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A reliability analysis method for rock slope controlled by weak structural surface

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ABSTRACT: Catastrophic landslides maybe occur in rock slope due to the effect of strong earthquakes or heavy rainfall. The stability of rock slope is usually controlled by different scales of weak structural surfaces, which are uncertain and randomly exist in the rock slope. According to the geological characteristics of rock slope, two typical failure modes – plane and wedge are possible. A second-order second-moment (SOSM) method is presented to calculate the reliability index and the failure probability of rock slope, which is an improvement over the first-order second-moment (FOSM) method, and performance functions are built up with the classic limit equilibrium method. The presented method is applied to analyze the failure probability of two rock slopes at the Jinping I Hydropower Station and is compared with the Monte Carlo method and the FOSM method. The computed results show that for plane failure, the reliability index and the failure probability determined by the presented method are 0.563 and 28.7%, respectively, and the reliability index and the failure probability determined by Monte Carlo method are 0.677 and 24.9%, respectively. However, for the FOSM method, the reliability index and failure probability are −0.025 and 51.0%, respectively. For both plane failure and wedge failure, the difference between the presented method and the Monte Carlo method is very small, but the failure probability of plane failure determined by FOSM method is larger than that of the other two methods. The presented method can provide a useful tool to evaluate the failure probability of rock slope.

Key words: rock slopes, stability evaluation, failure probability, Taylor expansion, reliability index

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1. INTRODUCTION

With the increase in human activity (such as urban construction, transportation construction, hydropower development, and mineral exploitation) and global climate change (gradually rising temperature, increasing frequency of extreme weather), increasing numbers of geological disasters have occurred (Liu and Chen, 2007; Zhou et al., 2010). Landslides are one of the major geological disasters worldwide, causing heavy losses of life and property every year (Ni et al., 2014; Chu et al., 2015). Due to unique topography and geological conditions, different scales of landslides often happen in mountainous areas in Southwest China (Zhou et al., 2013). Strong earthquakes, heavy rainfall, reservoir storage and artificial disturbance are the four primary triggering factors for landslides in Southwest China (Li et al., 2011; Jiang et al., 2014; Yang et al., 2015). Landslides are more common in soil slopes and deposition slopes; however, several catastrophic landslides have occurred in rock slope due to the effect of strong earthquakes or heavy rainfall (Zhou et al., 2013; Ghosh et al., 2014; Pourghasemi et al., 2014). There are apparent differences in the geographical characteristics of rock slope and soil slope. The failure of soil slopes is usually not determined by the slip surface, but the failure of rock slopes usually slips along the weak structural surfaces. Because rock slope generally consists of rock masses and different scales of weak structural surfaces (such as joints, fault, or weak interlayers), the stability of rock slope is controlled by these weak structural surfaces (William et al., 2008; KhaloKakaie and Naghadehi, 2012; Youssef et al., 2015).

Previous studies generally believe that the primary reasons for the failure of rock slope controlled by a weak structural surface under rainfall conditions include two aspects: the increasing hydrostatic pressure or hydrodynamic pressure in rock slopes generated by seepage flow and the decreasing shear strength

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of the weak structural surface generated by a saturated softening effect. These two aspects disturb the force balance of the rock slope and cause landslides (Park and West, 2001; Topal, 2007; Tan et al., 2013; Zeng et al., 2015; Pinheriro et al., 2015). The strength reduction of the weak structural surface induced by rainfall is the major physical and mechanical response of rock slope, and we should consider this aspect during the stability evaluation of rock slope under rainfall conditions (Park, 2005; Low, 2007). The relationship between the water content and the shear strength of the weak structural surface is always difficult to determine (Kourosh et al., 2011; Li et al., 2013). However, deterministic methods are often used to study the stability of rock slope, such as the limit equilibrium method, the finite element method, the finite difference method, and the discrete element method (Li and Chu, 2012; Reale et al., 2015). These methods can assess only the determined safety factors and stress and deformation distribution characteristics of the rock slope. Furthermore, several uncertainty conditions and parameters exist in the stability evaluation process of rock slope (Park et al., 2005; Park et al., 2012a; Li et al., 2014), such as the mechanical parameters, the rainfall condition and infiltration process, and the softening of weak structural surfaces. The deterministic methods are unable to handle the uncertainties during the stability evaluation process of rock slope (Yang et al., 2009), and the determined safety factor is not reasonable to evaluate the stability of the slope as an index (Jiang et al., 2014; Li et al., 2015a). Reliability analysis based on probability theory has been introduced by certain scholars to analyze the slope stability problem, and several advances have been made relative to the methods of the safety factor (Pathak and Nilsen, 2004; Gravanis et al., 2014; Park et al., 2016).

Previous studies on the reliability analysis of slopes are focused on the homogeneous slopes (such as soil slopes), and different computational methods are introduced (Rodriguez and Sitar, 2007; Wang et al., 2013; Wei et al., 2014). Most of these studies used three calculation methods: the Rosenbleuth method, the Monte Carlo method and the first-order second-moment (FOSM) method (Duzgun et al., 2003; Ge et al., 2011; Ganji and Jowkarshorijeh, 2012). The Rosenbleuth method yields the reliability index for slope stability analysis by several samples at points with prescribed rules (Sun et al., 2008; Park et al., 2012b). The Monte Carlo method is rarely adopted to locate the critical reliability slip surface due to its huge calculation time (Li et al., 2015b). For the FOSM method, the partial derivative of the performance function is needed to calculate the reliability index. However, the partial derivatives of performance functions are complex and difficult to obtain because the performance function in slope stability analysis is usually implicit (Li and Chu, 2012). The FOSM method was defined as an approximate method since the statistical parameters of the performance function were obtained by the approximation (Harr, 1987; Baecher and Christian, 2003), significant errors may be introduced by neglecting higher-order terms when the performance function is nonlinear (Cho, 2013). Some improvements have been made by several scholars. For example, Duzgun et al. (2003) proposed an advanced first-order second-moment method to conduct a reliability assessment for the case of the plane sliding failure of the slope; Xu et al. (2013) improved the FOSM method so that a nonlinear function could be considered using maximum entropy.

