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Modeling and Qualitative Dynamics of the Effects of Internal and External Storage device in a Discrete Fractional Computer Virus

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Abstract
In this work, we focus on the application of epidemic approaches to computer viruses and investigate the dynamic transmission of multiple viruses, aiming to reduce computer destruction. Our goal is to create and examine computer viruses using the Atangana-Baleanu sense, which is employed in the fractional difference model for the spread of computer viruses. It included removable storage devices and external computer peripherals that were infected with computer viruses. The applications of fixed-point theory and iterative techniques are employed to analyze the existence and uniqueness results concerning the suggested model. Moreover, we extend several kinds of Ulam's stability results for this discrete model. To demonstrate the implications of changing the fractional order in this instance of numerical simulation, we employed the Atanagana–Baleanu technique. The graphical outcomes validate our theoretical findings, which we used to evaluate the impact of infected external computers and removable storage devices on computer viruses.

Keywords Discrete nabla calculus · Computer virus · Atangana-Baleanu sense · Fixed point theory · Existence and uniqueness results · Ulam stability of solution

Mathematics Subject Classification 34A12 · 34D20 · 39A12 · 65P40 · 68M07

1 Introduction

A mathematical model typically explains a system using a collection of variables along with a set of equations that construct interactions among the variables. The ordinary differential equations are an essential type of such models. It explains how variables and their derivatives relate to one another. These models are widely available. An illustration would be population fluctuations in biology and ecology, chemical reactions in the field of chemistry, economics, particle mechanics in physics, etc.

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It is crucial to understand the theory of ordinary differential equations since it is a fundamental branch of research and a useful tool for mathematical modelling.

In epidemiology, mathematical modelling helps to identify the factors that affect a disease's spread and makes recommendations for prevention measures [1]. One of the first successes of mathematical epidemiology [2] was a formula to anticipate how a disease would behave. The overall population is separated into three classes in this model such as suspended, infected, and recovered and is considered to be constant. Greater complexity has been obtained in models over the years. It has been discovered that for a while, a portion of the infected class does not exhibit the signs of several diseases. SEIR models are employed to simulate these disorders [3]. It has only been possible to simulate the dynamics of epidemiology diseases using integer order differential equations, despite the fact that this research has been extensively studied [4–9]. Recent studies have shown that models utilising fractional order differential equations can successfully explain a wide variety of occurrences in several disciplines [10–18].

Recent years have seen the advent of incredibly efficient methods for solving problems using mathematical models thanks to fractional analysis, which has given mathematics and applied sciences a new lease on life. In terms of the process effect or problem areas that classical methods are unable to adequately describe, fractional analysis orders offers greater effectiveness than classical analysis techniques. As a result of the development of novel derivative and integral operators, it has blossomed into an area that is intensively researched nowadays. For the sake of their vast range of applications, fractional calculus are quickly obtaining the popularity and have attracted the consideration of numerous research initiatives. Since consequently, this topic has attracted the focus of mathematicians from various disciplines [19–24].

Now a day, the mathematicians [25–31] established the fundamental theory of fractional differential and difference inequalities with the help of fractional derivatives (FDs) and difference of the Riemann-Liouville (RL) and Caputo operators. Also researchers [27, 28, 32–37] studied if there exist local, global, extremal solutions, existence and uniqueness and stability analysis to nonlinear fractional differential equations (FDEs) and discrete fractional equations utilizing the analyzed fractional inequalities and the comparison results. However, in order to eliminate singular kernels in the traditional FD, Caputo and Fabrizio [38] introduced the FD employing an exponential kernel.

In [39], Atangana and Baleanu suggested a novel derivative as generalization of the Caputo-Fabrizio derivative (C-FD). They used the generalized Mittag-Leffler function to build the non-local and non-singular kernel. Their fractional operator has all advantages of C-FD, RL and Caputo derivatives. Some of the advantages of Atangana-Baleanu derivative (A-BD) appear in the differences between fractional operators [40]. The RL and Caputo derivatives are Markovian, C-FD is non-Markovian, while the A-BD has both Markovian and non-Markovian aspects. The RL and Caputo derivatives have power low kernel, and C-FD has exponential decay kernel, while the A-BD has a Mittag-Leffler function as a kernel which is power low and stretched exponential kernel. The ABC FD, also known as the new FD in the meaning of Caputo, was developed by Atangana and Baleanu in [39]. Its kernel is the Mittag-Leffler function. Given that this operator is nonlocal equipped with a kernel having the nonsingularity, the

ABC-fractional differential operator is more suitable for providing a more accurate description of events that occur in the real world. It can be applied in many different contexts to represent a various type of real life problems [41–53].

With establishing an excellent idea from the other investigations, we constructed this research article in fractional derivative form: We examined some essential lemmas and definitions in Sect. 2 that will serve as the foundation for the present research. Section 3 establishes the comprehensive formulation of the suggested mathematical model and provides a thorough model description in both integer and discrete fractional meaning. The description of the proposed model (3.2) is focused to exploring the existence criteria in Sect. 4 and use limit points and iterative expressions to demonstrate its uniqueness in Sect. 5. Various criteria of Ulam stability of the model (3.2) is addressed in Sect. 6. The behavior of this physical phenomenon is simulated to see how it will actually behave in Sect. 7. The research work is concluded with findings in Sect. 8.

2 Preliminaries

The following notations are offered in this section together with the definitions and lemma for discrete fractional calculus:

$$\mathbb{N}_{\xi_0} = \{\xi_0, \xi_0 + 1, \xi_0 + 2, \cdots\}, \quad \mathbb{N}^{T}_{\xi_0+1} = \{\xi_0 + 1, \xi_0 + 2, \cdots, T\}.$$

Let $\mathbb{B}_* : \mathbb{C}\left(\mathbb{N}^{T}_{\xi_0+1}, \mathbb{R}\right)$ be a Banach space with the norm

$$\|F\| = \max\left\{|F(\xi)| : \xi \in \mathbb{N}^{T}_{\xi_0+1}\right\}.$$

Definition 2.1 [63, 64]. Let $F : \mathbb{N}_{\xi_0} \to \mathbb{R}$ and $0 < \vartheta \leq 1$ be given. The nabla fractional sum ϑ of F is given as follows

$${}_{\xi_0}\nabla^{-\vartheta}_{\xi} F(\xi) = \frac{1}{\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} F(\zeta),$$

for all $\xi \in \mathbb{N}_{\xi_0+1}$, $\sigma(\zeta) = \zeta - 1$ and $\xi^{\widehat{\vartheta}} := \dfrac{\Gamma(\xi + \vartheta)}{\Gamma(\xi)}$.

