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ORIGINAL PAPER

A light robust model for aggregate production planning with consideration of environmental impacts of machines

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Abstract
In the present study, a multi-period multi-product aggregate production planning model is developed under uncertainty, considering some important aspects of real-world production systems. In order to apply environmental concerns and control the pollution arising from machines, environmental improvement planning is included as a periodic decision variable. Also, the pollution caused by the production is restricted to an allowable level. A light robust optimization approach is employed in which demands and processing times of operations are uncertain parameters. An illustrative example is presented to demonstrate the model validity and some test problems are designed to analyze the impact of uncertainty on the objective function. Several sensitivity analyses are carried out to provide useful managerial insights.

Keywords Aggregate production planning · Environmental concerns · Light robust optimization · Uncertainty

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1 Introduction

Aggregate production planning (APP) is a capacity planning often conducted in a medium-term horizon of 2–18-months to meet the fluctuating demand. APP is focused on determining the optimal quantity of raw material, production, workforce and inventory levels for each considered planning period with limited resources capacities (Wang and Fang 2001; Gomes da Silva et al. 2006).

Several challenges exist in production planning of manufacturing systems. First, in practice, the demand and processing times of manufacturing operations are often uncertain. Second, production processes are sources of air pollution and greenhouse gas emissions (GHG). In addition, inefficient scheduling of manufacturing processes in different shifts and the usage of older equipment, tools, and machines result in considerable environmental pollution. In response to this issue, it is necessary to consider a specific budget in each period of time for machines' improvement in terms of the environmental protection level, technology upgrade, or even machines' replacement. To overcome these problems, we proposed a light robust APP model under uncertainty with the aim of improving the environmental protection level in facilities. To the best of our knowledge, it is the first model that addresses the decision on the environmental investment in a medium-term uncertain aggregate production planning to provide greener facilities under the concept of "green production".

The rest of the paper is organized as follows: In Sect. 2, a literature review is provided. In Sect. 3, a multi-period, multi-product aggregate production planning model is proposed. Also, the robust optimization model based on light robust approach is demonstrated in this section. Computational results are highlighted in Sect. 4. A numerical example is provided and several analyses are conducted for the robust optimization model. Finally, Sect. 5 draws some conclusions and discusses future studies.

2 Literature review

Since the introduction of the first aggregate production planning model by Holt et al. (1955), various models have been developed to solve APP problems of differing complexity degrees. Many studies on APP problems have been performed over the past two decades (David 1974; Lim et al. 2005; Masud and Hwang 2007; Kazemi et al. 2009; Orcun et al. 2009; Xue et al. 2011; Zhang et al. 2012). Nam and Logendran (1992) provided a comprehensive review of a wide range of APP models from 14 books and 140 journal papers. To name a few, Gholamian et al. (2015) and Chakrabortty et al. (2015) developed a possibilistic Environment-based Particle Swarm optimization (PE-PSO) approach that includes escalating factors for all imprecise parameters. Escalating factors for various operational costs were also taken into consideration in their research.

As production operations result into lots of pollution sources, recently researchers have focused on improving production systems by considering the environmental factors. For example, Entezaminia et al. (2016) provided a multi-objective model for multi-product, multi-site aggregate production planning in a green supply chain by considering collection and recycling centers. In the proposed model, products are scored in terms of environmental criteria such as recyclability, biodegradability, energy consumption and product risk, using analytical hierarchy process (AHP). Some other green indicators, including waste management, greenhouse gas emissions arising from production methods and transportation are embedded in the model. The limited number of potential collection and recycling centers can be opened in order to produce the second-class goods.

Wang et al. (2011) proposed a multi-objective optimization model that relied on the facility location. They presented a strategic planning model for green chain network design. Their study had a different perspective on the "greenness." They considered the environmental investment decision making in the network design phase and stressed taking precautions against environmental pollution. They believed that an initial investment in environmental protection equipment or techniques should be determined in the design phase. This investment can impact on the environmental indicators in the operations phase. Jayaraman et al. (2017) developed a stochastic goal programming model for strategic planning decisions and investment allocations to fulfill sustainable developmental goals.

Fang et al. (2016) presented an integer linear programming model for optimal production planning in a hybrid manufacturing and recovering system with green principles. They considered recovering and disposal decisions to enhance the environmental protection. Their proposed model considers three recovery options, several levels of returns and, the value deterioration during the processing time period. Their model tries to enhance the environmental protection level and avoid defiance of relevant legislation. Choi and Xirouchakis (2015) presented a model for a holistic production planning in a reconfigurable manufacturing system by focusing on environmental impacts and energy consumption. Their suggested production planning model evaluates dynamically the environmental effects performance in terms of different holistic measures such as energy consumption at different planning horizon. Bournaris et al. (2015) developed a mathematical programming model for the support of irrigation water use and eco-friendly decision process in agricultural production planning. They considered different environmental measures such as different levels of chemicals or water consumption per crop. Their model can help a decision maker get alternative production plans and agricultural land uses considering the social, economic and environmental impact of different policies.

As mentioned above, another challenge in real-world planning problems is the point that operational data are often imprecise, incomplete, and uncertain. According to Al-e-Hashem et al. (2011a), four primary approaches have been developed to consider uncertainty of one form or another in production planning: (1) stochastic programming approach (Leung et al. 2006; Zanjani et al. 2013; Al-e-hashem et al. 2013), (2) stochastic dynamic programming approach (Li et al. 2009), (3) fuzzy programming approach (Iris and Cevikcan 2014; Tien-Fu Liang et al. 2011; Gholamian et al. 2016), and (4) robust optimization approach (Al-e-hashem et al. 2011a; Rahmani et al. 2013; Niknamfar et al. 2014). In the stochastic approach, input data are considered as random variables with related known probability density functions. In the fuzzy approach, some variables are quantified as fuzzy numbers. For example, da Silva and Marins (2014) presented a Fuzzy Goal Programming (FGP) model for aggregate production planning under uncertainty in sugar and ethanol milling industries. The proposed model provides reliable and precise outputs, taking into account technical and economic perspectives. Jamalnia and Soukhakian (2009) developed a hybrid fuzzy multi-objective nonlinear programming (H-FMONLP) model with different goal priorities for the multi-product multi-period aggregate production planning. Hybrid means that the model included quantitative and qualitative objectives.

In the stochastic dynamic approach, random variables can be used in dynamic programming to formulate the uncertainty in the multi-stage decision making. Finally, the robust optimization is a strong methodology to find a robust solution and to manage the risks arising from noisy data (Al-e-hashem et al. 2011b).

