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Fuzzy arithmetic DEA approach for fuzzy multi-objective transportation problem

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Abstract

In this paper, a transportation problem (TP) with fuzzy costs in the presence of multiple and conflicting objectives is investigated. In fact, a fuzzy data envelopment analysis (DEA) approach is proposed to solve the fuzzy multi-objective TP (FMOTP). To this end, each arc in FMOTP will be considered as a decision-making unit (DMU). Next, those objective functions that needs to be maximized will be used to define the outputs of DMU and those that needs to be minimized will be used to define the inputs of DMU. Consequently, two different fuzzy efficiency scores will be derived for each arc by solving fuzzy DEA models. So, a unique fuzzy attribute will be defined for each arc by combining the resulting fuzzy efficiency scores. Therefore, the FMOTP will be converted into a single objective fuzzy TP that can be solved using the standard algorithms. Finally, using a numerical example the proposed approach has been illustrated.

Keywords Fuzzy multi-objective transportation problem · Data envelopment analysis · Fuzzy arithmetic

1 Introduction

The transportation problem (TP) is one of the important topics in the context of operations research which is a specific case of linear programming (LP) problems. The central concept in TP is to determine the minimum total transportation cost of a commodity to satisfy the demand at destinations using the available

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supply at the origins (Ebrahimnejad 2014; Alexiou and Katsavounis 2019; Ebrahimnejad 2016). In a standard TP, in the problem formulation only the cost or profit from each origin to each destination is considered. For example, Ebrahimnejad and Verdegay (2018) investigated a TP which utilizes uncertainty as well as hesitation in predicting the transportation cost, availabilities and demands of products. Additionally, Kundu et al. (2014b) formulated and solved two fixed charge transportation problems with type-2 fuzzy parameters. However, in many real-world transportation problems, there are situations where several objectives are to be considered and optimized at the same time. Such problems are called multi-objective transportation problems (MOTPs). In fact, the MOTP deals with the distribution of goods considering several objectives, such as transportation cost, delivery time and quantity of goods delivered, simultaneously.

Intensive investigations on MOTP have been made by researchers (Amirteimoori 2011; Li and Lai 2000). For example, a bicriteria transportation problem model was presented by Aneja and Nair (1979). In another study, Lee and Moore (1973) studied optimizing transportation problems with multiple objectives. Climaco et al. (1993) developed interactive algorithms to solve MOTP and Kundu et al. (2014a) investigated multi-objective solid transportation problems (MOSTP) under various uncertain environments. By designing an uncertain multi-objective multi-item fixed charge solid transportation problem with budget constraint at each destination, Majumder et al. (2019) presented a profit maximization and time minimization scheme which considers the existence of possible indeterminacy. Based on studies, data envelopment analysis (DEA) approaches appear to be more feasible with the aim of achieving optimal efficient solution to a MOTP. As an example, Zarafat Angiz et al. (2003) proposed a DEA model to evaluate the efficiency of each assignment subject using non-homogeneous costs. In another research, Chen and Lu (2007) extended the assignment problem by considering multiple inputs and outputs and solved it with the help of DEA. They modeled the assignment problem as a classical integer LP problem. Additionally, using the DEA approach, Amirteimoori (2011, 2012) proposed new methods to solve both the transportation problem and the shortest path problem in the case that multiple attributes are considered along the arcs. For the assignment problem with multiple attributes, Shirdel and Mortezaee (2015) solved the multi-criterion assignment problem using additive DEA model.

Due to insufficient data, lack of evidence, etc. data for a MOTP is not always exact; in fact, it can be fuzzy, arbitrary or combination of both. A MOTP which has at least one of its parameters in terms of fuzzy numbers, called fuzzy MOTP (FMOTP). To the best of our knowledge, there is very limited literature available in this area, including the following. In one study, Li and Lai (2000) proposed a fuzzy approach to solve the MOTP, and Ammar and Youness (2005) investigated efficiency of solutions and stability of the MOTP with fuzzy parameters in another study. Kundu et al. modeled and solved a multi-objective multi-item solid transportation problem with fuzzy coefficients for the objectives and constraints in Kundu et al. (2013) and presented the nearest interval approximation for continuous type-2 fuzzy variable in Kundu et al. (2015). Lastly, Kocken et al. (2014) proposed a compensatory fuzzy approach to solve multi-objective linear TP with fuzzy parameters.

In this paper, the DEA approach (Amirteimoori 2011) has been extended to solve FMOTPs. To this end, each arc in a FMOTP has been associated with multiple fuzzy attributes. The idea behind the proposed approach is to convert the multiple fuzzy attributes with each arc into a unique fuzzy attribute; those objective functions that needs to be maximized have been used to define output indexes and those that need to be minimized have been used to define input indexes. Based on this idea, two different fuzzy efficiency scores have been derived for each arc by solving fuzzy DEA (FDEA) models. In the last phase, the resulting fuzzy efficiency scores are combined to get a unique fuzzy attribute. In other words, FMOTP will be converted into a single objective fuzzy TP that can be solved by any standard algorithm for fuzzy TPs.

In recent years, many researchers have formulated fuzzy DEA models to deal with the situations where some of the input and output data are imprecise (Babazadeh et al. 2016; Tavana and Khalili-Damghani 2014). In other words, since the original study by Sengupta (1992a, (1992b), a continuous interest and increased development in fuzzy DEA literature has been appeared. The classical DEA models with fuzzy input and output data can be classified into general groups such as the tolerance approach (Sengupta 1992a; Kahraman and Tolga 1998), the α-level approach (Kao and Liu 2012; Saati et al. 2002; Hatami-Marbini and Saati 2018), the fuzzy ranking approach (Guo and Tanaka 2001; Guo 2009; Leon et al. 2003), the possibility approach (Lertworasirikul et al. 2003; Ruis and Sirvent 2017), the fuzzy arithmetic (Wang et al. 2009; Azar et al. 2016) and finally the multi-objective linear programming (MOLP) approach (Hatami-Marbini et al. 2017). A comprehensive review of the fuzzy DEA methods can be found in Hatami-Marbini et al. (2011). However, the proposed method in this paper is an improved method of the fuzzy arithmetic approach introduced by Wang et al. (2009). This approach evaluates the fuzzy efficiency of decision-making units (DMUs) by using three linear programming problems according to fuzzy arithmetic without making any assumptions or doing too much computational effort.

On such motivation basis, the main contribution of this study are summarized as follows: (1) To the best of our knowledge, this study is the first attempt to solve fuzzy multi-objective transportation problems using fuzzy data envelopment analysis approach; (2) The proposed fuzzy DEA approach provides an efficient way to convert FMOTP into a single objective fuzzy transportation problem; (3) In contrast to the existing multi-objective optimization approaches such as goal programming and fuzzy linear programming, the proposed technique keeps the structure of transportation problem intact for the ease of solution and implementation; and finally (4) The use of the fuzzy DEA approach provides the simultaneous use of maximization and minimization functions in modeling fuzzy multi-objective transportation problem.

The rest of the paper is organized as following. In Sect. 2, some preliminaries about fuzzy set theory, FMOTP and FDEA will be presented. Section 3 will propose the extended DEA approach to solve the FMOTP by converting it into a standard fuzzy TP. The main advantages of the proposed fuzzy DEA approach will be explained in Sect. 4. Following by that, in Sect. 5, a numerical example will be presented to illustrate the proposed approach. Results will be compared with the existing methods in Sect. 6 and finally, Sect. 7 will provide some concluding remarks.

2 Preliminaries

In this section, first some basic definitions and arithmetic operations on fuzzy numbers will be presented (Ebrahimnejad and Verdegay 2018; Ebrahimnejad 2019) and then the mathematical models of FMOTP and FDEA will be formulated (Wang et al. 2009). It is worth mentioning that without loss of generality, in this paper, fuzzy data, assumed to include uncertainty of triangular membership form.

2.1 Basic definitions on fuzzy numbers

Definition 2.1.1 A fuzzy number $\tilde{A}$, denoted by $\tilde{A} = (a_l, a_m, a_u)$, is called a triangular fuzzy number if its membership function is given by relation (1)

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_l}{a_m-a_l} & for \;\; a_l \le x \le a_m \\ \\ \frac{a_u-x}{a_u-a_m} & for \;\; a_m \le x \le a_u \end{cases} \tag{1}$$

Definition 2.1.2 A triangular fuzzy number $\tilde{A} = (a_l, a_m, a_u)$ is said to be a non-negative (positive) triangular fuzzy number if and only if $a_l \ge 0 (a_l > 0)$.

Definition 2.1.3 Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be two positive triangular fuzzy numbers. Consequently, basic fuzzy arithmetic operations on these fuzzy numbers can be defined using the following relation (2):

$$\begin{array}{ll} \text{Addition:} & \tilde{A} + \tilde{B} = (a_l + b_l, a_m + b_m, a_u + b_u) \\ \text{Multiplication:} & \tilde{A} \times \tilde{B} = (a_l.b_l, a_m.b_m, a_u.b_u) \\ \text{Division:} & \tilde{A}/\tilde{B} = (a_l/b_u, a_m/b_m, a_u/b_l). \end{array} \tag{2}$$

2.2 Mathematical model of FMOTP

Fuzzy MOTP (FMOTP) is a special type of multi-objective linear programming problem in which some of the parameters are represented in terms of fuzzy numbers. Suppose that m sources contain different amounts of a commodity which must be distributed into n destinations. Associated with each link (i, j) from source i to destination j, there are h fuzzy attributes $\tilde{c}_{ij}^k (k = 1, \ldots, h)$ for transportation. The problem is to determine a feasible shipping plan from sources to destinations in order to optimize the objective functions. Let s_i be the supply of the commodity at source i and d_j be the demand for the commodity at destination j. Then, the FMOTP with h fuzzy objectives can be formulated as the model (3); where x_{ij} represents the amount of commodity that is being transported from source i to destination j:

$$\begin{aligned} &\text{Optimize } z = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^{1} x_{ij}, \ldots, \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^{h} x_{ij} \right) \\ &\quad s.t. \sum_{j=1}^{n} x_{ij} = s_i \qquad i = 1, \ldots, m, \\ &\quad \sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, \ldots, n, \\ &\quad x_{ij} \geq 0, \qquad i = 1, \ldots, m,\ j = 1, \ldots, n. \end{aligned} \tag{3}$$