Definition 2.2 [63, 64]. For $0 < \vartheta < \frac{1}{2}$, $\xi \in \mathbb{N}_{\xi_0}$ and a function $F : \mathbb{N}_{\xi_0} \to \mathbb{R}$, the left nabla ABC-fractional difference is

$$\left({}^{ABC}_{\xi_0}\nabla^{\vartheta}_{\xi} F\right)(\xi) = \frac{G(\vartheta)}{1 - \vartheta} \sum_{\zeta=\xi_0+1}^{\xi} E_{\widehat{\vartheta}}\left(\frac{-\vartheta}{1 - \vartheta}, \xi - \sigma(\zeta)\right) \nabla F(\xi),$$

where $G(\vartheta) = 1 - \vartheta + \dfrac{\vartheta}{\Gamma(1-\vartheta)}$ and the discrete Mittag-Leffler function for nabla can be expressed by

$$E_{\widehat{a,b}}(\eta, z) = \sum_{k \geq 0} \eta^k \frac{z^{\widehat{ka+b-1}}}{\Gamma(ak+b)},$$

for $|\eta| < 1, a, b, z \in \mathbb{C}$ and $Re(a) > 0$.

Definition 2.3 [63, 64]. For $\vartheta \in (0, 1)$, the left ABC discrete nabla fractional sum of order ϑ is given by

$$\left({}^{ABC}_{\xi_0}\nabla_{\xi}^{-\vartheta} F\right)(\xi) = \frac{1-\vartheta}{G(\vartheta)} F(\xi) + \frac{\vartheta}{G(\vartheta)} {}_{\xi_0}\nabla_{\xi}^{-\vartheta} F(\xi).$$

Lemma 2.4 [64] *Let ξ and ϑ be positive. Then* $\sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} = \dfrac{1}{\vartheta}(\xi - \xi_0)^{\widehat{\vartheta}}$.

3 On the discrete fractional model of a computer virus

A computer connected to a network can quickly spread a virus to other associated computers in the network due to the interconnectedness of various networks of computers and the large number of users on these networks. The host computer's general or some data can be destroyed, or it can have unauthorized access to sensitive user data-such as bank account details and other personal data without the user's knowledge. Another way to cause damage is to prevent the host computer from performing its functions by taking up some mainframe memory or by turning off this portion of the system. A number of strategies can be suggested with the aid of epidemics to lessen the threat of viruses. We can look to [54–56, 62] for more information on how viruses operate in computer networks.

Users need antivirus software to guard against virus disturbances. Due of the significance of this, numerous academics and researchers have examined how viruses operate in computer networks and produced models of how viruses behave in these networks. Many mathematicians [55, 57–59] have developed a model of how computer viruses operate before assessing the model's viability. The propagation and transmission capabilities of computer viruses are comparable to those of biological viruses. Computer viruses are distributed over the network in the same manner that biological viruses are passed from one animal to another [60]. We can use the SIR model to assess the effectiveness of computer viruses because they act similarly to biological viruses due to their similarities. The prospective applications of fractional calculus in engineering and science have sparked a lot of interest recently [29]. Pinto and Machado [61] have provided a fractional order-based analysis of the spread of computer viruses. In their approach, interactions between computers and removable storage devices are taken into account.

The traditional computer virus model described in [65, 66] has four compartments labelled $S(\xi)$, $L(\xi)$, $B(\xi)$ and $R(\xi)$ respectively, to represent susceptible, latent computers, computers that are breaking out of their infection state, and computers that have recovered following infection involving both internal and external viruses at time ξ, as follows:

$$\begin{cases} S^{'}(\xi) = \delta_1 + (\sigma_2 - \beta_1 S(\xi))\, L(\xi) \\ + (\sigma_3 - \beta_2 S(\xi))\, B(\xi) + \eta R(\xi) - (\sigma_1 + \mu + \theta) S(\xi), \\ L^{'}(\xi) = \delta_2 + (\beta_1 L(\xi) + \beta_2 B(\xi) + \theta)\, S(\xi) - (\sigma_1 + \sigma_2 + \mu + \alpha)\, L(\xi), \\ B^{'}(\xi) = \delta_3 + \alpha L(\xi) - (\sigma_1 + \sigma_3 + \mu)\, B(\xi), \\ R^{'}(\xi) = \delta_4 + (S(\xi) + L(\xi) + B(\xi))\, \sigma_1 - (\eta + \mu)\, R(\xi), \end{cases} \tag{3.1}$$

where the parameters $\delta_1, \delta_2, \delta_3, \delta_4, \sigma_1, \sigma_2, \sigma_3, \beta_1, \beta_2, \eta, \mu, \theta, \alpha$ are positive constants and the assumptions of these parameters are considered in [65, 66]. The aforementioned system (3.1) can be expressed in ABC discrete fractional order nabla form as follows:

$$\begin{cases} {}^{ABC}_{\xi_0}\nabla^{\vartheta} S(\xi) = \delta_1 + (\sigma_2 - \beta_1 S(\xi))\, L(\xi) \\ + (\sigma_3 - \beta_2 S(\xi))\, B(\xi) + \eta R(\xi) - (\sigma_1 + \mu + \theta) S(\xi), \\ {}^{ABC}_{\xi_0}\nabla^{\vartheta} L(\xi) = \delta_2 + (\beta_1 L(\xi) + \beta_2 B(\xi) + \theta)\, S(\xi) \\ \quad - (\sigma_1 + \sigma_2 + \mu + \alpha)\, L(\xi), \\ {}^{ABC}_{\xi_0}\nabla^{\vartheta} B(\xi) = \delta_3 + \alpha L(\xi) - (\sigma_1 + \sigma_3 + \mu)\, B(\xi), \\ {}^{ABC}_{\xi_0}\nabla^{\vartheta} R(\xi) = \delta_4 + (S(\xi) + L(\xi) + B(\xi))\, \sigma_1 - (\eta + \mu)\, R(\xi), \end{cases} \tag{3.2}$$

where the initial conditions are $S_0 \geq 0$, $L_0 \geq 0$, $B_0 \geq 0$, $R_0 \geq 0$. Here, ${}^{ABC}_{\xi_0}\nabla^{\vartheta}$ is used for the ABC discrete nabla difference operator of order $\vartheta \in (0, 1]$.

