Int J Soc Robot (2010) 2: 109–120
DOI 10.1007/s12369-010-0049-8

Adaptive Dynamic Coupling Control of Hybrid Joints of Human-Symbiotic Wheeled Mobile Manipulators with Unmodelled Dynamics

Zhijun Li · Yipeng Yang · Shiping Wang

Accepted: 2 March 2010 / Published online: 9 March 2010

Abstract In this paper, adaptive dynamic coupling control is considered for hybrid joints, which could be switched to either active (actuated) or passive (under-actuated) mode, for human-symbiotic wheeled mobile manipulators with unmodelled dynamics. This social robot can be used in the house, the office, etc, which can flexibly interact with the human and would not injure the people. The constraints for such social robots consist of kinematic constraints and dynamic constraints. Based on Lyapunov synthesis, adaptive coupling control using physical properties of mobile social robot is proposed for passive hybrid joints, which ensures that the system outputs track the given bounded reference signals within a small neighborhood of zero, and guarantee semi-global uniform boundedness of all closed loop signals. The effectiveness of the proposed controls is verified through extensive simulations.

Keywords Mobile manipulators · Hybrid joint · Unmodelled dynamics

1 Introduction

Today, robots are expected to provide various services directly to humans in environments. This situation has led to the idea of teams consisting of humans and robots working cooperatively on the same task [1]. Various names for this type of human-robot cooperation system have emerged including human-friendly robots, personal robots, assistant robots and symbiotic robots. These robots will continue to be employed also in the 21st century to cope with the increase in the elder and handicapped, the decrease in the birth rate and working population, and it will be introduced into non-industrial areas such as homes and offices to make a rich and comfortable life. One of the specific situations in the non-industrial areas is that the robots coexist and help humans in their life environments. The robots, therefore, must be with the capability of human-robot coexistence. They can be called "social robots".

In developing social robots, people expect the human-like, flexible interaction between human and robots [2]. Flexibility in human-adaptable motion can provide important benefits, including improved situation awareness, more safety, more balanced interaction workload, increased user acceptance, and improved overall performance.

A social robot needs to work and move among humans, it is required that special concerns should made on the mechanical compliance and its' control. A social robot should weigh not significantly more than a human, but the mechanical compliance is also a necessity. Most previous passive compliance approaches were based on the robot's structural compliance using special mechanical devices such as springs and dampers. With the passive compliance, the robot hardware could achieve more reliable compliance compared with the active compliance approaches. Therefore, in our previous work [3], a novel compliant passive mechanism—the hybrid joint was proposed for mobile manipulators, which is different from the traditional spring-damper system, as shown in Fig. 1. The hybrid joint has one clutch, when the clutch is released, the link is free, and the passive link is directly controlled by the dynamics coupling, as shown in Fig. 2. In [3], the switching logic of the hy-

This work is supported by Shanghai Pujiang Program under grant No. 08PJ1407000 and Natural Science Foundation of China under Grant Nos. 60804003 and 60935001, and New Faculty Foundation under Grant No. 200802481003.

Z. Li (✉) · Y. Yang · S. Wang
Department of Automation, Shanghai Jiao Tong University, Shanghai, China
e-mail: zjli@sjtu.edu.cn

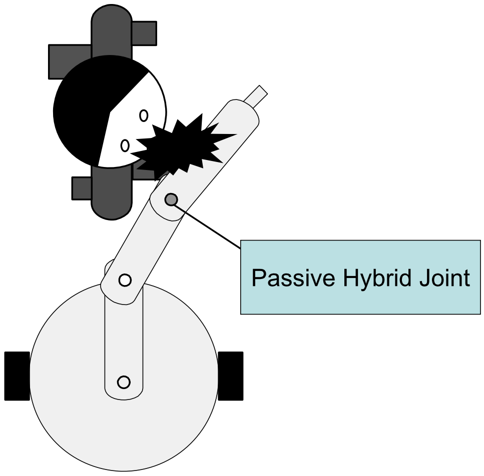

Fig. 1 The social robot

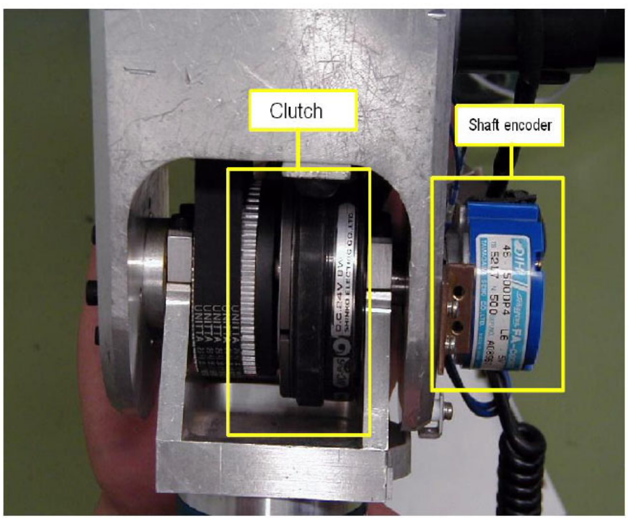

Fig. 2 The hybrid joint

brid joints capable of compliantly adapting to human's motion and force was realized by switching the hybrid joints to the active mode or passive mode as needed. The operational modes of the hybrid joints need to be changed depending on the requirement of a given task. For the human-robot cooperation, i.e., [4], a robot helps human to carry a big or long object, which is demanded in home, office and welfare site, as well as factory. However, the internal force of the carried object is inevitably produced, which would damage the human collaborator. If hybrid joints, especially in the passive mode, are introduced, the internal forces are decreased and human safety is improved. On the other hand, the hybrid joint in the actuated mode is equivalent to the full-actuated robots.

Most previous works have been done to investigate mobile manipulators with full actuated joints, i.e., [5–7], which could be applied to the hybrid joint in the active mode. However, for the hybrid joint in the unactuated mode, that is, the manipulator with the passive hybrid joints mounted on the two-wheeled driven mobile platform, where one or several hybrid joints in the passive mode may appear anywhere in the manipulator, makes it difficult to apply conventional approaches of robotics based on Euler-Lagrange description of the systems, due to simultaneously integrating both kinematic constraints and dynamic constraints. On the other hand, the mobile manipulator with passive hybrid joints is more complex than the mobile wheeled inverted pendulums [8] or pendulums [9, 10], whose dynamic balances in the vertical plane are achieved due to the existent gravity. For these reasons, increasing efforts need to be made towards control design that guarantees stability and robustness for mobile hybrid actuated manipulators. These systems are intrinsically nonlinear and their dynamics will be described by nonlinear differential equations.

In general, nonholonomic constraints include: (i) only kinematic constraints which geometrically restrict the direction of mobility, i.e., wheeled mobile robot [11, 12]; (ii) only dynamic constraints due to dynamic balance at passive degrees of freedom where no force or torque is applied, i.e., the manipulators with passive links [13, 14]; (iii) not only kinematic constraints but also dynamic constraints [3], i.e., mobile manipulator with one passive hybrid joint, shall be investigated in this paper. The common feature of these systems is governed by under-actuated configuration, i.e., the number of control inputs is less than the number of degrees of freedom to be stabilized [15], which makes it difficult to apply the conventional control approaches.

The hybrid joint in the passive mode [3], is a typical example of the second-order nonholonomic system as [13, 14], which can rotate freely and can be indirectly driven by the effect of the dynamic coupling between the active and passive joints. The coordination of multi-manipulators using passive joints was proposed in [16] to decrease the undesired internal forces.

Since the coupling between the actuated and the passive joints depends on the dynamic parameters, and is subject to errors if there are uncertainties on the values of these parameters, as in [13, 16], it is seldom found how to handle the situations in the presence of the unmodelled dynamics and external unknown disturbances. For example, under-actuated manipulators, however, are impossible to control the system with simple control schemes, because of the coupling between the joints.

Since the mobile manipulator is a strong dynamic coupled nonlinear system, there exists strong dynamic coupling between the mobile platform and the manipulator. How to utilize this dynamic coupling to control the manipulator with passive hybrid joints is one of the important issues and has not been investigated until now. On the other hand, mobile manipulators with hybrid joints is characterized by unstable balance and unmodelled dynamics, and is subject to time varying external disturbances, which are generally difficult to model accurately. Therefore, traditional model-based control may not be the ideal approach since it generally works best when the dynamic model is known exactly. The presence of uncertainties and disturbances could disrupt the function of the traditional model-based feedback control and lead to the unstable balance.