Note that the best possible solution for FMOTP (3) would be the ideal solution $\tilde{f} = \left(\tilde{f}_1^*, \tilde{f}_2^*, \ldots, \tilde{f}_h^*\right)$ given by:

$$\begin{aligned} &\tilde{f}_k^{\,*} = Optimize \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^{k} x_{ij} \\ &\quad s.t. \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1, \ldots, m, \\ &\quad \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1, \ldots, n, \\ &\quad x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m,\ j = 1, \ldots, n. \end{aligned} \tag{4}$$

As the objective functions of FMOTP are conflicting, it is impossible to obtain the ideal values. In this case, distance between a feasible solution in the objective space and the ideal solution can be minimized with the aim of seeking a solution as close as possible to the ideal solution for each choice of a norm. In this case, we use the following weighted compromise programming problems based on ℓ_1 norm and ℓ_2 norm:

$$\begin{aligned} &\text{Min} \sum_{k=1}^{h} w_k \left| \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^{k} x_{ij} - \tilde{f}_k^* \right| \\ &s.t. \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1, \ldots, m, \\ &\sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1, \ldots, n, \\ &x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m,\ j = 1, \ldots, n. \end{aligned} \tag{5}$$

$$
\begin{aligned}
& \text{Min} \left[\sum_{k=1}^{h} w_k \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^k x_{ij} - \tilde{f}_k^* \right)^2 \right]^{\frac{1}{2}} \\
& s.t. \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1, \ldots, n, \\
& x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m, j = 1, \ldots, n.
\end{aligned} \tag{6}
$$

2.3 Mathematical model of FDEA

Data envelopment analysis is a mathematical methodology which is used to determine the relative efficiencies of DMUs with multiple inputs and outputs (Banker et al. 1984; Charnes et al. 1978).

Suppose the efficiencies of n homogeneous DMUs is to be evaluated. Each $DMU_j (j = 1, \ldots, n)$ produces s different fuzzy outputs $\tilde{y}_j = (\tilde{y}_{1j}, \ldots, \tilde{y}_{sj})$, using m different fuzzy inputs $\tilde{x}_j = (\tilde{x}_{1j}, \ldots, \tilde{x}_{mj})$. The multiplier form of the CCR model for evaluating the relative efficiency of DMU_p is as the model (7):

$$
\begin{aligned}
& \text{Max } \tilde{\theta}_p = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rp}}{\sum_{i=1}^{m} v_i \tilde{x}_{ip}} \\
& s.t. \ \tilde{\theta}_j = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \leq 1, \qquad j = 1, \ldots, n, \\
& u_r, v_i \geq 0, \qquad r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{aligned} \tag{7}
$$

where $u_r (r = 1, \ldots, s)$ and $v_i (i = 1, \ldots, m)$ are the assigned weights to the outputs and inputs, respectively.

There are many approaches designed to solve fuzzy DEA models which either come from the direct defuzzification of fuzzy DEA models or optimistic and pessimistic DEA models. The former ignores the fact that a fuzzy fractional programming cannot be transformed into a linear programming model in the traditional way that we do for a crisp fractional programming. While the latter requires the solution of a series of linear programming models based on different alpha cut sets; in other words, it requires considerable computational efforts to compute fuzzy efficiencies of DMUs. Considering this limitation, the fuzzy arithmetic approach that has been used to solve fuzzy DEA model in this paper, does not include the aforementioned transformations. According to this approach, the fuzzy DEA model is formulated as LP models without the need of making any assumptions and too much computational effort. Moreover, according to this approach, the exact mathematical form of membership functions related to the fuzzy efficiencies of DMUs can be achieved.

Therefore, to solve the FDEA model (7), a revised version of the fuzzy arithmetic approach proposed by Wang et al. (2009) has been used in this study. To briefly explain this approach, without loss of generality, all input and output data are assumed to be characterized by positive triangular fuzzy numbers. Suppose that the positive triangular fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$ and $\tilde{y}_{ij} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$ represent the input and output data of $DMU_j\,(j = 1, \dots, n)$, respectively, for all $i = 1, \dots, m$ and $r = 1, \dots, s$. Then, according to fuzzy arithmetic, the fuzzy efficiency of DMU_p can be evaluated from the following FDEA model (8):

$$\begin{aligned}
&\text{Maximize } \tilde{\theta}_p \approx (\theta_p^l, \theta_p^m, \theta_p^u) = \left[\frac{\sum_{r=1}^{s} u_r y_{rp}^l}{\sum_{i=1}^{m} v_i x_{ip}^u}, \frac{\sum_{r=1}^{s} u_r y_{rp}^m}{\sum_{i=1}^{m} v_i x_{ip}^m}, \frac{\sum_{r=1}^{s} u_r y_{rp}^u}{\sum_{i=1}^{m} v_i x_{ip}^l}\right] \\
&s.t. \quad \tilde{\theta}_j \approx (\theta_j^l, \theta_j^m, \theta_j^u) = \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^l}{\sum_{i=1}^{m} v_i x_{ij}^u}, \frac{\sum_{r=1}^{s} u_r y_{rj}^m}{\sum_{i=1}^{m} v_i x_{ij}^m}, \frac{\sum_{r=1}^{s} u_r y_{rj}^u}{\sum_{i=1}^{m} v_i x_{ij}^l}\right] \le 1 \quad j = 1, \dots, n, \\
&u_r \ge 0\ r = 1, \dots, s,\ v_i \ge 0\ i = 1, \dots, m.
\end{aligned} \tag{8}$$

Note that as long as θ_j^u is less than or equal to one, θ_j^l and θ_j^m will be automatically satisfied. To determine the fuzzy efficiency of DMU_p, Wang et al. (2009) transformed model (8) into three LP models. However, these three LP models compute the values of θ_p^l, θ_p^m and θ_p^u separately. In other words, each model is computed with a different set of weight (u, v) without considering the other sets. While, as it can be seen in the model (8), $\tilde{\theta}_p \approx (\theta_p^l, \theta_p^m, \theta_p^u)$ must be obtained using a same set of weights (u, v) . Consequently, to avoid this problem, firstly, the value of the θ_p^l is obtained using the model (9) as follows:

$$\begin{aligned}
&\text{Max } \theta_p^l = \frac{\sum_{r=1}^{s} u_r y_{rp}^l}{\sum_{i=1}^{m} v_i x_{ip}^u} \\
&s.t.\ \theta_j^u = \frac{\sum_{r=1}^{s} u_r y_{rj}^u}{\sum_{i=1}^{m} v_i x_{ij}^l} \le 1, \quad j = 1, \dots, n, \\
&u_r \ge 0\ r = 1, \dots, s,\ v_i \ge 0\ i = 1, \dots, m.
\end{aligned} \tag{9}$$

Using the optimal weights of model (9), θ_p^m will be computed from the model (10) as following:

$$\begin{aligned} \text{Max } \theta_p^m &= \frac{\sum_{r=1}^s u_r y_{rp}^m}{\sum_{i=1}^m v_i x_{ip}^m} \\ s.t. \quad & \frac{\sum_{r=1}^s u_r y_{rp}^l}{\sum_{i=1}^m v_i x_{ip}^u} = \theta_p^{l*} \\ \theta_j^u &= \frac{\sum_{r=1}^s u_r y_{rj}^u}{\sum_{i=1}^m v_i x_{ij}^l} \leq 1, \quad j = 1, \ldots, n, \\ & u_r \geq 0 \; r = 1, \ldots, s, \; v_i \geq 0 \; i = 1, \ldots, m. \end{aligned} \tag{10}$$

where, θ_p^{l*} is the optimum value of the model (9). Finally, using the optimal weights of the models (9) and (10), θ_p^u is computed from the model (11) as follows:

$$\begin{aligned} \text{Max } \theta_p^u &= \frac{\sum_{r=1}^s u_r y_{rp}^u}{\sum_{i=1}^m v_i x_{ip}^l} \\ s.t. & \frac{\sum_{r=1}^s u_r y_{rp}^l}{\sum_{i=1}^m v_i x_{ip}^u} = \theta_p^{l*}, \; \frac{\sum_{r=1}^s u_r y_{rp}^m}{\sum_{i=1}^m v_i x_{ip}^m} = \theta_p^{m*} \\ \theta_j^u &= \frac{\sum_{r=1}^s u_r y_{rj}^u}{\sum_{i=1}^m v_i x_{ij}^l} \leq 1, \qquad j = 1, \ldots, n, \\ & u_r \geq 0 \; r = 1, \ldots, s, \; v_i \geq 0 \; i = 1, \ldots, m. \end{aligned} \tag{11}$$

where, θ_p^{l*} and θ_p^{m*} are the optimum values of the models (9) and (10) respectively. In this way, each of the θ_p^{l*}, θ_p^{m*} and θ_p^{u*} is computed with a same set of weights.