The proof for the existence of a solution to the proposed model (3.2) will be achieved by employing a successive iterative method. For this, we used Definition 2.1 and Definition 2.3 to help us create the model (3.2), and we obtain

$$\begin{aligned} S(\xi) - S_0 = {} & \frac{(1-\vartheta)}{G(\vartheta)}\Bigg[\delta_1 + \Big(\sigma_2 - \beta_1 S(\xi)\Big)L(\xi) \\ & + \Big(\sigma_3 - \beta_2 S(\xi)\Big)B(\xi) + \eta R(\xi) - (\sigma_1 + \mu + \theta)S(\xi)\Bigg] \\ & + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} \\ & \times\Bigg[\delta_1 + \Big(\sigma_2 - \beta_1 S(\zeta)\Big)L(\zeta) \\ & + \Big(\sigma_3 - \beta_2 S(\zeta)\Big)B(\zeta) + \eta R(\zeta) - (\sigma_1 + \mu + \theta)S(\zeta)\Bigg], \\ L(\xi) - L_0 = {} & \frac{(1-\vartheta)}{G(\vartheta)}\Bigg[\delta_2 + \Big(\beta_1 L(\xi) + \beta_2 B(\xi) + \theta\Big)S(\xi) \end{aligned}$$

$$
\begin{aligned}
&-\Big(\sigma_1+\sigma_2+\mu+\alpha\Big)L(\xi)\Big]\\
&+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\Big[\delta_2+\Big(\beta_1 L(\zeta)+\beta_2 B(\zeta)+\theta\Big)S(\zeta)\\
&-\Big(\sigma_1+\sigma_2+\mu+\alpha\Big)L(\zeta)\Big],
\end{aligned}
$$

$$
\begin{aligned}
B(\xi)-B_0 =\ &\frac{(1-\vartheta)}{G(\vartheta)}\Big[\delta_3+\alpha L(\xi)-\Big(\sigma_1+\sigma_3+\mu\Big)B(\xi)\Big]\\
&+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\Big[\delta_3+\alpha L(\zeta)-\Big(\sigma_1+\sigma_3+\mu\Big)B(\zeta)\Big],\\
R(\xi)-R_0 =\ &\frac{(1-\vartheta)}{G(\vartheta)}\Big[\delta_4+\Big(S(\xi)+L(\xi)+B(\xi)\Big)\sigma_1-\Big(\eta+\mu\Big)R(\xi)\Big]\\
&+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\Big[\delta_4\\
&+\Big(S(\zeta)+L(\zeta)+B(\zeta)\Big)\sigma_1-\Big(\eta+\mu\Big)R(\zeta)\Big].
\end{aligned}
$$

Consider the following functions W_i for $i = 1, 2, 3, 4$

$$
\begin{cases}
W_1(\xi,S)=\delta_1+(\sigma_2-\beta_1 S(\xi))\,L(\xi)\\
+(\sigma_3-\beta_2 S(\xi))\,B(\xi)+\eta R(\xi)-(\sigma_1+\mu+\theta)S(\xi),\\
W_2(\xi,L)=\delta_2+(\beta_1 L(\xi)+\beta_2 B(\xi)+\theta)\,S(\xi)-(\sigma_1+\sigma_2+\mu+\alpha)\,L(\xi),\\
W_3(\xi,B)=\delta_3+\alpha L(\xi)-(\sigma_1+\sigma_3+\mu)\,B(\xi),\\
W_4(\xi,R)=\delta_4+(S(\xi)+L(\xi)+B(\xi))\,\sigma_1-(\eta+\mu)\,R(\xi).
\end{cases}
$$

4 Existence Results

We consider some hypotheses before stating and discussing on the main theorems of the present section:

($\mathcal{H}_1$) If $S(\xi)$, $S^*(\xi)$, $L(\xi)$, $L^*(\xi)$, $B(\xi)$, $B^*(\xi)$, $R(\xi)$, $R^*(\xi)$ $\in \mathbb{C}\left(\mathbb{N}^T_{\xi_0+1}, \mathbb{R}\right)$ are continuous and $\epsilon_1, \epsilon_2, \epsilon_3 > 0$ such that $\|L\| \leq \epsilon_1$, $\|B\| \leq \epsilon_2$ and $\|S\| \leq \epsilon_3$.

($\mathcal{H}_2$) If $\ell_1 > 0$ such that for all $S, S^* \in \mathbb{B}_*$ and each $\xi \in \mathbb{N}^T_{\xi_0+1}$, we have $|W_1(\xi, S) - W_1(\xi, S^*)| \leq \ell_1 |S - S^*|$.

($\mathcal{H}_3$) If $\phi \in \mathbb{C}\left(\mathbb{N}^T_{\xi_0+1}, \mathbb{R}^+\right)$ is non decreasing function and $\lambda > 0$, for $\xi \in \mathbb{N}^T_{\xi_0+1}$ such that $\dfrac{\epsilon}{\Gamma(\vartheta)} \displaystyle\sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}}\phi(\zeta + \vartheta - 1) \leq \lambda\epsilon\phi(\xi + \vartheta - 1)$.

Theorem 4.1 *Under the assumptions ($\mathcal{H}_1$), ($\mathcal{H}_2$) and $\ell_i < 1$, for $i = 1, 2, 3, 4$, the kernels W_i satisfy Lipschitz condition.*

