then to 2015 CAD using an inflation

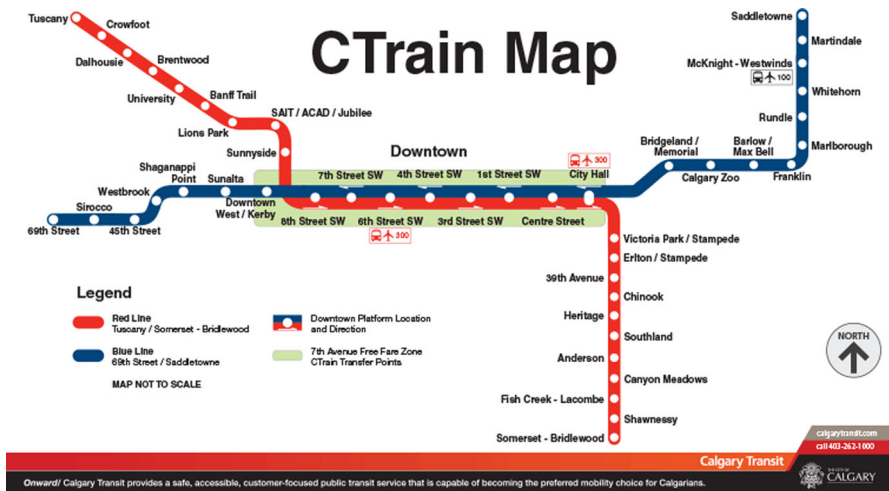


Fig. 4 Calgary transit’s C-Train network as of February 2015 (Calgary Transit 2014)

Table 1 Description of common values across Calgary’s four LRT arms, used in Sect. 5

Symbol	Description	Value
h	Train headway	0.08 h
c	Design capacity per car	162 spaces/car
γ_α	Passenger cost of crowding	\$22.85/h
λ_c	Hourly cost of car operation	\$50/h/car
A_c	Discounted cash flow of car ownership	\$45/h/car

calculator (Bank of Canada 2015) to arrive at a value of crowding of \$22.85. This value can be adjusted based on local variations and surveys conducted by transit agencies or other research.

5.1.2 *Passenger demand and headway*

Peak period uni-directional passenger counts were obtained from a maximum load point study by Calgary Transit (2014). The numbers used in this paper are an average passenger volume inbound to the CBD for each of the four arms for the entire reported peak period (4:01 a.m. to 9:00 a.m.).

At the time of writing, Calgary Transit advertises peak period headways on each arm as averaging 5 min, or 0.083 h. The logistical minimum for the system is 2 min (0.033 h), and for this reason two values for each model are listed in Table 3 the 5 min headway optimal train length and the 2 min headway optimal train length.

5.1.3 *Platform and vehicle costs*

Station platform cost is derived from the estimate by the Alberta Urban Municipalities Association (2007) of the cost of platform extension. At a total cost of \$44 million, this implies a \$1.4 million cost per station to extend the platform. This is discounted at 8 % over a 40-year estimated station life-cycle to obtain an hourly cost of \$13.50 per station. There are additional costs for maintenance and repair of station platforms, however these are not included in the model. The values reported in Table 2 for platform costs have been multiplied by n to report the overall platform cost for each arm individually.

The cost of acquiring light rail vehicles is estimated at \$4 million for this model (Davies 2011; Alberta Urban Municipalities Association 2007). This cost is also

Table 2 Calgary’s four radial LRT lines, with typical peak period values

Symbol	Northwest	South	Northeast	West
n	9	11	10	7
T	0.38	0.40	0.50	0.25
p	3052	4210	2760	1608
A_p	121.50	148.50	135.00	94.50
β_1	1.12	1.10	1.11	1.16
β_2	1.13	1.10	1.12	1.18

discounted at a rate of 8 % over 40 years, at a 20 h service day, to obtain an hourly cost of approximately \$45/h. Again, costs of maintenance and repair, and the labour associated with it are not included.

5.2 Maximum crowding

To ensure that the model does not provide impractical levels of crowding along the route, the model should be checked at the endpoint of the train’s run to ensure that the maximum crowding level is not beyond crush capacity. The parameter $\alpha(t)$ developed in Sect. 3.2 is evaluated at its most crowded point, $t = T = (n - 1)\tau$. At this point the crowding on the train becomes

$$\alpha_1(T) = \frac{ph}{L_1^*c} = \sqrt{\frac{3(\lambda_c + A_c + (h/T)A_p)}{\beta_1\gamma_\alpha c}} \tag{15a}$$

for the linear model, and

$$\alpha_2(T) = \left(\frac{ph}{L_2^*c}\right)^2 = \left(\frac{4(\lambda_c + A_c + (h/T)A_p)}{\beta_2\gamma_\alpha c}\right)^{2/3} \tag{15b}$$

for the quadratic model. Both are independent of demand. The values for the maximum crowding for a 5 and 2 min headway on each C-Train arm are listed in Table 3. Calgary Transit quotes a *theoretical* design capacity of 256 passengers/car which is a crowding factor of 4.26, and a *practical* design capacity of 162 passengers/car with a crowding factor of 2.7. All of the values tabulated fall well below that threshold.