Although the FOSM method can calculate the reliability index (Farah et al., 2011; Jun et al., 2013) and has certain accuracy, the precision of this method is limited because the method considers only the Taylor series expansion first-order item of the performance function. According to the rock slope controlled by weak structural surfaces, two failure modes are considered: plane failure and wedge failure. For the reliability analysis of rock slope under different conditions, a second-order second-moment (SOSM) method was introduced to calculate the reliability index, which can improve the calculation precision of the reliability index (β) relative to the FOSM method. The presented reliability analysis method is applied to study the stability of the left bank slope at the Jinping I Hydropower Station, and a comparative analysis is conducted between the Monte Carlo method and the FOSM method. Several useful conclusions are presented for understanding the stability and reliability analysis of rock slope controlled by weak structural surfaces.

2. BACKGROUND

2.1. Failure of Rock Slope

Along with the development of the economy and life needs in China, many infrastructure projects are under construction or planned in Southwest China, such as railways, highways, and large hydropower stations (Li et al., 2011; Zheng et al., 2014). These infrastructure projects have introduced several engineering problems, such as ecological, environmental and geological hazards. Construction safety is one of the key issues during the engineering construction process. Due to the special topography and geomorphology conditions, high steep slopes are widely distributed in Southwest China. The stability of high rock slope is common in different infrastructure projects, particularly the large hydropower station. The reasonable stability evaluation of high rock slope is crucial for the development of engineering measures and for hazard prevention and mitigation.

Rock mass is generally constituted by rock blocks and different scales of structural surfaces. The slope rock masses are located at

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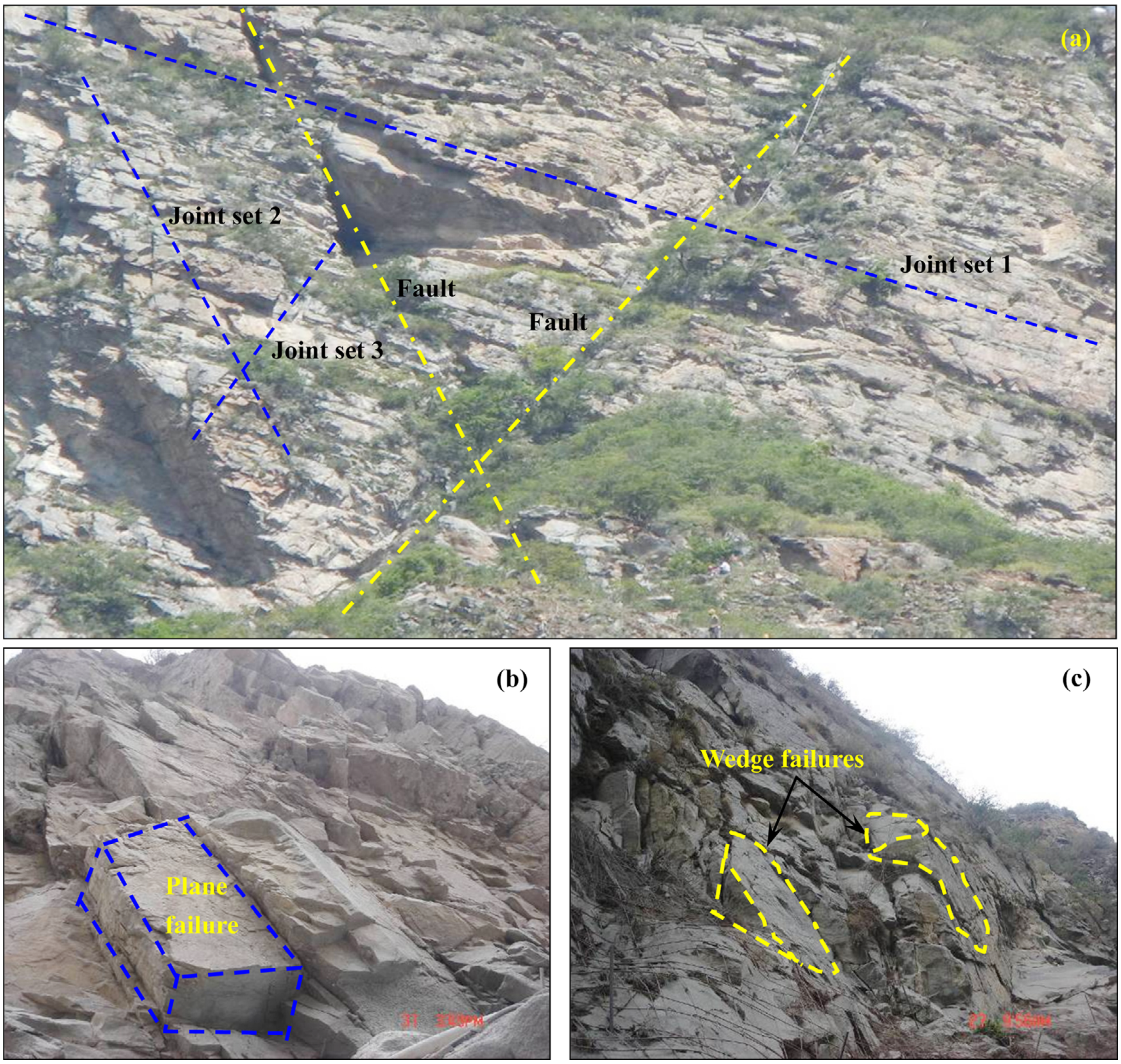

