

ORIGINAL PAPER

# Goal programming for financial portfolio management: a state-of-the-art review

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**Abstract** Over the last decades, the Goal Programming (GP) model has been applied to financial portfolio management and/or selection problem in decision-making contexts where several conflicting and incommensurable objectives are simultaneously aggregated. The aim of this paper is to identify the research trends and publication outlets for the application of GP model to portfolio management. We point out an increasing interest and affirmation of more sophisticated models. We present a characterization of the existing GP variants and provide historical data and statistical analysis.

Keywords Goal programming · Financial portfolio selection · Typology

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## 1 Introduction

During the last two decades we have noticed a rapid increase of publications using different variants of the Goal Programming (GP) model for portfolio selection (Lin and O'Leary 1993; Aouni 2009, 2010; Azmi and Tamiz 2010; Aouni et al. 2014). The GP model enables the Financial Decision Maker (FDM) to aggregate several financial dimensions in order to select the best compromise portfolio. The FDM can be an investor, a portfolio manager, a financial analyst or a financial councillor. The FDM is requested to choose the appropriate GP variant to deal with a specific portfolio selection situation depending on the nature of the available information within the specific financial decision-making situation and the market performance.

In this paper we focus on portfolio selection and management that involve the construction of a portfolio of securities (such as stocks, bonds, treasury bills and mutual funds), that maximizes the FDM utility and accommodate his/her preferences. The conducted literature review reveals that there are at least one hundred and fifty publications applying the GP model to financial portfolio management. These publications are dealing with decision making situations in which the type of information can be (a) deterministic (ninety-two papers), (b) stochastic (twenty-nine papers), and (c) fuzzy or imprecise (thirty-four articles). The aim of this paper is to review the main theoretical developments and applications of different GP model variants to financial portfolio selection and management.

The paper is structured as follows: Section 2 provides a brief review of the most important GP variants that have been developed and applied to portfolio management and points out the advantages of each variant. Section 3, which is the core of the paper, describes how data have been collected and then classified. Section 4 is devoted to the presentation of our analysis and to the discussion of the main results. Finally, Sect. 5 concludes the paper and provides some recommendations for future research.

#### 2 Goal programming for financial portfolio selection

The bi-criteria financial portfolio selection model was developed by Markowitz in 1952 and published in a fundamental paper published in the Journal of Finance. His model aggregates simultaneously the expected return and the risk of a given portfolio. These two dimensions are incommensurable since both criteria are measured through different scales and units. The security return and risk are also conflicting in a situation where high returns are correlated to high risks and vice versa. The aggregation of both dimensions requires the FDM to provide some tradeoffs (compromises) based on his/her preferences and value system.

The mathematical formulation of Markowitz (1952) model reads as follows:

1. Attribute 1: The expected return of the portfolio,  $\sum_{j=1}^{n} E_{j} x_{j}$ , to be maximized,

2. Attribute 2: The portfolio risk,  $\sum_{j=1}^{n} \sum_{k=1}^{n} x_j \sigma_{jk} x_k$ , to be minimized.

Subject to:

$$\sum_{j=1}^{n} x_j = 1,$$
$$x \in F,$$

where:  $x_j$ , proportion to be invested in the security *j*;  $E_j$ , expected return of security *j*;  $\sigma_{ik}$ , covariance of the returns of securities *j* and *k*; *F*, set of feasible solutions.

The aggregation of both attributes 1 and 2 can be done either by determining the minimum variance portfolio subject to an expected return  $E^*$ :

Minimize 
$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j$$

Subject to:

$$\sum_{i=1}^{n} E_{i} x_{i} = E^{*}$$
$$\sum_{i=1}^{n} x_{i} = 1$$
$$x \in F$$

or by maximizing the expected portfolio return subject to a maximum level of sustainable and affordable risk  $R^*$ :

Maximize 
$$\sum_{i=1}^{n} E_i x_i$$

Subject to:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_i \sigma_{ij} x_j \le R^*$$
$$\sum_{i=1}^{n} x_i = 1$$
$$x \in F.$$

Since the introduction of Markowitz model, several other attempts have been proposed to consider more sophisticated portfolio models able to capture more investment features and improve the overall performance. In fact, empirical evidence demonstrates that in order to select the best financial portfolio it is required to aggregate more than two dimensions. The FDM may want to optimize simultaneously several incommensurable and conflicting attributes such as: (a) return rate; (b) risk; (c) liquidity; (d) gross book value per share; (e) capitalization ratio; and (f) stock market value of each company. Zopounidis et al. (1999) identified fifteen criteria and they grouped them into the following three categories: corporate validity; acceptability of stocks by the investors, and financial vigor. Within each category, five attributes have been listed (see Table 1).