Theorem 2.1 *The fuzzy efficiency of DMU_p, derived by solving models (9)–(11), forms a triangular fuzzy number.*

Proof Let $(u^*, v^*) = (u_1^*, \ldots, u_s^*, v_1^*, \ldots, v_m^*)$ be the optimal solution of model (11). The last constrains of this model imply that $(u^*, v^*) \geq 0$. On the other hand, by considering the form of the non-negative fuzzy input $\tilde{x}_{ip} = (x_{ip}^l, x_{ip}^m, x_{ip}^u)$ and fuzzy output $\tilde{y}_{rp} = (y_{rp}^l, y_{rp}^m, y_{rp}^u)$ we will have:

$$\begin{aligned} 0 \leq x_{ip}^l \leq x_{ip}^m \leq x_{ip}^u, \quad i = 1, \ldots, m. \\ 0 \leq y_{rp}^l \leq y_{rp}^m \leq y_{rp}^u, \quad r = 1, \ldots, s. \end{aligned}$$

Therefore,

$$\begin{aligned} 0 \leq \sum_{i=1}^m v_i^* x_{ip}^l \leq \sum_{i=1}^m v_i^* x_{ip}^m \leq \sum_{i=1}^m v_i^* x_{ip}^u. \\ 0 \leq \sum_{r=1}^s u_r^* y_{rp}^l \leq \sum_{r=1}^s u_r^* y_{rp}^m \leq \sum_{r=1}^s u_r^* y_{rp}^u. \end{aligned}$$

Consequently,

$$0 \le \theta_p^{l*} = \frac{\sum_{r=1}^{s} u_r^* y_{rp}^l}{\sum_{i=1}^{m} v_i^* x_{ip}^u} \le \theta_p^{m*} = \frac{\sum_{r=1}^{s} u_r^* y_{rp}^m}{\sum_{i=1}^{m} v_i^* x_{ip}^m} \le \theta_p^{u*} = \frac{\sum_{r=1}^{s} u_r^* y_{rp}^u}{\sum_{i=1}^{m} v_i^* x_{ip}^l}.$$

This means that $\frac{\sum_{r=1}^{s} u_r^* \tilde{y}_{rp}}{\sum_{i=1}^{m} v_i^* \tilde{x}_{ip}} = \left(\theta_p^{l*}, \theta_p^{m*}, \theta_p^{u*}\right)$maintains the form of non-negative triangular fuzzy number and the proof is complete. □

Note that the models (9), (10) and (11) can be easily linearized into the models (12), (13) and (14), respectively:

$$\begin{aligned}
\text{Max} \quad & \theta_p^l = \sum_{r=1}^{s} u_r y_{rp}^l \\
& s.t. \sum_{i=1}^{m} v_i x_{ip}^u = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^u - \sum_{i=1}^{m} v_i x_{ij}^l \le 0, \quad j = 1, \ldots, n, \\
& u_r \ge 0 \; r = 1, \ldots, s, \; v_i \ge 0 \; i = 1, \ldots, m.
\end{aligned} \tag{12}$$

$$\begin{aligned}
\text{Max} \quad & \theta_p^m = \sum_{r=1}^{s} u_r y_{rp}^m \\
& s.t. \sum_{i=1}^{m} v_i x_{ip}^m = 1, \\
& \sum_{r=1}^{s} u_r y_{rp}^l - \theta_p^{l*} \sum_{i=1}^{m} v_i x_{ip}^u = 0, \\
& \sum_{r=1}^{s} u_r y_{rj}^u - \sum_{i=1}^{m} v_i x_{ij}^l \le 0, \quad j = 1, \ldots, n, \\
& u_r \ge 0 \; r = 1, \ldots, s, \; v_i \ge 0 \; i = 1, \ldots, m.
\end{aligned} \tag{13}$$

$$
\begin{aligned}
\text{Max} \quad \theta_p^u = & \sum_{r=1}^{s} u_r y_{rp}^u \\
s.t. & \sum_{i=1}^{m} v_i x_{ip}^l = 1, \\
& \sum_{r=1}^{s} u_r y_{rp}^l - \theta_p^{l*} \sum_{i=1}^{m} v_i x_{ip}^u = 0, \\
& \sum_{r=1}^{s} u_r y_{rp}^m - \theta_p^{m*} \sum_{i=1}^{m} v_i x_{ip}^m = 0, \\
& \sum_{r=1}^{s} u_r y_{rj}^u - \sum_{i=1}^{m} v_i x_{ij}^l \le 0, \;\; j = 1, \dots, n, \\
& u_r \ge 0 \; r = 1, \dots, s, \;\; v_i \ge 0 \; i = 1, \dots, m.
\end{aligned}
\tag{14}
$$

3 Fuzzy efficient transportation plan

In this section, a FDEA approach will be used to explore the optimal solution for FMOTP (7). This FMOTP has h fuzzy objectives that need to be minimized and maximized. Every arc (i, j) is associated with h fuzzy attributes. For arc (i, j) attributes corresponding to the fuzzy objectives that should be minimized are considered as fuzzy input attributes and denoted by $\tilde{X}_{ij} = (\tilde{x}_{ij}^1, \dots, \tilde{x}_{ij}^k)$ in where $\tilde{x}_{ij}^t = (x_{ij}^{t,l}, x_{ij}^{t,m}, x_{ij}^{t,u})$ for all $t = 1, \dots, k$. Similarly, for arc (i, j) attributes corresponding to the fuzzy objectives that should be maximized are considered as fuzzy output attributes and denoted by $\tilde{Y}_{ij} = (\tilde{y}_{ij}^1, \dots, \tilde{y}_{ij}^s)$ in where $\tilde{y}_{ij}^r = (y_{ij}^{r,l}, y_{ij}^{r,m}, y_{ij}^{r,u})$ for all $r = 1, \dots, s$. It means that for every arc (i, j) there exist k fuzzy inputs and s fuzzy outputs where $h = k + s$ (see Fig. 1). The main idea behind the proposed approach is to convert the FMOTP into a single objective fuzzy TP based on DEA approach. To this end, first an interesting plan is proposed to define an efficiency score on every arc by solving FDEA models and then an efficient transportation plan is found by solving a crisp TP maximizing the resulting efficiency scores on arcs.

For every arc (i, j) , two fuzzy efficiency scores are achieved as a criteria for the relative performance of the unit transportation objective from source i $(i = 1, \dots, m)$ to destination j $(j = 1, \dots, n)$. With this in mind, every arc is considered as a DMU. In first step, by considering the source i as a target, the relative performance of the unit transportation objective from source i to destination j is given by solving the FDEA model (15) by changing j:

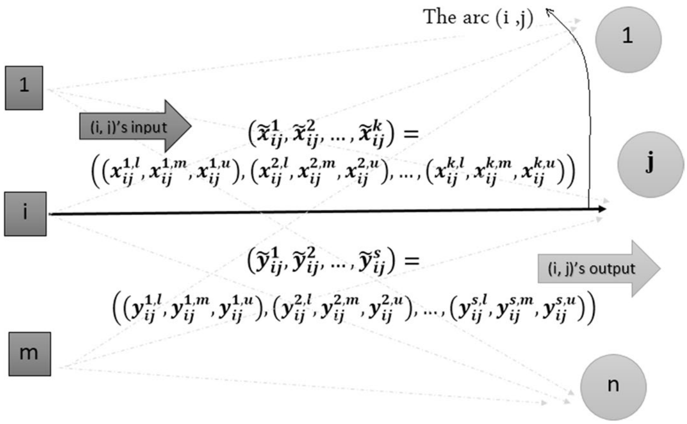

Fig. 1 Arc (i, j) with k fuzzy inputs and s fuzzy outputs

$$
\begin{aligned}
\tilde{E}^{(1*)}_{ij} &= \text{Max } \tilde{E}^{(1)}_{ij} = \frac{\sum_{r=1}^{s} u_r \tilde{y}^r_{ij}}{\sum_{t=1}^{k} v_t \tilde{x}^t_{ij}} \\
s.t.\ \tilde{E}^{(1)}_{if} &= \frac{\sum_{r=1}^{s} u_r \tilde{y}^r_{if}}{\sum_{t=1}^{k} v_t \tilde{x}^t_{if}} \le 1 \quad f = 1, \ldots, n, \\
& u_r \ge 0,\ r = 1, \ldots, s,\ v_t \ge 0,\ t = 1, \ldots, k.
\end{aligned} \tag{15}
$$

Note that model (15) tries to maximize the fuzzy efficiency of the arc (i, j) , provided that the fuzzy efficiency of the arcs (i, f) $(f = 1, \ldots, n)$ cannot exceed 1. To obtain the optimum value of the model (15), i.e. $\tilde{E}^{(1*)}_{ij} = (E^{(1*),l}_{ij}, E^{(1*),m}_{ij}, E^{(1*),u}_{ij})$, each component of the $\tilde{E}^{(1*)}_{ij}$ should be obtained based on the revised form of the fuzzy arithmetic approach mentioned in the previous section. In the other words, firstly, $E^{(1*),l}_{ij}$ should be computed from the model (16) as following:

$$
\begin{aligned}
E^{(1*),l}_{ij} = \ & \text{Max } \frac{\sum_{r=1}^{s} u_r y^{r,l}_{ij}}{\sum_{t=1}^{k} v_t x^{t,u}_{ij}} \\
& s.t.\ \frac{\sum_{r=1}^{s} u_r y^{r,u}_{if}}{\sum_{t=1}^{k} v_t x^{t,l}_{if}} \le 1,\ f = 1, \ldots, n, \\
& u_r \ge 0,\ r = 1, \ldots, s,\ v_t \ge 0,\ t = 1, \ldots, k.
\end{aligned} \tag{16}
$$

In this step, both $E^{(1*),l}_{ij}$, $E^{(1*),m}_{ij}$ can be obtained from the model (17) as it is shown in the following:

$$
\begin{aligned}
E_{ij}^{(1*),m} = \ & \text{Max}\ \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,m}}{\sum_{t=1}^{k} v_t x_{ij}^{t,m}} \\
& s.t.\ \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,l}}{\sum_{t=1}^{k} v_t x_{ij}^{t,u}} = E_{ij}^{(1*),l} \\
& \frac{\sum_{r=1}^{s} u_r y_{if}^{r,u}}{\sum_{t=1}^{k} v_t x_{if}^{t,l}} \le 1,\ \ f = 1, \dots, n, \\
& u_r \ge 0,\ r = 1, \dots, s,\ v_t \ge 0,\ t = 1, \dots, k.
\end{aligned}
\tag{17}
$$

Finally, in order to achieve the value of $E_{ij}^{(1*),u}$, the following model is solved using the optimal weights achieved from the models (16) and (17).

$$
\begin{aligned}
E_{ij}^{(1*),u} = \ & \text{Max}\ \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,u}}{\sum_{t=1}^{k} v_t x_{ij}^{t,l}} \\
& s.t.\ \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,l}}{\sum_{t=1}^{k} v_t x_{ij}^{t,u}} = E_{ij}^{(1*),l},\ \ \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,m}}{\sum_{t=1}^{k} v_t x_{ij}^{t,m}} = E_{ij}^{(1*),m}, \\
& \frac{\sum_{r=1}^{s} u_r y_{if}^{r,u}}{\sum_{t=1}^{k} v_t x_{if}^{t,l}} \le 1,\ \ f = 1, \dots, n, \\
& u_r \ge 0,\ r = 1, \dots, s,\ v_t \ge 0,\ t = 1, \dots, k.
\end{aligned}
\tag{18}
$$