Developing a highly accurate model could be much more complex than the design of a control mechanism. Modeling errors always exist, which undermine the dynamic coupling control performance. Based on the previous works [3, 7], different from the known dynamics in [3], in this paper, the dynamic uncertainty has been considered. By developing adaptive motion control with gain adaptation [17] and [18] for two-wheeled driven mobile manipulator with one hybrid joint, we attempt to utilize the dynamic coupling to control the passive hybrid joint with unknown modeling errors and external disturbances.

2 System Description of Mobile Manipulator for Social Environments

The proposed social robotic manipulator using a hybrid joint can passively deform in order to attenuate collision forces using the passive joint and the mobile platform [3]. However, the deformation of the passive joint and the movement of the platform cause the end-effector to be out of position, so we need to propose the control approach for the passive joint. In this section, we first introduce the dynamics model for the proposed social robots.

Lemma 1 [19] *Let $e = H(s)r$ with $H(s)$ representing an $(n \times m)$-dimensional strictly proper exponentially stable transfer function, r and e denoting its input and output, respectively. Then $r \in L_2^m \cap L_\infty^m$ implies that $e, \dot{e} \in L_2^n \cap L_\infty^n$, e is continuous, and $e \to 0$ as $t \to \infty$. If, in addition, $r \to 0$ as $t \to \infty$, then $\dot{e} \to 0$.*

2.1 Dynamics of Mobile Manipulators using Hybrid Joints

Consider an n DOF mobile manipulator, the dynamics can be described as

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f \tag{1}$$

where $q = [q_v^T, q_a^T, q_h^T]^T \in R^n$ with $q_v = [x, y, \theta]^T \in R^{n_v}$ denoting the generalized coordinates for the mobile platform and $q_a \in R^{n_a}$ denoting the coordinates of the active joints, and $q_h \in R^{n_h}$ denoting the coordinates of the hybrid joints, in this paper, we focus on $n_h = 1$, and $n = n_v + n_a + n_h$. The symmetric positive definite inertia matrix $M(q) \in R^{n\times n}$, the Centripetal and Coriolis torques $V(\dot{q}, q) \in R^{n\times n}$, the gravitational torque vector $G(q) \in R^n$, the external disturbances $d(t) \in R^n$, the known input transformation matrix $B(q) \in R^{n\times m}$, the control inputs $\tau \in R^m$ and the generalized constraint forces $f \in R^n$ could be represented as, respectively

$$M(q) = \begin{bmatrix} M_v & M_{va} & M_{vh} \\ M_{av} & M_a & M_{ah} \\ M_{hv} & M_{ha} & M_h \end{bmatrix},$$

$$V(q,\dot{q}) = \begin{bmatrix} V_v & V_{va} & V_{vh} \\ V_{av} & V_a & V_{ah} \\ V_{hv} & V_{ha} & V_h \end{bmatrix}, \qquad f = \begin{bmatrix} J_v^T \lambda_n \\ 0 \\ 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} G_v \\ G_a \\ G_h \end{bmatrix}, \qquad d(t) = \begin{bmatrix} d_v \\ d_a \\ d_h \end{bmatrix}, \qquad B(q)\tau = \begin{bmatrix} \tau_v \\ \tau_a \\ \tau_h \end{bmatrix}$$

where M_v, M_a, M_h describe the inertia matrices for the mobile platform, the active links and the hybrid links, respectively, M_{va}, M_{vh} and M_{ah} are the coupling inertia matrices of the mobile platform, the active links, and the hybrid links; V_v, V_a and V_h denote the Centripetal and Coriolis torques for the mobile platform, the active links and the hybrid links, respectively; V_{va}, V_{vh} and V_{ah} are the coupling Centripetal and Coriolis torques of the mobile platform, the active links and the hybrid links. G_v, G_a and G_h are the gravitational torque vectors for the mobile platform, the active links and the hybrid links, respectively; τ_v is the input vector associated with the left driven wheel and the right driven wheel, respectively; and τ_a are the control input vectors for the active joints of the manipulator; and τ_h are the control input vectors for the hybrid joints of the manipulator, where $\tau_h \neq 0$ if the hybrid joints switched to the actuated mode, while $\tau_h = 0$ if the hybrid joints are in the passive mode; d_v, d_a and d_h denote the external disturbances on the mobile platform, the active links and the hybrid links, respectively; $J_v \in R^{l\times n_v}$ is the kinematic constraint matrix related to nonholonomic constraints; $\lambda_n \in R^l$ is the associated Lagrangian multipliers with the generalized nonholonomic constraints. We assume that the mobile manipulator is subject to known nonholonomic constraints. In actual implementation, we can adopt the methods of producing enough friction between the wheels of the mobile platform and the ground.

2.2 Reduced System

The vehicle subject to nonholonomic constraints can be expressed as

$$J_v\dot{q}_v = 0. \tag{2}$$

The effect of the constraints can be viewed as a restriction of the dynamics on the manifold Ω_n as $\Omega_n = \{(q_v, \dot{q}_v) \mid J_v\dot{q}_v = 0\}$.

Assume that the annihilator of the co-distribution spanned by the covector fields $J_{v1}^T(q_v), \dots, J_{vl}^T(q_v)$ is an $(n_v - l)$-dimensional smooth nonsingular distribution Δ on R^{n_v}. This distribution Δ is spanned by a set of $(n_v - l)$ smooth and linearly independent vector fields $H_1(q_v), \dots, H_{n_v-l}(q_v)$, i.e., $\Delta = \text{span}\{H_1(q_v), \dots, H_{n_v-l}(q_v)\}$, which satisfy, in local coordinates, the following relation [20]

$$H^T(q_v)J_v^T(q_v) = 0 \tag{3}$$

where $H(q_v) = [H_1(q_v), \dots, H_{n_v-l}(q_v)] \in R^{n_v\times(n_v-l)}$. Note that $H^T H$ is of full rank. The constraint (2) implies the existence of vector $\dot{\eta} \in R^{n_v-l}$, such that

$$\dot{q}_v = H(q_v)\dot{\eta}. \tag{4}$$

Considering (4) and its derivative, the dynamics of mobile manipulator can be expressed as

$$\mathcal{M}(\zeta)\ddot{\zeta} + \mathcal{V}(\zeta,\dot{\zeta})\dot{\zeta} + \mathcal{G}(\zeta) + \mathcal{D}(t) = \mathcal{U} \tag{5}$$

where

$$\mathcal{M}(\zeta) = \begin{bmatrix} H^T M_v H & H^T M_{va} & H^T M_{vh} \\ M_{av}H & M_a & M_{ah} \\ M_{hv}H & M_{ha} & M_h \end{bmatrix},$$

$$\zeta = \begin{bmatrix} \eta \\ q_a \\ q_h \end{bmatrix}, \qquad \mathcal{G}(\zeta) = \begin{bmatrix} H^T G_v \\ G_a \\ G_h \end{bmatrix},$$

$$\mathcal{D}(t) = \begin{bmatrix} H^T d_v \\ d_a \\ d_h \end{bmatrix},$$

$$\mathcal{V}(\zeta,\dot{\zeta}) = \begin{bmatrix} H^T M_v\dot{H} + H^T V_v H & H^T V_{va} & H^T V_{vh} \\ M_{av}\dot{H} + V_{av}H & V_a & V_{ah} \\ M_{hv}\dot{H} + V_{hv}H & V_{ha} & V_h \end{bmatrix},$$

$$\mathcal{U} = \begin{bmatrix} \tau_v^T H & \tau_a^T & \tau_h \end{bmatrix}^T.$$

Remark 1 In this paper, we choose $\dot{\zeta} = [\omega, v, \dot{q}_a^T, \dot{q}_h]^T$, and $\dot{\eta} = [\omega, v]^T$, where v is the forward velocity of the mobile platform; and ω is the rotation velocity of the mobile platform.

Considering the property of the above mechanical system, we list the following properties [21] for the active hybrid joints:

Property 1 The inertia matrix $\mathcal{M}(\zeta)$ is symmetric and positive definite.

Property 2 The matrix $\dot{\mathcal{M}} - 2\mathcal{V}$ is skew-symmetric.