As presented in Table 1, solving a portfolio selection problem requires partial or total attribute aggregation. The GP model is one of the aggregation procedures that has been widely utilized in portfolio management. Its methodological framework is based on the Distance Function (DF) model. In general the DF model aims at minimizing the following quantity

$$\sum_{i=1}^{p} [w_i | g_i - f_i(x) |^r]^{1/r}$$

that expresses the weighted sum of the Euclidean distance between  $g_i$  and  $f_i(x)$  for any x belonging to the feasible set F. The coefficients  $w_i$  represent the relative importance of each objective  $f_i(x)$  and r defines the family type of the Euclidean distance functions. The absolute deviation  $|g_i - f_i(x)|$  measures the distance between the achievement levels  $f_i(x)$  and the aspiration levels  $(goals)g_i$ . The linear formulation of the DF model, known as Goal Programming model, was introduced by Charnes et al. (1955) and Charnes and Cooper (1961). Their formulations are characterized by the presence of positive and negative deviations, both to be

Table 1 Different dimensions for portfolio selection

Category	Attributes
Corporate validity criteria	Gross book value per share
	Capitalization ratio
	Stock market value of each firm
	The marketability of each share
	Financial position progress
Acceptability of stocks by the investors	Dividend yield
	Capital gain
	Exchange flow ratio
	Round lots traded per day
	Transaction value per day
Financial vigor criteria	Equity ratio
	Price/earnings ratio
	Structure ratio
	Equity/debt ratio
	Return on equity

minimized: both underachievement and overachievement of objectives are unwanted according to the satisfying philosophy. In other words, the decision maker will penalize both positive and negative deviations.

The standard formulation of the GP introduced by Charnes and Cooper (1961) reads as follows:

$$\operatorname{Min} \sum_{i=1}^{n} \left( \delta_i^- + \delta_i^+ \right)$$

Subject to:

$$f_i(x) + \delta_i^- - \delta_i^+ = g_i (i = 1, 2, ..., p)$$
$$x \in F$$
$$\delta_i^-, \delta_i^+ \ge 0 (i = 1, 2, ..., p)$$

where  $\delta_i^-$  and  $\delta_j^+$  are the negative and positive deviations, respectively. Since the objectives are conflicting, the obtained solution can be qualified as the solution of the best compromise or the most satisfactory solution. According to Martel and Aouni (1990) and Aouni et al. (2009), the GP model is simpler and easier to understand and to apply than other Multiple Criteria Decision Aid (MCDA) and/or Multiple Objective Programming (MOP) techniques. GP models can be easily implemented by using some powerful commercial software such as AMPL, Lingo, and CPLEX. The large number of its applications in several domains including portfolio selection demonstrates its potential and shows its applicability and effectiveness in practice. Furthermore, Aouni et al. (2009) showed that the GP model allows an explicit integration of the DM's preferences and it requires limited information during the process of preference elucidation with respect to other models used within the MOP paradigm.

For more than 60 years, the GP model has become the most popular model in MCDA and MOP and widely implemented in financial portfolio management (Aouni et al. 2014). Indeed, different GP variants have been applied and they range from Weighted Goal Programming (WGP) to Lexicographic Goal programming (LGP), from Polynomial Goal Programming (PGP) to Stochastic Goal Programming (SGP), and finally to Fuzzy Goal Programming (FGP). The GP variants were applied according to the nature of the decision-making situation and the available information about the objective functions, decision variables and decision-making parameters (Azmi and Tamiz 2010; Aouni et al. 2014).

