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A preemptive priority retrial queue with two classes of customers and general retrial times

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Abstract In this paper, we consider a continuous-time retrial queue with two classes of customers: priority customers and ordinary customers, where priority customers don't queue and have an exclusive preemptive priority to receive their services over ordinary customers. If an arriving ordinary customer finds the server busy, it enters a retrial group (called orbit) according to FCFS discipline. Only the ordinary customer at the head of the retrial queue is allowed to access the server. Firstly, we obtain the necessary and sufficient condition for the system to be stable by embedded Markov chain approach. Secondly, using supplementary variable method, we obtain the stationary probability distribution and some performance measures of interest. Thirdly, we give the analysis of the sojourn time in the system of an arbitrary ordinary customer. Lastly, numerical examples are given to show the effect of system parameters on several performance measures.

Keywords Retrial queue · Preemptive priority · Embedded Markov chain · Supplementary variable method

1 Introduction

Recently, a significant number of papers have dealt with retrial queueing systems, since retrial queues can be extensively used to stochastically model many problems arising in computer networks, telecommunication, telephone systems and in daily life, see, e.g. Falin and Templeton (1997), Artalejo and Gómez-Corral (2008), Gómez-Corral (2006) and Artalejo (1999, 2010).

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In the past years, the study of retrial queueing systems with priorities have been reinvigorated because different classes of customers need different quality of service (QoS). High priority customers are queued or not queued and served according to preemptive or non-preemptive discipline. If blocked, low priority customers (called as ordinary customers) leave the system and join the retrial group to retry until they find the server free. Choi and Park (1990) first investigated a nonpreemptive retrial queue with priority and ordinary customers, in which priority customers have nonpreemptive priority over ordinary customers and are queued in FCFS discipline, however, ordinary customers behave as a classical M/G/1 retrial queue with classical retrial policy, and two types of customers have the same service time distributions. Later on, Falin et al. (1993) considered the retrial queue model of Choi and Park (1990) in the case that two types of customers may have different service time distributions. Langaris and Moutzoukis (1995) considered a retrial queue with structured batch arrivals, preemptive priorities and server vacations, where the retrial times of each low priority customers in the orbit follow exponential distributions, that is the classical retrial policy is adopted. Krishna Kumar et al. (2002) considered an M/G/1 retrial queue with two-phase service and possible preemptive resume at the first phase of service. Wang (2008) studied an unreliable $\mathrm{M}_1, \mathrm{M}_2/\mathrm{G}_1, \mathrm{G}_2/1$ retrial queue with priority subscribers. Recently, Wu et al. (2013) presented a discrete-time Geo/G/1 retrial queue with preferred customers and impatient customers, where the arriving customer may push out the customer in service to commence his own service with some probability. Dimitriou (2013) considered a retrial queueing model accepting two types of positive customers and negative arrivals, where an arriving P_1 customer can preempt the service of a P_2 customer and force the server to start his service. Further, Dimitriou (2013) considered an unreliable single server retrial queue with two classes of customers and negative customers. Different from above literatures, Krishna Kumar and Pavai Madheswari (2004) investigated a retrial queueing system with two classes of customers, in which the class-1 customers are blocked if the server is not available and leave the system forever, while class-2 customers may be obliged to leave the service area and join the retrial group/orbit, to retry for their service after a random interval of time. Later, Liu and Gao (2011) extended Krishna Kumar and Pavai Madheswari's results to a discrete-time $\mathrm{Geo}_1, \mathrm{Geo}_2^X/\mathrm{G}_1, \mathrm{G}_2/1$ retrial queue with two classes of customers and feedback. However, there is no work that deals with retrial queueing system with two classes of customers, preemptive resume and general retrial times. This motivates us to investigate such queueing system in this work.

The rest of this paper is organized as follows. In Sect. 2, we give the model description of the preemptive retrial queue. Section 3 presents the stable condition of the system, and deals with steady-state analysis including the joint distribution of number of ordinary customers in the orbit and the server state at a random epoch and some system characteristics. In Sect. 4, we study the distribution of the sojourn time in the system of any arbitrary ordinary customer. Some numerical examples are provided in Sect. 5.

2 System model

In this section, we consider a continuous-time single server retrial queue with priority customers and ordinary customers. Priority customers have preemptive priorities over ordinary customers in the use of the server. We assume that priority customers and ordinary customers arrive according to two independent Poisson processes with rates δ and λ, respectively. The service times of priority customers are assumed to be arbitrary distributed with distribution function (d.f.) $A(x)$, probability density function (p.d.f) $a(x)$, finite first two moments α_1, α_2. The service times of ordinary customers follow arbitrary distribution with d.f. $B(x)$, p.d.f $b(x)$, finite first two moments β_1, β_2.

On the arrival of a priority customer, if the server is found to be idle, the arriving priority customer occupies the server and begins its service. If the server is busy serving a priority customer, the newly arriving priority customer will depart the system directly without service. If the server is being occupied by an ordinary customer, the arriving priority customer will interrupt the service of the ordinary customer and occupy the server to begin its service immediately. We assume that when an ordinary customer is preempted by a priority customer, it will wait in the service area until the server completes the service of the priority customer and continues to complete its remaining service. On the arrival of an ordinary customer, if the server is idle, the ordinary customer immediately begins receiving its service and leaves the system after the completion of the service. If the server is being busy, the arriving ordinary customer will join the retrial orbit.

We assume that only the ordinary customer at the head of the retrial queue is allowed to retry to access the server at a service completion instant. The retrial time is assumed to be generally distributed with d.f. $R(x)$, p.d.f $r(x)$. Then measured from the instant the server becomes idle, an external potential priority customer or ordinary customer and a retrial ordinary customer compete to access the server. The retrial ordinary customer is required to give up the attempt for service if an external priority customer or ordinary customer arrives first. In that case, the retrial ordinary customer goes back to its position in the retrial queue. About literatures on retrial queues with general retrial times, readers may refer to Gómez-Corral (1999), Wang (2006), Gao and Wang (2014) and references therein.

We assume that all the random variables defined above are independent mutually.

Throughout the rest of the paper, for a d.f. $F(x)$, we define $\overline{F}(x) = 1 - F(x)$ to be the tail of $F(x)$, $\widetilde{F}(s) = \int_0^\infty e^{-sx} dF(x)$, the Laplace–Stieltjes transform of $F(x)$ and $F^*(s) = \int_0^\infty e^{-sx} F(x) \mathrm{d}x$ to be the Laplace transform of function $F(x)$, and we adopt the notation $\overline{F}^*(s) = \frac{1-\widetilde{F}(s)}{s}$.