The linear form of the models (16), (17) and (18) can be written as the models (19), (20) and (21), respectively:

$$
\begin{aligned}
E_{ij}^{(1*),l} = \ & \text{Max} \sum_{r=1}^{s} u_r y_{ij}^{r,l} \\
& s.t. \sum_{t=1}^{k} v_t x_{ij}^{t,u} = 1, \\
& \sum_{r=1}^{s} u_r y_{if}^{r,u} - \sum_{t=1}^{k} v_t x_{if}^{t,l} \le 0,\ \ f = 1, \dots, n, \\
& u_r \ge 0,\ r = 1, \dots, s,\ v_t \ge 0,\ t = 1, \dots, k.
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
E_{ij}^{(1*),m} = \quad & \operatorname{Max} \sum_{r=1}^{s} u_r y_{ij}^{r,m} \\
& s.t. \sum_{t=1}^{k} v_t x_{ij}^{t,m} = 1, \\
& \sum_{r=1}^{s} u_r y_{ij}^{r,l} - \sum_{t=1}^{k} v_t x_{ij}^{t,u} E_{ij}^{(1*),l} = 0, \\
& \sum_{r=1}^{s} u_r y_{if}^{r,u} - \sum_{t=1}^{k} v_t x_{if}^{t,l} \leq 0, \ f = 1, \ldots, n, \\
& u_r \geq 0, \ r = 1, \ldots, s, \ v_t \geq 0, \ t = 1, \ldots, k.
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
E_{ij}^{(1*),u} = \quad & \operatorname{Max} \sum_{r=1}^{s} u_r y_{ij}^{r,u} \\
& s.t. \sum_{t=1}^{k} v_t x_{ij}^{t,l} = 1, \\
& \sum_{r=1}^{s} u_r y_{ij}^{r,l} - \sum_{t=1}^{k} v_t x_{ij}^{t,u} E_{ij}^{(1*),l} = 0, \\
& \sum_{r=1}^{s} u_r y_{ij}^{r,m} - \sum_{t=1}^{k} v_t x_{ij}^{t,m} E_{ij}^{(1*),m} = 0, \\
& \sum_{r=1}^{s} u_r y_{if}^{r,u} - \sum_{t=1}^{k} v_t x_{if}^{t,l} \leq 0, \ f = 1, \ldots, n, \\
& u_r \geq 0, \ r = 1, \ldots, s, \ v_t \geq 0, \ t = 1, \ldots, k.
\end{aligned} \tag{21}
$$

In the second step, by considering the destination j as a target, the relative performance of the unit transportation objective from source i to destination j is given by solving the FDEA model (22) by changing i:

$$
\begin{aligned}
\tilde{E}_{ij}^{(2*)} &= \operatorname{Max} \tilde{E}_{ij}^{(2)} = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{ij}^{r}}{\sum_{t=1}^{k} v_t \tilde{x}_{ij}^{t}} \\
s.t.\ \tilde{E}_{fj}^{(2)} &= \frac{\sum_{r=1}^{s} u_r \tilde{y}_{fj}^{r}}{\sum_{t=1}^{k} v_t \tilde{x}_{fj}^{t}} \leq 1 \qquad f = 1, \ldots, m, \\
& u_r \geq 0, \ r = 1, \ldots, s, \ v_t \geq 0, \ t = 1, \ldots, k.
\end{aligned} \tag{22}
$$

Similar to model (15), the optimum value of the model (22), i.e. $\tilde{E}_{ij}^{(2*)} = (E_{ij}^{(2*),l}, E_{ij}^{(2*),m}, E_{ij}^{(2*),u})$, can be obtained from the models (23), (24) and (25), respectively.

$$\begin{aligned} E_{ij}^{(2*),l} = \ & \text{Max} \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,l}}{\sum_{t=1}^{k} v_t x_{ij}^{t,u}} \\ & s.t. \frac{\sum_{r=1}^{s} u_r y_{fj}^{r,u}}{\sum_{t=1}^{k} v_t x_{fj}^{t,l}} \le 1, \ f = 1, \dots, m, \\ & u_r \ge 0, \ r = 1, \dots, s, \ v_t \ge 0, \ t = 1, \dots, k. \end{aligned} \tag{23}$$

Existing difference between the models (16) and (23) seems to be worth noticing. As mentioned, model (16) evaluates the arc (i, j) in comparison with the arcs $(i,f)\, f = 1, \dots, n$, while, model (23) evaluates this arc in compare with the arcs $(f,j)\, f = 1, \dots, m$.

$$\begin{aligned} E_{ij}^{(2*),m} = \ & \text{Max} \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,m}}{\sum_{t=1}^{k} v_t x_{ij}^{t,m}} \\ & s.t. \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,l}}{\sum_{t=1}^{k} v_t x_{ij}^{t,u}} = E_{ij}^{(2*),l} \\ & \frac{\sum_{r=1}^{s} u_r y_{fj}^{r,u}}{\sum_{t=1}^{k} v_i x_{fj}^{t,l}} \le 1, \ f = 1, \dots, m, \\ & u_r \ge 0, \ r = 1, \dots, s, \ v_t \ge 0, \ t = 1, \dots, k. \end{aligned} \tag{24}$$

The same difference which was mentioned between the models (16) and (23) which exists between the models (17) and (24) and also the models (18) and (25).

$$\begin{aligned} E_{ij}^{(2*),u} = \ & \text{Max} \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,u}}{\sum_{t=1}^{k} v_t x_{ij}^{t,l}} \\ & s.t. \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,l}}{\sum_{t=1}^{k} v_t x_{ij}^{t,u}} = E_{ij}^{(2*),l}, \quad \frac{\sum_{r=1}^{s} u_r y_{ij}^{r,m}}{\sum_{t=1}^{k} v_t x_{ij}^{t,m}} = E_{ij}^{(2*),m} \\ & \frac{\sum_{r=1}^{s} u_r y_{fj}^{r,u}}{\sum_{t=1}^{k} v_t x_{fj}^{t,l}} \le 1, \ f = 1, \dots, m, \\ & u_r \ge 0, \ r = 1, \dots, s, \ v_t \ge 0, \ t = 1, \dots, k. \end{aligned} \tag{25}$$

The linear form of the models (23), (24) and (25) can be written as the models (26), (27) and (28), respectively:

$$
\begin{aligned}
E_{ij}^{(2*),l} = \quad & \text{Max} \sum_{r=1}^{s} u_r y_{ij}^{r,l} \\
& s.t. \sum_{t=1}^{k} v_t x_{ij}^{t,u} = 1, \\
& \sum_{r=1}^{s} u_r y_{fj}^{r,u} - \sum_{t=1}^{k} v_t x_{fj}^{t,l} \le 0, \;\; f = 1, \dots, m, \\
& u_r \ge 0, \; r = 1, \dots, s, \; v_t \ge 0, \; t = 1, \dots, k.
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
E_{ij}^{(2*),m} = \quad & \text{Max} \sum_{r=1}^{s} u_r y_{ij}^{r,m} \\
& s.t. \sum_{t=1}^{k} v_t x_{ij}^{t,m} = 1, \\
& \sum_{r=1}^{s} u_r y_{ij}^{r,l} - \sum_{t=1}^{k} v_t x_{ij}^{t,u} E_{ij}^{(2*),l} = 0, \\
& \sum_{r=1}^{s} u_r y_{fj}^{r,u} - \sum_{t=1}^{k} v_t x_{fj}^{t,l} \le 0, \;\; f = 1, \dots, m, \\
& u_r \ge 0, \; r = 1, \dots, s, \; v_t \ge 0, \; t = 1, \dots, k.
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
E_{ij}^{(2*),u} = \quad & \text{Max} \sum_{r=1}^{s} u_r y_{ij}^{r,u} \\
& s.t. \sum_{t=1}^{k} v_t x_{ij}^{t,l} = 1, \\
& \sum_{r=1}^{s} u_r y_{ij}^{r,l} - \sum_{t=1}^{k} v_t x_{ij}^{t,u} E_{ij}^{(2*),l} = 0, \\
& \sum_{r=1}^{s} u_r y_{ij}^{r,m} - \sum_{t=1}^{k} v_t x_{ij}^{t,m} E_{ij}^{(2*),m} = 0, \\
& \sum_{r=1}^{s} u_r y_{fj}^{r,u} - \sum_{t=1}^{k} v_t x_{fj}^{t,l} \le 0, \;\; f = 1, \dots, m, \\
& u_r \ge 0, \; r = 1, \dots, s, \; v_t \ge 0, \; t = 1, \dots, k.
\end{aligned}
\tag{28}
$$

As a result, for every arc (i, j) two fuzzy efficiency scores $\tilde{E}_{ij}^{(1*)}$ and $\tilde{E}_{ij}^{(2*)}$ can be achieved. Now, the mean of $\tilde{E}_{ij}^{(1*)}$ and $\tilde{E}_{ij}^{(2*)}$ are used to derive a new fuzzy efficiency for arc (i, j) as it is shown in relation (29):

$$\tilde{E}_{ij} = \frac{\tilde{E}_{ij}^{(1*)} + \tilde{E}_{ij}^{(2*)}}{2} \tag{29}$$

The h fuzzy attributes is converted into a positive fuzzy attribute $\tilde{E}_{ij}$ in the FMOTP under consideration. In this way, the FMOTP (3) is converted into the fuzzy single objective TP (30):

$$\begin{aligned} \text{Max } z = & \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{E}_{ij} x_{ij} \\ s.t. & \sum_{j=1}^{n} x_{ij} = s_i \qquad i = 1, \dots, m, \\ & \sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, \dots, n, \\ & x_{ij} \geq 0, \qquad i = 1, \dots, m,\ j = 1, \dots, n. \end{aligned} \tag{30}$$

Finally, by solving the model (30), a transportation plan with maximum fuzzy efficiency will be achieved.

Theorem 3.1 *The optimal solution of FTP* (30) *is an efficient solution of FMOTP* (3).

Proof It is worth mentioning that the objective function of FTP (30) is the weighted sum objective function of the FMOTP (3). As the optimal solution of the weighted sum problem is known already with positive weights, it will always be efficient solution of the MOLP under consideration (Ehrgott 2005).