Proof First, we examine $W_1(\xi, S)$. By utilising $S(\xi)$ and $S^*(\xi)$, we calculate

$$
\begin{aligned}
\left\| W_1(S) - W_1(S^*) \right\| &= \|\delta_1 + (\sigma_2 - \beta_1 S)\, L + (\sigma_3 - \beta_2 S)\, B + \eta R - (\sigma_1 + \mu + \theta) S \\
&\quad - \left[\delta_1 + \left(\sigma_2 - \beta_1 S^*\right) L + \left(\sigma_3 - \beta_2 S^*\right) B + \eta R - (\sigma_1 + \mu + \theta) S^*\right]\| \\
&\leq [\beta_1 \|L\| + \beta_2 \|B\| + \sigma_1 + \mu + \theta] \left\| S - S^* \right\| \\
\left\| W_1(S) - W_1(S^*) \right\| &\leq \ell_1 \left\| S - S^* \right\|,
\end{aligned}
\tag{4.1}
$$

where $\ell_1 = \beta_1\epsilon_1 + \beta_2\epsilon_2 + \sigma_1 + \mu + \theta$. As a result, the Lipschitz constant ℓ_1 makes W_1 satisfy the criterion. We verify this requirement for $W_2(\xi, L)$ in the following. Due to this, we have

$$
\begin{aligned}
\left\| W_2(L) - W_2(L^*) \right\| &= \|\delta_2 + (\beta_1 L + \beta_2 B + \theta)\, S - (\sigma_1 + \sigma_2 + \mu + \alpha)\, L \\
&\quad - \left[\delta_2 + \left(\beta_1 L^* + \beta_2 B + \theta\right) S - (\sigma_1 + \sigma_2 + \mu + \alpha)\, L^*\right]\| \\
&\leq [\beta_1 \|S\| + \sigma_1 + \sigma_2 + \mu + \alpha] \left\| L - L^* \right\| \\
\left\| W_2(L) - W_2(L^*) \right\| &\leq \ell_2 \left\| L - L^* \right\|,
\end{aligned}
\tag{4.2}
$$

where $\ell_2 = \beta_1\epsilon_3 + \sigma_1 + \sigma_2 + \mu + \alpha$. With the help of the Lipschitz constant ℓ_2, W_2 fulfills the criteria of Lipschitz. On $W_3(\xi, B)$, we can write

$$
\begin{aligned}
\left\| W_3(B) - W_3(B^*) \right\| &= \left\| \delta_3 + \alpha L - (\sigma_1 + \sigma_3 + \mu)\, B - \left[\delta_3 + \alpha L - (\sigma_1 + \sigma_3 + \mu)\, B^*\right] \right\| \\
&\leq [\sigma_1 + \sigma_3 + \mu] \left\| B - B^* \right\| \\
\left\| W_3(B) - W_3(B^*) \right\| &\leq \ell_3 \left\| B - B^* \right\|,
\end{aligned}
\tag{4.3}
$$

where $\ell_3 = \sigma_1 + \sigma_3 + \mu$. Using the Lipschitz constant ℓ_3, it follows that W_3 satisfies Lipschitz condition. Now $W_4(\xi, R)$, we have

$$
\begin{aligned}
\left\| W_4(R) - W_4(R^*) \right\| &= \left\| \delta_4 + (S + L + B)\, \sigma_1 - (\eta + \mu)\, R - \left[\delta_4 + (S + L + B)\, \sigma_1 - (\eta + \mu)\, R^*\right] \right\| \\
&\leq [\eta + \mu] \left\| R - R^* \right\| \\
\left\| W_4(R) - W_4(R^*) \right\| &\leq \ell_4 \left\| R - R^* \right\|,
\end{aligned}
\tag{4.4}
$$

where $\ell_4 = \eta + \mu$. Hence, W_4 is also meets Lipschitzian with constant ℓ_4. As a findings of (4.1)–(4.4), the outcome is accomplished since W_i, where $i = 1, 2, 3, 4$, meet the Lipschitz property. □

Assume

$$\begin{aligned}
S(\xi)-S_0 &= \frac{(1-\vartheta)}{G(\vartheta)}W_1(\xi,S(\xi))+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}W_1(\zeta,S(\zeta)),\\
L(\xi)-L_0 &= \frac{(1-\vartheta)}{G(\vartheta)}W_2(\xi,L(\xi))+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}W_2(\zeta,L(\zeta)),\\
B(\xi)-B_0 &= \frac{(1-\vartheta)}{G(\vartheta)}W_3(\xi,B(\xi))+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}W_3(\zeta,L(\zeta)),\\
R(\xi)-R_0 &= \frac{(1-\vartheta)}{G(\vartheta)}W_4(\xi,R(\xi))+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}W_4(\zeta,R(\zeta)).
\end{aligned}\tag{4.5}$$

Theorem 4.2 *If* $\Delta=\max[\ell_1,\ell_2,\ell_3,\ell_4]<1$, *then the system* (3.2) *at least has a solution.*

Proof Consider

$$\begin{cases}\Omega_{1_n}(\xi)=S_n(\xi)-S(\xi), & \Omega_{2_n}(\xi)=L_n(\xi)-L(\xi),\\ \Omega_{3_n}(\xi)=B_n(\xi)-B(\xi), & \Omega_{4_n}(\xi)=R_n(\xi)-R(\xi).\end{cases}$$

Following this, we conclude that

$$\begin{aligned}
\left|\Omega_{1_n}(\xi)\right| &= \frac{(1-\vartheta)}{G(\vartheta)}\left|W_1(\xi,S_n(\xi))-W_1(\xi,S(\xi))\right|+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\\
&\quad\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\left|W_1(\zeta,S_n(\zeta))-W_1(\zeta,S(\zeta))\right|.
\end{aligned}\tag{4.6}$$

The condition $(\mathcal{H}_2)$ and Lemma 2.4 in (4.6) give

$$\begin{aligned}
\left\|\Omega_{1_n}(\xi)\right\| &\le \frac{\ell_1(1-\vartheta)}{G(\vartheta)}\|S_n-S\|+\frac{\ell_1\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\|S_n-S\|\\
&\le \frac{\ell_1(1-\vartheta)}{G(\vartheta)}\|S_n-S\|+\frac{\ell_1\vartheta\,(\xi-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)\vartheta}\|S_n-S\|\\
&\le \left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_1\|S_n-S\|\\
\left\|\Omega_{1_n}(\xi)\right\| &\le \left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]^n\ell_1^n\|S_n-S\|,
\end{aligned}\tag{4.7}$$

in which we have $S_n \to S$ for $\ell_1 < 1$ and as $n \to \infty$. In a similar way

$$\left\|\Omega_{2_n}(\xi)\right\| \leq \left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]^n \ell_2^n \left\|L_n - L\right\|, \tag{4.8}$$

$$\left\|\Omega_{3_n}(\xi)\right\| \leq \left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]^n \ell_3^n \left\|B_n - B\right\|, \tag{4.9}$$

$$\left\|\Omega_{4_n}(\xi)\right\| \leq \left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]^n \ell_4^n \left\|R_n - R\right\|, \tag{4.10}$$

According to (4.7)–(4.10), when $n \to \infty$, then $\Omega_{i_n} \to 0$ and $\ell_i < 1$ for $i = 2, 3, 4$. Finally, a solution exists for the system (3.2). □

5 Unique Solution

We will demonstrate the uniqueness of solutions for our proposed model (3.2).