Fig. 1. Rock slope cut by several weak structural surfaces, an example of the high rock slope at the Changheba Hydropower Station, Sichuan Province, Southwest China: (a) rock slope cut by three joint sets and two main faults, (b) potential plane failure of the slope, and (c) potential wedge failures of the slope.

the shallow part of the mountain. The structural surfaces are well developed, and rock masses are fractured due to long-term weathering and unloading effects. Figure 1 shows an example of the high rock slope cutting by several weak structural surfaces at the Changheba Hydropower Station, which is a large hydropower station located on the Dadu River in the Sichuan Province, Southwest China. As shown in Figure 1a, the high rock slope is cut by three sets of joints and two main faults, and the slope stability is controlled by the weak structural surfaces. For this high rock slope during the construction process, two main failure modes exist: plane failure (Fig. 1b) and wedge failure (Fig. 1c). These figures show that although the potential failures are on a small scale, they are all controlled by the weak structural surfaces (in this case, several joints). In this paper, these two typical failure modes are used as the computational basis of the reliability analysis of the rock slope. Figure 2 shows the schematic diagram of two typical failure modes for the rock slope. As shown in Figure 2a, for plane failure, the slope stability is generally controlled by one or two primary weak structural surfaces, which follow the trend of the slope; however, for wedge failure, several different tendencies of weak structural surfaces form unstable rock blocks (Fig. 2b). The corresponding slope stability of the plane failure and the wedge failure are influenced by the shear strength parameters of these weak structural surfaces. However, accurate values for these parameters are difficult to determine, and some uncertainty and randomness exist under different conditions.

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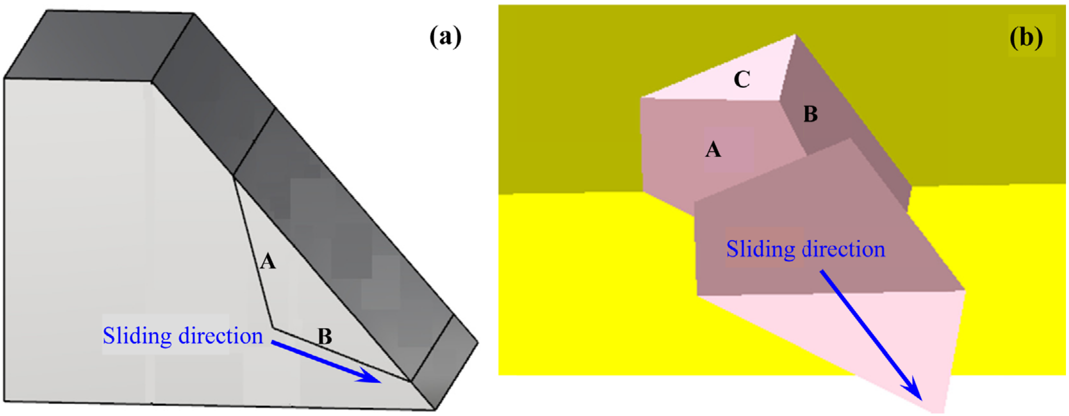

Fig. 2. Schematic diagram of two typical failure modes for the rock slope controlled by a weak structural surface: (a) plane failure, and (b) wedge failure.

2.2. Stability Evaluation for Rock Slope

According to the stability evaluation of rock slope, the limit equilibrium method (LEM) is commonly used to determine the safety factor of the slope. Force balance and Mohr-Coulomb friction criteria are often used for LEM, and the safety factor is defined as shear resistance divided by shear stress. Figure 3 shows the force analysis of two typical failure modes for the rock slopes using the limit equilibrium method. For the plane failure shown in Figure 3a, the potential sliding block is resisted by the shear resistance of the bottom slip surface B and the residual tensile strength (σ_t) of the rear edge slip surface A. Using force balance analysis and the Mohr-Coulomb friction criteria, the safety factor (F_s) of the rock slope with plane failure can be determined as follows:

$$F_s = \frac{cA_2 + (W\cos\beta + A_1\sigma_t\cos\alpha)\tan\varphi}{W\sin\beta - A_1\sigma_t\sin\alpha}, \quad (1)$$

where A_1 and A_2 are the areas of rear edge slip surface A and bottom slip surface B, respectively; c is the cohesion of slipping surface B; φ is the internal friction angle of the whole sliding block; W is the mass weight of the sliding block; α is the intersection angle between slip surface B and the horizontal plane; and β is the intersection angle between slip surfaces A and B.