As the values of $\tilde{E}_{ij}$ given in (29) are considered as the weights of the weighted sum problem(3), it is sufficient to show that $\tilde{E}_{ij} > 0$. According to definition of $\tilde{E}_{ij} = \frac{\tilde{E}_{ij}^{(1*)} + \tilde{E}_{ij}^{(2*)}}{2}$ we should prove that $\tilde{E}_{ij}^{(1*)} = \left(\tilde{E}_{ij}^{(1*),l}, \tilde{E}_{ij}^{(1*),m}, \tilde{E}_{ij}^{(1*),u}\right) > 0$ and $\tilde{E}_{ij}^{(2*)} = \left(\tilde{E}_{ij}^{(2*),l}, \tilde{E}_{ij}^{(2*),m}, \tilde{E}_{ij}^{(2*),u}\right) > 0$. To do so, we need to prove $\tilde{E}_{ij}^{(1*),l} > 0$ and $\tilde{E}_{ij}^{(2*),l} > 0$ according to definition of a positive fuzzy number. As $\tilde{E}_{ij}^{(1*),l}$ and $\tilde{E}_{ij}^{(2*),l}$ are the optimal values (efficiency scores) of the crisp input-oriented models of DEA, we have $0 < \tilde{E}_{ij}^{(1*),l} \leq 1$ and $0 < \tilde{E}_{ij}^{(2*),l} \leq 1$. □

Regarding model (30), it should be noted that the only uncertainty is about the precise values of the $\tilde{E}_{ij}$; it means there is no uncertainty about the supply and demand of the product. There are some convenient methods to deal with these kinds of problems. One of them is the proposed method by Kaur and Kumar (2012) based on a ranking function. They modified the existing methods to find the initial basic feasible solution and proposed the generalized fuzzy modified distribution method to find the fuzzy optimal solution with the help of basic feasible solutions. Another method which has been used in current study is the proposed approach by Ebrahimnejad (2014), in which the author pointed out that it is possible to find an optimal solution of the problem without solving any fuzzy TP. To this end, it is enough to use any arbitrary linear ranking function, the rank of each fuzzy number be substituted instead of the corresponding fuzzy number in the fuzzy TP under consideration. In this way, the fuzzy transportation problem is converted into crisp one which is easily solved by the standard transportation algorithms. Their results are independent of the choice of the linear ranking function. In other words, although the obtained solution may be different but the results are still valid for the new solution. As a result, to achieve the crisp form of the fuzzy TP under discussion, the ranking index $\Re(\tilde{A}) = (a_l + 4a_m + a_u)/6$ can be used in which $\tilde{A} = (a_l, a_m, a_u)$ is a triangular fuzzy number.

In this section, another popular technique called fuzzy programming method for solving FTP (30) will be explored. To do so, consider $\tilde{E}_{ij} = \left(E^l_{ij}, E^m_{ij}, E^u_{ij}\right)$, the FTP (30) is simplified to the following multi-objective problem:

$$\begin{aligned} \text{Min } z_1 &= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(E^m_{ij} - E^l_{ij}\right)x_{ij} \\ \text{Max } z_2 &= \sum_{i=1}^{m}\sum_{j=1}^{n} E^m_{ij} x_{ij} \\ \text{Max } z_3 &= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(E^u_{ij} - E^m_{ij}\right)x_{ij} \\ & s.t. \quad Constraints\ of\ Model\ (30). \end{aligned} \tag{31}$$

to solve model (31), the positive ideal solution (PIS) and negative ideal solution (NIS) are obtained by solving the following linear programming problems:

$$\begin{array}{ll} z_1^{PIS} = Min\ \sum_{i=1}^{m}\sum_{j=1}^{n}\left(E^m_{ij} - E^l_{ij}\right)x_{ij} & z_1^{NIS} = Max\ \sum_{i=1}^{m}\sum_{j=1}^{n}\left(E^m_{ij} - E^l_{ij}\right)x_{ij} \\ s.t.\quad Constraints\ of\ Model\ (30). & s.t.\quad Constraints\ of\ Model\ (30). \\ z_2^{PIS} = Maz\ \sum_{i=1}^{m}\sum_{j=1}^{n} E^m_{ij}x_{ij} & z_2^{NIS} = Min\ \sum_{i=1}^{m}\sum_{j=1}^{n} E^m_{ij}x_{ij} \\ s.t.\quad Constraints\ of\ Model\ (30). & s.t.\quad Constraints\ of\ Model\ (30). \\ z_3^{PIS} = Maz\ \sum_{i=1}^{m}\sum_{j=1}^{n}\left(E^u_{ij} - E^m_{ij}\right)x_{ij} & z_3^{NIS} = Min\ \sum_{i=1}^{m}\sum_{j=1}^{n}\left(E^u_{ij} - E^m_{ij}\right)x_{ij} \\ s.t.\quad Constraints\ of\ Model\ (30). & s.t.\quad Constraints\ of\ Model\ (30). \end{array} \tag{32}$$

Hence, the linear membership functions of $\tilde{z}_1$, $\tilde{z}_2$ and $\tilde{z}_3$ are given as below:

$$\mu_{\tilde{z}_1}(z_1) = \begin{cases} 1, & z_1 < z_1^{PIS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}}, & z_1^{PIS} < z_1 < z_1^{NIS} \\ 0, & z_1 > z_1^{NIS} \end{cases} \tag{33}$$

$$\mu_{\tilde{z}_2}(z_2) = \begin{cases} 1, & z_2 > z_2^{PIS} \\ \frac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}}, & z_2^{NIS} < z_2 < z_2^{PIS} \\ 0, & z_2 < z_2^{NIS} \end{cases} \tag{34}$$

$$\mu_{\tilde{z}_3}(z_3) = \begin{cases} 1, & z_3 > z_3^{PIS} \\ \frac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}}, & z_3^{NIS} < z_3 < z_3^{PIS} \\ 0, & z_3 < z_3^{NIS} \end{cases} \tag{35}$$

Finally, according to the fuzzy programming approach, the following model is solved:

$$\begin{aligned} &\text{Max } \alpha \\ &s.t.\ \mu_{\tilde{z}_i}(z_i) \geq \alpha, \qquad i = 1, 2, 3, \\ &\quad \textit{Constraints of Model } (30). \end{aligned} \tag{36}$$

By substituting the membership functions of (33)–(35) into the problem (36), the following problem is obtained:

$$\begin{aligned} &\text{Max } \alpha \\ &s.t.\ z_1 \leq z_1^{NIS} - \left(z_1^{NIS} - z_1^{PIS}\right)\alpha, \\ &\quad z_2 \geq z_2^{NIS} + \left(z_2^{PIS} - z_2^{NIS}\right)\alpha, \\ &\quad z_3 \geq z_3^{NIS} + \left(z_3^{PIS} - z_3^{NIS}\right)\alpha, \\ &\quad \textit{Constraints of Model } (30). \end{aligned} \tag{37}$$

4 Main advantages of the fuzzy DEA approach

The proposed algorithm provides a novel framework of analysis to formalize and solve FMOTP problems with fuzzy transportation costs. The algorithm builds on the fuzzy data envelopment analysis approach allowing us to select the most favorable weights FMOTP is converted into a weighted sum scalarization version. The procedure consists of firstly selecting the most desirable weights of weighted sum problem of FMOTP and then refines such weights through the corresponding fuzzy DEA models defined by the subsequent linear programming problems. To this end, first for each arc two fuzzy efficiencies are derived by solving two fuzzy DEA models and then the resulting fuzzy efficiencies are combined to define a total fuzzy efficiency of that arc. This fuzzy efficiency is considered as the weight of that arc showing how efficient this arc is to transport every unit of commodity from source to destination.

One of the main advantages of the algorithm relies on its computational simplicity, allowing for its implementation within complex network structures ; i.e. the set of linear maximization problems constitutes an intuitive framework to build the DEA structure to be used to solve the FMOTP problem which is defined by the fuzzy arc weights of the network.

The ideal way to solve a fuzzy multi-objective transportation problem as a special case of fuzzy multi-objective linear programming problem would be to optimize the decision maker's fuzzy utility function. However, it is not possible to obtain a mathematical representation of the decision maker's fuzzy utility function in many problems. The fuzzy DEA approach proposed in this paper introduces an efficient plan for the FMOTP without explicit the knowledge of the decision maker's fuzzy utility function.

A frequently discussed method in FMOTP is the weighted sum approach , in which each fuzzy objective function is multiplied by a strictly positive scalar weight. The weighted fuzzy objective functions are summed to form a composite fuzzy objective function. Then, all that is needed is to choose appropriate weights which are determined by decision makers based on their preference and according to relative importance of objective functions. Since in real-world applications, the decision maker has little information about the data of the problem, it is difficult for them to establish the appropriate weights. Therefore, the determined weights of the fuzzy objective functions and the derived solution based on this approach are questionable. However, according to fuzzy DEA approach proposed in this study, the most favorable weights are obtained based on a strong mathematical theory and regarding the data of the problem.

5 Numerical example

To illustrate the proposed approach, the numerical example provided in Amirteimoori (2011) will be used in this section by replacing its original inputs/outputs for every arc with fuzzy ones.

Consider an automobile manufacturer which has assembly plants located in eight towns: A, B, C, D, E, F, G and H. The manufacturer chooses the shipping cost as only objective that should be minimized and the value of shipment along with the profit as objectives that should be maximized to assemble and distribute the cars to major markets in three towns: I, J and K. All appropriate data of the shipping costs, value of shipments and profits are available as triangular fuzzy sets, denoted by ordered triple (a_l, a_m, a_u), which are listed in Table1. The precise quantities of the supplies $s_i\,(i = 1, \dots, m)$ and demands $d_j\,(j = 1, \dots, n)$ are also listed in Table 1.

To solve the current FMOTP, as mentioned in Sect. 3, all three existing fuzzy attributes related to each arc (i, j) should be converted into a positive fuzzy attribute $\tilde{E}_{ij}$. To this end, each arc must be considered as a DMU with one input and two outputs. In fact, the shipping cost will have an input role and the value of the shipment together with the profit play the outputs roles.