Theorem 5.1 *If $(\mathcal{H}_1)$ is satisfied and the following is true, then the ABC model (3.2) has exactly one solution*

$$\left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_i \leq 1, \tag{5.1}$$

for $i = 1, 2, 3, 4$.

Proof From the assumption (4.5), for $\mathbb{N}^T_{\xi_0+1}$, it follows that

$$\begin{aligned}\left|S(\xi) - S^*(\xi)\right| &= \frac{1-\vartheta}{G(\vartheta)}\left|W_1(\xi, S(\xi)) - W_1(\xi, S^*(\xi))\right| \\ &+ \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\left|W_1(\zeta, S(\zeta)) - W_1(\zeta, S^*(\zeta))\right|.\end{aligned} \tag{5.2}$$

The condition $(\mathcal{H}_2)$ and Lemma 2.4 in (5.2) follow that

$$\begin{aligned}\left\|S - S^*\right\| &\leq \frac{\ell_1(1-\vartheta)}{G(\vartheta)}\left\|S - S^*\right\| + \frac{\ell_1\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\left\|S - S^*\right\| \\ &\leq \frac{\ell_1(1-\vartheta)}{G(\vartheta)}\left\|S - S^*\right\| + \frac{\ell_1\,(\xi-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\left\|S - S^*\right\| \\ \left\|S - S^*\right\| &\leq \left[\frac{\Gamma(\vartheta)(1-\vartheta)+(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_1\left\|S - S^*\right\|\end{aligned}$$

and so

$$\|S - S^*\| \left[1 - \left(\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right)\ell_1\right] \le 0. \tag{5.3}$$

The relation (5.3) can only be justified if $\|S - S^*\| = 0$, so we have $S = S^*$. Also, together with

$$\|L - L^*\| \le \left[\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_1 \|L - L^*\|,$$

As we approach

$$\|L - L^*\| \left[1 - \left(\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right)\ell_1\right] \le 0.$$

This means that $\|L - L^*\| = 0$ and $L = L^*$. In addition

$$\|B - B^*\| \le \left[\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_1 \|B - B^*\|,$$

from this

$$\|B - B^*\| \left[1 - \left(\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right)\ell_1\right] \le 0.$$

is true if $\|B - B^*\| = 0$, which results in $B = B^*$. In similar manner

$$\|R - R^*\| \le \left[\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_1 \|R - R^*\|,$$

from the above relation, we obtain

$$\|R - R^*\| \left[1 - \left(\frac{\Gamma(\vartheta)(1-\vartheta) + (T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right)\ell_1\right] \le 0. \tag{5.4}$$

The relation (5.4) satisfy when $\|R - R^*\| = 0$. Thus $R = R^*$. So, our model (3.2) admits a unique solution. □

6 Hyers-Ulam Stability

To begin this section, we are going to focus on some necessary inequalities and notions for our model (3.2) to meet the hypotheses of different kinds of the Ulam's stability.

Now, focus on an IVP (3.2) and these inequalities [33, 35]

$$\begin{cases} \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{S}(\xi) - W_1\left(\xi, \hat{S}(\xi)\right)\right| \le \epsilon_1, & \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{L}(\xi) - W_2\left(\xi, \hat{L}(\xi)\right)\right| \le \epsilon_2, \\ \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{B}(\xi) - W_3\left(\xi, \hat{B}(\xi)\right)\right| \le \epsilon_3, & \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{R}(\xi) - W_4\left(\xi, \hat{R}(\xi)\right)\right| \le \epsilon_4 \end{cases} \tag{6.1}$$

and

$$\begin{cases} \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{S}(\xi) - W_1\left(\xi, \hat{S}(\xi)\right)\right| \le \epsilon_1\phi_1(\xi), & \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{L}(\xi) - W_2\left(\xi, \hat{L}(\xi)\right)\right| \le \epsilon_2\phi_2(\xi), \\ \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{B}(\xi) - W_3\left(\xi, \hat{B}(\xi)\right)\right| \le \epsilon_3\phi_3(\xi), & \left|{}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{R}(\xi) - W_4\left(\xi, \hat{R}(\xi)\right)\right| \le \epsilon_4\phi_4(\xi), \end{cases} \tag{6.2}$$

where $\xi \in \mathbb{N}^T_{\xi_0+1}$.

Definition 6.1 The IVP (3.2) is Hyers-Ulam (HU) stable if $\mathcal{A}_i > 0$, $\epsilon_i > 0$ for $\mathbb{N}^4_1$ and for every solution $\hat{S}(\xi), \hat{L}(\xi), \hat{B}(\xi), \hat{R}(\xi), \in \mathbb{B}_*$ of (6.1), there is a solution $S(\xi), L(\xi), B(\xi), R(\xi) \in \mathbb{B}_*$ of (3.2) with

$$\begin{cases} \left|\hat{S}(\xi) - S(\xi)\right| \le \mathcal{A}_1\epsilon_1, & \left|\hat{L}(\xi) - L(\xi)\right| \le \mathcal{A}_2\epsilon_2, \\ \left|\hat{B}(\xi) - B(\xi)\right| \le \mathcal{A}_3\epsilon_3, & \left|\hat{R}(\xi) - R(\xi)\right| \le \mathcal{A}_4\epsilon_4, \quad \xi \in \mathbb{N}^T_{\xi_0+1}. \end{cases}$$

Definition 6.2 The IVP (3.2) is Hyers-Ulam-Rassias (HUR) stable if $\mathcal{D}_i > 0$, $\epsilon_i > 0$ for $\mathbb{N}^4_1$ and for each $\hat{S}(\xi), \hat{L}(\xi), \hat{B}(\xi), \hat{R}(\xi) \in \mathbb{B}_*$ of (6.2), there is a solution $S(\xi), L(\xi), B(\xi), R(\xi) \in \mathbb{B}_*$ of (3.2) with

$$\begin{cases} \left|\hat{S}(\xi) - S(\xi)\right| \le \mathcal{D}_1\epsilon_1\phi_1(\xi), & \left|\hat{L}(\xi) - L(\xi)\right| \le \mathcal{D}_2\epsilon_2\phi_2(\xi), \\ \left|\hat{B}(\xi) - B(\xi)\right| \le \mathcal{D}_3\epsilon_3\phi_3(\xi), & \left|\hat{R}(\xi) - R(\xi)\right| \le \mathcal{D}_4\epsilon_4\phi_4(\xi), \quad \xi \in \mathbb{N}^T_{\xi_0+1}. \end{cases}$$