Define the functions $\alpha(x)$ and $\beta(x)$, respectively, as the conditional completion rates for services of a priority customer and an ordinary customer, and $\gamma(x)$ as the conditional completion rate for retrial attempt, i.e.,

$$\alpha(x) = \frac{a(x)}{\overline{A}(x)}, \quad \beta(x) = \frac{b(x)}{\overline{B}(x)}, \quad \gamma(x) = \frac{r(x)}{\overline{R}(x)}$$

Remark 1 If the arriving priority customer find the server busy no matter with a priority customer or an ordinary customer, it will leave the system forever, i.e., no preemption occurs during the service of an ordinary customer, the model becomes to be a non-preemptive priority queue with two class customers and general retrial times, which is an extension of the paper (Krishna Kumar and Pavai Madheswari 2004).

As a practical application of the queue model considered in this paper, we can take a cognitive network with a single licensed channel as an example. In cognitive networks (see Akyildiz et al. 2006; Yucek and Arslan 2009), there are two classes of users, called as primary users (PUs, corresponding to priority customers) and secondary users (SUs, corresponding to ordinary customers). PUs have exclusive preemptive priority to occupy a certain spectrum band (called licensed channel, corresponding to the single server) and their access is generally controlled by a Primary Operator (PO). SUs have no spectrum license, they implement additional functionalities to share the licensed channel without interfering with PUs, i.e., they can access the licensed channel when there are no PUs occupying it and the message transmissions of the SU can be randomly interrupted by the arriving PU and the interrupted SU can continue its message transmission immediately after the service of the PU. If the channel is busy with a PU, the arriving PU leaves the system forever, however, the new arriving SU enters the retrial group and retries its luck to get service after some time if it finds the channel busy upon arrival. Obviously, this cognitive radio work can be modelled as our retrial queue.

3 Stability condition and steady state performance analysis of the system

In this section, we will focus on the discussion of the stability condition of the system by using embedded Markov chain technique and steady state performance analysis of the system by using supplementary variable method.

3.1 Stability condition

First, we introduce some notations used in the future.

Let $g_k(k \geq 0)$ denote the probability that there are k ordinary customers enter the retrial queue during the service time of a priority customer and its probability generating function be $G(z) = \sum_{k=0}^{\infty} z^k g_k$, then

$$g_k = \int_0^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} a(t) dt,$$

and

$$G(z) = \widetilde{A}(\lambda(1-z)).$$

Let S be the generalized service time of an ordinary customer from the epoch that it begins its service to the epoch at which the ordinary customer's service is completed, that is the server is ready for the next new service, and let A_S be the number of ordinary customers that enter the retrial queue during S. The probability generating function of A_S is denoted as $H(z) \triangleq \sum_{k=0}^{\infty} z^k h_k = \sum_{k=0}^{\infty} z^k P(A_S = k)$, and we define $\widetilde{S}(s) = E[e^{-sS}]$ to be the Laplace transform of S. Denote

$$h(k,t)dt = P(t < S \leq t + dt, A_S = k),$$
$$\widetilde{H}(z,s) = \sum_{k=0}^{\infty} z^k \int_0^{\infty} e^{-st} h(k,t)dt.$$

Then we have

$$h(k,t) = \left(\sum_{n=0}^{\infty} p_n(t) b(t) * a^{(n)}(t) \right) q_k(t),$$
$$h_k = \int_0^{\infty} h(k,t)dt,$$

where $*$ denotes convolution, $p_k(t) = \frac{(\delta t)^k}{k!} e^{-\delta t}$, $q_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$, $a^{(k)}(t)$ denotes k-convolution of $a(t)$. After algebra convolutions, we can obtain

$$\widetilde{H}(z,s) = \widetilde{B}(\Phi(s,z)),$$

where $\Phi(s,z) = s + \lambda(1 - z) + \delta(1 - \widetilde{A}(s + \lambda(1 - z)))$.

Putting $\phi(z) = \Phi(0,z) = \lambda(1 - z) + \delta(1 - G(z))$, hence

$$\widetilde{S}(s) = \widetilde{H}(1,s) = \widetilde{B}(s + \delta(1 - \widetilde{A}(s))),$$
$$H(z) = \widetilde{H}(z,0) = \widetilde{B}(\phi(z)),$$
$$H'(1) = \frac{dH(z)}{dz}\bigg|_{z=1} = \lambda\beta_1(1 + \delta\alpha_1).$$

Let $T_k (T_0 = 0)$ be the time epoch at which the server becomes idle for the kth time, $N_k = N(T_k)$ be the number of ordinary customers in the orbit at the time T_k, then the process $\{N_k, k \geq 0\}$ is an embedded Markov chain with state space $\mathbb{N}$.

Before further development, we first give a lemma.

Lemma 1 (Foster's criterion, see Pakes 1969) *An irreducible and aperiodic Markov chain $\{Y_k, k \geq 0\}$ with state space $\mathbb{S}$ is ergodic if there exists a non-negative test function $f(i), i \in \mathbb{S}$, and $\varepsilon > 0$ such that the mean drift $y_i = E[f(Y_{k+1}) - f(Y_k) | Y_k = i]$ is finite for all $i \in \mathbb{S}$ and $y_i \leq -\varepsilon$ for all $i \in \mathbb{S}$ except perhaps a finite number.*

Then we have the following theorem.

Theorem 1 *The embedded Markov chain $\{N_k, k \geq 0\}$ is ergodic if and only if the following inequality holds:*

$$\left(\widetilde{R}(\delta + \lambda) + \lambda\overline{R}^*(\delta + \lambda)\right)\lambda\beta_1(1 + \delta\alpha_1) + \delta\overline{R}^*(\delta + \lambda)\lambda\alpha_1 < \widetilde{R}(\delta + \lambda).$$

Proof From the assumptions of our model, the one-step transition probabilities are given as follows:

$$P(N_{k+1}=j|N_k=i)$$
$$=\begin{cases} \dfrac{\delta}{\delta+\lambda}g_j+\dfrac{\lambda}{\delta+\lambda}h_j, & i=0,\ j\geq 0,\\ \widetilde{R}(\delta+\lambda)h_0, & i>0,\ j=i-1,\\ \delta\overline{R}^*(\delta+\lambda)g_{j-i}+\lambda\overline{R}^*(\delta+\lambda)h_{j-i}+\widetilde{R}(\delta+\lambda)h_{j-i+1}, & i>0,\ j>i-1,\\ 0, & \text{otherwise.} \end{cases}$$

Obviously, the Markov chain $\{N_k, k\geq 0\}$ is irreducible and aperiodic. And the mean drift

$$x_i=E[N_{k+1}-N_k|N_k=i]$$
$$=\begin{cases} \dfrac{\delta}{\delta+\lambda}\lambda\alpha_1+\dfrac{\lambda}{\delta+\lambda}\lambda\beta_1(1+\delta\alpha_1), & i=0,\\ \big(\widetilde{R}(\delta+\lambda)+\lambda\overline{R}^*(\delta+\lambda)\big)\lambda\beta_1(1+\delta\alpha_1)+\delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1-\widetilde{R}(\delta+\lambda), & i>0. \end{cases}$$

Then from Foster's criterion, by taking the function $f(i)=i$, we know that the inequality $\big(\widetilde{R}(\delta+\lambda)+\lambda\overline{R}^*(\delta+\lambda)\big)\lambda\beta_1(1+\delta\alpha_1)+\delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1<\widetilde{R}(\delta+\lambda)$ is a sufficient condition for the system to be stable.