Among the above-discussed approach, robust optimization models have been proposed in the literature to handle uncertainty. A scenario-based robust optimization approach is proposed by Mulvey et al. (1995). Leung and Wu (2004) focused on a robust optimization model for APP problem including uncertain parameters. Leung et al. (2007) considered a robust model for multi-site APP problem. Zanjani et al. (2010) studied the robust optimization method to deal with multi-period, multi-product production planning problem for a manufacturing environment with random yield. Al-e-hashem et al. (2011a, b) proposed a multi-objective robust optimization model for multi-site APP problem under uncertainty. Other robust approaches for linear quadratic problems and conic quadratic problems are developed by Ben-Tal and Nemirovski (1998, 2002, 2004). A linear robust optimization approach in continuous spaces for interval uncertain parameters is also addressed by Bertsimas and Sim (2003). The limitation of the previous studies was that these robust optimization approaches lead to very conservative solutions. To overcome this limitation, Fischetti and Monaci (2009) investigated a new way to model the uncertainty, leading to a modeling framework titled 'Light Robustness'. This approach creates a balance between the feasibility due to uncertain parameters (the robustness of solutions) and the quality of the solution. They proposed two different models for the light robust approach, where their first model mainly relies on Bertsimas and Sim's definition of robustness.

2.1 The Bertsimas and Sim approach

Consider the following generic linear programming (LP) model:

$$\min \sum_{j \in N} c_j x_j \tag{1}$$

$$\sum_{j \in N} \tilde{a}_{ij} x_j \leq b_i \qquad i \in M \tag{2}$$

$$x_j \geq 0 \qquad j \in N \tag{3}$$

Soyster (1973) made the first attempt to deal with the uncertainty of parameters in mathematical models. The formula for considering uncertainty is as follows:

$$\min\left\{\sum_{j\in N} c_j x_j \,|\, \sum_{j\in N} \tilde{a}_{ij} x_j \le b_i, \forall \tilde{a}_{ij} \in K_j, j \in N\right\} \tag{4}$$

where K_j are convex sets related to "column-wise" uncertainty. Assuming that the $\tilde{a}_{ij}$ is the uncertain coefficient, it may take any value in $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ range. In this definition, a_{ij} is the nominal value and $\hat{a}_{ij}$ is the maximum deviation from the nominal value. In the Soyster approach, all uncertain parameters are considered to be fixed at the worst-case value. This, in turn, results in less optimal solutions as well as over-conservative models. To overcome this limitation, Ben-Tal and Nemirovski (1999, 2000, 2002) developed less conservative models to allow a tradeoff between performance and robustness. As shown by Bertsimas and Sim (2003, 2004), in a real situation, it is unlikely that all uncertain parameters take their worst-case value. Bertsimas and Sim (2003) introduced a parameter as the robust optimization coefficient called Γ_i (Budget of uncertainty), which shows the level of risk aversion for each constraint. This parameter is not necessarily integer and may take any value in the $[0, |J_i|]$ range in which $|J_i|$ is the total number of uncertain parameters in constraint i. To define the robust counterpart of the main model, each row $i \in M$ is replaced with the new constraint as follows:

$$\sum_{j\in N} a_{ij} x_j + \beta(x, \Gamma_i) \le b_i \tag{5}$$

where $\beta(x, \Gamma_i)$ is the protection function that can be defined as follows:

$$\beta(x, \Gamma_i) = \max_{S\subseteq N:|S|\le \Gamma_i} \sum_{j\in S} \hat{a}_{ij} x_j \tag{6}$$

The input parameter Γ_i is used to control the robustness of the solution: $\Gamma_i = n$ corresponds to Soyster's formulation as a worst-case approach, whereas $\Gamma_i = 0$ shows the nominal case in which the robustness is not considered. Furthermore, linear programming duality is used to reformulate the robust model as follows:

$$\min \sum_{j\in N} c_j x_j \tag{7}$$

$$\sum_{j\in N} a_{ij} x_j + \Gamma_i z_i + \sum_{j\in N} p_{ij} \le b_i \quad i \in M \tag{8}$$

$$-\hat{a}_{ij} x_j + z_i + p_{ij} \ge 0 \quad i \in M, j \in N \tag{9}$$

$$z_i \ge 0 \quad i \in M \tag{10}$$

$$p_{ij} \ge 0 \quad i \in M, j \in N \tag{11}$$

$$x_j \geq 0 \quad j \in N \tag{12}$$

Variables z_i and p_{ij} are necessary dual variables in this approach. The solutions provided by Bertsimas and Sim (BS) approach are feasible on condition that more than Γ_i coefficients are allowed to take the worst-case.

2.2 The basic light robustness approach

A closer look at the BS definition indicates that with regard to the objective function value, the optimal solution related to the BS approach can be worse than the nominal optimal solution. It should be noted that the definition of Light Robustness (LR) is focused on balancing the quality of the solution with the objective function and the robustness of the solution with the uncertainty of coefficients. This LR approach, as a new modeling framework, produces the most robust solution among those solutions which are not too far in terms of optimality for the nominal problem. The LR counterpart of (1)–(3) can be indicated as:

$$\min \sum_{i \in M} \omega_i \gamma_i \tag{13}$$

$$\sum_{j \in N} a_{ij} x_j + \beta(x, \Gamma_i) - \gamma_{i \leq} b_i \quad i \in M \tag{14}$$

$$\sum_{j \in N} a_{ij} x_j \leq b_i \quad i \in M \tag{15}$$

$$\sum_{j \in N} c_j x_j \leq (1 + \rho) Z * \tag{16}$$

$$x_j \geq 0 \quad j \in N \tag{17}$$

$$\gamma_i \geq 0 \quad i \in M \tag{18}$$

Objective function (13) is used to minimize the weighted sum of slack variables γ_i acting as second-storage resource variables. When it comes to the uncertainty of parameters, each variable γ_i demonstrates the robustness level of the solution in row $i \in M$. Particularly, γ_i takes a positive value on the condition that the - robust constraint i is violated. The input parameter ρ is defined to balance the feasibility and the optimality of the solution. $\rho = \infty$ means that the nominal objective function fails to be taken into account at all, and $\rho = 0$ denotes the nominal problem. Using constraint (16), a maximum deviation of the value of the objective function is imposed with regard to the optimal objective value of the nominal problem.

In the objective function (13), weights ω_i are taken into account to contravene for various scales of constraints. Therefore, it is assumed that all constraints are given in a comparable unit and $\omega_i = 1$ for all i. The LP counterpart of (1)–(3) can be shown as follows:

$$\min \sum_{i \in M} \gamma_i \tag{19}$$

$$\sum_{j \in N} a_{ij} x_j + \Gamma_i z_i + \sum_{j \in N} p_{ij} - \gamma_i \leq b_i \quad i \in M \tag{20}$$

$$-\hat{a}_{ij} x_j + z_i + p_{ij} \geq 0 \quad i \in M, j \in N \tag{21}$$

$$z_i \geq 0 \quad i \in M \tag{22}$$

$$p_{ij} \geq 0 \quad i \in M, j \in N \tag{23}$$

$$\sum_{j \in N} a_{ij} x_j \leq b_i \quad i \in M \tag{24}$$

$$\sum_{j \in N} c_j x_j \leq (1 + \rho) Z* \tag{25}$$

$$x_j \geq 0 \quad j \in N \tag{26}$$

$$\gamma_i \geq 0 \quad j \in N \tag{27}$$

It should be noted that LR is a modeling framework to achieve robustness, and is not a rigid technique. In this method, the robustness can be obtained by simultaneous consideration of optimality and possible infeasibility.