By considering the source i as a target, the fuzzy efficiency scores $\tilde{E}_{ij}^{(1*)}$s should be obtained using the model (15). The model corresponds to the arc (A, K) , as an instance, is as the model (38):

Table 1 Data of the example

		I	J	K	s_i
	Shipping cost	(525, 531, 540)	(425, 431, 436)	(390, 395, 410)	
A ⇒	Value of shipment	(3400, 3500, 3550)	(372, 380, 390)	(3800, 3950, 4050)	10
	Profit	(480, 500, 510)	(590, 600, 615)	(380, 400, 410)	
	Shipping cost	(386, 394, 411)	(410, 418, 425)	(505, 512, 520)	
B ⇒	Value of shipment	(2800, 2850, 2940)	(2380, 2395, 2410)	(2500, 2590, 2620)	13
	Profit	(590, 600, 615)	(685, 700, 710)	(480, 485, 500)	
	Shipping cost	(400, 405, 412)	(505, 512, 520)	(400, 412, 420)	
C ⇒	Value of shipment	(305, 310, 320)	(400, 409, 415)	(380, 390, 405)	11
	Profit	(790, 800, 815)	(970, 1000, 1050)	(1000, 1100, 1150)	
	Shipping cost	(350, 355, 365)	(490, 493, 500)	(560, 570, 590)	
D ⇒	Value of shipment	(275, 290, 295)	(370, 385, 400)	(412, 419, 425)	7
	Profit	(700, 705, 715)	(600, 617, 630)	(500, 518, 525)	
	Shipping cost	(294, 299, 304)	(380, 398, 412)	(300, 315, 320)	
E ⇒	Value of shipment	(410, 415, 425)	(500, 512, 520)	(250, 255, 270)	9
	Profit	(580, 585, 595)	(475, 490, 500)	(365, 380, 390)	
	Shipping cost	(314, 319, 324)	(458, 464, 472)	(430, 435, 450)	
F ⇒	Value of shipment	(508, 512, 520)	(202, 215, 220)	(345, 355, 360)	9
	Profit	(480, 488, 496)	(300, 305, 320)	(505, 512, 520)	
	Shipping cost	(614, 619, 625)	(480, 490, 510)	(350, 354, 365)	
G ⇒	Value of shipment	(606, 612, 620)	(500, 510, 520)	(500, 550, 580)	4
	Profit	(614, 619, 625)	(490, 505, 512)	(480, 490, 510)	
	Shipping cost	(450, 456, 462)	(382, 394, 402)	(430, 439, 445)	
H ⇒	Value of shipment	(290, 299, 305)	((500, 512, 520)	(490, 499, 510)	6
	Profit	(595, 601, 605)	(424, 432, 440)	(505, 519, 530)	
	d_j	30	25	14	

$$
\begin{aligned}
E_{AK}^{(1*)} = \ & \text{Max}\,[u_1(3800,3950,4050)+u_2(380,400,410)]\diagup v_1(390,395,410)\\
& s.t.\ [u_1(3800,3950,4050)+u_2(380,400,410)]\diagup v_1(390,395,410)\le 1,\\
& \quad [u_1(3400,3500,3550)+u_2(480,500,510)]\diagup v_1(525,531,540)\le 1,\\
& \quad [u_1(372,380,390)+u_2(590,600,615)]\diagup v_1(425,431,436)\le 1,\\
& \quad u_1,u_2,v_1\ge 0.
\end{aligned}
\tag{38}
$$

To solve the model (38), three linear programming models (39), (40) and (41) should be solved, respectively:

$$
\begin{aligned}
E_{AK}^{(1*),l} = \ & \text{Max}\, 3800u_1 + 380u_2 \\
& s.t.\, 410v_1 = 1, \\
& \quad 4050u_1 + 410u_2 - 390v_1 \le 0, \\
& \quad 390u_1 + 615u_2 - 425v_1 \le 0, \\
& \quad 3550u_1 + 510u_2 - 525v_1 \le 0, \\
& \quad u_1, u_2, v_1 \ge 0.
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
E_{AK}^{(1*),m} = \ & \text{Max}\, 3950u_1 + 400u_2 \\
& s.t.\, 395v_1 = 1, \\
& \quad 3800u_1 + 380u_2 - 410E_{AK}^{(1*),l}v_1 = 0, \\
& \quad 4050u_1 + 410u_2 - 390v_1 \le 0, \\
& \quad 390u_1 + 615u_2 - 425v_1 \le 0, \\
& \quad 3550u_1 + 510u_2 - 525v_1 \le 0, \\
& \quad u_1, u_2, v_1 \ge 0.
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
E_{AK}^{(1*),u} = \ & \text{Max}\, 4050u_1 + 410u_2 \\
& s.t.\, 390v_1 = 1, \\
& \quad 3800u_1 + 380u_2 - 410E_{AK}^{(1*),l}v_1 = 0, \\
& \quad 3950u_1 + 400u_2 - 395E_{AK}^{(1*),m}v_1 = 0, \\
& \quad 4050u_1 + 410u_2 - 390v_1 \le 0, \\
& \quad 390u_1 + 615u_2 - 425v_1 \le 0, \\
& \quad 3550u_1 + 510u_2 - 525v_1 \le 0, \\
& \quad u_1, u_2, v_1 \ge 0.
\end{aligned} \tag{41}
$$

Similarly, the values of the $\tilde{E}_{ij}^{(1*)}$s can be obtained for the other arcs. The optimum value of the models (39), (40) and (41), together with the obtained results for other $\tilde{E}_{ij}^{(1*)}$ can be seen in Table 2.

Similarly, by considering the destination j as a target, the fuzzy efficiency scores $\tilde{E}_{ij}^{(2*)}$s should be computed. Again, considering the arc (A, K) as an example, the $E_{AK}^{(2*)}$ can be obtained from the model (42):

Table 2 Values of the $\tilde{E}_{ij}^{(1*)}$ indecies for the arcs

	I	J	K
A	(0.78,0.82,0.84)	(0.94,0.96,1.00)	(0.89,0.96,1.00)
B	(0.90,0.95,1.00)	(0.94,0.97,1.00)	(0.63,0.66,0.68)
C	(0.73,0.76,0.79)	(0.76,0.79,0.81)	(0.89,0.93,1.00)
D	(0.94,0.97,1.00)	(0.88,0.93,0.97)	(0.83,0.87,0.90)
E	(0.94,0.97,1.00)	(0.84,0.89,0.95)	(0.56,0.60,0.64)
F	(0.95,0.97,1.00)	(0.40,0.42,0.44)	(0.71,0.74,0.77)
G	(0.67,0.69,0.70)	(0.66,0.71,0.73)	(0.90,0.95,1.00)
H	(0.96,0.98,1.00)	(0.91,0.95,1.00)	(0.92,0.96,1.00)

$$
\begin{aligned}
E_{AK}^{(2*)} = \; & \text{Max}\, [u_1(3800,3950,4050) + u_2(380,400,410)] / v_1(390,395,410)] \\
& s.t.\, [u_1(3800,3950,4050) + u_2(380,400,410)] / v_1(390,395,410) \le 1, \\
& \quad [u_1(2500,2590,2620) + u_2(480,485,500)] / v_1(505,512,520) \le 1, \\
& \quad [u_1(380,390,405) + u_2(1000,1100,1150)] / v_1(400,412,420) \le 1, \\
& \quad [u_1(412,419,425) + u_2(500,518,525)] / v_1(560,570,590) \le 1, \\
& \quad [u_1(250,255,270) + u_2(365,380,390)] / v_1(300,315,320) \le 1, \\
& \quad [u_1(345,355,360) + u_2(505,512,520)] / v_1(430,435,450) \le 1, \\
& \quad [u_1(500,550,580) + u_2(480,490,510)] / v_1(350,354,365) \le 1, \\
& \quad [u_1(490,499,510) + u_2(505,519,530)] / v_1(430,439,445) \le 1, \\
& \quad u_1, u_2, v_1 \ge 0
\end{aligned} \tag{42}
$$

According to model (42), values of the $E_{AK}^{(2*),l}$, $E_{AK}^{(2*),m}$ and $E_{AK}^{(2*),u}$ can be obtained by solving the linear programming models (43), (44) and (45), respectively:

$$
\begin{aligned}
E_{AK}^{(2*),l} = \; & \text{Max}\, 3800u_1 + 380u_2 \\
& s.t.\, 410v_1 = 1, \\
& \quad 4050u_1 + 410u_2 - 390v_1 \le 0, \\
& \quad 2620u_1 + 500u_2 - 505v_1 \le 0, \\
& \quad 405u_1 + 1150u_2 - 400v_1 \le 0, \\
& \quad 425u_1 + 525u_2 - 560v_1 \le 0, \\
& \quad 270u_1 + 390u_2 - 300v_1 \le 0, \\
& \quad 360u_1 + 520u_2 - 430v_1 \le 0, \\
& \quad 580u_1 + 510u_2 - 350v_1 \le 0, \\
& \quad 510u_1 + 530u_2 - 430v_1 \le 0, \\
& \quad u_1, u_2, v_1 \ge 0.
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
E_{AK}^{(2*),m} = \ & \text{Max}\, 3950u_1 + 400u_2 \\
& s.t.\ 395v_1 = 1, \\
& \quad 3800u_1 + 380u_2 - 410E_{AK}^{(2*),l}v_1 = 0, \\
& \quad 4050u_1 + 410u_2 - 390v_1 \le 0, \\
& \quad 2620u_1 + 500u_2 - 505v_1 \le 0, \\
& \quad 405u_1 + 1150u_2 - 400v_1 \le 0, \\
& \quad 425u_1 + 525u_2 - 560v_1 \le 0, \\
& \quad 270u_1 + 390u_2 - 300v_1 \le 0, \\
& \quad 360u_1 + 520u_2 - 430v_1 \le 0, \\
& \quad 580u_1 + 510u_2 - 350v_1 \le 0, \\
& \quad 510u_1 + 530u_2 - 430v_1 \le 0, \\
& \quad u_1, u_2, v_1 \ge 0.
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
E_{AK}^{(2*),u} = \ & \text{Max}\, 4050u_1 + 410u_2 \\
& s.t.\ 390v_1 = 1, \\
& \quad 3800u_1 + 380u_2 - 410E_{AK}^{(2*),l}v_1 = 0, \\
& \quad 3950u_1 + 400u_2 - 395E_{AK}^{(2*),m}v_1 = 0, \\
& \quad 4050u_1 + 410u_2 - 390v_1 \le 0, \\
& \quad 2620u_1 + 500u_2 - 505v_1 \le 0, \\
& \quad 405u_1 + 1150u_2 - 400v_1 \le 0, \\
& \quad 425u_1 + 525u_2 - 560v_1 \le 0, \\
& \quad 270u_1 + 390u_2 - 300v_1 \le 0, \\
& \quad 360u_1 + 520u_2 - 430v_1 \le 0, \\
& \quad 580u_1 + 510u_2 - 350v_1 \le 0, \\
& \quad 510u_1 + 530u_2 - 430v_1 \le 0, \\
& \quad u_1, u_2, v_1 \ge 0.
\end{aligned}
\tag{45}
$$

Similarly, the values of the $\tilde{E}_{ij}^{(2*)}$s can be obtained for the other arcs. The results of the models (43), (44) and (45) are shown in Table 3 together with the other $\tilde{E}_{ij}^{(2*)}$s.