Remark 6.3 A function $\hat{S}(\xi) \in \mathcal{B}_*$ is a solution of (6.1) and (6.2) if $\exists\ f : \mathbb{N}^T_{\xi_0+1} \to \mathbb{R}$ satisfying, for $\xi \in \mathbb{N}^T_{\xi_0+1}$,

(i) $|f_1(\xi)| \le \epsilon_1$,
(ii) $|f_1(\xi)| \le \epsilon_1\phi_1(\xi)$,
(iii) ${}^{ABC}_{\xi_0}\nabla^{\vartheta}\hat{S}(\xi) = W_1(\xi, \hat{S}(\xi)) + f_1(\xi)$.

In a similar manner, we may define for other classes in the model (3.2) for some $f_i(\xi)$ with $i = 2, 3, 4$.

Theorem 6.4 *If the inequality (5.1) and ($\mathcal{H}_2$) hold, then the model (3.2) is HU stable.*

Proof According to Remark 6.3 with Definitions 2.1 and 2.3, we obtain the solution $\hat{S}(\xi)$ is given by

$$\hat{S}(\xi) = \hat{S}_0 + \frac{(1-\vartheta)}{G(\vartheta)} W_1(\xi, \hat{S}(\xi)) + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} W_1(\zeta, \hat{S}(\zeta)),$$
$$+ \frac{(1-\vartheta)}{G(\vartheta)} f_1(\xi) + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} f_1(\zeta). \tag{6.3}$$

From this it follows that

$$\begin{aligned} &\left| \hat{S}(\xi) - \hat{S}_0 - \frac{(1-\vartheta)}{G(\vartheta)} W_1(\xi, \hat{S}(\xi)) - \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} W_1(\zeta, \hat{S}(\zeta)) \right| \\ &\quad \leq \frac{(1-\vartheta)}{G(\vartheta)} |f_1(\xi)| + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} |f_1(\zeta)| \\ &\quad \leq \left[\frac{(1-\vartheta)}{G(\vartheta)} + \frac{(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)} \right] \epsilon_1. \end{aligned} \tag{6.4}$$

From solution (4.5), for $\xi \in \mathbb{N}_\ell$, it follows that

$$\begin{aligned} \left| \hat{S}(\xi) - S(\xi) \right| \leq &\left| \hat{S}(\xi) - \hat{S}_0 - \frac{(1-\vartheta)}{G(\vartheta)} W_1(\xi, \hat{S}(\xi)) - \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \right. \\ &\left. \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} W_1(\zeta, \hat{S}(\zeta)) \right| \\ &+ \left[\frac{(1-\vartheta)}{G(\vartheta)} + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} \right] \\ &\left| W_1(\zeta, \hat{S}(\zeta)) - W_1(\xi, S(\xi)) \right|. \end{aligned}$$

Using Lemma 2.4 and inequality (6.4) along with hypothesis ($\mathcal{H}_2$), we have that

$$\begin{aligned} \left\| \hat{S} - S \right\| \leq &\left[\frac{(1-\vartheta)}{G(\vartheta)} + \frac{(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)} \right] \epsilon_1 \\ &+ \left[\frac{(1-\vartheta)}{G(\vartheta)} + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{\zeta=\xi_0+1}^{\xi} (\xi - \sigma(\zeta))^{\widehat{\vartheta-1}} \right] \ell_1 \left\| \hat{S} - S \right\| \\ \leq &\left[\frac{(1-\vartheta)}{G(\vartheta)} + \frac{(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)} \right] \left(\epsilon_1 + \ell_1 \left\| \hat{S} - S \right\| \right). \end{aligned} \tag{6.5}$$

Inequality (6.5) yields $\left\|\hat{S}-S\right\| \leq \mathcal{A}_1\epsilon_1$, where $\mathcal{A}_1 = \dfrac{\left[\dfrac{(1-\vartheta)}{G(\vartheta)} + \dfrac{(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]}{1-\ell_1\left[\dfrac{(1-\vartheta)}{G(\vartheta)} + \dfrac{(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]}$.

Similarly, we have $\left\|\hat{L}-L\right\| \leq \mathcal{A}_2\epsilon_2$, $\left\|\hat{B}-B\right\| \leq \mathcal{A}_3\epsilon_3$ and $\left\|\hat{R}-R\right\| \leq \mathcal{A}_4\epsilon_4$ Thus, the model (3.2) is HU stable. □

Theorem 6.5 *If the inequality* (5.1) *and ($\mathcal{H}_3$) hold, the model* (3.2) *is HUR stable.*

Proof According to Remark 6.3 and Eq. (6.3), we get

$$\begin{aligned}
&\left|\hat{S}(\xi)-\hat{S}_0-\frac{(1-\vartheta)}{G(\vartheta)}W_1(\xi,\hat{S}(\xi))-\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}W_1(\zeta,\hat{S}(\zeta))\right|\\
&\qquad\leq \frac{(1-\vartheta)}{G(\vartheta)}|f_1(\xi)|+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}|f_1(\zeta)|\\
&\qquad\leq \frac{(1-\vartheta)}{G(\vartheta)}\epsilon_1\phi_1(\xi)+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\epsilon_1\phi_1(\zeta)\\
&\qquad\leq \left[\frac{1+(\lambda-1)\vartheta}{G(\vartheta)}\right]\epsilon_1\phi_1(\xi).
\end{aligned}\tag{6.6}$$

In view of Lemma 2.4 and inequality (6.6) with the aid of Theorem 6.4, we get

$$\begin{aligned}
\left\|\hat{S}-S\right\| &\leq \left[\frac{1+(\lambda-1)\vartheta}{G(\vartheta)}\right]\epsilon_1\phi_1(\xi)\\
&\quad+\left[\frac{(1-\vartheta)}{G(\vartheta)}+\frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)}\sum_{\zeta=\xi_0+1}^{\xi}(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}\right]\ell_1\left\|\hat{S}-S\right\|\\
&\leq \left[\frac{1+(\lambda-1)\vartheta}{G(\vartheta)}\right]\epsilon_1\phi_1(\xi)+\left[\frac{(1-\vartheta)}{G(\vartheta)}+\frac{(T-\xi_0)^{\widehat{\vartheta}}}{G(\vartheta)\Gamma(\vartheta)}\right]\ell_1\left\|\hat{S}-S\right\|.
\end{aligned}$$