2.3 Research gaps and major contributions

Upon reviewing the recent literature, it seems that there are two important research gaps in production planning problems.

First, as de Oliveira Neto and Lucato (2016) stated, there are several initiatives in the literature to reduce the environmental impacts of production operations; nevertheless, the use of production planning activities to achieve these goals has rarely been considered. In fact, most studies on aggregate production planning have overlooked the environmental issues. In those few previous studies focusing on the environmental issues, the only strategy was to limit the generated pollution to the maximum amount of total pollution, while including some constraints (Entezaminia et al. 2016; Porkar et al. 2018). As production planning is a periodic decision method, our proposed approach has a different viewpoint on green production planning. In fact, it is helpful to consider a specific budget in each period of time for improving machinery or even replacing them in terms of their environmental protection levels. The more improvement level in machinery, the less pollution is generated. Second, to present robust approaches, some researchers have tried to find a solution that is still feasible for worst-case scenario (Soyster 1973). This approach has presented a straightforward way to model uncertainty, but they can lead to overconservative solutions that are quite bad in terms of objective function (Indeed, a feasible solution may not exist at all). As Bertsimas and Sim (2003) have stated, it is unreal to assume that all coefficients take their worst-case value, at the same time, in real situations (Fischetti and Monaci 2009). In fact, the presented robust models in the literature sometimes lead to very conservative solutions that have little practical applications. These models often tend to lead to poor solutions in relation to optimality. To overcome this matter, it is necessary using a less conservative approach such as a light robust approach (Fischetti and Monaci 2009). To the best of our knowledge, this is the first attempt at modeling the uncertain aggregate production planning via LR approach by focusing on improving the environmental level of machines. The major contributions of this paper can be summarized as follows:

- Developing a light robust optimization model for a multi-period, multi-product aggregate production planning with environmental concerns under uncertainty.
- Considering the environmental protection improvement in machinery to address green production planning.
- Considering escalating factors for cost parameters to make a more precise light robust production plan.

3 Model description

In this section, a multi-product and a multi-period aggregate production planning is developed over a given planning horizon. The model is designed to determine the optimal level of production at the regular time and overtime as well as a subcontracting volume. The optimal level of inventories, workers, and backorders are also considered as decision variables. Moreover, escalating factors for different operational costs with rising trend are considered in the proposed model. As mentioned before, in addition to the production planning, the improvement of the environmental protection level in machinery is also included in the proposed model.

In each factory, different machines produce various amounts of pollution depending on their age (old or new), their level of depreciation, the level of applied technology, the energy consumption and other features. Therefore, deciding about the amount and timing of assignment of the budget for improving or replacing old machinery is an important strategic decision for production managers. In order to simplify the model, based on Wang et al. (2011), the initial environmental level of machine j is assumed to be IL_{jo}. Indeed, it is assumed that a machine produces a specific amount of CO_2 pollution based on its features such as age, technological level, and energy consumption. This amount of pollution is divided into some intervals. For instance, if the pollution rate of machine j in period t is in a special interval, the machine will be considered to be in the protection level of IL_{jt}. More increase in protection level of a machine through budget allocation, more reduction in amount of produced CO_2. In addition, based on standard rules defined by experts for CO_2 emissions, it is possible to determine the best level that a machine can be improved. We assume that a series of machines are selected to be improved based on the model choice in each period. This improvement level for each selected machine is also considered to be a discrete number which occurs at the interval of $\left[IL_{jo}, BLj\right]$. When the budget is unlimited, the best strategy to improve the environmental levels is the replacement of the old machinery with new advanced and eco-friendly ones. However, in most real situations, it is assumed that the amount of the budget specified for the improvement of machinery is limited. The allocated budget would be gradually available to the manager to assign. Hence, in each planning period, the managers need to select some machinery in order to be upgraded to a specific level or to be replaced.

Making simultaneous decisions about the production planning and improvements of environmental levels of machines, in an integrated model, brings potential benefits. Determining the quantity of production at regular time and overtime merely based on resource capacities and the total cost may lead to an unallowable level of total pollution. Nonetheless, in an integrated model, the way in which production quantities are determined at the regular time and overtime results in keeping the amount of pollution in an allowable level. Hence, in the proposed model, selecting the most effective machinery and determining the best level of their improvement due to available budget generate better solutions. It reduces the pollution level as well as the total cost while satisfying the demands. The pollution level is inversely related to the protection level of machinery. The improvement of the environmental protection level in machinery leads to a reduction in the related pollution level. Actually, in each period of time, a tradeoff should be made between paying for machinery upgrade and pollution level reduction. On the other hand, the pollution arising from the factories and related to regular and overtime production is embedded in the model. Section 3.1 describes the notations used in the proposed model including parameters and decision variables.

3.1 Notations

3.1.1 Indices

i	Index of products $\{i=1,2,\ldots, I\}$
j	Index of machines $\{j=1,2,\ldots, J\}$
t	Index of time periods $\{t=1,2,\ldots, T\}$
l_j	Index of environmental protection levels of machine j $\{l_j = IL_{j0}, \ldots, BL_j\}$

3.2 Parameters

cp_{it}	Regular time production cost of each unit of product i in period t
co_{it}	Overtime production cost of each unit of product i in period t
cc_{it}	Subcontracting cost of each unit of product i in time period t
cr_{ijt}	Setup cost of machine j to manufacture each unit of product i in time period t

D_{it}	Demand for product i in time period t
b_{it}	Backorder cost of each unit of product i in time period t
h_{it}	Inventory holding cost of each unit of product i in time period t
g_i	Labor hours required to manufacture each unit of product i in regular time
g'_i	Labor hours required to manufacture each unit of product i in overtime
a_{ij}	Machining time required for machine j to manufacture each unit of product i in regular time
a'_{ij}	Machining time required for machine j to manufacture each unit of product i in overtime
ch_t	Hiring cost of each worker in period t
cl_t	Firing cost of each worker in period t
cw_t	Labor cost of each worker in period t
I_{i0}	The initial inventory level for product i at the start of the planning horizon
W_0	The initial workforce level
$W_t\max$	Maximum workforce level available in period t
$C_{it}\max$	Maximum subcontracting volume of the product i in period t
R_{jt}	Regular time capacity available for machine j in period t
β_{jt}	The fraction of regular time capacity for machine j available for overtime in period t
α_t	The fraction of regular time workforce available for overtime in period t
f	The working hours for each worker in each period of time
e_{ch}	Escalating factor for hiring cost
e_l	Escalating factor for firing cost
e_w	Escalating factor for labor cost
e_r	Escalating factor for setup cost
e_c	Escalating factor for subcontracting cost
e_o	Escalating factor for production cost in overtime
e_p	Escalating factor for production cost in regular time
e_b	Escalating factor for backorder cost
e_h	Escalating factor for inventory cost
IL_{j0}	Initial environmental protection level of machine j
BL_j	The best environmental protection level of machine j
IC_j	The cost of one level improvement in machine j
BUD_t	The budget related to the improvement of the machinery in period t
AE_t	Maximum allowable level of pollution arising from the factory in period t
WL_{jl_j}	The pollution level related to the machine j with environment protection level l_j in order to produce one product unit
M	A large number