2.3 Physical Properties

When the hybrid joints are switched to the active mode, we partition ζ into $\dot{\zeta}_1 = \omega \in R$, $\dot{\zeta}_2 = [v, \dot{q}_a^T]^T$ and $\zeta_3 = q_h \in R$, according to the above partitions, corresponding to the definition of (5), we can rewrite the structure of the dynamics of mobile actuated manipulators as:

$$\mathcal{M}(\zeta) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix},$$

$$\begin{aligned} \mathcal{V}(\zeta,\dot{\zeta})\dot{\zeta} &= \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_{11}\dot{\zeta}_1 + V_{12}\dot{\zeta}_2 + V_{13}\dot{\zeta}_3 \\ V_{21}\dot{\zeta}_1 + V_{22}\dot{\zeta}_2 + V_{23}\dot{\zeta}_3 \\ V_{31}\dot{\zeta}_1 + V_{32}\dot{\zeta}_2 + V_{33}\dot{\zeta}_3 \end{bmatrix}, \\ \mathcal{G} &= \begin{bmatrix} G_1 & G_2^T & G_3 \end{bmatrix}^T, \\ \mathcal{D} &= \begin{bmatrix} d_1 & d_2^T & d_3 \end{bmatrix}^T, \\ \mathcal{U} &= \begin{bmatrix} u_1 & u_2^T & u_3 \end{bmatrix}^T. \end{aligned} \tag{6}$$

Following [14], for the control design of mobile manipulators with hybrid joint, where $u_3 = 0$, it is easy to obtain $n_v + n_a - l > n_h$. It is apparent that even if $n_a = 0$, the above is also achieved.

Assumption 1 In order to make ζ_3 controllable, especially in the passive mode, we assume that matrices M_{13} and M_{31} are larger than zero and M_{11}^{-1} exists.

Remark 2 If M_{13} and M_{31} are equal to zero, while M_{12} and M_{21} are larger than zero, which means that ζ_3 will be coupled with one vector of ζ_2, we only need to exchange ζ_1 with the vector of ζ_2. In this paper, we focus on $M_{13} = M_{31} > 0$.

After some simple manipulations, we can obtain three dynamics as

$$M_{11}\ddot{\zeta}_1 = u_1 - V_1 - G_1 - d_1 - M_{12}\ddot{\zeta}_2 - M_{13}\ddot{\zeta}_3, \tag{7}$$

$$\begin{aligned} &(M_{22} - M_{21}M_{11}^{-1}M_{12})\ddot{\zeta}_2 + (M_{23} - M_{21}M_{11}^{-1}M_{13})\ddot{\zeta}_3 \\ &\quad + V_2 + G_2 + d_2 - M_{21}M_{11}^{-1}V_1 - M_{21}M_{11}^{-1}G_1 \\ &\quad - M_{21}M_{11}^{-1}d_1 = u_2 - M_{21}M_{11}^{-1}u_1, \end{aligned} \tag{8}$$

$$\begin{aligned} &(M_{32} - M_{31}M_{11}^{-1}M_{12})\ddot{\zeta}_2 + (M_{33} - M_{31}M_{11}^{-1}M_{13})\ddot{\zeta}_3 \\ &\quad + V_3 + G_3 + d_3 - M_{31}M_{11}^{-1}V_1 - M_{31}M_{11}^{-1}G_1 \\ &\quad - M_{31}M_{11}^{-1}d_1 = -M_{31}M_{11}^{-1}u_1. \end{aligned} \tag{9}$$

Let $\mathcal{A} = M_{22} - M_{21}M_{11}^{-1}M_{12}$, $\mathcal{B} = M_{23} - M_{21}M_{11}^{-1}M_{13}$, $\mathcal{C} = M_{32} - M_{31}M_{11}^{-1}M_{12}$, $\mathcal{D} = M_{33} - M_{31}M_{11}^{-1}M_{13}$, $\mathcal{E} = (V_{22} - M_{21}M_{11}^{-1}V_{12})\dot{\zeta}_2 + (V_{23} - M_{21}M_{11}^{-1}V_{13})\dot{\zeta}_3$, $\mathcal{F} = (V_{32} - M_{31}M_{11}^{-1}V_{12})\dot{\zeta}_2 + (V_{33} - M_{31}M_{11}^{-1}V_{13})\dot{\zeta}_3$, $\mathcal{H} =$

$(V_{21} - M_{21}M_{11}^{-1}V_{11})\dot{\zeta}_1 + G_2 + d_2 - M_{21}M_{11}^{-1}G_1 - M_{21}M_{11}^{-1}d_1$, $\mathcal{K} = (V_{31} - M_{31}M_{11}^{-1}V_{11})\dot{\zeta}_1 + G_3 + d_3 - M_{31}M_{11}^{-1}G_1 - M_{31}M_{11}^{-1}d_1$. Then, we can rewrite (7), (8) and (9) as

$$M_{11}\ddot{\zeta}_1 = u_1 - V_1 - G_1 - d_1 - M_{12}\ddot{\zeta}_2 - M_{13}\ddot{\zeta}_3, \quad (10)$$

$$\mathcal{A}\ddot{\zeta}_2 + \mathcal{B}\ddot{\zeta}_3 + \mathcal{E} + \mathcal{H} = -M_{21}M_{11}^{-1}u_1 + u_2, \quad (11)$$

$$\mathcal{C}\ddot{\zeta}_2 + \mathcal{D}\ddot{\zeta}_3 + \mathcal{F} + \mathcal{K} = -M_{31}M_{11}^{-1}u_1. \quad (12)$$

Let $\xi = [\zeta_3^T, \zeta_2^T]^T$, considering (5) and (6), equations (11) and (12) become

$$\mathcal{M}_1(\zeta)\ddot{\xi} + \mathcal{V}_1(\zeta, \dot{\zeta})\dot{\xi} + \mathcal{D}_1 = \mathcal{B}_1\mathcal{U}_1 \quad (13)$$

where

$$\mathcal{M}_1(\zeta) = \begin{bmatrix} \mathcal{D} & \mathcal{C} \\ \mathcal{B} & \mathcal{A} \end{bmatrix}, \qquad \mathcal{D}_1 = \begin{bmatrix} \mathcal{K} \\ \mathcal{H} \end{bmatrix},$$

$$\mathcal{B}_1 = \begin{bmatrix} M_{31}M_{11}^{-1} & 0 \\ M_{21}M_{11}^{-1} & I \end{bmatrix}, \qquad \mathcal{U}_1 = \begin{bmatrix} -u_1 \\ u_2 \end{bmatrix},$$

$$\mathcal{V}_1(\zeta, \dot{\zeta}) = \begin{bmatrix} V_{33} - M_{31}M_{11}^{-1}V_{13} & V_{32} - M_{31}M_{11}^{-1}V_{12} \\ V_{23} - M_{21}M_{11}^{-1}V_{13} & V_{22} - M_{21}M_{11}^{-1}V_{12} \end{bmatrix}.$$

Decompose $\mathcal{V}_1(\zeta, \dot{\zeta}) = \hat{\mathcal{V}}_1 + \tilde{\mathcal{V}}_1$ such that

$$\dot{\mathcal{M}}_1 - 2\tilde{\mathcal{V}}_1 = 0. \quad (14)$$

Property 3 The inertia matrix $\mathcal{M}_1$ is symmetric and positive definite.

Remark 3 Since $v, \omega, \dot{q}_h \in R$, $M_{11}, M_{31} \in R$.

Property 4 The eigenvalues of the inertia matrix $\mathcal{B}_1$ are positive.

Remark 4 There exist the minimum and maximum eigenvalues $\lambda_{min}(\mathcal{B}_1)$ and $\lambda_{max}(\mathcal{B}_1)$, such that $\forall x \in R^{n-n_h}$, the known positive parameter b satisfying $b \le \lambda_{min}(\mathcal{B}_1)$.

For the hybrid joints, we give the following assumptions for the actuated and passive modes, respectively,

Assumption 2 (Actuated Hybrid Joints) [22, 23] The desired trajectories $\zeta_{1d}(t), \zeta_{2d}(t), \zeta_{3d}(t)$ and their time derivatives up to the 3rd order are continuously differentiable and bounded for all $t \ge 0$.

Remark 5 Since a lot of works has been done for the full-actuated mobile manipulators, such as [5–7], therefore, in this paper, we focus on the mobile manipulators with passive hybrid joints. Moreover, we give the following assumption.

Assumption 3 For the hybrid joints in the actuated mode, we could adopt the controllers, such as in [7], that ensure the tracking errors for the variables $\zeta_1, \zeta_2, \zeta_3$ from any $(\zeta_j(0), \dot{\zeta}_j(0)) \in \Omega$, where $j = 1, 2, 3$, $\zeta_j, \dot{\zeta}_j$ converge to a manifold Ω_{ad} specified as

$$\Omega_{ad} = \{(\zeta_j, \dot{\zeta}_j) \mid |\zeta_j - \zeta_{jd}| \le \epsilon_{j1}, |\dot{\zeta}_j - \dot{\zeta}_{jd}| \le \epsilon_{j2}\} \quad (15)$$

where $\epsilon_{ji} > 0, i = 1, 2$. Ideally, ϵ_{ji} should be the threshold of measurable noise. At the same time, all the closed loop signals are to be kept bounded.