The same inequality is also the necessary condition for ergodicity. Assume that $\big(\widetilde{R}(\delta+\lambda)+\lambda\overline{R}^*(\delta+\lambda)\big)\lambda\beta_1(1+\delta\alpha_1)+\delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1\geq\widetilde{R}(\delta+\lambda)$, which implies that $x_i\geq 0$ for all $i\geq 0$. Furthermore, according to the one-step transition probabilities, we know that the down drift

$$D_i=\sum_{j<i}(j-i)P(N_{k+1}=j|N_k=i)=\begin{cases} 0, & i=0,\\ -\widetilde{R}(\delta+\lambda)h_0, & i>0, \end{cases}$$

which implies that the Markov chain $\{N_k, k\geq 0\}$ satisfies Kaplan's condition namely if the sequence $\{D_m, m\geq 0\}$ is bounded below. Thus the Markov chain $\{N_k, k\geq 0\}$ is not ergodic, and then the necessity of the ergodicity is proven. □

3.2 Steady state performance analysis of the system

By the description of the model, we know that at time t, the state of the system considered in this paper can be described by the Markov process $X(t)=\{N(t),J(t),\xi_0(t),\xi_1(t),\xi_2(t),\xi_3(t)\}$, where $N(t)$ denotes the number of ordinary customers in the orbit at time t, $J(t)$ represents the server state, defined as follows

$$J(t)=\begin{cases}0, & \text{the server is idle at time t,}\\ 1, & \text{the server is busy with a priority customer without preempting}\\ & \text{an ordinary customer at time t,}\\ 2, & \text{the server is busy with a priority customer with preempting}\\ & \text{an ordinary customer at time t,}\\ 3, & \text{the server is busy with an ordinary customer at time t.}\end{cases}$$

When $J(t)=0$ and $N(t)>0$, $\xi_0(t)$ is the elapsed retrial time; when $J(t)=1$, $\xi_1(t)$ is the elapsed service time of the priority customer; when $J(t)=2$, $\xi_1(t)$ is the elapsed service time of the priority customer and $\xi_2(t)$ is the elapsed service time of the interrupted ordinary customer; when $J(t)=3$, $\xi_3(t)$ denotes the elapsed service time of the ordinary customer.

It is easy to see that the Markov process $X(t)=\{N(t),J(t),\xi_0(t),\xi_1(t),\xi_2(t),\xi_3(t)\}$ is a semi-regenerative process with embedded Markov chain $\{N_k,k\geq 0\}$, from Burke's theorem that the steady state probabilities of the Markov process exist and are positive if and only if the embedded Markov chain $\{N_k,k\geq 0\}$ is ergodic. Therefore, the inequality $\left(\widetilde{R}(\delta+\lambda)+\lambda\overline{R}^*(\delta+\lambda)\right)\lambda\beta_1(1+\delta\alpha_1)+\delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1<\widetilde{R}(\delta+\lambda)$ is a sufficient and necessary condition for the system to be stable. Let $X=\{N,J,\xi_0,\xi_1,\xi_2,\xi_3\}$ be the stationary limit of the Markov process $X(t)=\{N(t),J(t),\xi_0(t),\xi_1(t),\xi_2(t),\xi_3(t)\}$.

Henceforth, we assume that the system is stable. Define the following limiting probabilities and limiting probability densities:

$$\begin{aligned}
p_{0,0}&=P(N=0,J=0)=\lim_{t\to\infty}P(N(t)=0,J(t)=0),\\
p_{k,0}(x)\mathrm{d}x&=P(N=k,J=0,x<\xi_0\leq x+\mathrm{d}x)\\
&=\lim_{t\to\infty}P(N(t)=k,J(t)=0,x<\xi_0(t)\leq x+\mathrm{d}x),x\geq 0,k\geq 1,\\
p_{k,1}(x)\mathrm{d}x&=P(N=k,J=1,x<\xi_1\leq x+\mathrm{d}x)\\
&=\lim_{t\to\infty}P(N(t)=k,J(t)=1,x<\xi_1(t)\leq x+\mathrm{d}x),x\geq 0,k\geq 0,\\
p_{k,2}(x,y)\mathrm{d}x\mathrm{d}y&=P(N=k,J=2,x<\xi_1\leq x+\mathrm{d}x,y<\xi_2\leq y+\mathrm{d}y)\\
&=\lim_{t\to\infty}P(N(t)=k,J(t)=2,x<\xi_1(t)\leq x+\mathrm{d}x,y<\xi_2(t)\leq y+\mathrm{d}y),\\
&\quad x\geq 0,y\geq 0,k\geq 0,\\
p_{k,3}(x)\mathrm{d}x&=P(N=k,J=3,x<\xi_3\leq x+\mathrm{d}x)\\
&=\lim_{t\to\infty}P(N(t)=k,J(t)=3,x<\xi_3(t)\leq x+\mathrm{d}x),x\geq 0,k\geq 0.
\end{aligned}$$

By the method of the supplementary variable, we easily obtain the system of equilibrium equations:

$$(\delta+\lambda)p_{0,0}=\int_0^\infty p_{0,1}(x)\alpha(x)\mathrm{d}x+\int_0^\infty p_{0,3}(x)\beta(x)\mathrm{d}x, \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}p_{k,0}(x) = -(\delta + \lambda + \gamma(x))p_{k,0}(x),\ k \geq 1, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}p_{k,1}(x) = -(\lambda + \alpha(x))p_{k,1}(x) + (1 - \delta_{k,0})\lambda p_{k-1,1}(x),\ k \geq 0, \tag{3}$$

$$\frac{\partial}{\partial x}p_{k,2}(x,y) = -(\lambda + \alpha(x))p_{k,2}(x,y) + (1 - \delta_{k,0})\lambda p_{k-1,2}(x),\ k \geq 0, \tag{4}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}p_{k,3}(x) = &-\ (\delta + \lambda + \beta(x))p_{k,3}(x) + (1 - \delta_{k,0})\lambda p_{k-1,3}(x)\\ &+ \int_0^\infty p_{k,2}(y,x)\alpha(y)dy,\ k \geq 0,\end{aligned} \tag{5}$$

where $\delta_{k,0}$ is the Kronecker's symbol. The boundary conditions are

$$p_{k,0}(0) = \int_0^\infty p_{k,1}(x)\alpha(x)\mathrm{d}x + \int_0^\infty p_{k,3}(x)\beta(x)\mathrm{d}x,\ k \geq 1, \tag{6}$$