From above it follows $\left\|\hat{S}-S\right\| \leq \mathcal{D}_1\epsilon_1\phi_1(\xi)$, where

$$\mathcal{D}_1 = \frac{[1+(\lambda-1)\vartheta]}{G(\vartheta)-\ell_1\left[(1-\vartheta)+\dfrac{(T-\xi_0)^{\widehat{\vartheta}}}{\Gamma(\vartheta)}\right]}.$$

Similarly, $\left\|\hat{L}-L\right\| \leq \mathcal{D}_2\epsilon_2\phi_2(\xi)$, $\left\|\hat{B}-B\right\| \leq \mathcal{D}_3\epsilon_3\phi_3(\xi)$ and $\left\|\hat{R}-R\right\| \leq \mathcal{D}_4\epsilon_4\phi_4(\xi)$. So, the model (3.2) is HUR stable. □

7 Numerical Simulations

The mathematical analysis of the epidemic computer virus model with the effect of external and internal storage media is proposed with the new fractional operator with given parameters details in [65, 66]. This section demonstrates the numerical computation findings based on the mathematical study of the computer epidemic model (3.2) by using the ABC fractional difference operator.

Assume that $\xi_0 = 0$, $\dfrac{(\xi-\sigma(\zeta))^{\widehat{\vartheta-1}}}{\Gamma(\vartheta)} = \dfrac{\Gamma(\xi-\zeta+\vartheta)}{\Gamma(\vartheta)\Gamma(\xi-\zeta+1)}$, $\xi = m$ and $\zeta = k$ in Eq. (4.5) gives the numerical formulations explicitly in the form of the model (3.2) is given by

$$
\begin{aligned}
S(m) &= S_0 + \frac{(1-\vartheta)}{G(\vartheta)}\big[\delta_1 + (\sigma_2 - \beta_1 S(m))\, L(m) \\
&\quad + (\sigma_3 - \beta_2 S(m))\, B(m) + \eta R(m) - (\sigma_1 + \mu + \theta) S(m)\big] \\
&\quad + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{k=1}^{m} \frac{\Gamma(m-k+\vartheta)}{\Gamma(m-k+1)} \big[\delta_1 + (\sigma_2 - \beta_1 S(k))\, L(k) \\
&\quad + (\sigma_3 - \beta_2 S(k))\, B(k) + \eta R(k) - (\sigma_1 + \mu + \theta) S(k)\big], \\
L(m) &= L_0 + \frac{(1-\vartheta)}{G(\vartheta)} \left[\delta_2 + (\beta_1 L(m) + \beta_2 B(m) + \theta)\, S(m) - (\sigma_1 + \sigma_2 + \mu + \alpha)\, L(m)\right] \\
&\quad + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{k=1}^{m} \frac{\Gamma(m-k+\vartheta)}{\Gamma(m-k+1)} \big[\delta_2 + (\beta_1 L(k) + \beta_2 B(k) + \theta)\, S(k) \\
&\quad - (\sigma_1 + \sigma_2 + \mu + \alpha)\, L(k)\big], \\
B(m) &= B_0 + \frac{(1-\vartheta)}{G(\vartheta)} \left[\delta_3 + \alpha L(m) - (\sigma_1 + \sigma_3 + \mu)\, B(m)\right] \\
&\quad + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{k=1}^{m} \frac{\Gamma(m-k+\vartheta)}{\Gamma(m-k+1)} \left[\delta_3 + \alpha L(k) - (\sigma_1 + \sigma_3 + \mu)\, B(k)\right], \\
R(m) &= R_0 + \frac{(1-\vartheta)}{G(\vartheta)} \left[\delta_4 + (S(m) + L(m) + B(m))\, \sigma_1 - (\eta + \mu)\, R(m)\right] \\
&\quad + \frac{\vartheta}{G(\vartheta)\Gamma(\vartheta)} \sum_{k=1}^{m} \frac{\Gamma(m-k+\vartheta)}{\Gamma(m-k+1)} \left[\delta_4 + (S(k) + L(k) + B(k))\, \sigma_1 - (\eta + \mu)\, R(k)\right].
\end{aligned}
$$

With the novel fractional operator, a mathematical study of the computer virus epidemic model is proposed, taking into account the impact of both internal and exterior media for storage. The new ABC fractional difference operator has been utilized in a numerical simulation for the computer virus model. Consider the following parameter values $\delta_1 = 0.25$, $\delta_2 = 0.28$, $\delta_3 = 0.27$, $\delta_4 = 0.23$, $\alpha = 0.033$, $\beta_1 = 0.0043$, $\beta_2 = 0.0063$, $\sigma_1 = 0.021$, $\sigma_2 = 0.01$, $\sigma_3 = 0.018$, $\eta = 0.015$, $\mu = 0.005$, $\theta = 0.0038$ and the initial conditions are $(S_0, L_0, B_0, R_0) = (50, 5, 4, 5)$ with different fractional order $\vartheta = 0.8, 0.85, 0.9, 0.95, 1$ in system (3.2). Figures 1, 2, 3 and 4 demonstrate the memory effect using graphs of the solutions that are approximate in various fractional orders ϑ. It is evident that both internal and external storage devices are an extensive source of viral transmission. Furthermore, we discovered that external

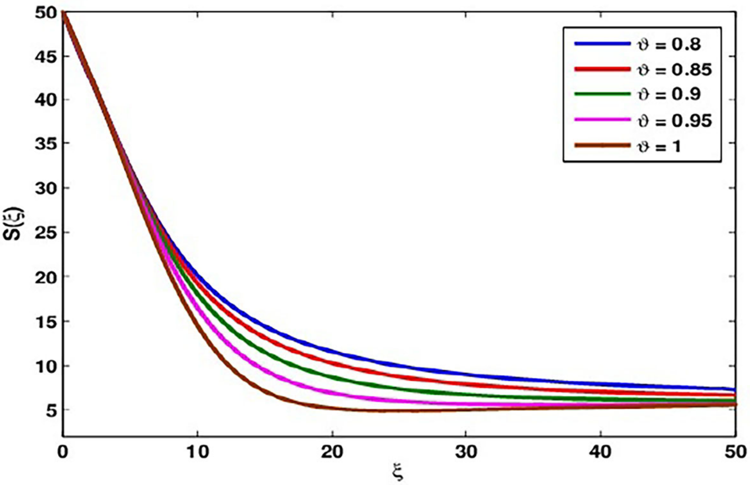

Fig. 1 Nature of the obtained solution for $S(\xi)$ with different fractional orders