Through the WGP, the FDM may express his/her preferences by assigning weights to positive and negative deviations. Pendaraki et al. (2005); Bilbao-Terol et al. (2015) and Bravo et al. (2010) have utilized the WGP for financial portfolio selection where the relative importance of both risk and return objectives were expressed through the weights  $w_i^+$  and  $w_i^-$  associated with positive and negative deviations, respectively. In fact the weights have a double role, namely: (a) standardization of the units and scales of measurement and (b) valorization of the Decision-Maker's preferences (Kettani et al. 2004).

The LGP, also known as pre-emptive GP, allows the FDM to rank the objectives according to a lexicographic order based on their relative importance. The deviations of a higher level of priority are introduced as constraints within the subsequent mathematical programs related to the objectives of lower levels of priority. As a result, the objectives in the lower priority levels play a marginal role in the decision-making process. Lee (1972) has developed the first formulation of LGP for portfolio selection. This formulation aggregated simultaneously three dimensions, namely: (a) dividends, (b) the growth of earnings; and (c) 50% dividend payout ratio. Lee and Lerro (1973) have extended Lee (1972) formulation for mutual funds and they concluded that their model allowed to obtain quite similar solutions to those resulting from Markowitz (1952, 1959) and Sharpe (1967) models. Kumar et al. (1978) applied the LGP for dual-purpose funds managed by an investment company issuing two types of shares: (a) income shares and (b) capital shares. In fact, the LGP has been widely applied in financial portfolio selection since the 1980s.

Incorporating skewness into the decision-making process in the context of portfolio selection may cause major change in composition of the financial portfolio comparatively to portfolio based only on the mean–variance model. The PGP model proposed by Lai (1991) allowed incorporating preferences regarding skewness and other objectives and he claimed that this model was more efficient than LGP model. In their paper, Canela and Collazo (2007) have reformulated the different PGP models proposed by Lai (1991), Chunhachinda et al. (1997), Prakash et al. (2003) and Sun and Yan (2003) and claimed that these formulations may lead to unfeasible solutions.

Several financial decision-making contexts are characterized by uncertainty in which the decision-making parameters are random variables. The SGP model considers goals as stochastic values with a specific probability distribution. Our literature review reveals that several SGP formulations have been proposed for financial portfolio selection by using the notion of deterministic equivalent formulation. However, Aouni and La Torre (2010) introduced the concept of scenario generation in formulating a SGP model applied to portfolio selection. Through this model, probabilities were associated with all possible events or scenarios and the corresponding goals depended on the specific scenario based on the state of nature.

The FGP model was developed to deal with some financial decisional context in which the FDM can only provide vague or imprecise goal values (Arenas-Parra et al. 2001). The FGP is based on the concept of membership function that was introduced by Zimmerman (1976, 1978, 1983) and Freeling (1980) for modeling the fuzziness related to the decision-making parameters. In their paper, Bilbao-Terol et al. (2006) provided a formulation based on Sharpe (1967) model by considering ambiguous and vague parameters and calculating the betas using past observations. Their model was applied to Spanish mutual funds. Moreover, Mansour et al. (2007) formulated an imprecise GP model for portfolio selection based on the satisfaction function where the FDM's preferences are explicitly incorporated into the decision-making process. Their model has been applied to Tunisian stock exchange market.

The goals associated with the rate of return, the liquidity and the risk were considered to be imprecise and expressed through an interval-value function.

Several other GP variants have been applied to portfolio management over the years. We can mention: the min-max GP variant (Deng et al. 2005), the interactive GP model (Spronk 1980; Perez et al. 2007), the nonlinear GP model (Li and Xu 2007), the integer GP (Harrington and Fisher 1980) or the mixed-integer GP model (Aouni et al. 2013) and its variants such as the mixed-integer stochastic GP in Stoyan and Kwon (2011).

Recently, the Compromised Programming (CP) model was applied to portfolio selection problem by Ballestero and Romero (1996) and Ballestero (1998). Further extensions include a Fuzzy Compromised GP model (Ballestero et al. 2007) where the distance between fuzzy ideal goal values and the achievement levels were to be minimized. Nowadays, the Chance Constrained Compromise Programming (CCCP) model for the portfolio selection problem (Boswarva and Aouni 2012) is quite popular as well. We remind that the CCCP and the SGP formulations are based on the assumption that aspiration levels of objectives are normally distributed with known mean and variance.