For the wedge failure shown in the Figure 3b, the potential sliding block is resisted by the shear resistance of two weak structural surfaces, but the residual tensile strength of the rear edge slip surface is not consider here. Using force balance analysis and Mohr-Coulomb friction criteria such as plane failure, the safety factor (F_s) of the rock slope with wedge failure can be determined as follows (modified form the safety factor evaluation model for wedge failure presented by Hoek and Bray (2005)):

$$F_s = \frac{N_1\tan\varphi_1 + N_2\tan\varphi_2 + c_1A_1 + c_2A_2}{W\sin\beta_s}, \quad (2)$$

where A_1 and A_2 are the area of these two weak structural surfaces (slip surfaces); c_1 and c_2 are the cohesions of slip surfaces A_1 and A_2, respectively; φ_1 and φ_2 are the internal friction angles of slip surfaces A_1 and A_2, respectively; W is the mass

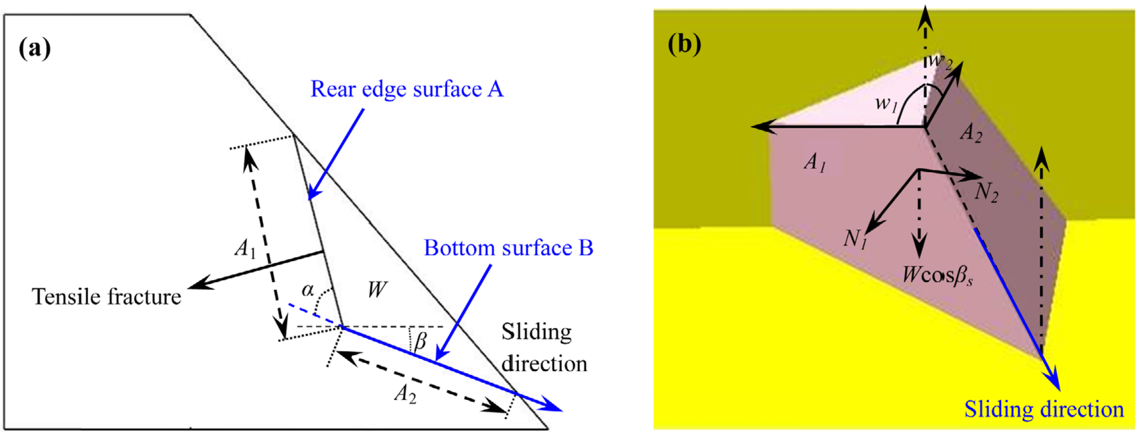

Fig. 3. Force analysis of two typical failure modes for the rock slopes using limit equilibrium method: (a) plane failure, and (b) wedge failure.

weight of the sliding wedge; N_1 and N_2 are the normal forces of slip surfaces A_1 and A_2, respectively; and β_s is the dip angle of the intersection line for these two slip surfaces, which is the sliding direction of the wedge.

According to the force balance analysis for mass weight W on the slip surfaces, the normal forces N_1 and N_2 can be computed as follows:

$$\begin{cases} N_1 = \dfrac{W\cos\beta_s\cos\chi_2}{\sin\chi_1\cos\chi_2 + \cos\chi_1\sin\chi_2}, \\ N_2 = \dfrac{W\cos\beta_s\cos\chi_1}{\sin\chi_1\cos\chi_2 + \cos\chi_1\sin\chi_2}, \end{cases} \tag{3}$$

where χ_1 and χ_2 are the angles between the normal line of the intersection and slip surfaces A_1 and A_2, respectively. χ_1 and χ_2 can be determined as follows:

$$\begin{cases} \sin\chi_1 = \sin\beta_A\sin\beta_s\sin(\psi_s - \psi_A) + \cos\beta_A\cos\beta_s, \\ \sin\chi_2 = \sin\beta_B\sin\beta_s\sin(\psi_s - \psi_B) + \cos\beta_B\cos\beta_s, \end{cases} \tag{4}$$

where β_A and ψ_A are the dip angle and the dip of slip surface A, respectively; β_B and ψ_B are the dip angle and the dip of slip surface B; and β_s and ψ_s are the dip angle and the dip of the intersection line of slip surface A and slip surface B (sliding direction).

The safety factors of the rock slope with plane failure and wedge failure can be determined by Equations (1)–(4), but there some uncertainty and randomness arise during the stability evaluation of slope. The uncertainty and randomness are generally caused by the random distribution of mechanical parameters, particularly the shear strength parameters of weak structural surfaces. By using of limit equilibrium method, three uncertain parameters are commonly used, which include rock density, cohesion and friction angle of weak structural surfaces. Each uncertain parameter is assumed within a range of values ($p_{\min}$, $p_{\max}$), then using a special distribution function to generate a series of random values for this parameter. Table 1 shows the calculation parameters used for the sensitivity analyses of mechanical parameters on the safety factor of a simple wedge failure. Figure 4 shows the uncertainty and randomness in the safety factor evaluation results for a simple wedge failure using Monte Carlo simulation based on the limit equilibrium method. When we use the limit equilibrium method to evaluate the slope stability, the safety factor of the slope is controlled by the used mechanical parameters. Figure 4a shows the stereographic projection analysis