$$p_{k,1}(0) = \delta_{k,0}\delta p_{0,0} + \delta(1 - \delta_{k,0})\int_0^\infty p_{k,0}(x)\mathrm{d}x,\ k \geq 0, \tag{7}$$

$$p_{k,2}(0,x) = \delta p_{k,3}(x),\ k \geq 0, \tag{8}$$

$$p_{k,3}(0) = \delta_{k,0}\lambda p_{0,0} + \lambda(1 - \delta_{k,0})\int_0^\infty p_{k,0}(x)\mathrm{d}x + \int_0^\infty p_{k+1,0}(x)\alpha(x)\mathrm{d}x,\ k \geq 0, \tag{9}$$

and the normalization condition is

$$p_{0,0} + \sum_{k=1}^{\infty}\int_0^\infty p_{k,0}(x)\mathrm{d}x + \sum_{k=0}^{\infty}\left(\int_0^\infty \big(p_{k,1}(x) + p_{k,3}(x)\big)\mathrm{d}x + \int_0^\infty p_{k,2}(x,y)\mathrm{d}x\mathrm{d}y\right) = 1. \tag{10}$$

By introducing the generating functions $P_0(x,z) = \sum_{k=1}^{\infty} z^k p_{k,0}(x)$, and $P_j(x,z) = \sum_{k=0}^{\infty} z^k p_{k,j}(x),\ j = 1,3$, and $P_2(x,y,z) = \sum_{k=0}^{\infty} z^k p_{k,2}(x,y)$, from Eqs. (2)–(5), we have

$$P_0(x,z) = P_0(0,z)\exp\{-(\delta + \lambda)x\}\overline{R}(x), \tag{11}$$

$$P_1(x,z) = P_1(0,z)\exp\{-\lambda(1 - z)x\}\overline{A}(x), \tag{12}$$

$$P_2(x,y,z) = P_2(0,y,z)\exp\{-\lambda(1 - z)x\}\overline{A}(x), \tag{13}$$

$$\frac{\partial}{\partial x}P_3(x,z) = -(\delta + \lambda(1 - z) + \beta(x))P_3(x,z) + P_2(0,x,z)G(z). \tag{14}$$

and then by (1), (6)–(9), we can obtain

$$P_0(0,z) = P_1(0,z)G(z) + \int_0^\infty P_3(x,z)\beta(x)\mathrm{d}x - (\delta+\lambda)p_{0,0}, \tag{15}$$

$$P_1(0,z) = \delta p_{0,0} + \delta P_0(0,z)\overline{R}^*(\delta+\lambda), \tag{16}$$

$$P_2(0,x,z) = \delta P_3(x,z), \tag{17}$$

$$P_3(0,z) = \lambda p_{0,0} + \lambda P_0(0,z)\overline{R}^*(\delta+\lambda) + \frac{1}{z}P_0(0,z)\widetilde{R}(\delta+\lambda). \tag{18}$$

Inserting (17) into (14) leads to

$$P_3(x,z) = P_3(0,z)\exp\{\phi(z)x\}\overline{B}(x), \tag{19}$$

Combining (15), (16), (18) and (19) and making some algebra calculations, we can obtain

$$P_0(0,z) = \frac{z[\delta+\lambda-(\delta G(z)+\lambda H(z))]}{\mathcal{D}(z)}p_{0,0}, \tag{20}$$

$$P_1(0,z) = \frac{\widetilde{R}(\delta+\lambda)(H(z)-z)}{\mathcal{D}(z)}\delta p_{0,0}, \tag{21}$$

$$P_2(0,x,z) = \frac{\delta\widetilde{R}(\delta+\lambda)\phi(z)}{\mathcal{D}(z)}\exp\{\phi(z)x\}\overline{B}(x)p_{0,0}, \tag{22}$$

$$P_3(0,z) = \frac{\widetilde{R}(\delta+\lambda)\phi(z)}{\mathcal{D}(z)}p_{0,0}, \tag{23}$$

where $\mathcal{D}(z) = H(z)\widetilde{R}(\delta+\lambda) + z\overline{R}^*(\delta+\lambda)(\delta G(z)+\lambda H(z)) - z$.

Applying the normalization condition (10), we obtain

$$p_{0,0} + \int_0^\infty P_0(x,1)\mathrm{d}x + \int_0^\infty (P_1(x,1)+P_3(x,1))\mathrm{d}x + \int_0^\infty\int_0^\infty P_2(x,y,1)\mathrm{d}x\mathrm{d}y = 1,$$

and by using (11)–(13) and (19)–(23), we can arrive at

$$p_{0,0} = \frac{\widetilde{R}(\delta+\lambda) - \left[\left(\widetilde{R}(\delta+\lambda)+\lambda\overline{R}^*(\delta+\lambda)\right)\lambda\beta_1(1+\delta\alpha_1) + \delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1\right]}{\widetilde{R}(\delta+\lambda)(1+\delta\alpha_1)}. \tag{24}$$

Now we summarize the above results in following theorem.

Theorem 2 *If the system is stable, the generating functions of the stationary joint distribution of the orbit size and the server state are given by:*

$$p_{0,0} = \frac{\widetilde{R}(\delta+\lambda) - \left[\left(\widetilde{R}(\delta+\lambda) + \lambda\overline{R}^*(\delta+\lambda)\right)\lambda\beta_1(1+\delta\alpha_1) + \delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1\right]}{\widetilde{R}(\delta+\lambda)(1+\delta\alpha_1)},$$

$$P_0(x,z) = \frac{z[\delta+\lambda-(\delta G(z)+\lambda H(z))]}{\mathcal{D}(z)}\exp\{-(\delta+\lambda)x\}\overline{R}(x)p_{0,0},$$

$$P_1(x,z) = \frac{\widetilde{R}(\delta+\lambda)(H(z)-z)}{\mathcal{D}(z)}\exp\{-\lambda(1-z)x\}\overline{A}(x)\delta p_{0,0},$$

$$P_2(x,y,z) = \frac{\delta\widetilde{R}(\delta+\lambda)\phi(z)}{\mathcal{D}(z)}\exp\{-\lambda(1-z)x\}\exp\{\phi(z)y\}\overline{A}(x)\overline{B}(y)p_{0,0},$$

$$P_3(x,z) = \frac{\widetilde{R}(\delta+\lambda)\phi(z)}{\mathcal{D}(z)}\exp\{\phi(z)x\}\overline{B}(x)p_{0,0}.$$