When the FDM implements a GP model, he/she is not passive and his/her preferences and opinions can be taken into consideration in the portfolio selection process. In particular, the concept of satisfaction function developed by Martel and Aouni (1990) has been utilized to explicitly incorporate the FDM's preferences. In general, satisfaction functions are taking values in the interval [0, 1]. Therefore, the satisfaction function gives a value of 1 when the FDM is totally satisfied with the proposed solution. Otherwise, it is monotonically decreasing according to his/her appreciation of the achievement level of each objective.

Finally, GP formulations are often combined with other methods and techniques to solve multiple criteria problems: the most used are the Analytic Network Process (ANP); the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method; and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).

# **3** Data collection

For the purpose of this paper we extended the dataset of a previous survey produced by Aouni et al. (2014) and deepened the analysis. The most important step in our literature retrieval process was a computer search of Web of Science and Scopus databases. Our search period was not temporally limited. Using the descriptors "Goal Programming", "Financial Portfolio" and "Portfolio Management", we retrieved approximately 21 (Web of Science) and 136 (Scopus) abstracts to be added to the 91 papers of our previous research for review. Our initial dataset accounted for 248 papers.

Then we cleaned the database by removing duplicate rows/works. Each publication was carefully reviewed before taking a decision on its inclusion in this study. We excluded survey papers like Azmi and Tamiz (2010) or works that did not deal with financial portfolio management. A final total of 151 outputs (see

Appendix) was considered to be acceptable for the purposes of this survey. In particular, we identified: 131 papers (87%), 3 books (2%), 5 book chapters (3%), 10 conference papers (7%) and 2 working papers (1%). We classified each output according to the following categories: (a) Year of publication, (b) Journal, (c) Journal area, (d) Country/institution affiliation of the author, (e) Application area, (f) GP variants, and (g) Decision type.

Obviously, most of the data were available on Scopus and Web of Science. The journal area was identified according to the journal citation report. The application area was mainly identified through keywords provided by the authors themselves. For each paper we indicated the country/countries and whether this country belonged to the Organization for Economic Co-operation and Development (OECD), as a proxy for being a developed country. The institution affiliation of the author(s) was also used to describe the kind of collaboration: we were interested to identify if the output was due to an academic collaboration or a bridge collaboration with industry practitioners or governmental officials.

Finally, we were able to distinguish among three different families of information, namely: (a) deterministic, (b) stochastic and (c) fuzzy or imprecise. The remaining categories are self-explanatory.

#### 4 Bibliographical analysis

This section summarizes the results and discusses the findings for each of our classified categories. As shown in Fig. 1, there is an increasing interest on the application of the GP model to financial portfolio selection. We notice ten publications in the 1970s, forty-nine papers during 2000s and fifty-eight

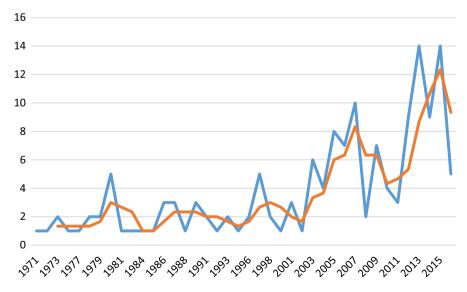


Fig. 1 Time evolution of the publications

publications during 2010s. The establishment of international conferences (e.g. MOPGP<sup>1</sup> conferences) might have impacted on this positive trend. The number of papers published in each year ranges from 1 to 14 (with an average equal to 3.8). Due to the possible time lag in reviewing and revising the submitted manuscripts and the scheduling of journal publications, it is justifiable to look at a three-year simple moving average for the publications (red line of Fig. 1). As expected, the values of the moving averages clearly confirm a steadily increasing trend.