Assumption 4 (Passive Hybrid Joints) [22, 23] The desired trajectories $\zeta_{2d}(t), \zeta_{3d}(t)$ and their time derivatives up to the 3rd order are continuously differentiable and bounded for all $t \ge 0$.

The control objective for the motion of the system with the unactuated hybrid joint is to design, if possible, controllers that ensure the tracking errors for the variables ζ_2, ζ_3 from any $(\zeta_j(0), \dot{\zeta}_j(0)) \in \Omega$, where $j = 2, 3$, $\zeta_j, \dot{\zeta}_j$ converge to a manifold Ω_{ud} specified as Ω where

$$\Omega_{ud} = \{(\zeta_j, \dot{\zeta}_j) \mid |\zeta_j - \zeta_{jd}| \le \epsilon_{j1}, |\dot{\zeta}_j - \dot{\zeta}_{jd}| \le \epsilon_{j2}\} \quad (16)$$

where $\epsilon_{ji} > 0, i = 1, 2, j = 2, 3$. Ideally, ϵ_{ji} should be the threshold of measurable noise. At the same time, all the closed loop signals are to be kept bounded.

In the following, we can analyze and design the control for each subsystem. For clarity, define the tracking errors and the filtered tracking errors as $e_j = \zeta_j - \zeta_{jd}$, and $r_j = \dot{e}_j + \Lambda_j e_j$ where Λ_j is positive definite, $j = 2, 3$. Then, based on Lemma 1, to study the stability of e_j and $\dot{e}_j$, we only need to study the properties of r_j. In addition, the following computable signals are defined:

$$\dot{\zeta}_{jr} = \dot{\zeta}_{jd} - \Lambda_j e_j, \quad (17)$$

$$\ddot{\zeta}_{jr} = \ddot{\zeta}_{jd} - \Lambda_j \dot{e}_j. \quad (18)$$

3 Adaptive Dynamic Coupling Control

3.1 ζ_2 and ζ_3-subsystems

In reality, the physical model of a system cannot be exactly known, i.e., there exist model uncertainties, which would cause the dynamics uncertainties. In addition, external disturbances may also affect the performance of the system. In this section, we take both factors into consideration to develop adaptive control scheme to deal with uncertainties as well as external disturbances.

Since $\dot{\xi} = \dot{\xi}_r + r$, $\ddot{\xi} = \ddot{\xi}_r + \dot{r}$, (13) become

$$\mathcal{M}_1\dot{r} + \tilde{\mathcal{V}}_1 r = -\mathcal{M}_1\ddot{\xi}_r - \hat{\mathcal{V}}_1 r - \mathcal{V}_1\dot{\xi}_r - \mathcal{D}_1 + \mathcal{B}_1\mathcal{U}_1 \quad (19)$$

where $r=[r_3^T,r_2^T]^T,\ddot{\xi}_r=[\ddot{\zeta}_{3r}^T,\ddot{\zeta}_{2r}^T]^T$. Since $\mathcal{V}_1$ is the matrix function of $\dot{\zeta},\zeta$, and $\mathcal{M}_1$ is the matrix consisting of the submatrices of $\mathcal{M}$, therefore, the following assumption is listed as:

Assumption 5 The nominal $\mathcal{B}_{10}$ for $\mathcal{B}_1$ is a known positive definite matrix, satisfying $\mathcal{B}_1=\mathcal{B}_{10}+\Delta\mathcal{B}$ with an unknown matrix $\Delta\mathcal{B}$.

Assumption 6 There exist some finite positive constants $c_i>0$ $(1\le i\le 8)$ such that $\forall\zeta\in R^{2+n_a+1},\forall\dot{\zeta}\in R^{2+n_a+1}$, $\|\mathcal{M}_1\|\le c_1,\|\mathcal{V}_1\|\le c_2+c_3\|\dot{\zeta}\|,\|\hat{\mathcal{V}}_1\|\le c_4+c_5\|\dot{\zeta}\|$, $\|\mathcal{D}_1\|\le c_6+c_7\|\dot{\zeta}\|,\|\Delta\mathcal{B}\|\le c_8$.

Remark 6 Although we assume some finite positive constants $C=[c_1,\ldots,c_8]^T$ in Assumption 6, these constants are unknown beforehand, therefore, we propose the following adaptive law to approximate them.

Assumption 7 There is time varying positive function ϖ which converges to zero as $t\to\infty$ and satisfies $\lim_{t\to\infty}\int_0^t\varpi(s)ds=\rho<\infty$ with finite constant ρ.

Consider the following control laws and the adaptive law as

$$\mathcal{U}_1=-\mathcal{B}_{10}^{-1}K_Pr-\frac{1}{b}\sum_{i=1}^{8}\frac{r\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i}, \tag{20}$$

$$\dot{\hat{c}}_i=-\sigma_i\hat{c}_i+\frac{\gamma_i\Psi_i^2\|r\|^2}{\|r\|\Psi_i+\delta_i} \tag{21}$$

where K_P is positive definite, $\gamma_i>0$ and $\delta>0$ and $\sigma_i>0$, $1\le i\le 8$, satisfying Assumption 7: $\int_0^\infty\delta_i(s)ds=\rho_{i\delta}<\infty$, $\int_0^\infty\sigma_i(s)ds=\rho_{i\sigma}<\infty$ with the constants $\rho_{i\delta}$ and $\rho_{i\sigma}$. Let $\hat{C}=[\hat{c}_1,\ldots,\hat{c}_8]^T$ and $\Psi=[\|\ddot{\xi}_r\|,\|\dot{\xi}_r\|,\|\dot{\zeta}\|\|\dot{\xi}_r\|,\|r\|,\|r\|\|\dot{\zeta}\|,1,\|\dot{\zeta}\|,\|\mathcal{B}_{10}^{-1}K_Pr\|]^T$, and $\Phi=C^T\Psi$. Note that adaptive gains in sliding mode control laws have been applied in [17], and [18], it is used here in a more generalized sense.

To analyze closed loop stability for the ζ_2 and ζ_3-subsystem, consider the following Lyapunov function candidate

$$\mathbb{V}_1=\frac{1}{2}r^T\mathcal{M}_1r+\frac{1}{2}\tilde{C}^T\Gamma^{-1}\tilde{C} \tag{22}$$

where $\Gamma=\mathrm{diag}[\gamma_1,\ldots,\gamma_8]$, and $\tilde{C}=C-\hat{C}$. Its time derivative is given by

$$\dot{\mathbb{V}}_1=r^T\left(\frac{1}{2}\dot{\mathcal{M}}_1r+\mathcal{M}_1\dot{r}\right)+\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}}. \tag{23}$$

Considering (14), and substituting (19) into (23), and integrating (20), we have

$$\begin{aligned}\dot{\mathbb{V}}_1&=r^T(\mathcal{B}_1\mathcal{U}-\mathcal{M}_1\ddot{\xi}_r-\hat{\mathcal{V}}_1r-\mathcal{V}_1\dot{\xi}_r-\mathcal{D}_1)+\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}}\\
&=r^T\Bigg[(\mathcal{B}_1-\mathcal{B}_{10})(-\mathcal{B}_{10}^{-1}K_Pr)-\frac{1}{b}\mathcal{B}_1\sum_{i=1}^{8}\frac{r\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i}\\
&\quad+\mathcal{B}_{10}\mathcal{B}_{10}^{-1}(-K_Pr)-\mathcal{M}_1\ddot{\xi}_r-\hat{\mathcal{V}}_1r-\mathcal{V}_1\dot{\xi}_r-\mathcal{D}_1\Bigg]\\
&\quad+\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}}\\
&=-r^TK_Pr-\frac{1}{b}r^T\mathcal{B}_1\sum_{i=1}^{8}\frac{r\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i}\\
&\quad-r^T(\mathcal{B}_1-\mathcal{B}_{10})\mathcal{B}_{10}^{-1}K_Pr-r^T\mathcal{M}_1\ddot{\xi}_r\\
&\quad-r^T\mathcal{V}_1\dot{\xi}_r-r^T\hat{\mathcal{V}}_1r-r^T\mathcal{D}_1+\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}}.\end{aligned} \tag{24}$$