Remark 2 If no preemption occurs during the service of an ordinary customer, the state of the server $J(t)$ only takes values of $0, 1, 3$. In this case, similarly we can prove that the stationary condition is

$$\left(\widetilde{R}(\delta+\lambda) + \lambda\overline{R}^*(\delta+\lambda)\right)\lambda\beta_1 + \delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1 < \widetilde{R}(\delta+\lambda),$$

and in steady-state,

$$p_{0,0} = \frac{\widetilde{R}(\delta+\lambda) - \left[\left(\widetilde{R}(\delta+\lambda) + \lambda\overline{R}^*(\delta+\lambda)\right)\lambda\beta_1 + \delta\overline{R}^*(\delta+\lambda)\lambda\alpha_1\right]}{\widetilde{R}(\delta+\lambda)(1+\delta\alpha_1)},$$

$$P_0(x,z) = \frac{z\left[\delta+\lambda-(\delta\widetilde{A}(\lambda(1-z))+\lambda\widetilde{B}(\lambda(1-z)))\right]\exp\{-(\delta+\lambda)x\}\overline{R}(x)}{\widetilde{R}(\delta+\lambda)\widetilde{B}(\lambda(1-z)) + z\overline{R}^*(\delta+\lambda)\left[\delta\widetilde{A}(\lambda(1-z))+\lambda\widetilde{B}(\lambda(1-z))\right] - z}p_{0,0},$$

$$P_1(x,z) = \frac{\widetilde{R}(\delta+\lambda)(\widetilde{B}(\lambda(1-z))-z)\exp\{-\lambda(1-z)x\}\overline{A}(x)}{\widetilde{R}(\delta+\lambda)\widetilde{B}(\lambda(1-z)) + z\overline{R}^*(\delta+\lambda)\left[\delta\widetilde{A}(\lambda(1-z))+\lambda\widetilde{B}(\lambda(1-z))\right] - z}\delta p_{0,0},$$

$$P_3(x,z) = \frac{\widetilde{R}(\delta+\lambda)\phi(z)\exp\{-\lambda(1-z)x\}\overline{B}(x)}{\widetilde{R}(\delta+\lambda)\widetilde{B}(\lambda(1-z)) + z\overline{R}^*(\delta+\lambda)\left[\delta\widetilde{A}(\lambda(1-z))+\lambda\widetilde{B}(\lambda(1-z))\right] - z}p_{0,0}.$$

In the following corollary, we focus on the marginal generating functions of the number of ordinary customers in the orbit for different server states and the generating functions of the number of ordinary customers in the orbit and in the system.

Corollary 1

1. *The marginal generating function of the number of ordinary customers in the orbit when the server is idle but the system is not empty is*

$$P_0(z) = \int_0^\infty P_0(x,z)\mathrm{d}x = \frac{z[\delta+\lambda-(\delta G(z)+\lambda H(z))]}{\mathcal{D}(z)}\overline{R}^*(\delta+\lambda)p_{0,0}.$$

2. *The marginal generating function of the number of ordinary customers in the orbit when the server is busy serving a priority customer without preempting an ordinary customer is*

$$P_1(z) = \int_0^\infty P_1(x,z)\mathrm{d}x = \frac{\widetilde{R}(\delta+\lambda)(H(z)-z)}{\mathcal{D}(z)}\overline{A}^*(\lambda(1-z))\delta p_{0,0}.$$

3. *The marginal generating function of the number of ordinary customers in the orbit when the server is busy serving a priority customer with preempting an ordinary customer is*

$$P_2(z) = \int_0^\infty\int_0^\infty P_2(x,y,z)\mathrm{d}x\mathrm{d}y = \frac{\delta\widetilde{R}(\delta+\lambda)(1-H(z))\overline{A}^*(\lambda(1-z))}{\mathcal{D}(z)}p_{0,0}.$$

4. *The marginal generating function of the number of ordinary customers in the orbit when the server is busy serving an ordinary customer is*

$$P_3(z) = \int_0^\infty P_3(x,z)\mathrm{d}x = \frac{\widetilde{R}(\delta+\lambda)(1-H(z))}{\mathcal{D}(z)}p_{0,0}.$$

5. *The generating function of the number of ordinary customers in the orbit, $P(z)$, is given by*

$$P(z) = p_{0,0} + P_0(z) + P_1(z) + P_2(z) + P_3(z) = \frac{\widetilde{R}(\delta+\lambda)\phi(z)}{\mathcal{D}(z)}\frac{p_{0,0}}{\lambda}.$$

6. *The generating function of the number of ordinary customers in the system, $\Psi(z)$, is given by*

$$\begin{aligned}\Psi(z) &= p_{0,0} + P_0(z) + P_1(z) + zP_2(z) + zP_3(z)\\ &= \frac{\widetilde{R}(\delta+\lambda)H(z)(1-z)(1+\delta\overline{A}^*(\lambda(1-z)))}{\mathcal{D}(z)}p_{0,0}.\end{aligned}$$

From the above results, we can get some performance measures of the system in steady state.

Corollary 2

1. *The probability that the server is idle but the system is not empty, denoted by P_0, is given by*

$$P_0 = P_0(1) = \frac{\lambda\overline{R}^*(\delta+\lambda)(\delta\alpha_1 + \lambda\beta_1(1+\delta\alpha_1))}{\widetilde{R}(\delta+\lambda)(1+\delta\alpha_1)}.$$

2. *The probability that the server is busy serving a priority customer without preempting an ordinary customer, denoted by P_1, is given by*

$$P_1 = P_1(1) = \frac{\delta\alpha_1(1 - \lambda\beta_1(1 + \delta\alpha_1))}{1 + \delta\alpha_1}.$$

3. *The probability that the server is busy serving a priority customer with preempting an ordinary customer, denoted by* P_2*, is given by*

$$P_2 = P_2(1) = \delta\alpha_1\lambda\beta_1.$$

4. *The probability that the server is busy serving an ordinary customer, denoted by* P_3*, is given by*

$$P_3 = P_3(1) = \lambda\beta_1.$$

5. *The mean number of ordinary customers in the orbit,* L_q*, is given by*

$$L_q = P'(1) = P'_0(1) + P'_1(1) + P'_2(1) + P'_3(1).$$

6. *The mean number of ordinary customers in the system,* L_s*, is given by*

$$L_s = \Psi'(1) = L_q + P_2(1) + P_3(1).$$

4 Analysis of the sojourn time in the system of an arbitrary ordinary customer

In this section, we discuss the distribution of the sojourn time in the system of an arbitrary tagged ordinary customer, denoted by W, which is defined as the time period from the instant that the ordinary customer arrives at the system to the instant that the ordinary customer is completely served. Let $\widetilde{W}(s) = E[e^{-sW}]$ be the Laplace–Stieltjes transform of W, then we have the following result.