Finally, by averaging the fuzzy efficiency scores $\tilde{E}_{ij}^{(1*)}$ and $\tilde{E}_{ij}^{(2*)}$, values of the $\tilde{E}_{ij}$s $(i = 1, \dots, m, j = 1, \dots, n)$ can be seen in Table 4.

By having the fuzzy efficiency scores $\tilde{E}_{ij}$s, all that needs to be done is solving the relevant fuzzy single objective model (46):

Table 3 Values of the $\tilde{E}_{ij}^{(2*)}$ indecies for the arcs

	I	J	K
A	(0.83,0.87,0.89)	(0.66,0.68,0.71)	(0.89,0.96,1.00)
B	(0.90,0.95,1.00)	(0.95,0.97,1.00)	(0.61,0.63,0.65)
C	(0.94,0.97,1.00)	(0.90,0.94,1.00)	(0.83,0.93,1.00)
D	(0.94,0.97,1.00)	(0.59,0.61,0.63)	(0.32,0.34,0.35)
E	(0.94,0.97,1.00)	(0.58,0.62,0.66)	(0.42,0.44,0.48)
F	(0.75,0.77,0.80)	(0.31,0.32,0.34)	(0.41,0.43,0.45)
G	(0.49,0.50,0.51)	(0.48,0.52,0.53)	(0.51,0.55,0.58)
H	(0.63,0.65,0.66)	(0.53,0.56,0.58)	(0.44,0.46,0.48)

Table 4 Values of the $\tilde{E}_{ij}$ indecies for the arcs

	I	J	K
A	(0.80,0.84,0.87)	(0.80,0.82,0.85)	(0.89,0.96,1.00)
B	(0.90,0.95,1.00)	(0.94,0.97,1.00)	(0.62,0.65,0.67)
C	(0.83,0.86,0.89)	(0.83,0.86,0.91)	(0.86,0.93,1.00)
D	(0.94,0.97,1.00)	(0.73,0.77,0.80)	(0.57,0.61,0.63)
E	(0.94,0.97,1.00)	(0.71,0.75,0.80)	(0.49,0.52,0.56)
F	(0.85,0.87,0.90)	(0.36,0.37,0.39)	(0.56,0.59,0.61)
G	(0.58,0.59,0.61)	(0.57,0.61,0.63)	(0.71,0.75,0.79)
H	(0.79,0.81,0.83)	(0.72,0.75,0.79)	(0.68,0.71,0.74)

$$\begin{aligned}
\text{Max}\ & \tilde{E}_{AI}x_{AI} + \tilde{E}_{AJ}x_{AJ} + \tilde{E}_{AK}x_{AK} + \tilde{E}_{BI}x_{BI} + \tilde{E}_{BJ}x_{BJ} + \tilde{E}_{BK}x_{BK} + \tilde{E}_{CI}x_{CI} + \tilde{E}_{CJ}x_{CJ} \\
& + \tilde{E}_{CK}x_{CK} + \tilde{E}_{I}x_{DI} + \tilde{E}_{DJ}x_{DJ} + \tilde{E}_{DK}x_{DK} + \tilde{E}_{EI}x_{EI} + \tilde{E}_{EJ}x_{EJ} + \tilde{E}_{EK}x_{EK} + \tilde{E}_{FI}x_{FI} \\
& + \tilde{E}_{FJ}x_{FJ} + \tilde{E}_{FK}x_{FK} + \tilde{E}_{GI}x_{GI} + \tilde{E}_{GJ}x_{GJ} + \tilde{E}_{GK}x_{GK} + \tilde{E}_{HI}x_{HI} + \tilde{E}_{HJ}x_{HJ} + \tilde{E}_{HK}x_{HK} \\
s.t.\ & x_{AI} + x_{AJ} + x_{AK} = 10, \\
& x_{BI} + x_{BJ} + x_{BK} = 13,\ \ , x_{EI} + x_{EJ} + x_{EK} = 9, \\
& x_{CI} + x_{CJ} + x_{CK} = 11\ , x_{FI} + x_{FJ} + x_{FK} = 9, \\
& x_{DI} + x_{DJ} + x_{DK} = 7,\ x_{GI} + x_{GJ} + x_{GK} = 4, \\
& x_{HI} + x_{HJ} + x_{HK} = 6, \\
& x_{AI} + x_{BI} + x_{CI} + x_{DI} + x_{EI} + x_{FI} + x_{GI} + x_{HI} = 30, \\
& x_{AJ} + x_{BJ} + x_{CJ} + x_{DJ} + x_{EJ} + x_{FJ} + x_{GJ} + x_{HJ} = 25, \\
& x_{AK} + x_{BK} + x_{CK} + x_{DK} + x_{EK} + x_{FK} + x_{GK} + x_{HK} = 14, \\
& x_{ij} \geq 0 \ \text{ for all i,j.}
\end{aligned} \tag{46}$$

To solve the problem (46), as mentioned in the section 3, we follow the approach proposed by Ebrahimnejad (2014). Therefore, each fuzzy efficiency should be replaceed with its corresponding rank obtained from the linear ranking function $\Re(\tilde{A}) = (a_l + 4a_m + a_u)/6$ in which $\tilde{A} = (a_l, a_m, a_u)$ is a triangular fuzzy number. The related results are shown in Table 5.

Table 5 The crisp values of the efficiency scores $\tilde{E}_{ij}$s

	$\Re(\tilde{E}_{ij}) = (\tilde{E}^l_{ij} + 4\tilde{E}^m_{ij} + \tilde{E}^u_{ij})/6$		
	I	J	K
A	0.839347	0.822687	0.957496
B	0.952678	0.97182	0.644977
C	0.862361	0.865448	0.931813
D	0.971213	0.767434	0.604772
E	0.968221	0.754571	0.522251
F	0.871747	0.37111	0.587937
G	0.594691	0.608342	0.749353
H	0.813214	0.755946	0.708504

According to Table 5, the crisp form of the fuzzy single objective model will be as follows:

$$\begin{aligned}
\text{Max}\,& 0.84x_{AI} + 0.82x_{AJ} + 0.96x_{AK} + 0.95x_{BI} + 0.97x_{BJ} + 0.64x_{BK} + 0.86x_{CI} \\
&+ 0.87x_{CJ} + 0.93x_{CK} + 0.97x_{DI} + 0.77x_{DJ} + 0.60x_{DK} + 0.97x_{EI} + 0.75x_{EJ} \\
&+ 0.52x_{EK} + 0.87x_{FI} + 0.37x_{FJ} + 0.59x_{FK} + 0.59x_{GI} + 0.61x_{GJ} + 0.75x_{GK} \\
&+ 0.81x_{HI} + 0.76x_{HJ} + 0.71x_{HK} \\
s.t.\,& x_{AI} + x_{AJ} + x_{AK} = 10, \\
& x_{BI} + x_{BJ} + x_{BK} = 13,\ x_{EI} + x_{EJ} + x_{EK} = 9, \\
& x_{CI} + x_{CJ} + x_{CK} = 11,\ x_{FI} + x_{FJ} + x_{FK} = 9, \\
& x_{DI} + x_{DJ} + x_{DK} = 7,\ x_{GI} + x_{GJ} + x_{GK} = 4, \\
& x_{HI} + x_{HJ} + x_{HK} = 6, \\
& x_{AI} + x_{BI} + x_{CI} + x_{DI} + x_{EI} + x_{FI} + x_{GI} + x_{HI} = 30, \\
& x_{AJ} + x_{BJ} + x_{CJ} + x_{DJ} + x_{EJ} + x_{FJ} + x_{GJ} + x_{HJ} = 25, \\
& x_{AK} + x_{BK} + x_{CK} + x_{DK} + x_{EK} + x_{FK} + x_{GK} + x_{HK} = 14, \\
& x_{ij} \geq 0 \ \text{ for all i,j.}
\end{aligned} \tag{47}$$

Finally, at the end, by solving the model (47), a fuzzy transportation plan with the maximum fuzzy efficiency is determined as follows:

$$\begin{gathered}
x_{AK} = 10,\ x_{BJ} = 13,\ x_{CJ} = 11,\ x_{DI} = 7, \\
x_{EI} = 9,\ x_{FI} = 9, x_{GJ} = 4,\ x_{HI} = 5, x_{HJ} = 1.
\end{gathered}$$

6 Results and discussions

Considering the resulted efficient transportation plan, 10, 13, 11, 7, 9, 9, 5 and 1 cars should be transported from the assembly plants located in towns *A, B, C, D, E, F, G, H, H* to the major markets located in towns *K, J, J, I, I, I, J, I,* and *J*, respectively. This efficient transportation plan to transport the cars from the assembly plants to the major markets is illustrated in Fig. 2.