Consider Property 4, we have

$$\begin{aligned}\dot{\mathbb{V}}_1&\le-r^TK_Pr-\sum_{i=1}^{8}\frac{r^Tr\hat{c}_i\Psi_i^2}{\Psi_i\|r\|+\delta_i}+\|r\|\|\mathcal{B}_1\\
&\quad-\mathcal{B}_{10}\|\|\mathcal{B}_{10}^{-1}K_Pr\|+\|r\|\|\mathcal{M}_1\|\|\ddot{\xi}_r\|\\
&\quad+\|r\|\|\mathcal{V}_1\|\|\dot{\xi}_r\|+\|\hat{\mathcal{V}}_1\|\|r\|^2+\|r\|\|\mathcal{D}_1\|\\
&\quad+\tilde{C}^T\Gamma^{-1}\dot{\tilde{C}}.\end{aligned} \tag{25}$$

Since $\|\mathcal{M}_1\|\le c_1,\|\mathcal{V}_1\|\le c_2+c_3\|\dot{\zeta}\|,\|\hat{\mathcal{V}}_1\|\le c_4+c_5\|\dot{\zeta}\|$, $\|\mathcal{D}_1\|\le c_6+c_7\|\dot{\zeta}\|,\|\mathcal{B}_1-\mathcal{B}_{10}\|\le c_8$, we have

$$\begin{aligned}\dot{\mathbb{V}}_1&\le-r^TK_Pr+\|r\|\Phi-\sum_{i=1}^{8}\frac{\hat{c}_i\Psi_i^2\|r\|^2}{\|r\|\Psi_i+\delta_i}\\
&\quad+\hat{C}^T\Sigma\Gamma^{-1}\tilde{C}-\sum_{i=1}^{8}\frac{\|r\|^2\tilde{c}_i\Psi_i^2}{\|r\|\Psi_i+\delta_i}\\
&\le-r^TK_Pr+C^T\Delta+\hat{C}^T\Sigma\Gamma^{-1}(C-\hat{C})\\
&\le-r^TK_Pr+C^T\Delta+\frac{1}{4}C^T\Sigma\Gamma^{-1}C\end{aligned} \tag{26}$$

with $\Sigma=\mathrm{diag}[\sigma_1,\ldots,\sigma_8]$, $\Delta=[\delta_1,\ldots,\delta_8]^T$. Therefore, $\dot{\mathbb{V}}_1\le-\lambda_{min}(K_P)\|r\|^2+C^T\Delta+\frac{1}{4}C^T\Sigma\Gamma^{-1}C$. Since $C^T\Delta+\frac{1}{4}C^T\Sigma\Gamma^{-1}C$ is bounded, there exists $t>t_1$, $C^T\Delta+\frac{1}{4}C^T\Sigma\Gamma^{-1}C\le\rho_1$ with the finite constant ρ_1, when $\|r\|\ge\sqrt{\frac{\rho_1}{\lambda_{min}(K_P)}}$, then $\dot{\mathbb{V}}_1\le 0$, from above all, r converges to a small set Ω_1 containing the origin as $t\to\infty$,

$$\Omega_1:\|r\|\le\sqrt{\frac{\rho_1}{\lambda_{min}(K_P)}}. \tag{27}$$

Integrating both sides of the above equation gives

$$\begin{aligned}\mathbb{V}_1(t)-\mathbb{V}_1(0)&\le-\int_0^t\lambda_{min}(K_P)\|r\|^2ds\\
&\quad+\int_0^t\left(C^T\Delta+\frac{1}{4}C^T\Sigma\Gamma^{-1}C\right)ds.\end{aligned} \tag{28}$$

Since C and Γ are constant, $\int_0^\infty \Delta ds = \rho_\delta = [\rho_{1\delta}, \dots, \rho_{8\delta}]^T$, $\int_0^\infty \Sigma ds = \rho_\sigma = \mathrm{diag}[\rho_{1\sigma}, \dots, \rho_{8\sigma}]$, we can rewrite (28) as $\mathbb{V}_1(t) - \mathbb{V}_1(0) \le -\int_0^t \lambda_{min}(K_P)\|r\|^2 ds + C^T \rho_\delta + \frac{1}{4} C^T \rho_\sigma \Gamma^{-1} C < \infty$. Thus $\mathbb{V}_1$ is bounded, which implies that $r \in L_\infty$. From (28), we have $\int_0^t \lambda_{min}(K_P)\|r\|^2 ds \le \mathbb{V}_1(0) - \mathbb{V}_1(t) + C^T \rho_\delta + \frac{1}{4} C^T \rho_\sigma \Gamma^{-1} C$, which leads to $r \in L_2$. From $r_j = \dot{e}_j + \Lambda_j e_j$, it can be obtained that $e_j, \dot{e}_j \in L_\infty$. As we have established $e_j, \dot{e}_j \in L_\infty$, from Assumption 4, we conclude that $\zeta_j, \dot{\zeta}_j, \dot{\xi}_r, \ddot{\xi}_r \in L_\infty$. Therefore, all the signals on the right hand side of (19) are bounded, and we can conclude that $\dot{r}$ and therefore $\ddot{\zeta}_j$ are bounded. Thus, $r \to 0$ as $t \to \infty$ can be obtained. Consequently, we have $e_j \to 0, \dot{e}_j \to 0$ as $t \to \infty$. Since $r, \zeta_j, \dot{\zeta}_j, \zeta_{jr}, \dot{\zeta}_{jr}, \ddot{\zeta}_{jr}$ are all bounded it is easy to conclude that $\mathcal{U}$ is bounded from (20).

All the signals on the left hand side of (19) are bounded, therefore, $\mathcal{H}$ and $\mathcal{K}$ are also bounded, since $\mathcal{H}$ and $\mathcal{K}$ contain $\dot{\zeta}_1$, we can obtain $\dot{\zeta}_1$ is bounded.

3.2 ζ_1-subsystem

Finally, for system (7)–(9) under control laws (20), apparently, the ζ_1-subsystem (7) can be rewritten as

$$\dot{\varphi} = f(\nu, \varphi, \mathcal{U}) \tag{29}$$

where $\varphi = [\zeta_1^T, \dot{\zeta}_1^T]^T$, $\nu = [r^T, \dot{r}^T]^T$, $\mathcal{U} = [u_1, u_2^T]^T$. From ζ_2 and ζ_3 subsystem and their stability, the zero dynamics of ζ_1-subsystem can be addressed as [24]

$$\dot{\varphi} = f(0, \varphi, \mathcal{U}^*(0, \varphi)) \tag{30}$$

where $\mathcal{U}^*$ is the input vector at $\nu = 0$.

Assumption 8 [24] System (7), (8) and (9) is hyperbolically minimum-phase, i.e. zero dynamics (30) is exponentially stable. In addition, assume that the control input $\mathcal{U}$ is designed as a function of the states (ξ, φ) and the reference signal satisfying Assumption 4, and the function $f(\nu, \varphi, \mathcal{U})$ is Lipschitz in ν, i.e., there exists Lipschitz constants L_ν and L_f for $f(\nu, \varphi, \mathcal{U})$ such that

$$\|f(\nu, \varphi, \mathcal{U}) - f(0, \varphi, \mathcal{U}_\varphi)\| \le L_\nu \|\nu\| + L_f \tag{31}$$

where $\mathcal{U}_\varphi = \mathcal{U}^*(0, \varphi)$.

Considering Assumption 8 and the converse theorem of Lyapunov [25], there exists a Lyapunov function $\mathbb{V}_0(\varphi)$ with the following properties

$$\varsigma_2 \|\varphi\|^2 \le \mathbb{V}_0(\varphi) \le \varsigma_1 \|\varphi\|^2, \tag{32}$$

$$\frac{\partial \mathbb{V}_0}{\partial \varphi} f(0, \varphi, \mathcal{U}_\varphi) \le -\lambda_a \|\varphi\|^2, \tag{33}$$

$$\left\| \frac{\partial \mathbb{V}_0}{\partial \varphi} \right\| \le \lambda_b \|\varphi\| \tag{34}$$

where $\varsigma_1, \varsigma_2, \lambda_a$ and λ_b are positive constants.

From the previous stability analysis about the ζ_2-subsystem (8) and the ζ_3-subsystem (9), we know that $\zeta_2, \zeta_3, \dot{\zeta}_2, \dot{\zeta}_3$ are bounded. Accordingly, ν are bounded. We denote the upper bound of ν as

$$\|\nu\| \le \|\nu\|_{max} \tag{35}$$

where $\|\nu\|_{max}$ is a positive constant.