Theorem 3 *If the system is stable, Laplace–Stieltjes transform of W is given by*

$$\widetilde{W}(s) = (p_{0,0} + P_0)\widetilde{S}(s) + \widetilde{R}(\delta + \lambda)Q(s)p_{0,0}$$
$$\times \frac{\delta[H(Q(s)) - Q(s)]\left[\widetilde{A}(s) - G(Q(s))\right] + \phi(Q(s))\left[\widetilde{S}(s) - H(Q(s))\right]}{\mathcal{D}(Q(s))(\lambda(1 - Q(s)) - s)},$$

where

$$Q(s) = \frac{\widetilde{S}(s)\widetilde{R}(s + \delta + \lambda)}{1 - \overline{R}^*(s + \delta + \lambda)(\delta\widetilde{A}(s) + \lambda\widetilde{S}(s))}.$$

Proof By conditioning on the server's state when the tagged ordinary customer arrives, we have that

$$\begin{aligned}\widetilde{W}(s) = &\ (p_{0,0}+P_0)\widetilde{S}(s) + \sum_{k=0}^{\infty}\int_0^{\infty} p_{k,1}(x)E\left[e^{-sW}|N=k, J=1, \xi_1=x\right]\mathrm{d}x \\ &+ \sum_{k=0}^{\infty}\int_0^{\infty}\int_0^{\infty} p_{k,2}(x,y)E\left[e^{-sW}|N=k, J=2, \xi_1=x, \xi_2=y\right]\mathrm{d}x\mathrm{d}y \\ &+ \sum_{k=0}^{\infty}\int_0^{\infty} p_{k,3}(x)E\left[e^{-sW}|N=k, J=3, \xi_2=x\right]\mathrm{d}x \\ &\triangleq (p_{0,0}+P_0)\widetilde{S}(s) + I(s) + II(s) + III(s).\end{aligned}$$

If k ordinary customers are already in the retrial orbit and the server is busy when the tagged ordinary customer arrives and joins the orbit, then W is equal to $W_r^* + W^{(k+1)}$, where W_r^* denotes the residual (generalized) service time of the message of priority customer (or ordinary customer) being transmitted from the instant the ordinary customer arrives, and $W^{(k+1)}$ represents the total sojourn time of the $(k+1)$th ordinary customer in the retrial queue spent in the system from the instant that the server becomes idle for the first time after the tagged ordinary customer arrives. Then we obtain that

$$E\left[e^{-sW}|N=k, J=1, \xi_1=x\right] = E\left[e^{-sW_r^*}|N=k, J=1, \xi_1=x\right]E\left[e^{-sW^{(k+1)}}\right] \tag{25}$$

$$E\left[e^{-sW}|N=k, J=2, \xi_1=x, \xi_2=y\right] = E\left[e^{-sW_r^*}|N=k, J=2, \xi_2=x, \xi_2=y\right]E\left[e^{-sW^{(k+1)}}\right], \tag{26}$$

$$E\left[e^{-sW}|N=k, J=3, \xi_2=x\right] = E\left[e^{-sW_r^*}|N=k, J=2, \xi_2=x\right]E\left[e^{-sW^{(k+1)}}\right]. \tag{27}$$

Using the well-known formulas

$$P\left(y < \xi_1^r \le y+\mathrm{d}y|\xi_1^\star > x\right) = \frac{a(x+y)\mathrm{d}y}{\overline{A}(x)},$$
$$P\left(y < \xi_2^r \le y+\mathrm{d}y|\xi_2^\star > x\right) = \frac{b(x+y)\mathrm{d}y}{\overline{B}(x)},$$

where $\xi_1^\star$ and $\xi_2^\star$ denote the service times of priority customer and ordinary customer, respectively; ξ_1^r and ξ_2^r denote, respectively, the corresponding residual service times.

For convenience, we denote $\varsigma(s) = s + \delta(1-\widetilde{A}(s))$, then we have

$$E\left[e^{-sW_r^*}|N=k, J=1, \xi_1=x\right] = \int_0^{\infty}\frac{a(x+y)}{\overline{A}(x)}e^{-sy}\mathrm{d}y = \frac{1}{\overline{A}(x)}\int_x^{\infty} a(u)e^{-s(u-x)}\mathrm{d}u, \tag{28}$$

$$\begin{aligned}
&E\left[e^{-sW_r^*}|N=k,J=2,\xi_2=x,\xi_2=y\right]\\
&=\sum_{n=0}^{\infty}\frac{1}{\overline{A}(x)\overline{B}(y)}\int_0^{\infty}\int_0^{\infty}a(x+u)b(y+v)e^{-s(u+v)}\frac{(\delta v\widetilde{A}(s))^n}{n!}e^{-\delta v}\mathrm{d}v\mathrm{d}u\\
&=\frac{1}{\overline{A}(x)\overline{B}(y)}\int_x^{\infty}\int_y^{\infty}a(u)b(v)e^{-s(u-x)-\varsigma(s)(v-y)}\mathrm{d}v\mathrm{d}u.
\end{aligned}\tag{29}$$

$$\begin{aligned}
E\left[e^{-sW_r^*}|N=k,J=3,\xi_2=x\right]&=\sum_{n=0}^{\infty}\int_0^{\infty}\frac{b(x+y)}{\overline{B}(x)}e^{-sy}\frac{(\delta y)^n}{n!}e^{-\delta y}(\widetilde{A}(s))^n\mathrm{d}y\\
&=\frac{1}{\overline{B}(x)}\int_x^{\infty}b(u)e^{-\varsigma(s)(u-x)}\mathrm{d}u.
\end{aligned}\tag{30}$$

Because the general retrial policy is adopted, we know that

$$E\left[e^{-sW^{(k+1)}}\right]=\left(E\left[e^{-sW^{(1)}}\right]\right)^{k+1}.\tag{31}$$

Note that after the server becomes idle, there exists competition for service among an external priority customer, an external ordinary customer and the ordinary customer at the head of the orbit. Then we arrive at

$$\begin{aligned}
Q(s)&\triangleq E\left[e^{-sW^{(1)}}\right]\\
&=\widetilde{S}(s)\int_0^{\infty}e^{-st}e^{-(\delta+\lambda)t}r(t)\mathrm{d}t+\int_0^{\infty}e^{-st}\delta e^{-(\delta+\lambda)t}\overline{R}(t)\widetilde{A}(s)Q(s)\mathrm{d}t\\
&\quad+\int_0^{\infty}e^{-st}\lambda e^{-(\delta+\lambda)t}\overline{R}(t)\widetilde{S}(s)Q(s)\mathrm{d}t\\
&=\widetilde{S}(s)\widetilde{R}(s+\delta+\lambda)+Q(s)\overline{R}^*(s+\delta+\lambda)(\delta\widetilde{A}(s)+\lambda\widetilde{S}(s)),
\end{aligned}$$

which yields

$$Q(s)=\frac{\widetilde{S}(s)\widetilde{R}(s+\delta+\lambda)}{1-\overline{R}^*(s+\delta+\lambda)(\delta\widetilde{A}(s)+\lambda\widetilde{S}(s))}.$$