3.3 Decision variables

P_{it}	Quantity of regular time production of product i in time period t
O_{it}	Quantity of overtime production of product i in time period t

C_{it} Quantity of subcontracting for product i in time period t
Y_{it} Binary variable equals to 1 if the productI i is produced in period t, 0 otherwise
W_t The workforce level in time period t.

3.4 Deterministic model

The green multi-period and multi-product aggregate production planning model for considering environmental protection can be formulated as follows:

$$\begin{aligned} Min\,Z = & \sum_i \sum_t [cp_{it}P_{it}(1+e_p)^t + co_{it}O_{it}(1+e_o)^t + cc_{it}C_{it}(1+e_c)^t] \\ & + \sum_i \sum_j \sum_t cr_{ijt}Y_{it}(1+e_r)^t + \sum_t [cw_tW_t(1+e_w)^t + ch_tHH_t(1+e_{ch})^t \\ & + cl_tL_t(1+e_l)^t] + \sum_i \sum_t h_{it}I_{it}(1+e_h)^t + \sum_i \sum_t b_{it}B_{it}(1+e_b)^t \end{aligned} \tag{28}$$

Subject to:

$$\sum_i a_{ij}P_{it} \le R_{jt} \quad \forall j, \forall t \tag{29}$$

$$\sum_i a'_{ij}O_{it} \le \beta_{jt}R_{jt} \quad \forall j, \forall t \tag{30}$$

$$P_{it} + O_{it} + C_{it} + B_{it} - B_{it-1} + I_{it-1} - I_{it} = D_{it} \quad \forall j, \forall t \tag{31}$$

$$P_{it} + O_{it} \le M.Y_{it} \quad \forall j, \forall t \tag{32}$$

$$W_t = W_{t-1} + HH_t - L_t \quad \forall t \tag{33}$$

$$\sum_i g_iP_{it} \le fW_t \quad \forall t \tag{34}$$

$$\sum_i g'_iO_{it} \le f\alpha_tW_t \quad \forall t \tag{35}$$

$$W_t \le W_t\mathrm{max} \tag{36}$$

$$C_{it} \le C_{it}\mathrm{max} \tag{37}$$

$$IL_{jt} = IL_{j(t-1)} + X_{jt} \tag{38}$$

$$\sum_j IC_jX_{jt} \le BUD_t \tag{39}$$

$$\sum_{i} (P_{it} + O_{it}) \le AE_t \sum_{j} \sum_{l_j=IL_{j0}}^{BL_j} \frac{YL_{jljt}}{WL_{jlj}} \tag{40}$$

$$\sum_{l_j=IL_{j0}}^{BL_j} YL_{jl_jt} = 1 \tag{41}$$

$$IL_{jt} = \sum_{l_j=IL_{j0}}^{BL_j} l_j.YL_{jl_jt} \tag{42}$$

$$P_{it}, O_{it}, C_{it}, B_{it}, I_{it} \ge 0 \tag{43}$$

$$YL_{jl_jt}, Y_{it} = \{0, 1\}; IL_{jt}, X_{jt}, L_t, HH_t, W_t \in Integer \tag{44}$$

Objective function (28) shows the total nominal cost that should be minimized. The total cost consists of production cost, subcontracting cost, setup cost, labor cost, inventory cost as well as backorder cost. Constraints (29) and (30) indicate the limited capacity of machines in regular time and overtime. Constraint (31) is considered to satisfy the demand of products. Constraint (32) guarantees that if a product is not selected for production in period t, no quantities will be produced in regular time and overtime. Constraint (33) shows that the level of workforce in each period needs to be equal to the workforce level that exists in the previous period with respect to the number of hired and fired workers. The workforce capacity constraints in regular time and overtime at each period of time are respectively shown in Constraints (34) and (35).

Constraint (36) is related to the maximum capacity of the workforce and Constraint (37) is related to the maximum volume of the subcontracting. Constraint (38) implies that the environmental protection level of each machine at period t needs to be equal to the environmental protection level of each machine at period $t-1$ in addition to the improvement level in that period. Constraint (39) implies that the cost required for the improvement level of machinery cannot exceed the available budget. Constraint (40) ensures that the amount of pollution arising from machinery in regular time and overtime is limited to an allowable level. This constraint provides an integration of decisions about determining the production levels in regular time as well as overtime and decisions about machine improvement with the aim of reducing the pollution level. In this constraint, it is assumed that all products are processed on all machines.

Constraint (41) restricts each machine to be in a specific environmental protection level at each period. Constraint (42) determines the environmental protection level of each machine in each period of time. Finally, the types of decision variables are indicated via Constraints (43) and (44).

3.5 The proposed light robust model

Often, in production systems, demand and processing time of activities are uncertain parameters. In this section, the light robust approach proposed by Fischetti and Monaci (2009) is implemented to cover uncertainties of these parameters. This approach provides a compromise between the robustness of the solution due to uncertain parameters and the quality of the solution (with respect to the objective function). LR is a flexible approach that can generate less conservative solutions in terms of the objective function, as compared with other interval robust approaches. The uncertain parameters included in the proposed model are as follows:

D_{it} The nominal value of demand for product i in time period t
$\hat{D}_{it}$ The maximum deviation from the demand for product i in time period t
a_{ij} The nominal machining time required for machine j to manufacture each unit of product i in regular time
a'_{ij} The nominal machining time required for machine j to manufacture each unit of product i in overtime
$\hat{a}_{ij}$ The maximum deviation from the machining time required for machine j to manufacture each unit of product i in regular time
$\hat{a}'_{ij}$ The maximum deviation from the machining time required for machine j to manufacture each unit of product i in overtime
ρ Balancing factor in LR to control the maximum worsening of the optimal nominal solution
HH_t The number of hired workers in time period t
L_t The number of fired workers in time period t
B_{it} The backorder level of product i in time period t
I_{it} The inventory level of product i in time period t
X_{jt} The amount of level improvement in machine j in time period t
IL_{jt} The environment protection level in machine j in period t
YL_{jl_jt} Binary variable equals to 1 if the machine j is in the level l_j of environmental protection in period t, 0 otherwise
H_{it} The cost of the net inventory.