Lemma 2 [24] *For the internal dynamics $\dot{\varphi} = f(\nu, \varphi, \mathcal{U})$ of the system, if Assumptions 4 and 6 are satisfied, then there exist positive constants L_φ and T_0, such that*

$$\|\varphi(t)\| \le L_\varphi, \quad \forall t > T_0. \tag{36}$$

Proof Considering Assumption 8, there exists a Lyapunov function $\mathbb{V}_0(\varphi)$. Differentiating $\mathbb{V}_0(\varphi)$ along (7), (8), (9) yields

$$\begin{aligned}\dot{\mathbb{V}}_0(\varphi) &= \frac{\partial \mathbb{V}_0}{\partial \varphi} f(\nu, \varphi, \mathcal{U}) \\ &= \frac{\partial \mathbb{V}_0}{\partial \varphi} f(0, \varphi, \mathcal{U}_\varphi) \\ &\quad + \frac{\partial \mathbb{V}_0}{\partial \varphi}[f(\nu, \varphi, \mathcal{U}) - f(0, \varphi, \mathcal{U}_\varphi)].\end{aligned} \tag{37}$$

Noting (31)–(34), (37) can be written as

$$\dot{\mathbb{V}}_0(\varphi) \le -\lambda_a \|\varphi\|^2 + \lambda_b \|\varphi\|(L_\nu \|\nu\| + L_f). \tag{38}$$

Noting (35), we have

$$\dot{\mathbb{V}}_0(\varphi) \le -\lambda_a \|\varphi\|^2 + \lambda_b \|\varphi\|(L_\nu \|\nu\|_{max} + L_f). \tag{39}$$

Therefore, $\dot{\mathbb{V}}_0(\varphi) \le 0$, φ converges to a small set containing the origin when

$$\|\varphi\| \ge \frac{\lambda_b}{\lambda_a}(L_\nu \|\nu\|_{max} + L_f). \tag{40}$$

By letting $L_\varphi = \frac{\lambda_b}{\lambda_a}(L_\nu \|\nu\|_{max} + L_f)$, we conclude that there exists a positive constant T_0, such that (36) holds. □

Theorem 1 *Consider the system (7)–(9) with Assumptions 4 and 6, under the action of control laws (20) and adaptation laws (21). For compact set Ω_1, where $(\zeta(0), \dot{\zeta}(0), \hat{C}(0)) \in \Omega$, the tracking error r converges to the compact set Ω_1 defined by (27), and all the signals in the closed loop system are bounded.*

Proof From the results (28), it is clear that the tracking errors r_j converges to the compact set Ω_1 defined by (27). In addition, the signal $\tilde{C}$ is bounded. From Lemma 1, we can know $e_2, \dot{e}_2, e_3, \dot{e}_3$ are also bounded. From the boundedness of ζ_{2d}, ζ_{3d} in Assumption 4, we know that ζ_2, ζ_3 are bounded. Since $\dot{\zeta}_{2d}, \dot{\zeta}_{3d}$ are also bounded, it follows that $\dot{\zeta}_2, \dot{\zeta}_3$ are bounded. With C constant, we know that $\hat{C}$ is also bounded. From Lemma 2, we know that the ζ_1-subsystem (7) is stable, and $\zeta_1, \dot{\zeta}_1$ are bounded. This completes the proof. □

3.3 Zero-Dynamics Stability Analysis

First, we briefly mention the method used to analyze the zero-dynamics stability. By substituting the control inputs $\mathcal{U}$ into the zero-dynamics and after some simplifications, (7) in terms of $\dot{\zeta}_1, \ddot{\zeta}_1$ was obtained. The analysis of the stability of the zero-dynamics ζ_1-subsystem is can be carried out for the nominal system without uncertainty using a linearization approach. Thus, Assumption 8 is true at least locally. In this paper, we will verify, through the simulation results, that the zero dynamics are indeed stable.

3.4 Switching Stability

For the system switching stability between the actuated and passive mode, we give the remark as follows:

Remark 7 Consider the system (5) with the actuated mode (6) and the under-actuated mode (7)–(9), if the system is both stable before and after the switching phase using the Assumption 3 and (20), We may assume that there exists no external impact during the switching. Switching occurs only at well-defined and predetermined time-instances and each of the two employed control methods remains enabled for long enough to allow sufficient tracking results. This implies the time-switched control law remains overall stable.

4 Simulation

In this section, we demonstrate the effectiveness of the proposed control for the social robotics applications. Let us consider a wheeled hybrid-actuated manipulators, shown in Fig. 3.

The following variables have been chosen to describe the social robot (see also Fig. 3):

τ_l, τ_r: the torques of two wheels;
τ_1: the torques of the hybrid-actuated joint, that is, in the passive mode, $\tau_1 = 0$;
θ_l, θ_r: the rotation angle of the left wheel and the right wheel of the mobile platform;
v: the forward velocity of the mobile platform;
θ: the direction angle of the mobile platform;
ω: the rotation velocity of the mobile platform, and $\omega = \dot{\theta}$;
θ_1: the joint angle of the passive hybrid joint;
m_1, m_2: the mass of links of the manipulator;
I_{z1}, I_{z2}: the inertia moment of the link 1 and the link 2;
l_1, l_2: the link length of the manipulator;
r: the radius of the wheels;
$2l$: the distance of the wheels;
d: the distance between the manipulator and the driving center of the mobile base;
m_p: the mass of the mobile platform;
I_p: the inertia moment of the mobile platform;
I_w: the inertia moment of each wheel;
m_w: the mass of each wheel;
g: gravity acceleration;

The mobile hybrid-actuated manipulator is subjected to the following constraint: $\dot{x}\cos\theta - \dot{y}\sin\theta = 0$. Using Lagrangian approach, we can obtain the standard form with $q = [\theta_l, \theta_r, \theta_1]^T$, $\dot{\zeta} = [\dot{\zeta}_1, \dot{\zeta}_2, \dot{\zeta}_3]^T = [v, \dot{\theta}, \dot{\theta}_1]^T$, then we could obtain

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau$$

where

$$M(q) = \begin{bmatrix} p_0 - p_1\cos\theta_1 + p_2 + p_3\sin\theta_1 & q_0 + p_1\cos\theta_1 - p_2 & q_1\sin\theta_1 - q_2\cos\theta_1 + q_3^2 + q_4 \\ q_0 + p_1\cos\theta_1 - p_2 & p_0 - p_1\cos\theta_1 + p_2 - p_3\sin\theta_1 & q_1\sin\theta_1 + q_2\cos\theta_1 - q_3 - q_4 \\ q_1\sin\theta_1 - q_2\cos\theta_1 + q_3 + q_4 & q_1\sin\theta_1 + q_2\cos\theta_1 - q_3 - q_4 & I_{z1} + I_{z2} + m_2 l_2^2 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix},$$

$$p_0 = \frac{1}{4}(m_p + m_1 + m_2 + 2m_w)r^2 + \frac{1}{4}(I_p + 2Iw + m_1 d^2 + m_2 d^2 + 2m_w l^2)r^2 + (I_{z1} + I_{z2})r^2/4,$$

$$p_1 = m_2 l_2 d r^2/2,$$

$$p_2 = m_2 l_2^2 r^2/4,$$

$$p_3 = m_2 l_2 r^2/2,$$

$$p_4 = I_{z1} + I_{z2} + m_2 l_2^2,$$

$$q_0 = (m_p + m_1 + m_2 + 2m_w)r^2/4 - \frac{1}{4}(I_p + 2I_w + m_1 d^2 + m_2 d^2 + 2m_w l^2)r^2 - (I_{z1} + I_{z2})r^2/4,$$

$$q_1 = m_2 l_2 r/2,$$

$$q_2 = m_2 l_2 d r/2,$$

$$q_3 = m_2 l_2 r^2/2,$$

$$q_4 = (I_{z1} + I_{z2})r/2,$$

$$c_{11} = (p_1 \sin\theta_1 + p_3 \cos\theta_1)\dot{\theta}_1/2,$$

$$c_{12} = -p_1 \sin\theta_1 \dot{\theta}_1,$$

$$c_{13} = (p_1 \sin\theta_1 + p_3 \cos\theta_1)\dot{\theta}_r/2 - p_3 \sin\theta_1 \dot{\theta}_l + (q_1 \cos\theta_1 + q_2 \sin\theta_1)\dot{\theta}_1,$$

$$c_{21} = -p_1 \sin\theta_1 \dot{\theta}_1/2,$$

$$c_{22} = (p_1 \sin\theta_1 - p_3 \cos\theta_1)\dot{\theta}_1/2,$$

$$c_{23} = -p_1 \sin\theta_1 \dot{\theta}_r/2 + (p_1 \sin\theta_1 - p_3 \cos\theta_1)\dot{\theta}_l/2 + (q_1 \cos\theta_1 - q_2 \sin\theta_1)\dot{\theta}_1,$$

$$c_{31} = -(p_1 \sin\theta_1 + p_3 \cos\theta_1)\dot{\theta}_r/2 + p_1 \sin\theta_1 \dot{\theta}_l/2,$$