Substituting the above into (31) leads to

$$E\left[e^{-sW^{(k+1)}}\right]=(Q(s))^{k+1}.\tag{32}$$

Combining (25)–(30) and (32), one can obtain that

$$\begin{aligned}
I(s)&=\sum_{k=0}^{\infty}\int_0^{\infty}p_{k,1}(x)E\left[e^{-sW}|N=k,J=1,\xi_1=x\right]\mathrm{d}x\\
&=Q(s)\int_0^{\infty}P_1(x,Q(s))\frac{1}{\overline{A}(x)}\int_x^{\infty}a(u)e^{-s(u-x)}\mathrm{d}u\mathrm{d}x\\
&=Q(s)P_1(0,Q(s))\frac{\widetilde{A}(s)-\widetilde{A}(\lambda(1-Q(s)))}{\lambda(1-Q(s))-s},
\end{aligned}\tag{33}$$

$$
\begin{aligned}
II(s) &= \sum_{k=0}^{\infty}\int_0^{\infty}\int_0^{\infty} p_{k,2}(x,y)E\left[e^{-sW}|N=k,J=2,\xi_1=x,\xi_2=y\right]\mathrm{d}x\mathrm{d}y \\
&= Q(s)\int_0^{\infty}\int_0^{\infty} P_2(x,y,Q(s))\left(\frac{1}{\overline{A}(x)\overline{B}(y)}\int_x^{\infty}\int_y^{\infty} a(u)b(v)e^{-s(u-x)-\varsigma(s)(v-y)}\mathrm{d}v\mathrm{d}u\right)\mathrm{d}y\mathrm{d}x \\
&= \delta P_3(0,Q(s))Q(s)\frac{\widetilde{A}(s)-\widetilde{A}(\lambda(1-Q(s)))}{\lambda(1-Q(s))-s}\frac{\widetilde{S}(s)-H(Q(s))}{\phi(Q(s))-\varsigma(s)},
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
III(s) &= \sum_{k=0}^{\infty}\int_0^{\infty} p_{k,3}(x)E\left[e^{-sW}|N=k,J=3,\xi_2=x\right]\mathrm{d}x \\
&= Q(s)\int_0^{\infty} P_3(x,Q(s))\frac{1}{\overline{B}(x)}\int_x^{\infty} b(u)e^{-\varsigma(s)(u-x)}\mathrm{d}u\mathrm{d}x \\
&= Q(s)P_3(0,Q(s))\frac{\widetilde{S}(s)-H(Q(s))}{\phi(Q(s))-\varsigma(s)}.
\end{aligned}
\tag{35}
$$

with the help of (21) and (23), we can complete the proof of Theorem 3. □

5 Numerical examples

In this section, we present some numerical examples to show the effects of system parameters on the important performance measures, such as the probability that the system is busy, i.e., the utilization factor of the system, $1-p_{0,0}$, and the mean number of ordinary customers in the orbit L_q. The values of the parameters are chosen to satisfy the stable condition. In general, We assume that the service times

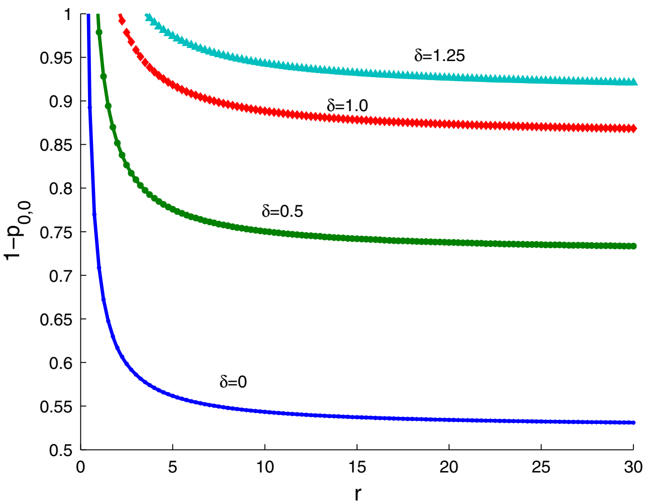


Fig. 1 The effect of r on $1-p_{0,0}$ for different δ

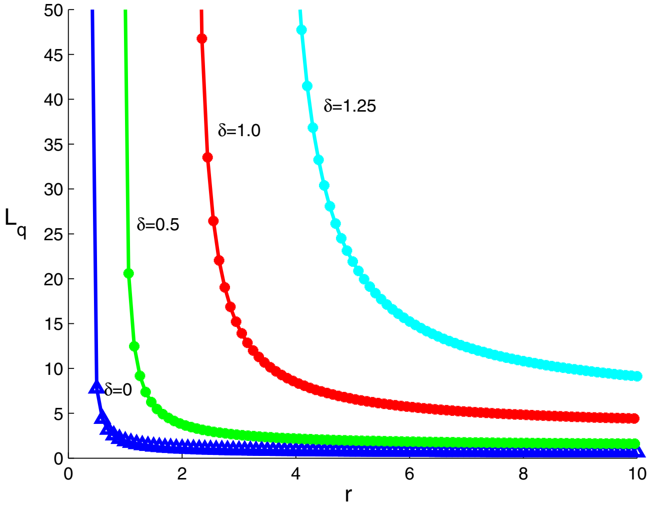


Fig. 2 The effect of r on L_q for different δ

of priority customers and ordinary customers, respectively, follow exponential distributions with mean $\alpha_1 = 0.5$ and $\beta_1 = 1.5$.

In Figs. 1 and 2, we assume that arrival rate of ordinary customers is $\lambda = 0.35$. In Figs. 3 and 4, we assume that arrival rate of priority customers $\delta = 2.0$. And in Figs. 1, 2, 3 and 4, the retrial times are governed by an exponential distribution with Laplace–Stieltjes transform $\widetilde{R}(s) = \frac{r}{r+s}$.

In Fig. 1, we plot $1 - p_{0,0}$ versus the retrial rate r for different values of $\delta = 0, 0.5, 1.0, 1.25$. In Fig. 3 we plot $1 - p_{0,0}$ versus the retrial rate r for different values of $\lambda = 0.05, 0.15, 0.2$. As expected, from Figs. 1 and 3, the utilization factor $1 - p_{0,0}$ decrease with increasing retrial rate r. The reason is that as r increases (i.e., the rate of customer in the orbit to the server increases), the number of customers in the orbit decreases and the probability $p_{0,0}$ that no customers in the orbit increases, which leads to the decrease of $1 - p_{0,0}$, and as r tends to infinity, $1 - p_{0,0}$ converges to a fixed value which corresponds to the quantity related to the queue without retrial ordinary customers. Additionally, Figs. 1 and 3 also show that with increasing of the arrival rate δ and λ, the utilization factor $1 - p_{0,0}$ increases. The same conclusion holds for Figs. 2 and 4, which illustrates the behavior of the mean number of ordinary customers in the orbit L_q as function of r for different values of δ and λ, respectively.