Assume that $\tilde{D}_{it}$,$\tilde{a}_{it}$ and $\tilde{a}'_{jt}$ are uncertain demand and processing times that take value in the range of $\tilde{D}_{it} = \left[D_{it} - \hat{D}_{it}, D_{it} + \hat{D}_{it}\right], \tilde{a}_{it} = \left[a_{it} - \hat{a}_{it}, a_{it} + \hat{a}_{it}\right]$, and $\tilde{a}'_{it} = \left[a'_{it} - \hat{a}'_{it}, a'_{it} + \hat{a}'_{it}\right]$. Based on the definition of robust optimization provided in Sect. 2.1, Γ^D_{it},Γ^a_{jt} and $\Gamma^{a'}_{jt}$ are the budgets of uncertainty in the uncertain demand and uncertain processing times, where the maximum possible values for them are $\Gamma^D_{it} \in [0, \left|J^D\right|]$, $\left|J^D\right| = |t|$ for each i and t, $\Gamma^a_{jt} \in [0, |J^a|]$, $|J^a| = N$ for each j and t and finally $\Gamma^{a'}_{jt} \in [0, \left|J^{a'}\right|], \left|J^{a'}\right| = N$ for each j and t. Furthermore, $Z^a_{jt}, P^a_{ijt}, Z^{a'}_{jt}, P^{a'}_{ijt} Z^D_{jt}$ and $P^D_{i\tau t}$ are new duality decision variables for developing the robust formulation (as discussed in Sect. 2.1). The slack variables γ_i and γ'_{jt} demonstrate the robustness levels of the solutions in the corresponding constraints and control the model infeasibility. The proposed LR model is as follows:

$$Min\,LROF = \sum_j \sum_t (\gamma_{jt} + \gamma'_{jt}) \tag{45}$$

Subject to:

$$\begin{aligned}&\sum_i \sum_t [cp_{it}P_{it}(1+e_p)^t + co_{it}O_{it}(1+e_o)^t + cc_{it}C_{it}(1+e_c)^t] \\ &+ \sum_i \sum_j \sum_t cr_{ijt}Y_{it}(1+e_r)^t + \sum_t [cw_tW_t(1+e_w)^t + ch_tHH_t(1+e_{ch})^t \\ &+ cl_tL_t(1+e_l)^t] + \sum_i \sum_t H_{it} \le (1+\rho)Z^*\end{aligned} \tag{46}$$

$$\sum_i a_{ij}P_{it} + \Gamma^a_{jt}Z^a_{jt} + \sum_i P^a_{ijt} - \gamma_{jt} \le R_{jt} \quad \forall j, \forall t \tag{47}$$

$$\sum_i a'_{ij}O_{it} + \Gamma^{a'}_{jt}Z^{a'}_{jt} + \sum_i P^{a'}_{ijt} - \gamma'_{jt} \le \beta_{jt}R_{jt} \quad \forall j, \forall t \tag{48}$$

$$Z^a_{jt} + P^a_{ijt} \ge \hat{a}_{ij}P_{it} \quad \forall i, \forall t, \forall j \tag{49}$$

$$Z^{a'}_{jt} + P^{a'}_{ijt} \ge \hat{a}'_{ij}O_{it} \quad \forall i, \forall t, \forall j \tag{50}$$

$$H_{it} \ge h_{it}(1+e_h)^t(\sum_{\tau\le t}(P_{i\tau} + O_{i\tau} + C_{i\tau} - D_{i\tau}) + \Gamma^D_{it}Z^D_{it} + \sum_{\tau\le t}P^D_{i\tau t}) \quad \forall i, \forall t \tag{51}$$

$$H_{it} \ge b_{it}(1+e_b)^t(-\sum_{\tau\le t}(P_{i\tau} + O_{i\tau} + C_{i\tau} - D_{i\tau}) + \Gamma^D_{it}Z^D_{it} + \sum_{\tau\le t}P^D_{i\tau t}) \quad \forall i, \forall t \tag{52}$$

$$Z^D_{it} + P^D_{i\tau t} \ge \hat{D}_{i\tau} \quad \forall i, \forall t, \forall \tau \le t \tag{53}$$

$$P_{it} + O_{it} \le M.Y_{it} \quad \forall i, \forall t \tag{54}$$

$$W_t = W_{t-1} + HH_t - L_t \quad \forall t \tag{55}$$

$$\sum_i g_iP_{it} \le fW_t \quad \forall t \tag{56}$$

$$\sum_i g'_iO_{it} \le f\alpha_tW_t \quad \forall t \tag{57}$$

$$W_t \le W_t\text{max} \quad \forall t \tag{58}$$

$$C_{it} \leq C_{it}\max \quad \forall i, \forall t \tag{59}$$

$$IL_{jt} = IL_{j(t-1)} + X_{jt} \quad \forall j, \forall t \tag{60}$$

$$\sum_{j} IC_j X_{jt} \leq BUD_t \quad \forall t \tag{61}$$

$$\sum_{i} (P_{it} + O_{it}) \leq AE_t \sum_{j} \sum_{l_j = IL_{j0}}^{BL_j} \frac{YL_{jl_jt}}{WL_{jl_j}} \quad \forall t \tag{62}$$

$$\sum_{l_j = IL_{j0}}^{BL_j} YL_{jl_jt} = 1 \quad \forall j, \forall t \tag{63}$$

$$IL_{jt} = \sum_{l_j = IL_{j0}}^{BL_j} l_j . YL_{jl_jt} \quad \forall j, \forall t \tag{64}$$

$$P_{it}, O_{it}, C_{it}, B_{it}, I_{it}, Z^a_{jt}, P^a_{ijt}, Z^{a'}_{jt}, P^{a'}_{ijt}, \gamma_{jt}, \gamma'_{jt}, Z^D_{jt}, P^D_{i\tau t} \geq 0 \quad \forall i, \forall t \tag{65}$$

$$Y_{it} = \{0, 1\}, YL_{jl_jt} = \{0, 1\}, IL_{jt}, X_{jt}, L_t, HH_t, W_t \in Integer \quad \forall i, \forall t, \forall l \tag{66}$$

Equation (45) represents the objective function. Reduction in γ_{jt} and γ'_{jt} leads to control the Constraints (47) and (48) and then the avoidance of increasing the production parameters P_{it}, O_{it}. In fact, in case that uncertain parameters $\tilde{a}_{it}$ and $\tilde{a}'_{it}$ significantly deviate from the nominal values, Eq. (45) avoids infeasibility. Hence, the objective function is formulated to decrease the infeasibility of the generated solutions under future fluctuations. Constraint (46) controls the quality of generated solutions. It ensures that the generated solutions have an acceptable deviation from the optimal solutions (optimal solution in the nominal model). Constraints (47) and (48) are related to the capacity of machines after applying the protection function and control variables.