$$c_{32} = p_1 \sin\theta_1 \dot{\theta}_r/2 + (p_3 \cos\theta_1 - p_1 \sin\theta_1)\dot{\theta}_l/2,$$

$$c_{33} = 0,$$

$$G(q) = 0,$$

$$B(q) = \mathrm{diag}[1].$$

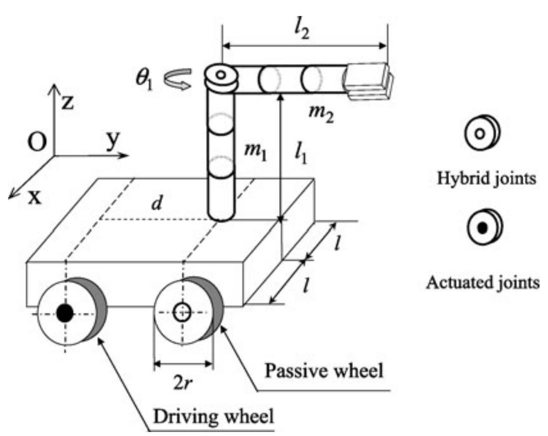

Fig. 3 The mobile hybrid-actuated manipulator in the simulation

In the simulation, we assume the parameters $p_0 = 6.0\ \mathrm{kg\,m^2}$, $p_1 = 1.0\ \mathrm{kg\,m^2}$, $p_2 = 0.5\ \mathrm{kg\,m^2}$, $p_3 = 1.0\ \mathrm{kg\,m^2}$, $p_4 = 2.0\ \mathrm{kg\,m^2}$, $q_0 = 4.0\ \mathrm{kg\,m^2}$, $q_1 = 0.5\ \mathrm{kg\,m^2}$, $q_2 = 1.0\ \mathrm{kg\,m^2}$, $q_3 = 1.0\ \mathrm{kg\,m^2}$, $q_4 = 0.5\ \mathrm{kg\,m^2}$, $d = 1.0$ m, $r = 0.5$ m, $\zeta(0) = [\pi/90, 0.2, 0.0]^T$, $\dot{\zeta}(0) = [0.5, 0.0, -0.5]^T$. The disturbances from environments on the system are introduced as $0.1\sin(t)$, $0.1\sin(t)$ and $0.1\sin(t)$ to the simulation model. Since the accurate model of the practical social robot is difficult to obtain, therefore, in the control design, we assume that we do not know the exact parameters beforehand, while, we know the initial bound of $\hat{C}(0) = [10.0, \ldots, 10.0]^T$. The control gains are selected as $K_P = 1.0$, $\delta_i = \sigma_i = 1/(1+t)^2$, $\gamma_i = 2.0$, $\mathcal{B}_{10} = [\{1.0, 0.0\}, \{0.0, 1.0\}]$, $b = 1.0$. The desired trajectories are chosen as $\zeta_{2d} = 0.3t$ m, $v_d = 0.3$ m/s, $\theta_{1d} = 0$ rad. The ζ_2 and ζ_3 positions tracking are shown in Figs. 4 and 5, respectively, and the corresponding velocities have shown in Fig. 7. From these figures, we know that the ζ_2 and ζ_3 tracking positions have converged to the desired trajectories, and the input torques are shown in Fig. 6, the bounded angular velocity ω is shown in Fig. 8, which satisfies the proposed control objectives in Sect. 2.3. From these figures, even if the nominal parameters of the system, which have 30%–50% model uncertainty, and the initial disturbances boundedness from the environment are unknown, we can get good performance by the proposed control.

5 Conclusion

In this paper, for human-robot coexistence and safety for human-robot interaction, a mobile manipulator with a hybrid joint was proposed in the previous work. In an unex-

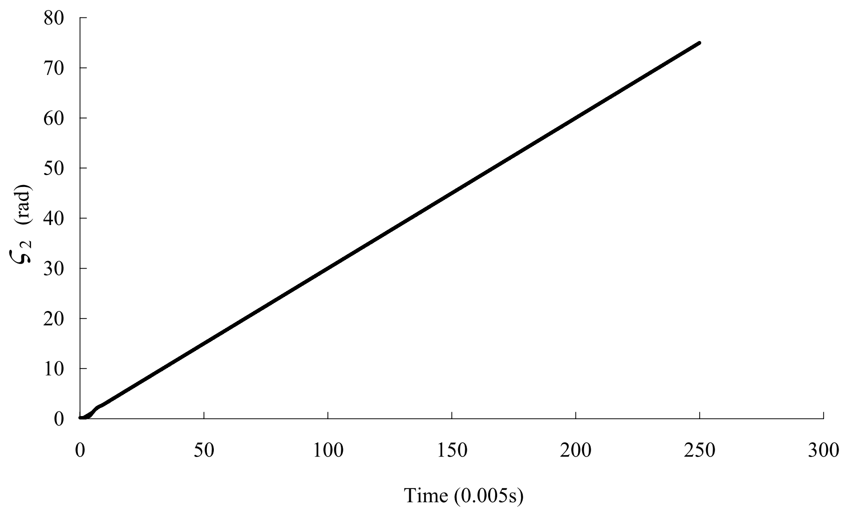

Fig. 4 Tracking the desired position ζ_2

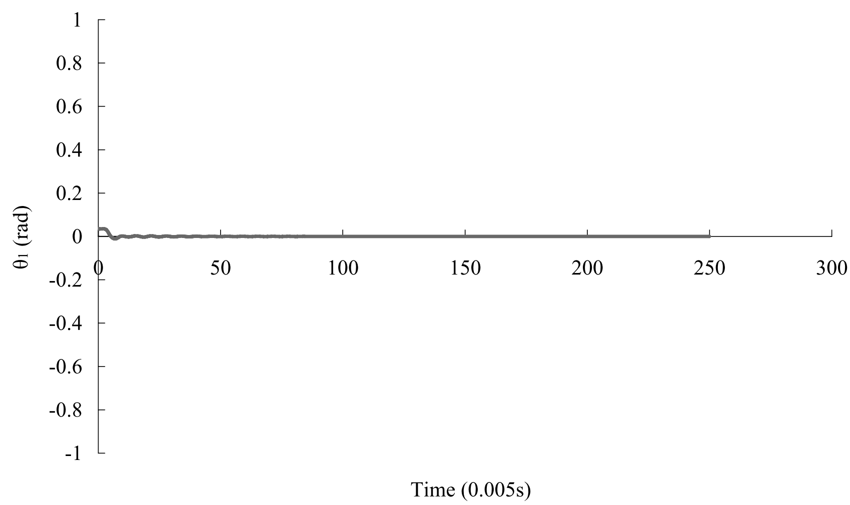

Fig. 5 Tracking the desired position of θ_1

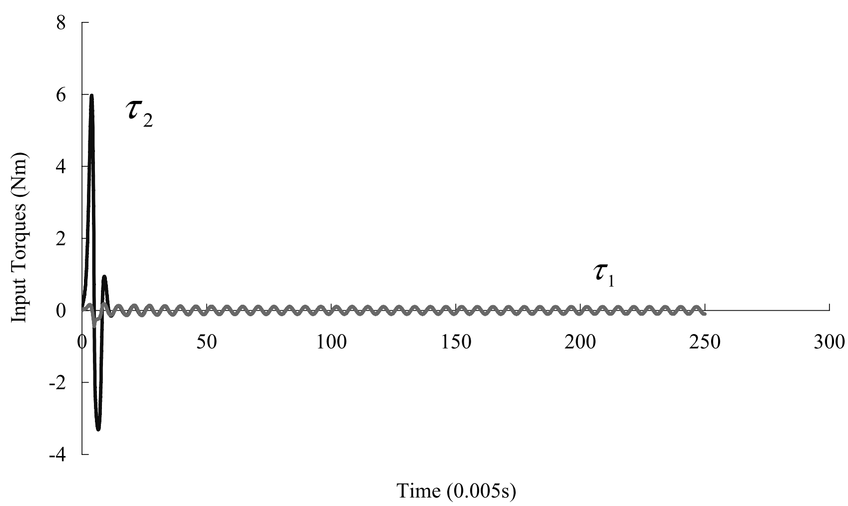

Fig. 6 Input torques

pected collision, this robot will not inflict seriously with humans due to the deformation of the compliant joint and the movement of the platform. However, the joint deformation and the platform movement cause the end-effector to deviate from a desired task. To solve the problem, adaptive dynamic coupling control designs are carried out for dy-