By substituting the efficient transportation plan in each objective function, the membership functions of fuzzy shipping cost ($\mu_1(x)$), fuzzy value of shipment ($\mu_2(x)$) and fuzzy profit ($\mu_3(x)$) are given as follows:

$$\mu_1(x) = \begin{cases} \frac{x-26739}{414}, & 26739 \le x \le 27153 \\ \frac{27724-x}{571}, & 27153 \le x \le 27724 \end{cases} \tag{48}$$

$$\mu_2(x) = \begin{cases} \frac{x-87477}{2237}, & 87477 \le x \le 89714 \\ \frac{91330-x}{1616}, & 89714 \le x \le 91330 \end{cases} \tag{49}$$

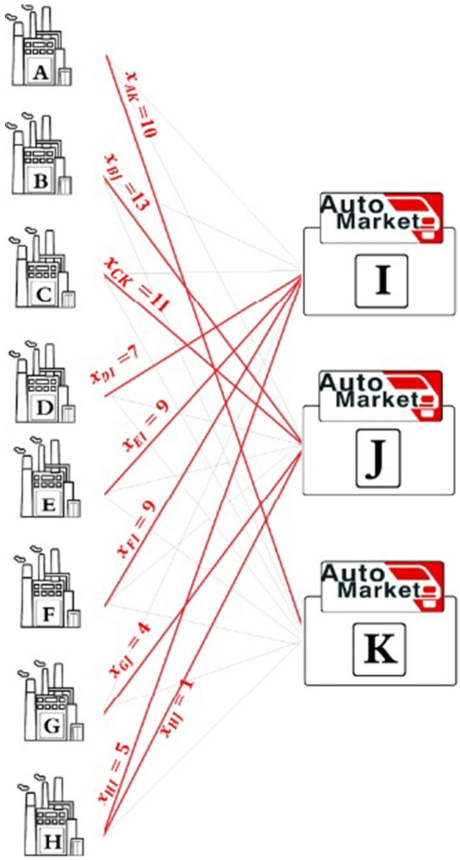

Fig. 2 Efficient transportation plan

$$\mu_3(x) = \begin{cases} \frac{x-43134}{955}, & 43134 \le x \le 44089 \\ \frac{45209-x}{1120}, & 44089 \le x \le 45209 \end{cases} \tag{50}$$

Figure 3 shows the membership functions of fuzzy shipping cost, fuzzy value of shipment as well as fuzzy profit.

It is worthy to note that by solving the single objective FTP (46) based on the fuzzy programing technique, the following transportation plan is obtained according to the model (37):

$$x_{AJ} = 10,\ x_{BI} = 6.52,\ x_{BJ} = 6.48,\ x_{CI} = 7.11, x_{CK} = 3.89,\ x_{DI} = 7,$$
$$x_{EI} = 2.89, x_{EK} = 6.11,\ x_{FI} = 6.48,\ x_{FJ} = 2.52,\ x_{GK} = 4, x_{HJ} = 6.$$

Based on the transportation plan, the fuzzy shipping cost, fuzzy value of shipment and fuzzy profit are given as (27837.55, 28360.41, 28907.11), (54483.75, 55542.26, 56843.99) and (40827.88, 41843.84, 42835.26), respectively. It can be seen that the fuzzy shipping cost derived based on the proposed approach is less than the one derived from the fuzzy programming technique. Additionally, the fuzzy value of shipment and fuzzy profit obtained based on the proposed technique are greater than those derived from the fuzzy programming technique. So, the proposed approach to solve FTP (30) is preferable considering the obtained transportation plan. In general, there are several important advantages of the proposed method compared with the fuzzy programing technique:

- ▹ The classical problem (37) applied for solving FTP (30) is not a transportation structured problem; whereas problem derived from the proposed approach to solve FTP (30) is a classical transportation problem.
- ▹ The optimal solution for the FTP (30) obtained using the proposed method has integer values, whereas the fuzzy programing technique yields fuzzy optimal solution with non-integer values in the fuzzy quantities of some products to be transported from origins to destinations, which have no physical meaning.
- ▹ The classical problem (37) utilized to solve FTP (30) with the fuzzy programing technique has more constraints and variables when compared with problem derived from the proposed approach. Therefore, utilizing the proposed technique to solve FTP (30) is highly economical in comparison with the fuzzy

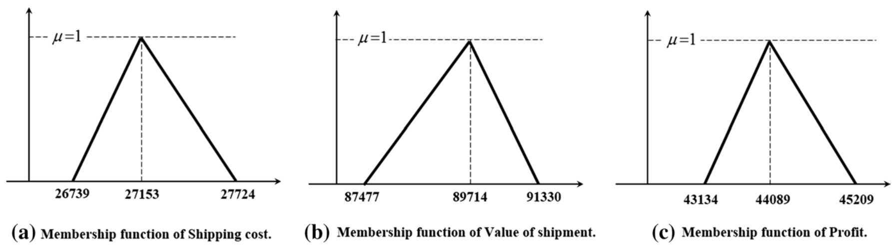

Fig. 3 Membership functions of the fuzzy objective functions

programming technique from a computational viewpoint, considering the number of constraints and variables.

- ⊳ Utilizing fuzzy DEA approach proposed in this study, the FMOTP (3) is converted into a single objective FTP without modifying the structure of the transportation problem, whereas using other popular techniques such as goal programming method and fuzzy approach would increase the number of constraints for the problem by adding new ones. Therefore, if the problem requires an integer optimal solution, then fuzzy DEA approach can be used by simply determining an optimal solution to the transportation problem obtained by ignoring the integrality restrictions, while goal programming method and fuzzy programming approach are not able to find integer solutions without adding the integrality restrictions.

Note that by solving the FMOTP based on the given data in Table 1, the following solution is obtained according to problem (5) as an approximation of the ideal solution:

$$x_{Ak} = 10,\ x_{BI} = 13,\ x_{CJ} = 7,\ x_{CK} = 4, x_{DI} = 7,\ x_{EI} = 1,\ x_{EJ} = 8, x_{FI} = 9,\ x_{GJ} = 4,\ x_{HJ} = 6.$$

According to this transportation plan, the fuzzy shipping cost, fuzzy value of shipment and fuzzy profit are given as (26875, 27467, 28286), (94627, 97234, 99775) and (40364, 41644, 42797), respectively. It should be noted that the fuzzy shopping cost (26739, 27153, 27724)) which is derived using the proposed method is less than the fuzzy shopping cost (26875, 27467, 28286) achieved by solving model (5). Additionally, the fuzzy profit (43134, 44089, 45209) which is achieved using the proposed method is greater than the fuzzy profit (40364, 41644, 42797) received by solving model (5). Therefore, the proposed method is preferable considering fuzzy shopping cost and fuzzy profit.

Similarly, by solving the FMOTP based on the given data in Table 1, the following solution is obtained according to problem (6) as an approximation of the ideal solution:

$$\begin{aligned} &x_{CK} = 10,\ x_{BI} = 9.48,\ x_{BJ} = 3.52,\ x_{CJ} = 7, x_{DI} = 7,\\ &x_{EI} = 0.52,\ x_{EJ} = 8.48, x_{FI} = 9,\ x_{GJ} = 4,\ x_{HJ} = 6. \end{aligned}$$

According to this transportation plan, the fuzzy shipping cost, fuzzy value of shipment and fuzzy profit are given as (27536.87, 28115.14, 28847.29), (93616.73, 96087.96, 98356.14) and (41143.64, 42406.05, 43537.45), respectively. It follows that the fuzzy shopping cost (26739, 27153, 27724) derived based on the proposed method is less than the fuzzy shopping cost (27536.87, 28115.14, 28847.29) achieved by solving model (6). Moreover, the fuzzy profit (43134, 44089, 45209) derived using the proposed method is greater than the fuzzy profit (41143.64, 42406.05, 43537.45) given by solving model (6). Therefore, the proposed method is preferable considering fuzzy shopping cost and fuzzy profit based on the results derived by ℓ_2 norm.

7 Conclusions

In the last few decades, the traditional TP considers only single objective function. Nowadays, when a homogeneous product is transferred from a source to different destinations in competitive economic condition, there would be more than single criterion including the transportation cost, average delivery time of product, deterioration rate of goods. Consequently, the traditional TP is not efficient to accommodate such real-life decision-making problems. Moreover, in traditional TP, it is assumed that all the relevant parameters like supply, demand and transportation cost, are precise. However, in most of the real-world situations, these parameters are imprecise due to impact of different reasons, such as incomplete information. To overcome such situations, TP with fuzzy multi-objective functions has been investigated in a way that functions would contradict to each other. Also, a FDEA approach is proposed to convert the FMOTP into a fuzzy single objective. To this end, existing fuzzy arithmetic approaches for solving FDEA models have been revised in order to find the same weights for each separate DEA model to obtain the fuzzy efficiency scores of each arc. Next, the resulting fuzzy efficiency scores of each arc have been aggregated into a unique one. The obtained unique fuzzy efficiency score has been considered as the final fuzzy cost of a single objective fuzzy TP. Finally, an existing ranking function-based approach for solving the single-objective fuzzy TP have been used to find the optimal solution.

The proposed technique does not require the goal and parametric approaches which are difficult to implement in real-life situations. By employing the proposed approach to find the efficient solution, there is no need for prior knowledge of the fuzzy programming and goal programming approaches which are challenging for a new decision maker. The proposed method to solve the FMOTP (3) is based on traditional transportation algorithms. Therefore, the existing and easily available software can be used for the same. Moreover, it is possible to assume a generic ranking index for comparing the fuzzy numbers involved in the single objective FTP (30), in such a way that each time that the decision maker wants to solve the FTP problem they can choose (or propose) the ranking index that best suits the FTP problem. It is worth noting that in the goal programming approach an aspiration level of the objective function should be given initially by the decision maker. Since in real-world application, the decision maker has little information about the data of the problem under consideration, it is difficult for them to establish the mentioned data. As a result, the derived solution using this approach is questionable. However, the proposed fuzzy DEA approach applied in this study not only gives the efficient transportation plan without any knowledge of decision maker, but also keeps the transportation structure of the problem.

Finally, it is worth mentioning that this study links the FMOTP and FDEA together. In this way, most of the existing concepts in FDEA can be transformed into the FMOTP. For example, as for future research it would be worthwhile to solve the FMOTP using other approaches from the FDEA such as common set of weights (CSW) (Contreras et al. 2019). As another direction for further study, there could be interest in examining a situation in which the attributes corresponding to each arc are only from the maximizing or minimizing type. In this situation, all DMUs will be without input or output which require the different models to be evaluated. Finally, the work is in progress to extend the DEA based approach (Shirdel and Mortezaee 2015) for solving multi-objective shortest path problem (Abbaszadeh Sori et al. 2020a, b; Ebrahimnejad 2020).

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