Constraint (49) and (50) are relevant to the definition of the robust optimization model. It is worth noting that because of the uncertain condition of demand in balancing Constraint (31), it is impossible to satisfy all possible amounts. Therefore, using the approach proposed by José Alem and Morabito (2012), we defined a new variable called net inventory as $I'_{it} = \sum_{\tau \leq t} (P_{i\tau} + O_{i\tau} + C_{i\tau} - D_{i\tau})$. Furthermore, H_{it} shows the cost of the net inventory. If the value of I'_{it} is positive, we have inventory for product i in period t and H_{it} determines the inventory holding cost and if I'_{it} is negative, there is backorder and H_{it} shows the backorder cost. Finally, using Bertsimas and Sim's robust optimization approach, Constraint (51) and (52) are generated. Constraint (51) indicates the total inventory holding cost and Constraint (52) indicates the total backorder cost. Constraint (53) relates to the robust modeling approach. Constraints (54)–(66) are similar to the constraints of the deterministic model.

4 Computational results

In this section, first, a numerical example of a small factory is simulated to validate the deterministic nominal model. Second, some examples are simulated in larger scales to further analyze the characteristics of the proposed model. The models are solved using GAMS 23.5/CPLEX 12.2 on a computer equipped with an *i*5 2.7 GHz processor and 4GBs of RAM.

4.1 Numerical example

In order to show the efficiency of the proposed model, a numerical example is generated similar to the production line of a refrigerator factory located in Hamadan, Iran. The initial data is collected based on the production process in this factory. However, due to unavailability of all the required data and simplification of the example, the required parameters are simulated based on a range of available data from this factory (Table 1). In this case, the machines of this factory were old with a low technological level; thus, a huge amount of CO_2 was generated in the factory. Like foam injection, the machines can be a source of harmful pollution. Hence, in this factory, producing GHG emissions, particularly CO_2, via manufacturing machines are considered. It should be noted that this model can be applied to any factory where manufacturing machines produce CO_2 gas. Thus, according to the managers' opinions and due the presence of aging machines, the initial environmental protection levels of machines are assumed to be 1 ($IL_{j0} = 1$) and the maximum improvement level is equal to 10 ($BL_j = 10$).

Table 1 Parameter design

Parameter	Setting (unit)
cp_{it}	Uniform [15, 25] ($/unit)
co_{it}	Uniform [40, 60] ($/unit)
cr_{ijt}	Uniform [50, 80] ($/unit)
cc_{it}	Uniform [200, 250] ($/unit)
b_{it}	Uniform [1400, 1500] ($/unit)
h_{it}	Uniform [4, 6] ($/unit)
IC_j	Uniform [5, 10] (100$/improvement level)
a_{ij}	Uniform [2, 4] (min)
a'_{ij}	Uniform [2, 4] (min)
D_{it}	Uniform [1000, 3000] (unit)
μ_j	Uniform [20, 30] (kg CO_2e/unit)
AE_t	Uniform [200,000, 300,000] (kg CO_2e)
BUD_t	Uniform [200, 300] (100$)

The production line manufactures two product types using two types of machines. The initial inventory of products is considered to be zero (units) and the initial level of the workforce is equal to 10 (man-hour). The working hours on each day equals to 400 min. Moreover, it is shown that $\alpha_t = 0.4$. Maximum workforce level is 20 (man-hour) and the escalating factor for all cost parameters is considered to be 0.01. The times required to produce each unit of product i at regular time and overtime are $g_i = g'_i = 1$(man-hour/unit). Costs related to the workforce in each period of time are $cw_t = 100$ (\$/man-hour), $ch_t = 40$ (\$/man-hour) and $cl_t = 50$ (\$/man-hour). To generate the pollution level related to each machine, we defined a formula as $WL_{jl_j} = \frac{\mu_j}{2^{l_j-a}}$ (kg CO_2e/unit) which is based on Wang et al. (2011). In the equation defined by Wang et al. (2011), the value of the numerator is generated uniformly in the range of 48–72 due to their considered machines. This range is not necessarily matched with any machines in different industrial environments. Therefore, in this paper, the parameter μ_j is defined in order to match Wang et al.'s (2011) equation with different machines. This parameter is determined based on the machine type and other features including age, technological level, energy consumption level and experts' opinions. Thus, considering types of machines and experts' experience, it is possible to determine the value of μ_j. On the other hand, in the proposed equation by Wang et al. (2011), the pollution amount has an inverse relationship with the environmental protection level. This relationship is defined by 2^{l_j-a} with $a=1$ in the denominator of the equation. In this paper, given the assumption that the machines do not show a high pollution level when they are at the initial environmental protection level, we set a equal to zero. It should be mentioned that, in different real case studies, the pollution level can be determined by setting fitting values for μ_j and a. The subcontracting volume in each period of time is limited and can be calculated as $C_{it}\max = 0.2\left(\frac{\sum_t D_{it}}{T}\right)$ (units) for all products in each period.

Table 2 Aggregate production plan obtained from solving the proposed model for numerical example

	Product (i)	Period (t)					
		1	2	3	4	5	6
Regular time production (minutes)	1	1729.228	1162.573	1144.33	1750.467	1494.864	1162.573
	2	1973.2	2437.427	2452.373	1955.8	2165.2	2437.427
Overtime production (minutes)	1	719.641	719.641	719.641	719.641	719.641	719.641
	2	0	0	0	0	0	0
Subcontracting products (units)	1	216.4392	500.1333	500.1333	500.1333	271.4945	253.7863
	2	558.8	558.8	558.8	558.8	558.8	548.5727
Inventory (units)	1	1329.308	1910.655	1197.759	0	0	0
	2	0	5.227337	347.4	0	0	0
Backorder (units)	1	0	0	0	0	0	0
	2	0	0	0	0	0	0

Table 3 Workforce and machine plan obtained from solving the proposed model for numerical example

	Period (t)					
	1	2	3	4	5	6
Labor (man-hour)						
Hiring level (man-hour)	0	0	0	1	0	0
Layoff level (man-hour)	0	1	0	0	0	1
Workforce level (man-hour)	10	9	9	10	10	9
Environmental protection level						
Machine 1	2	4	5	7	9	10
Machine 2	1	1	1	1	1	1
Improvement level						
Machine 1	1	2	1	2	2	1
Machine 2	0	0	0	0	0	0

Table 2 represents the production plan of this factory for both product types in each time period. Workforce planning and the schedule of each period can be observed in Table 3. The improvement level in machines and the required hiring and firing level are also presented. As shown in Table 3, the second machine has no improvement, since the budget is only sufficient for improving the first machine.

As can be seen from Table 2, the total production capacity in overtime is assigned to the Product 1. Hence, Product 2 is not produced in overtime and the remained demand is met by the subcontracting. None of the products are backlogged because the back order cost is high. On the other hand, the demand can be satisfied by subcontracting and keeping inventory in each period. Machine 2 is selected to be improved in all periods and this indicates that the budget was not sufficient to improve both machines. It should be noted that at the end of Period 6, Machine 1 reaches its best possible level. So, in the next periods, Machine 2 will be definitely chosen to be improved.