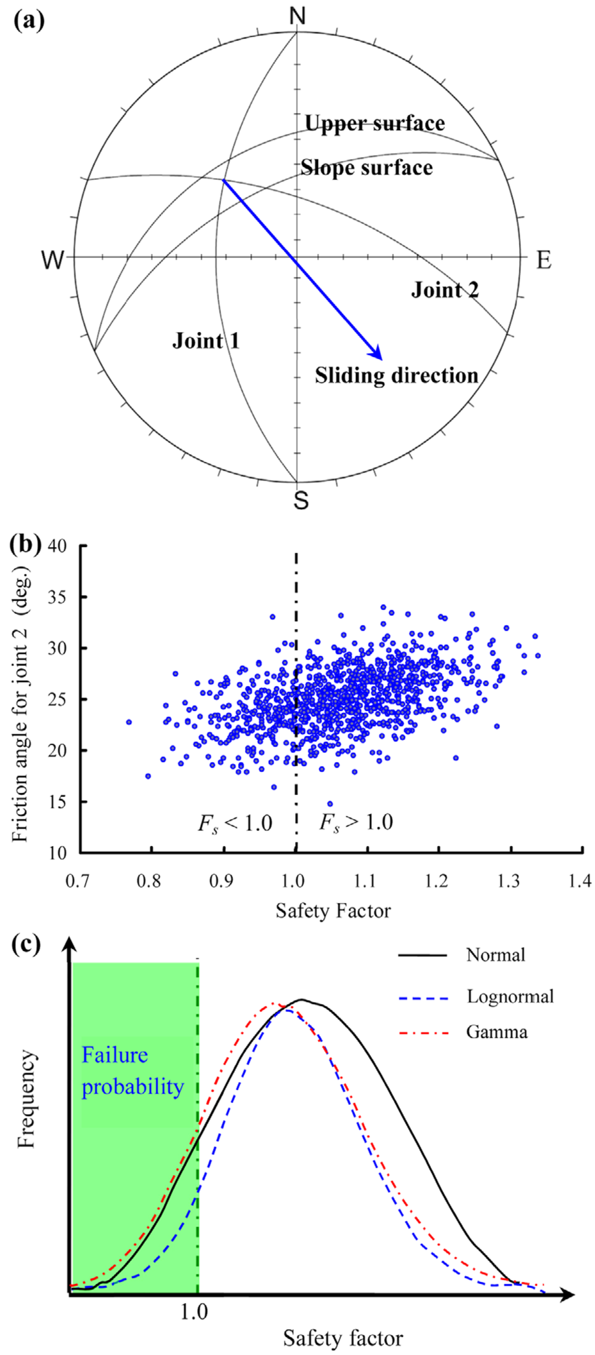

Fig. 4. Uncertainty and randomness in the safety factor evaluation results for a simple wedge failure using the Monte Carlo method based on the limit equilibrium method: (a) stereographic projection analysis of the simple wedge failure, (b) random mechanical parameters' effect on the safety factor of slope, and (c) different random distribution functions' effect on the failure probability of the slope.

Table 1. Calculation parameters used for the sensitivity analyses of mechanical parameters on the safety factor of a simple wedge failure

Surface	Dip (°)	Dip direction (°)	c (kPa)			φ (°)		
			Minimum	Maximum	Mean	Minimum	Maximum	Mean
Joint 1	50	270	10	30	20	14	36	25
Joint 2	62	20	10	30	20	14	36	25
Slope	53	335	Slope height (m)			Mass weight (kN/m^3)		
Upper	35	335	70			24		

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of the simple wedge failure. For determination one of the uncertain parameter effect on the safety factor of the slope, other uncertain parameters are commonly used the fixed values (such as mean values), then varied safety factors of the slope can be determined by using of different parameter combinations. As shown in Figure 4b, the safety factor of the wedge failure is varied using different mechanical parameters. According to the random distribution characteristics of the mechanical parameters of rock slope, the normal function, the exponential function, the Weibull function and the gamma function are commonly used random distribution functions for geo-materials. As shown in Figure 4c, the failure probability (often defined as the probability of a safety factor less than 1.0) of a slope is influenced by the random distribution functions, and there is a determined relationship between the failure probability and the reliability index. Different random distribution functions can achieve different results for the failure probability of the slope. Previous laboratory tests and statistical results indicated that the randomness for the shear strength parameters of weak structural surfaces is generally consistent with normal distribution (Park and West, 2001; Sun et al., 2008; Li et al., 2015a). Taking into account that the normal distribution function is the most commonly used for random variables, this random distribution function is used in this paper.

3. METHOD

For the reliability index and failure probability determined by Monte Carlo simulation based on the limit equilibrium method, large calculations should be performed for different parameter sets. If the number of simulated random variables is not large enough, the uncertainty and randomness of the mechanical parameters are not well reflected. However, for a certain randomly distributed mechanical parameter, some characteristic values of the random variables can be determined, such as the mean value and the variance for normally distributed parameters. The reliability index and failure probability can be directly determined by these characteristic values of random variables, and the calculation equations for the safety factor of rock slope under different conditions can be used to establish the performance function. Here, the second-order second-moment (SOSM) method was introduced to calculate the reliability index, which is an improvement over the first-order second-moment (FOSM) method.

3.1. Failure Probability

The safety factor (F_s) is commonly used to evaluate slope stability, which can be expressed by a function with a series of physical and mechanical parameters of geo-materials. This function is determined by the special calculation method, which can generally be expressed as follows:

$$F_s = f(x_1, x_2, \cdots, x_n)\,, \tag{5}$$

where x_1, x_2, ···, x_n is a series of physical and mechanical parameters, such as density, elastic modulus, cohesion, internal friction angle, and pore water pressure, and $f(\cdot)$ is an implicit function and depends on the specific calculation method.