Figure 2 summarizes the number of papers, related to financial portfolio selection through the GP model, by country. There are a total of 225 researchers affiliated with different institutions in 36 countries across the world. They have written an average of 1.7 output thus it is a fragmented production. However, we can identify some top authors, namely A. Bilbao Terol (13), B. Aouni (12), M. Arenas Parra (10) and F. Ben Abdelaziz (10). The largest number of authors are affiliated with an American university (22.6%), followed by affiliations with Spanish (12.4%), Canadian (6.5%), Tunisian (4.8%) and British universities (4.8%). Together, these five countries (CR5) account for 51.1%, the community is slightly concentrate. There are some considerations to be done in relation to authors' affiliation: during their academic life researchers change universities and countries, in other words the same author can contribute to different 'national' productions. Moreover, a researcher can have a double affiliation and this affects the final results, and makes impossible to provide an accurate measure of the contribution of a single author/country.

Grouping the publications related to the application of the GP model to financial portfolio selection by continent, we found that Europe (29.8%) is the most productive continent, followed by North America (23.8%), Asia (22.5%), Africa (3.3%) and Oceania (0.7%). Our literature review revealed that there are also several intercontinental collaborations (19.9%).

Most papers have been written by more than one author (88.1%) and the average number of authors per paper is 2.49. As regards the degree of development of a country, we distinguished between works written by authors affiliated with an institution in a developed (OECD members) or developing countries with a dummy variable that takes value 2 when all authors work in an institution of an OECD country or a set of OECD countries (but not one or more developing countries), 1 if at least one of the authors works an institution of an OECD country and 0 otherwise. 96 of the works (63.6%) are from the 21 developed countries, and 35 (23.2%) are from the 15 developing countries. Our review indicates that, 20 of the 151 research paper (13.2%) are collaborations between authors from OECD and non-OECD countries. It is clear the predominant role played by Institutions host in developed countries.

With regard to the type of affiliated institution, most of the papers (135 or 89.4%) were written by university professors and researchers from 129 different universities, 2 (1.4%) were authored by industry practitioners and government officials, and 14 (9.3%) were jointly written by authors from both sectors. The top universities in terms of output are the University of Oviedo (13 papers), followed by Laurentian University (10 papers), Technical University of Madrid, University of Milan, University of Rhode Island, and University of Tunis (6 papers each). These

<sup>&</sup>lt;sup>1</sup> The first MOPGP conference was held at the University of Portsmouth, United Kingdom, June, 1994.

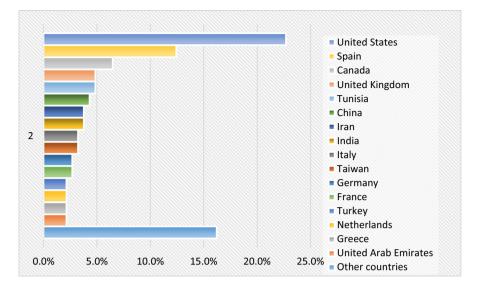


Fig. 2 Distribution by country

data confirm the key role played by European universities in this community and reflect the location of the top authors.

Moving to technical issues related to different GP variants, we noticed that the majority of papers using GP model for portfolio selection and portfolio management are using other variants of GP rather than its standard formulation. Moreover, 31.6% of the papers are using nonlinear GP, interactive GP and CP. Since 2000, fuzzy techniques are getting more and more popular and they account for 17.4% of the papers, whilst SGP is the most recent but the less used model (7.7%) in financial portfolio selection. The WGP variant is still quite popular (18.1%) over the years. LGP (11.0%) and PGP (14.2%) have been used in a limited manner over the decades. Table 2 shows the most commonly used models for each application area. These categories were developed in response to the papers that we found. The first categories refer to the specific type of securities (e.g. Bank portfolio, Dual-Purpose Funds or SRI). The other categories refer to the different aspects of investment decision (investment analysis, portfolio formation and portfolio management) or to the employed mathematical technique (mathematical modeling or optimization).