4.2 Analysis of the robust optimization model

In this section, first, some examples are generated to show the performance of the model. Table 4 summarizes the number of products, machines, and periods to generate differently sized problems. The rest of parameters are generated based on the uniform distributions listed in Table 1.

According to the definition in the robust approach, it is obvious that $\Gamma_{it}^{D} \in [0, t]$ for each i and $\Gamma_{jt}^{a} \in [0, N]$ for every j. These values are defined as $\Gamma_{it}^{D} = \psi t$ and $\Gamma_{jt}^{a} = \psi N$,

Table 4 Size of the problem for instance generation

Number of Products	Number of machines	Number of periods
5, 10, 30	5, 8, 10, 30	6, 10

where $0 \le \psi \le 1$. On the other hand, to simulate the uncertain range of parameters, it is assumed that $\hat{D}_{it} = \theta D_{it}, \hat{a}'_{ij} = \theta a'_{ij}$ and $\hat{a}_{ij} = \theta a_{ij}$ in which $0 \le \theta \le 1$.

The left-hand side of Constraint (46) is labeled as LRZ. The LRZ is the actual total cost determined by the solution of the light robust model and is as follows:

$$LRZ = \sum_i \sum_t [cp_{it}P_{it}(1+e_p)^t + co_{it}O_{it}(1+e_o)^t + cc_{it}C_{it}(1+e_c)^t] + \sum_i \sum_j \sum_t cr_{ijt}Y_{it}(1+e_r)^t + \sum_t [cw_tW_t(1+e_w)^t + ch_tHH_t(1+e_{ch})^t + cl_tL_t(1+e_l)^t] + \sum_i \sum_t H_{it}$$

Moreover, Z^* is the optimal value of the deterministic model and LROF is the objective function value of the light robust model (45).

First, two examples (Example 1 with five products, ten machines, and six time periods and Example 2 with ten products, eight machines, and six time periods) were generated to represent the impacts of the model parameters in details. The effects of changes in θ and ψ, as the degree of uncertainty and conservatism, on LRZ and LROF of Example 1 are shown in Table 5.

In these experiments, the maximum worsening of the optimal robust solution (with respect to the optimal nominal solution) is considered at the fixed level ($\rho = 0.2$). In order to do the sensitivity analysis, the value of parameter ψ is considered to be between 0 and 1. The zero value shows the problem without uncertainty. The worst value of the problem occurs when $\psi = 1$. This value indicates the model behavior at the highest level of uncertainty. Moreover, in Table 5, the value of θ is considered to be up to 40%. In this paper, five values are selected as the samples to analyze the model behavior. Figure 1 illustrates the effect of uncertainty and the degree of conservatism on the robust objective function.

The third column in Table 5 ($\psi = 0$) indicates the lack of consideration in terms of uncertainty, where LRZ becomes equal to Z* (the value of Z* is 6,536,666). In other columns, LRZ is equal to an upper limit of constraint (46)

Table 5 The effect of uncertainty in Example 1

θ	Functions	ψ					
		0	0.2	0.4	0.6	0.8	1
5%	LRZ	6,536,666	7,844,000	7,844,000	7,844,000	7,844,000	7,844,000
	LROF	0	21773.43	45398.91	64736.51	80176.36	91600.81
10%	LRZ	6,536,666	7,844,000	7,844,000	7,844,000	7,844,000	7,844,000
	LROF	0	129094.6	195,138	248615.8	295859.5	331881.4
20%	LRZ	6,536,666	7,844,000	7,844,000	7,844,000	7,844,000	7,844,000
	LROF	0	435491.5	613698.5	769882.2	903327.5	1001620
30%	LRZ	6,536,666	7,844,000	7,844,000	7,844,000	7,844,000	7,844,000
	LROF	0	787799.7	1,107,465	1,383,526	1,603,543	1,765,943
40%	LRZ	6,536,666	7,844,000	7,844,000	7,844,000	7,844,000	7,844,000
	LROF	0	1,164,326	1,655,226	2,056,386	2,375,089	2,610,000

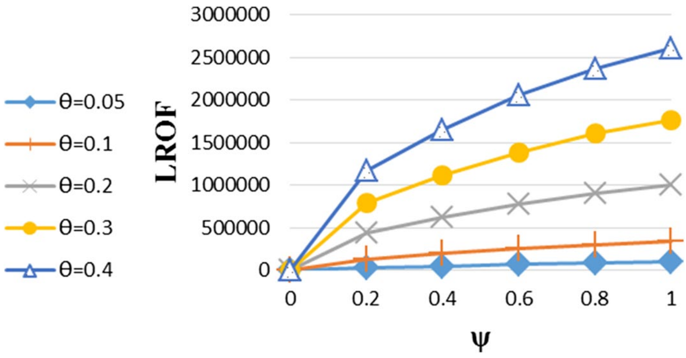

Fig. 1 The combined effect of uncertainty and conservatism degree of Example 1

Table 6 The effect of regret for Example 1

ρ	Functions	ψ					
		0	0.2	0.4	0.6	0.8	1
0.1	LRZ	6,536,666	7,190,333	7,190,333	7,190,333	7,190,333	7,190,333
	LROF	0	862709.2	1216459	1494649	1,715,921	1,879,434
0.2	LRZ	6,536,666	7,844,000	7,844,000	7,844,000	7,844,000	7,844,000
	LROF	0	787799.7	1107465	1383526	1603543	1765943
0.3	LRZ	6,536,666	8,497,666	8,497,666	8,497,666	8497666	8,497,666
	LROF	0	771886.8	1,061,432	1,293,652	1,494,280	1,655,289
0.4	LRZ	6,536,666	9,151,333	9,151,333	9,151,333	9,151,333	9,151,333
	LROF	0	756243.7	1,045,676	1,271,922	1,451,248	1,585,001

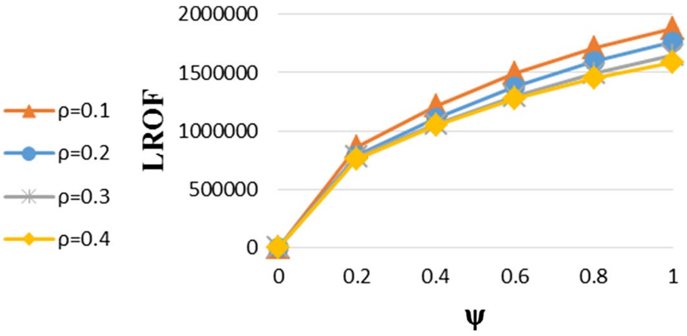

Fig. 2 The combined effect of the maximum deviation from optimality and the uncertainty for Example1