Performance function (G) can be defined according to the calculation method of the safety factor, which can be determined as follows:

$$G = g(X) = g(x_1, x_2, \cdots, x_n) = F_s - 1\,. \tag{6}$$

For this function, every parameter x_i is a random variable that obeys some random distribution. Hence resulted in the same randomness of the performance function G, and some calculation method should be adopted to solve this performance function. Here, according to the state of slope, failure will occur for the slope when G is less than 0. The slope is stable when G is larger than 0, and $G = 0$ is a critical state. Therefore, the failure probability (P_f) of the slope can be expressed when $G < 0$ or $F_s < 1$. Random variation G is assumed to obey a normal distribution, and the reliability index (β) can be calculated as follows:

$$\beta = \frac{\mu(G)}{\sigma(G)}\,, \tag{7}$$

where μ_G is the mean value, and σ_G is the standard deviation of the performance function $g(x)$.

The failure probability (P_f) of the slope can be expressed as follows:

$$P_f = \Phi(-\beta)\,, \tag{8}$$

where $\Phi(\cdot)$ is a normal function. This expression indicates that if the reliability index is a known value, the failure probability of the rock slope can be determined and can be used to evaluate the stability of the slope.

3.2. Reliability Index

The reliability index, β, is the key to calculating failure probability (Zheng et al., 2015), but the precise value of the reliability index is hard to calculate. Furthermore, the calculation work for some previous methods is large, so it is necessary to find an effective calculation method that can reflect the real situation and reduce the calculation work. Here, the second-order second-moment (SOSM) method was introduced to calculate the reliability index, which expanded the performance function with Taylor series expansion and added one more order than the FOSM method.

Here, standard normal variable z_i ($i = 1, 2, \cdots, n$) was used to transform the random variable x_i ($i = 1, 2, \cdots, n$) as follows:

$$z_i = \frac{x_i - \mu(x_i)}{\sigma(x_i)}, \tag{9}$$

where $\mu(x_i)$ and $\sigma(x_i)$ are the average value and standard deviation of the random variable x_i ($i = 1,2,\cdots,n$), respectively.

First, expanding performance function g(X) on limit point $X^* = (x_1^*, x_2^*, \cdots, x_n^*)$ with the Taylor formula and ignoring the items superior to the quadratic items, performance function g(X) can be expressed as follows:

$$g(X) = g(x_1^*, x_2^*, \cdots, x_n^*) + \sum_{i=1}^{n}(x_i - x_i^*)\left.\frac{\partial g}{\partial x_i}\right|_{x_i^*} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_i^*)(x_j - x_j^*)\left.\frac{\partial^2 g}{\partial x_i \partial x_j}\right|_{x_i^* x_j^*}. \tag{10}$$

The limit point represents the critical state of the rock slope, which means the performance function is equal to 0:

$$g(X) = g(x_1^*, x_2^*, \cdots, x_n^*) = 0. \tag{11}$$

Then, the expansion of performance function g(X) can be expressed by the standard normal variable z_i as follows:

$$g(X) = \sum_{i=1}^{n}(z_i - z_i^*)\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*} + \frac{1}{2}\sum_{i=1}^{n}(z_i - z_i^*)^2\left.\frac{\partial^2 g}{\partial z_i^2}\right|_{z_i^*} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n}(z_i - z_i^*)(z_j - z_j^*)\left.\frac{\partial^2 g}{\partial z_i \partial z_j}\right|_{z_i^* z_j^*}. \tag{12}$$

According to the upper part of Equation (12), reliability index β can be calculated using the distribution parameters of each random variable, and it can be divided into three situations, as follows.

For situation 1, the calculation parameters of the random variables are independent of each other. Because random variables x_i are all mutually independent, random variables z_i are also mutually independent and belong to the normal distribution, as in Equation (9). Then, the average value, μ_g, of performance function $g(X)$ can be derived as follows:

$$\mu_g = E[g(X)] = -\sum_{i=1}^{n}\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*} z_i^* + \frac{1}{2}\sum_{i=1}^{n}\left.\frac{\partial^2 g}{\partial z_i^2}\right|_{z_i^*} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left.\frac{\partial^2 g}{\partial z_i \partial z_j}\right|_{z_i^* z_j^*} z_i^* z_j^*. \tag{13}$$

The standard deviation, σ_g, of performance function $g(X)$ can be expressed as follows:

$$\begin{cases} D[g(X)] = E[g(X)^2] - \{E[g(X)]\}, \\ \sigma_g = \sqrt{D[g(X)]}, \end{cases} \tag{14}$$

where $D[g(X)]$ is the variance of performance function $g(X)$, and $E[\cdot]$ is the expected value of one variable.

However, standard deviation σ_g can not be calculated by a simple linear relationship because performance function $g(X)$ has expanded to the second-order of Taylor formula. Here, we use the square of performance function $g(X)$ to deduce standard deviation σ_g as follows:

$$g^2(X) = g^2(x_1, x_2, \cdots, x_n). \tag{15}$$

Expanding the square formula $g^2(X)$ of the performance function to the second order with the Taylor formula and ignoring the items superior to the quadratic items, function $g^2(X)$ can be transformed into another expression using Equations (9) and (11),

$$g^2(X) = \sum_{i=1}^{n}(z_i - z_i^*)^2\left(\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*}\right)^2 + \sum_{i=1}^{n}\sum_{j\neq i}^{n}(z_i - z_i^*)(z_j - z_j^*)\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*}\left.\frac{\partial g}{\partial z_j}\right|_{z_j^*}. \tag{16}$$