A wide variety of techniques are utilized in all different areas. Most works deal with portfolio selection and portfolio management, and the more innovative models are concentrated in these two area. A recent area of application concerns socially responsible investment (SRI): in recent years, sustainable development and social responsibility have become important issues around the globe, thus investment

Table 2	Table 2 GP variants by application area <sup>a</sup>	y applicatic	n area <sup>a</sup>											
GP models	GP Asset models liability management	Bank portfolio	Dual- purpose funds	Equity mutual funds	SRI	Stock market	Financial planning	Portfolio analysis	Portfolio selection	SRI Stock Financial Portfolio Portfolio Portfolio Portfolio market planning analysis selection management modelling optimization	Portfolio modelling	Portfolio optimization	Other Total	Total
WGP	2	2		3	3		2		10	3			2	27
LGP		1	2	3			1		9	5	1		1	17
PGP	2	1						3	6	7		5		22
SGP									6	3				12
FGP				1	7		1		18	4			1	27
Other GP	1	7		1	0	б	7	1	22	5	7	9	7	49
Total	5	9	5	8	٢	3	9	4	74	19	3	11	9	154
<sup>a</sup> Two I	Two papers present more GP	tore GP vai	variants											

strategy employs criteria (based on social, environmental and ethical screens<sup>2</sup>) other than financial risk and return when selecting firms in which to invest.

Another interesting aspect to explore is about the area and the journals publishing papers related to the application of GP to portfolio selection that may help to better understand the GP community active in this research field. The top journals are in the Operation Research area, namely: European Journal of Operational Research (11.5%), INFOR (6.9%), Journal of the Operational Research Society (4.6%), and Annals of Operations Research (3.8%). It seems that operational researchers tend to publish their results in OR journals rather than address specialized journals in Finance or Management. We also have some management journals, such as Decision Sciences, Journal of Banking and Finance, and Omega (3.8% each). All application areas are represented in the journals (Table 3).

Table 4 shows the classification of GP variants based on the type of information related to the parameters of the decision-making situation. We noticed that the GP model is largely applied in deterministic contexts. More recently, we count some applications related to stochastic and fuzzy decision-making contexts.

# 5 Concluding remarks

The aim of this paper is to provide a categorization of the applications of different GP variants for financial portfolio selection and portfolio management according to different characteristics (ranging from the type of information related to the decision-making situation to the application area or the demographic variables across the last decades). The performed literature review shows that the number of papers related to this subject has increased steadily especially over the past two decades, and this trend is expected to continue as the applicability of GP technique in financial portfolio management is fully recognized by researchers worldwide, with a focus in the European area, and in the developed countries in general. With regard to publication outlets, it seems that over the years most academicians have preferred to publish in top journals in operation research. This is due to an increasing use of more sophisticated models (SGP and FGP models) able to provide a more complete representation of complexity, or at least a more complete understanding of the real world (see Table 4).

We also notice that researchers within the field of Management Science and Operations Research are very active in applying the GP model to portfolio selection that was traditionally related to the field of Finance. The rapid increase in using GP model can be explained by the fact that it is an easy tool to be understood and implemented, and it is supported by commercial optimization software. Moreover, the GP model is more flexible than the other MCDA techniques. It is a learning process in which the FDM can interact and continuously adjust the parameters in order to improve the decision-making process through a progressive and evolving

<sup>&</sup>lt;sup>2</sup> The environment concern includes climate change and clean technologies or pollution. Under the social concerns we can look at human rights and labor relations for instance. Ethical or governance concerns relate to board issues. Popular negative screens refer to the sin screens (production of alcohol, tobacco or gambling products) or military weapons, just to mention a few.

Table 3	Table 3 Classification by journal	on by jour	mal											
GP ≜ models	ANOR Applications of management science		Applied mathematics and computation	Applied soft computing journal	Computers and g industrial engineering	Decision EJOR Expert sciences system with applica	EJOR	t Expert systems with application	ExpertFinancialSystemsSystemsSystemsMarkets andwithportfolioapplicationsapplications	Financial Markets and portfolio management	Fuzzy nd optimization and decision nt making		INFOR Information sciences	nation ces
WGP 1				1			4				1			
LGP						7	6							
PGP							2		2					
FGP 1	1			1		1	2	2		1	1		2 2	
SGP 1							1				4			
Other 2 GP	2 1		4		5	7	9	1		1	б		1	
Total 5	5 2		4	2	2	5	15	3	2	2	6	.,	3 2	
GP models	els IRJFE	ITOR	Journal of banking and finance		Journal of multicriteria decision analysis		JORS	LNEMS 1	Management science	Omega	Conference paper	WP	Book and chapter	Total
WGP			1	1		3		1	5	1		1	11	26
LGP		2										1	10	16
PGP		ю							1	4	1	1	8	22
FGP				2						3			8	26
SGP	1		1							2			2	11
Other GP	P 2			33				1	2	1	2	5	10	48
Total	3	5	2	9		3		5	5	6	3	5	38	151