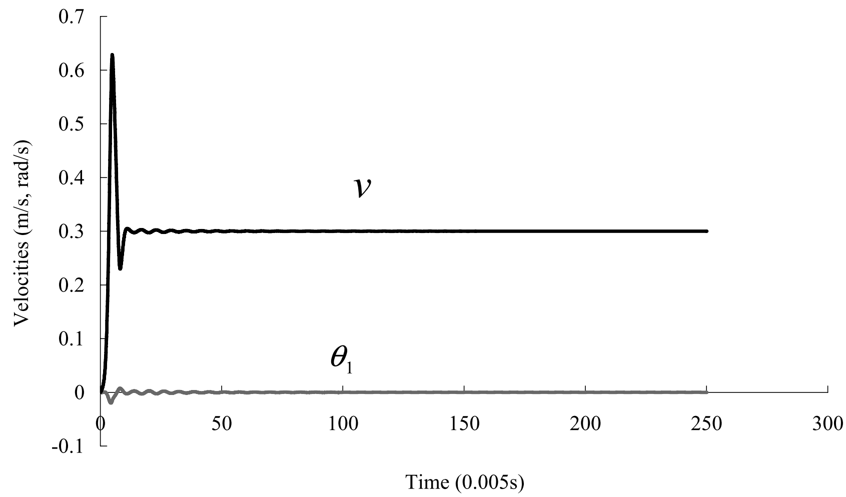

Fig. 7 The produced velocities of v and θ_1

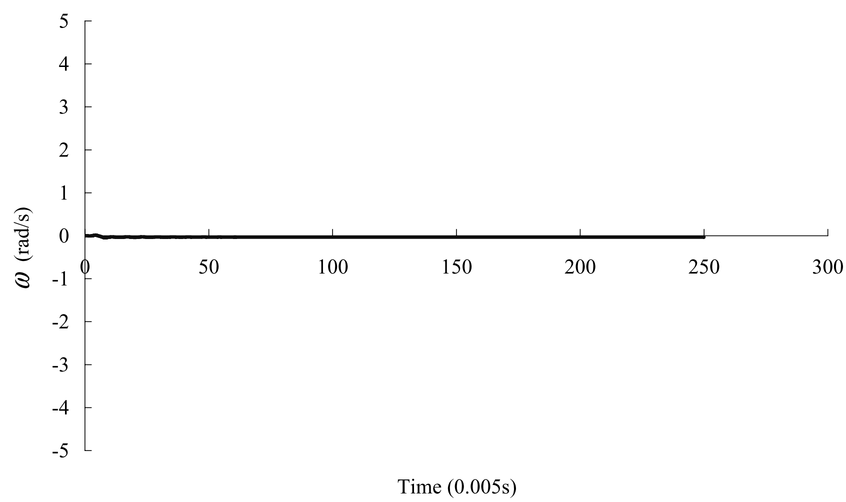

Fig. 8 The bounded ω

namic balance and tracking of desired trajectories of social mobile robot with passive hybrid joints in the presence of unmodelled dynamics, or parametric/functional uncertainties. The controllers are mathematically shown to guarantee semi-globally uniformly bounded stability, and the steady state compact sets to which the closed loop error signals converge are derived. The size of compact sets can be made small through appropriate choice of control design parameters. Simulation results demonstrate that the system is able to track reference signals satisfactorily, with all closed loop signals uniformly bounded.

References

1. Stirling WC, Nokleby MS (2009) Satisficing coordination and social welfare for robotic societies. Int J Soc Robot 1(1):53–69
2. Bartneck C, Kulic DD, Croft E, Zoghbi S (2009) Measurement instruments for the anthropomorphism, animacy, likeability, perceived intelligence, and perceived safety of robots. Int J Soc Robot 1(1):71–81
3. Li Z, Ming A, Xi N, Gu J, Shimojo M (2005) Development of hybrid joints for the complaint arm of human-symbiotic mobile manipulator. Int J Robot Autom 20(4):260–270
4. Takubo T, Arai H, Tanie K (2002) Control of mobile manipulator using a virtual impedance wall. In: Proceedings of IEEE international conference on robotics and automation, pp 3571–3576
5. Lin S, Goldenberg AA (2001) Neural-network control of mobile manipulators. IEEE Trans Neural Netw 12(5):1121–1133
6. Dong W (2002) On trajectory and force tracking control of constrained mobile manipulators with parameter uncertainty. Automatica 38:1475–1484
7. Li Z, Ge SS, Ming A (2007) Adaptive robust motion/force control of holonomic-constrained nonholonomic mobile manipulators. IEEE Trans Syst Man Cybern, Part B, Cybern 37(3):607–617
8. Brooks R, Aryanada L, Edsinger A, Fitzpatrick P, Kemp CC, O'Reilly U, Torres-jara E, Varshavskaya P, Weber J (2004) Sensing and manipulating built-for-human environments. Int J Humanoid Robot 1(1):1–28
9. Spong MW (1995) The swing up control problem for the acrobot. IEEE Control Syst 15:49–55
10. Zhang M, Tarn T (2002) Hybrid control of the pendubot. IEEE/ASME Trans Mechatron 7(1):79–86
11. Ge SS, Wang J, Lee TH, Zhou GY (2001) Adaptive robust stabilization of dynamic nonholonomic chained systems. J Robot Syst 18(3):119–133

12. Ge SS, Wang Z, Lee TH (2003) Adaptive stabilization of uncertain nonholonomic systems by state and output feedback. Automatica 39(8):1451–1460
13. De Luca A, Oriolo G (2002) Trajectory planning and control for planar robots with passive last joint. Int J Robot Res 21(5–6):575–590
14. Bergerman M, Lee C, Xu Y (1995) A dynamic coupling index for underactuated manipulators. J Robot Syst 12(10):693–707
15. Isidori A, Marconi L, Serrani A (2003) Robust autonomous guidance: an internal model approach. Springer, New York
16. Tinos R, Terra MH, Ishihara JY (2006) Motion and force control of cooperative robotic manipulators with passive joints. IEEE Trans Control Syst Technol 14(4):725–734
17. Ertugrul M, Kaynak O, Kerestecioglu F (2000) Gain adaptation in sliding mode control of robotic manipulators. Int J Syst Sci 31(9):1099–1106
18. Alwi H, Edwards C (2008) Detection and fault-tolerant control of a civil aircraft using a sliding-mode-based scheme fault. IEEE Trans Control Syst Technol 16(3):499–510
19. Ge SS, Hang CC, Lee TH, Zhang T (2002) Stable adaptive neural network control. Kluwer Academic, Boston
20. Su C, Stepanenko Y (1994) Robust motion/force control of mechanical systems with classical nonholonimic constraints. IEEE Trans Autom Control 39(3):609–614
21. Ge SS, Lee TH, Harris CJ (1998) Adaptive neural network control of robot manipulators. World Scientific, London
22. Wang ZP, Ge SS, Lee TH (2004) Robust motion/force control of uncertain holonomic/nonholonomic mechanical systems. IEEE/ASME Trans Mechatron 9(1):118–123
23. Chang YC, Chen BS (2000) Robust tracking designs for both holonomic and nonholonomic constrained mechanical systems: adaptive fuzzy approach. IEEE Trans Fuzzy Syst 8:46–66
24. Ge SS, Zhang J (2003) Neural-network control of nonaffine nonlinear system with zero dynamics by state and output feedback. IEEE Trans Neural Netw 14(4):900–918
25. Hahn W (1967) Stability of motion. Springer, New York

Zhijun Li received the Dr. Eng. degree in mechatronics, Shanghai Jiao Tong University, P. R. China, in 2002. From 2003 to 2005, he was a postdoctoral fellow in Department of Mechanical Engineering and Intelligent systems, The University of Electro-Communications, Tokyo, Japan. From 2005 to 2006, he was a research fellow in the Department of Electrical and Computer Engineering, National University of Singapore, and Nanyang Technological University, Singapore. Currently, he is an associate professor in the Department of Automation, Shanghai Jiao Tong University, P. R. China. Dr. Li is IEEE Senior Member and his current research interests are the adaptive/robust control, mobile manipulator, nonholonomic system, etc.

Yipeng Yang was born in China in 1977. He received the Bachelor's degree and Master's degree from the Department of Applied Mathematics and Depart of Control Theory and Engineering in Shanghai Jiao Tong University, China, in the year 2001 and 2003 respectively. In 2008 he received the Ph.D. degree on Operations Research from North Carolina State University, USA. Since then he has been an assistant professor in the Depart of Control Theory and Engineering in Shanghai Jiao Tong University, China. His main research interests include stochastic control, switching system control and optimization.

Shiping Wang received the Bachelor's degree and Master's degree from the Department of Applied Mathematics in Shanghai Jiao Tong University, China. Her main research interests include stochastic system and optimization.