Then, the average value of $g^2(X)$ can be simplified as follows:

$$E[g(X)] = \sum_{i=1}^{n}\left(\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*}\right)^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*}\left.\frac{\partial g}{\partial z_j}\right|_{z_j^*} z_i^* z_j^*. \tag{17}$$

Using Equations (13), (14), and (17), the standard deviation of $g(X)$ can be calculated as follows:

$$\sigma[g(X)] = \sqrt{\begin{aligned}&\sum_{i=1}^{n}\left(\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*}\right)^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*}\left.\frac{\partial g}{\partial z_j}\right|_{z_j^*} z_i^* z_j^* - \\ &\left(-\sum_{i=1}^{n}\left.\frac{\partial g}{\partial z_i}\right|_{z_i^*} z_i^* + \frac{1}{2}\sum_{i=1}^{n}\left.\frac{\partial^2 g}{\partial z_i^2}\right|_{z_i^*} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left.\frac{\partial^2 g}{\partial z_i \partial z_j}\right|_{z_i^* z_j^*} z_i^* z_j^*\right)^2\end{aligned}}. \tag{18}$$

Finally, reliability index β can be calculated according to Equation (7) as follows:

$$\beta = \frac{E[g(X)]}{\sigma[g(X)]}. \tag{19}$$

For situation 2, the calculation parameters of random variables are correlated with each other. The correlation coefficient of x_i and x_j is r_{ij}, and r_{ij} is equal to 1 when $i = j$. Then, the correlation coefficient of z_i and z_j can be determined as follows:

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$$\rho_{z_i,z_j} = \frac{COV(z_i, z_j)}{\sqrt{Dz_i}\sqrt{Dz_j}} = \frac{\sigma_{x_i}\sigma_{x_j}COV(z_i, z_j)}{\sqrt{\sigma_{x_i}^2 Dz_i}\sqrt{\sigma_{x_j}^2 Dz_j}} = \frac{COV(x_i, x_j)}{\sqrt{Dx_i}\sqrt{Dx_j}} = r_{ij}, \tag{20}$$

where $COV(\cdot)$ is a covariance function.

The average value $E[g(X)]$ and $E[g^2(X)]$ can be simplified as follows:

$$E[g(X)] = \sum_{i=1}^{n} z_i^* \frac{\partial g}{\partial z_i}\bigg|_{z_i^*} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\frac{\partial^2 g}{\partial z_i \partial z_j}\bigg|_{z_i^* z_j^*} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} z_i^* z_j^* \frac{\partial^2 g}{\partial z_i \partial z_j}\bigg|_{z_i^* z_j^*}, \tag{21}$$

$$E[g^2(X)] = \sum_{i=1}^{n}\sum_{i=1}^{n}(z_i^* z_j^* + r_{ij})\frac{\partial g}{\partial z_i}\bigg|_{z_i^*}\frac{\partial g}{\partial z_j}\bigg|_{z_j^*} z_i^* z_j^*. \tag{22}$$

Reliability index β can be calculated as follows:

$$\beta = \frac{E[g(X)]}{\sqrt{E[g^2(X)] - \{E[g(X)]\}^2}}. \tag{23}$$

For situation 3, the calculation parameters of random variables obey other distributions. It is necessary to transfer random variables to be an equivalent normal distribution when the original random variables x_i (i = 1, 2,⋯, n) are not normally distributed because the average value, μ_x, and the standard deviation, σ_x, of all random variables could be calculated in equivalent normal distribution. Next, reliability index β can be calculated using the above equations under an equivalent normal distribution.

Previous studies have often assumed the relationship between the original distribution and the equivalent normal distribution to be as follows (Duzgun et al., 2003):

$$\begin{cases} P(x \le x^*) = F_x(x^*), \\ P(x \le x^*) = \Phi\left(\dfrac{x^* - \mu_x^N}{\sigma_x^N}\right), \end{cases} \tag{24}$$

where $P(x < x^*)$ is the probability that random variable x_i is less than certain value x^*, and $F(x)$ is the probability function of random variables x_i.

The average value of equivalent normal variables can be calculated as follows:

$$\mu_x^N = x^* + \sigma_x^N \Phi^{-1}[F_{x_i}(x^*)]. \tag{25}$$

The probability density function $f_x(x_i^*)$ and the standard deviation, σ_x^N, of the equivalent normal variables can be determined as follows:

$$\sigma_x^N = \varphi\left(\frac{x^* - \mu_x^N}{\sigma_x^N}\right)/f_x(x^*) = \varphi\{\Phi^{-1}[F(x^*)]\}/f_x(x^*). \tag{26}$$

Then, reliability index β can also be calculated.

3.3. Calculation Process

The presented method for determining the reliability index and failure probability of rock slope with plane failure or wedge failure is fulfilled by Matlab and Excel VBA programs, and the calculation process is simply described as follows:

(1) First, expand performance function g(X) on limited point x^*_i (i = 1,2,⋯,n) with the Taylor formula to the second order.

(2) Second, ignore the items superior to quadratic items and introduce standardized normal variable z_i.

(3) Third, calculate the average value and standard deviation of the random variables, and derive the partial derivatives of the performance function.

(4) Finally, calculate failure probability P_f according to reliability index β in different situations using the presented method.