Table 7 The effect of uncertainty for Example 2

θ	Functions	ψ					
		0	0.2	0.4	0.6	0.8	1
5%	LRZ	12,378,945	14,854,735	14,854,735	14,854,735	14,854,735	14,854,735
	LROF	0	43977.23	88868.78	131926.6	167,509	195188.8
10%	LRZ	12,378,945	14,854,735	14,854,735	14,854,735	14,854,735	14,854,735
	LROF	0	239184.2	358102.9	458950.1	543498.7	702538.4
20%	LRZ	12,378,945	14,854,735	14,854,735	14,854,735	14,854,735	14,854,735
	LROF	0	709722.6	1,010,930	1,499,693	1,960,838	2,395,863
30%	LRZ	12,378,945	14,854,735	14,854,735	14,854,735	Infeasible	Infeasible
	LROF	0	1,263,239	2,043,673	2,896,973		
40%	LRZ	12,378,945	14,854,735	14,854,735	Infeasible	Infeasible	Infeasible
	LROF	0	1,906,269	3,212,802			

Table 8 The effect of regret for Example 2

ρ	Functions	ψ					
		0	0.2	0.4	0.6	0.8	1
0.1	LRZ	12,378,945	13,616,840	13,616,840	13,616,840	Infeasible	Infeasible
	LROF	0	1,411,963	2,226,178	3,114,323		
0.2	LRZ	12,378,945	14,854,735	14,854,735	14,854,735	Infeasible	Infeasible
	LROF	0	1,263,239	2,043,673	2,896,973		
0.3	LRZ	12,378,945	16,092,629	16,092,629	16,092,629	16,092,629	Infeasible
	LROF	0	1,217,910	1,868,610	2,699,086	3,521,625	
0.4	LRZ	12,378,945	17,330,524	17,330,524	17,330,524	17,330,524	Infeasible
	LROF	0	1,191,832	1,711,817	2,511,612	3,294,193	

i.e. $(1 + \rho)Z*$. Increasing the value of θ, as the range of uncertainty (considering a fixed value for ψ), leads to an increase in the LROF value. In each row, increasing the value of ψ, as the conservatism degree, leads to an increase in the LROF value. Due to the fixed value of ρ in this table, whenever LROF takes positive values, LRZ would be equal to $(1 + \rho)Z*$. If LROF = 0, then LRZ would take values between Z^* and $(1 + \rho)Z*$.

The effects of changing the value of ψ and ρ for $\theta = 30\%$ on LRZ and LROF in Example 1 are given in Table 6.

In Table 6, for an optional θ, different values of 10–40% are considered for ρ. Since there is no limitation to select ρ, it is possible to choose any other values. These numbers are just selected to analyze the system behavior. Figure 2 illustrates the simultaneous effects of the budget of uncertainty, as the degree of conservatism, and different values of maximum worsening of optimal solution on the robust objective function.

Due to the limited resource capacities (regular time, overtime and the capacity of the workforce) and the optimality regret constraint, i.e. Constraint (46), it is expected that increasing the uncertainty leads to the model infeasibility. In fact, while the light robust approach reduces the risk of infeasibility, however, the increasing amount of uncertainty and reducing the maximum worsening can cause infeasibility. Table 7 reveals that either increasing the uncertainty range θ or increasing the value of the budget of uncertainty ψ leads to model infeasibility in Example 2.

Table 8 shows that decreasing the value of ρ for a fixed ψ leads to infeasibility in Example 2. On the other hand, decreasing the value of ρ and increasing the value of ψ simultaneously results in infeasibility as well. These results show the feasible area of Example 2 for different levels of uncertainty. The results can be used to choose the appropriate budget of uncertainty to have a feasible and acceptable under-optimality solution.

In each column of Table 8, increasing the value of ρ and applying greater optimality worsening to the reliability constraint (considering a fixed value for uncertainty percentage) leads to a decrease in the value of LROF. Increasing both the value of ψ and the uncertainty percentage leads to an increase in LROF.

The performance of the model with different levels in terms of the number of products and periods is investigated in nine large size problems. The results are shown in Table 9.

In order to examine the performance of the proposed model in terms of the complexity and computational time, problems in different sizes were generated. It should be noted that in all generated problems,$\theta = 0.2$, $\psi = 0.3$ and $\rho = 0.3$. The results are provided in Table 9. The solution times reported in this table are the sum of the solution times for both deterministic and robust models of each problem. It can be concluded from Table 9 that an increase in the number of periods has more impacts on the solution time and complexity of the problem. However, compared with the two other factors, an increase in the number of machines has less influence on the model complexity.

Table 9 The performance of the model in different size problems

Problem ID	Number of products	Number of machines	Number of periods	LRZ	LROF	Time (s)
1	5	5	6	6232036.8	273072.1	3.4
2	5	5	10	14207899.5	567473.3	12.1
3	10	5	6	17717172.3	565562.9	9.3
4	10	5	10	31693058.6	1109127.7	28.5
5	10	10	10	25779856.1	2118027.5	44.2
6	30	5	6	972728848.4	320490.0	49.8
7	30	10	6	814981498.4	21174.4	111.2
8	30	10	10	2542541164.5	240661.6	364.3
9	30	30	10	1809293402.2	622290.8	592.7

5 Conclusion

A multi-period multi-product aggregate production planning model was introduced in a single production facility under uncertainty. Green principles were applied to control the production pollution associated with the machinery. The pollution related to regular time and overtime production in the factory is considered to be restricted. The improvement of the environmental protection level in machines was considered in the proposed model. The most important factor which separates this paper from other studies in the literature is its focus on the necessity of making management decisions about the improvement of manufacturing machines along with the production planning. To deal with the uncertainties in terms of demands and processing times, a light robust optimization approach was implemented. Through an illustrative example and other generated experiments, we demonstrated how the proposed model can be used to determine the production plan and the environmental improvement decisions.

The proposed model can introduce some significant managerial insights. First, the presented model helps production managers determine the production plan of the factory based on not only the costs and demands but also environmental impacts. Actually, if the environmental rules and regulations specify the standard limitations for producing GHG emissions in each factory, based on this model, managers can plan the production in such a way that the total pollution does not exceed the permitted limit. Secondly, this model helps managers achieve a specific plan for machines' improvement and pollution reduction. It helps a manager to decide to what extent the budget should be allocated to each machine. Thirdly, as the uncertainty in the manufacturing environments leads to difficulties in the planning process, the proposed robust model helps each manager recognize a feasible plan for the future based on the level of conservatism. Indeed, the results of the proposed model can be used by decision-makers to identify feasible areas of the problem for different levels of uncertainty. This matter helps them choose the appropriate budget of uncertainty in order to make acceptable production plans and decisions in improving green levels of machines.

Further studies can concentrate on enhancing the applicability of the proposed model to real-world situations and using real data to investigate the proposed model in other application domains. In addition, new solution methods are needed to solve the proposed model.

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