From Figs. 1, 2, 3 and 4, we can also observe that, as r approaches the ergodicity condition, the mean number of ordinary customers in the orbit L_q tends to infinite (due to the system becomes unstable) and, as a consequence, the utilization factor $1 - p_{0,0}$ converges to 1.

In Figs. 5 and 6, we assume that arrival rates of priority customers and ordinary customers are, respectively, $\delta = 2.0, \lambda = 0.05$, and the retrial times follow the

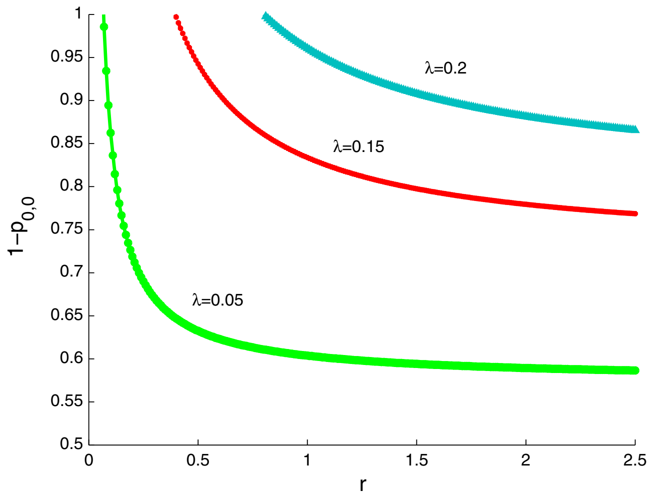
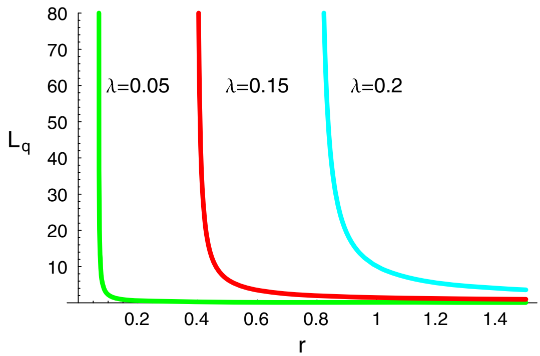


Fig. 3 The effect of r on $1 - p_{0,0}$ for different λ

Fig. 4 The effect of r on L_q for different λ



Erlang distribution with Laplace–Stieltjes transform $\widetilde{R}(s) = \left(\frac{r}{r+s}\right)^n$. In Fig. 5 we plot $1 - p_{0,0}$ versus the parameter r for different values of $n = 1, 2, 3, 4$. Figure 5 shows that the utilization factor $1 - p_{0,0}$ increases with increasing of n and also increases as the value of r, which is agree to our intuition. Similarly, the same effects are presented in Fig. 6, which shows the behavior of L_q as function of r.

In Fig. 7a, b, for the performance measure L_q, we aim to compare the preemptive rule with the non-preemptive case for different retrial times, respectively, follow exponential distribution with Laplace–Stieltjes transform $\widetilde{R}(s) = \frac{r}{r+s}$ and Erlang(2) distribution with Laplace–Stieltjes transform $\widetilde{R}(s) = \left(\frac{r}{r+s}\right)^2$. we assume that the arrival rates of priority customers and ordinary customers are, respectively, $\delta = 0.5$ and $\lambda = 0.025$ and their service times, respectively, follow exponential distributions with mean $\alpha_1 = 5.5$ and $\beta_1 = 4.5$. As expected, Fig. 7 shows that the orbit size L_q in

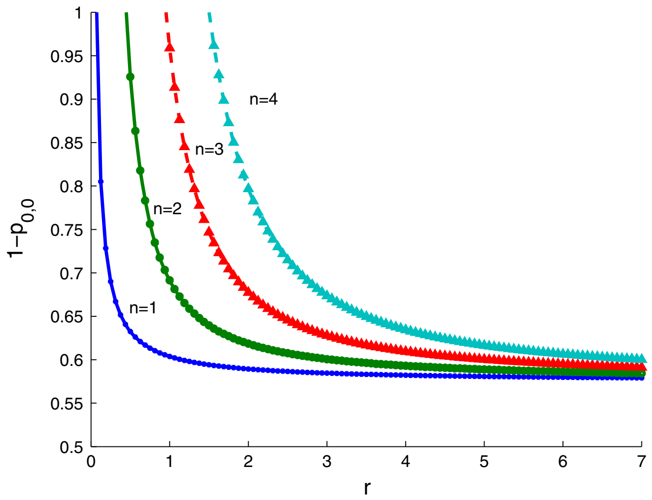
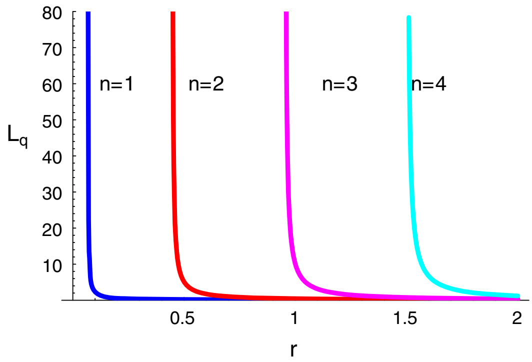


Fig. 5 The effect of r on $1 - p_{0,0}$ for different n

Fig. 6 The effect of r on L_q for different n



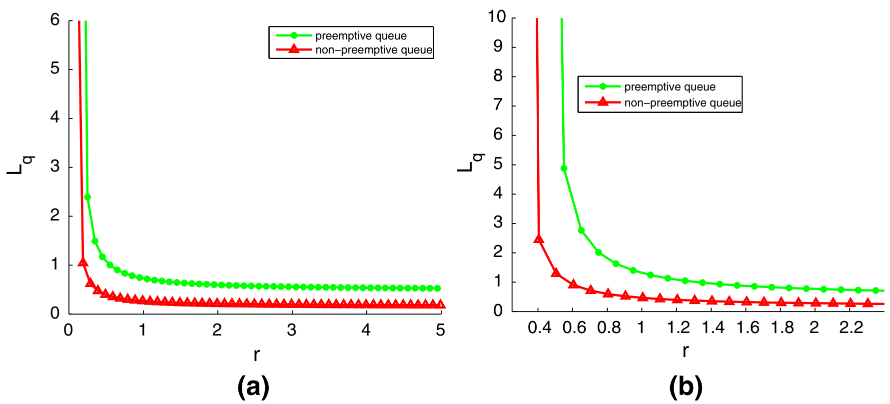


Fig. 7 Comparison between non-preemptive and preemptive queues for L_q versus r

the queue with preemptive rule is larger than that with non-preemptive priority, which shows the correctness of our theoretical analysis.

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