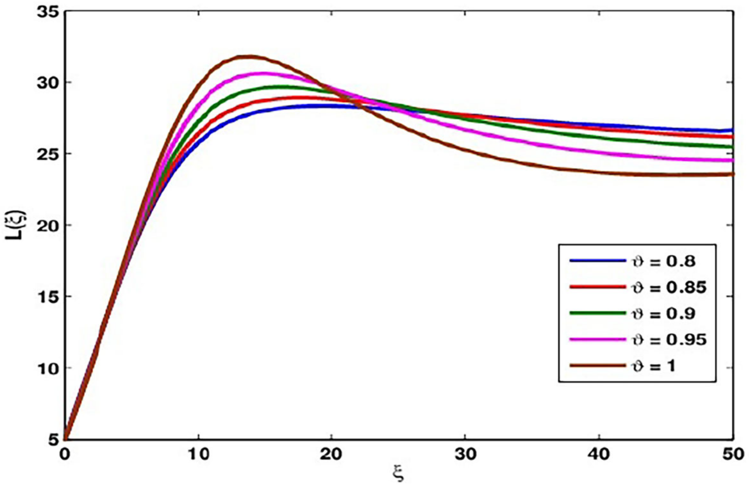

Fig. 2 Nature of the obtained solution for $L(\xi)$ with different fractional orders

storage devices had a sizable impact on viral infections. Figures 1, 2, 3 and 4 show that as latent, breaking-out, and recovered computers increase quickly, the number of susceptible computers declines. The simulation in Fig. 2 depicts how the interactions of latent computers change over time, increasing and then reducing. Figure 3 illustrates how quickly computers are being damaged by the break out, which is also expanding quickly. As a result of the influence of the fractional operator, the recovery rate of computing devices is growing smoothly in Fig. 4.

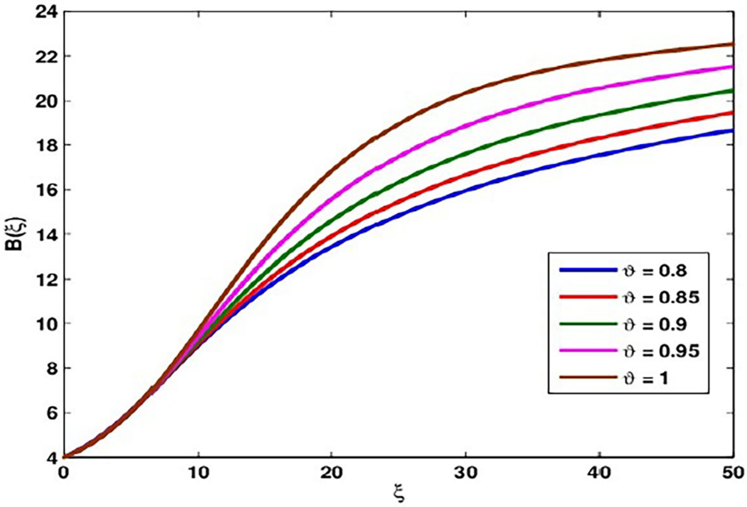

Fig. 3 Nature of the obtained solution for $B(\xi)$ with different fractional orders

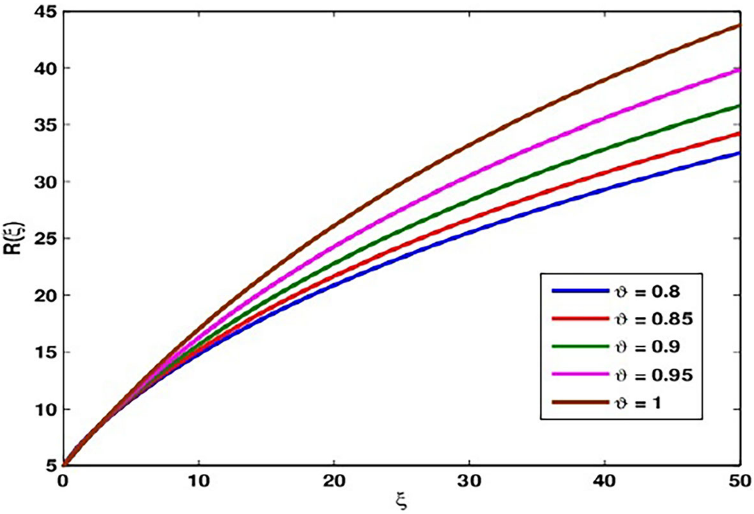

Fig. 4 Nature of the obtained solution for $R(\xi)$ with different fractional orders

8 Conclusion

Discrete fractional equations are gaining increasing importance due to their exceptional ability to accurately simulate real-world physical problems. In the context of the discrete fractional computer virus model employing the ABC fractional difference operator, our research work presents a novel finding. First and foremost, through the application of fixed point methodology, we have strengthened the existence hypothesis for the model, specifically focusing on the existence and uniqueness of the solution.

Following these findings, we took into account HU stability to determine the stability of the solution. Next, we examine numerical simulations for the proposed model. To validate the theoretical findings of the computer virus model, we conducted numerical simulations at various fractional orders, specifically $\vartheta = 0.8, 0.85, 0.9, 0.95, 1$ and obtained graphical outcomes. The proliferation of the virus has been demonstrated to be influenced by both internal and external storage devices. Additionally, we found that as the time variable ξ increases, the influence of all external storage media on viral infection intensifies. These findings are particularly valuable for preventing computer virus infections and mitigating risks from potential outcomes. This research work employs a novel strategy that can serve as a starting point for discussions on various real-world scenarios within the context of discrete behavior frameworks. In future work, we intend to extend the modeling of a computer virus to include stochastic fractional-order derivatives as well as partial differential equations.

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Data Availibility No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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