5.3 Numerical results

The current C-Train platform and train length is three cars, however at the time of writing an expansion is nearing completion to increase the train length to four cars along all routes. With the exception of the West arm, the 5-min linear model

Table 3 Numerical output for Calgary’s four radial LRT lines

Model Output	Northwest	South	Northeast	West
L_1^* (5 min)	5.3	7.1	4.8	2.8
L_2^* (5 min)	3.2	4.4	2.9	1.7
L_1^* (2 min)	2.3	3.1	2.1	1.2
L_2^* (2 min)	1.3	1.8	1.2	0.7
L_1^* (1 σ)	6.6	8.9	6.1	3.5
L_2^* (1 σ)	4.0	5.4	3.7	2.1
$\alpha_1(T)$ (5 min)	0.3	0.3	0.3	0.3
$\alpha_2(T)$ (5 min)	0.2	0.2	0.2	0.2
$\alpha_1(T)$ (2 min)	0.3	0.3	0.3	0.3
$\alpha_2(T)$ (2 min)	0.2	0.4	0.4	0.4

suggests expansion to four car trains is sorely needed, while the quadratic model indicates that at current demand the optimal train length falls between three and four cars, suggesting that an expansion to four car trains is a valid move to accommodate increasing demand. Low passenger counts on the West LRT route may be in part due to its relative newness in comparison with the other more established lines, and the model suggests that the current train length is sufficient in the short term. A reduction to 2 min headways, while feasible on exclusive right-of-ways, is difficult due to the two routes sharing the same track in the CBD, and may pose other logistical problems. These shorter headways would reduce the need for longer trains considerably. The maximum crowding levels for both models arrive at numbers that would have all seats filled, and a small number of passengers standing during the trip.

Section 4.4 outlined the response of optimal train length when there is variation in demand. This method was applied to both models for all four C-Train arms, where the demand was assumed to be normally distributed with a mean of p and a coefficient of variation of 25 %. One standard deviation was used to obtain a train length with 84 % reliability. These values are also reported in Table 2 as L_1^* (1σ) and L_2^* (1σ) for the linear and quadratic models respectively. These values could be used for planning purposes to accommodate potential increases in demand, while still accounting for the unpredictability in passenger demand models.

6 Conclusion

A cost-of-crowding model for a many-to-one commuter LRT route was developed in lieu of a traditional passenger waiting cost model, with the understanding that LRT headways are minimized during peak periods. Both a linear and a quadratic relationship between passenger crowding and valuation of travel time were considered. The expression was evaluated to produce an optimal operating train length for the network. Numerical analysis of the resulting formula produced train lengths similar to those in current and planned operation on the C-Train network in Calgary, Canada. Evaluation of maximum crowding on the model under these conditions produced figures that were well within acceptable values.

The developed model provides insight into the effects of crowding on light rail transit systems. The first-principles mathematical development of route progression can be applied to a number of different scenarios and provides a generalized look into LRT modeling. The model demonstrates that crowding can affect operational and planning decisions on such a transit system. Specifically, it was shown that train length should grow directly proportional to the demand of the system, but can be affected in a sub-linear way by capital and operating costs. The model produces feasible results when applied to a real-world scenario.

The model required various assumptions, including the symmetry of the route and passenger distribution, however some variability was included by allowing demand to vary according to a probability distribution. The simplification gained by the many-to-one model could potentially be removed or adjusted in future

treatments of the scenario. Adding the physical characteristics of the cars could allow for a secondary optimization over car length and configuration (and the associated capital costs), which would allow planners to choose rolling stock that is best suited for their situation. Further investigating and understanding the discomfort associated with crowding could increase the realism of the model.

Due to the massive capital investment required for the development of LRT networks, it is important that every possible step is taken to ensure that the developed route is designed to accommodate growth in population and technology. It is hoped that this paper provides groundwork for the careful consideration of train and platform length and its inherent link to on-board crowding during the design or expansion phase of a network. The versatility of LRT technology is its attractiveness, and should not be negated by sub-optimal initial design strategies.

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