<b>Table 4</b> Information type andGP model in portfolio		1970s	1980s	1990s	2000s	2010s	Total
management <sup>a</sup>	Deterministic	10	17	12	27	26	92
	WGP	2	5	5	6	8	26
	LGP	6	5	1	1	4	17
	PGP		1	3	11	7	22
	Other GP	2	6	3	9	7	27
	Fuzzy		1	1	12	20	34
	WGP					2	2
	FGP		1	1	8	16	26
	Other GP				4	2	6
	Stochastic		1	2	13	16	29
	FGP					1	1
	SGP				4	8	11
	Other GP		1	2	6	7	16
<sup>a</sup> Two papers present more GP variants	Grand total	10	19	15	49	62	155

sequence of actions. The investment decisions are taken by the FDM and the GP model is a tool to support and not to replace humans' decisions. The GP model allows the FDM to express his/her preferences based on his/her intuition, experience and knowledge. In addition, the behavior of the financial portfolio management depends on several external factors that are difficult to control and to predict during the modeling process. These factors are related to: (a) international economy; (b) national economy; (c) international political stability; (d) natural phenomena; and (e) the FDM psychology. Future avenues in GP theory and modeling include the formulation of more complex GP variants that will also try to model the effect of external factors as well as the subjectivity nature of the financial decision making process. In this perspective a more intense collaboration between academic researchers and industry practitioners will be beneficial.

# Appendix

Authors	Journal	Year
Aboul-Enein S., Dionne G., Papageorgiou N.	European Journal of Finance	2013
Alexander G.J., Resnick B.G.	Journal of Banking and Finance	1985
Allen, J., Bhattacharya, S., Smarandache, F.	International Journal of Social Economics	2003
Amiri, M., Ekhtiari, M., Yazdani, M.	Expert Systems with Applications	2011
Aouni B., Colapinto C., La Torre D.	Annals of Operations Research	2013
Aouni, A., Ben Abdelaziz, F., La Torre, D.	Journal of Multi-Criteria Decision Analysis	2012
Aouni, A., Colapinto, C., La Torre, D.	Journal of Financial Decision Making	2010

Authors	Journal	Year
Aouni, B., Ben Abdelaziz, F, El Fayedh, R.	Administrative Sciences Association of Canada, Management Science	2003
Aouni, B., Ben Abdelaziz, F., Martel, J. M.	European Journal of Operational Research	2005
Aouni, B., Colapinto, C., La Torre, D.	INFOR	2014
Arenas Parra M., Bilbao Terol A., Rodríguez Uría M.V.	European Journal of Operational Research	2001
Arenas-Parra, M., et al.	Applied Mathematics and Computation	2006
Bahloul, S. Abid, F.	International Journal of Multi-Criteria Decision Making	2013
Ballestero, E.	Journal of Operational Research Society	1998
Ballestero, E., Pla-Santamaria, D.	International Transactions in Operational Research	2003
Ballestero, E., Pla-Santamaria, D.	Omega	2004
Ballestero, E., Romero, C.	Journal of the Operational Research Society	1996
Ballestero E., Garcia-Bernabeu A.	INFOR	2012
Ballestero E., Garcia-Bernabeu A., Hilario A.	Book Chapter	2015
Ballestero, E., et al.	European Journal of Operational Research	2007
Ballestero, E., et al.	INFOR	2009
Batson, R.G.	Long Range Planning	1989
Ben Abdelaziz F., El Fayedh R., Rao A.	INFOR	2009
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