If the characteristic values of the random variables are known, the reliability index and failure probability can be quickly determined by the presented method. Furthermore, the presented method can improve the calculation accuracy of failure probability P_f rock slope controlled by a weak structural surface. In the following section, two rock slopes at the Jinping I Hydropower Station are used to verify the reasonability of the presented method. Furthermore, if several sets of random mechanical parameters are generated according to the normal distribution function, the corresponding safety factor using different mechanical parameters can also be computed.

4. RESULTS AND DISCUSSION

In this section, the presented reliability analysis method is applied to study the failure probability of two rock slopes: One is plane failure, and the other is wedge failure. Lower shear strength parameters are used in the following analyses (geo-materials of the weak structural surface are saturated), and no reinforcement measures are considered. The simulated failure probability and reliability index of the slope are compared with the Monte Carlo method and the FOSM method. Key issues related to the reliability analysis of rock slope are discussed at the end of this section.

4.1. Geological Setting

The Jinping I Hydropower Station is located at the lower stream of the Yalong River in Sichuan Province, Southwest

Fig. 5. The Jinping I Hydropower Station: (a) site location, and (b) regional topography and geomorphology conditions.

China. Figure 5a shows the site location of the Jinping I Hydropower Station. The catchment area of the reservoir is approximately 10.3×10^4 km^2, is stored behind a 305-m-high concrete arch dam. The power installation of the station is 3,600 MW, and the annual average power generation is approximately 166.2×10^8 kW·h (Zhou et al., 2014).

High and steep mountains with a height difference of 1,500–3,500 m are common in the project area because of the strong tectonic processes. Figure 5b shows the regional topography and geomorphology conditions of the Jinping I Hydropower Station. The inclinations of the slopes are mostly in the range of 55–75°, and long-term strong weathering and unloading effects, joints, fractures and faults are well developed in the slope. These special geological conditions that resulted in the slope stability problem are some of the key issues in the construction process of this hydropower station.

4.2. Plane Failure

A typical plane failure of the left bank slope at the Jinping I Hydropower Station is selected as the case study example, and the geological condition of this plane failure is shown in Figure 6a. As shown in Figure 6a, the elevation of this slope ranges from 1,960 m to 2,075 m, and the main type of rock mass is silty and sandy slate. The plane failure is cut by two faults, fLL1 and XL21. The slope excavation surface is with

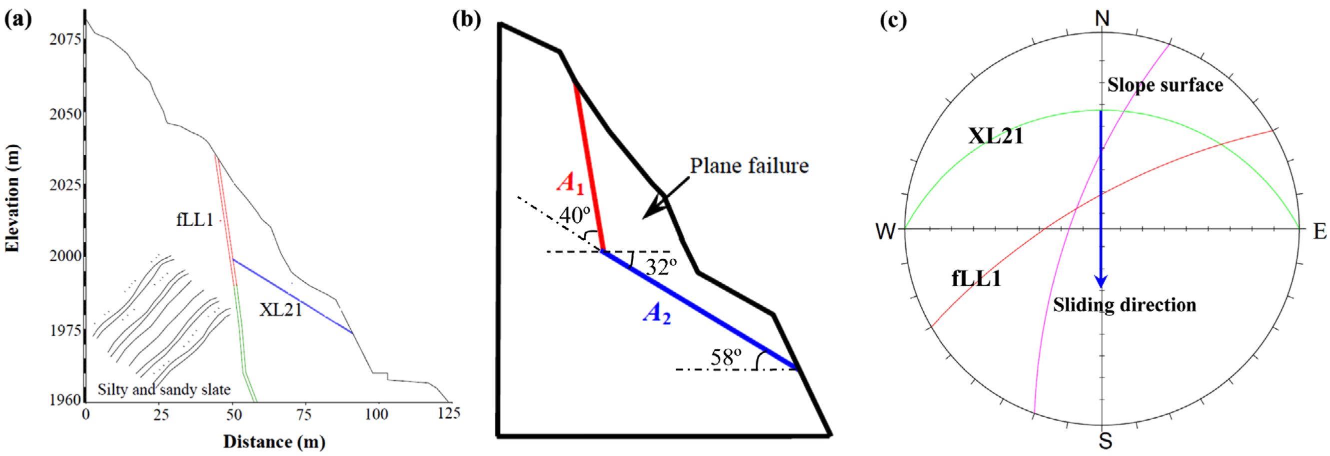

Fig. 6. An example of plane failure at the Jinping I Hydropower Station: (a) geological condition, (b) calculation diagram of this plane failure, and (c) stereographic projection analysis of the plane failure.

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the strike of N20°E, dip direction of SE and the dip angle of 52°. Fault fLL1 is the rear edge slip surface with the strike of N70°E, dip direction of SE and the dip angle of 72°. Fault XL21 is the bottom slip surface with the strike of EW, dip direction of S and the dip angle of 32°. Figure 6b shows the calculation diagram of this plane failure. This plane failure is controlled by bottom slip surface A_2 and rear edge slip surface A_1. The stereographic projection analysis of this plane failure is shown in Figure 6c. Through geometric analyses and mechanical calculations, the basic parameters for the reliability analysis are determined. For example, the total mass weight is 17,420 kN/m. According to the mechanical parameters provided by the designer, the average value and standard deviation of the residual tensile strength of fault fLL1 are 3.05 kPa and 0.394 kPa, respectively; the average value and standard deviation of the cohesion of fault XL21 are 21.52 kPa and 1.97 kPa, respectively. Table 2 summarizes all calculation parameters used for the reliability analysis of plane failure at the Jinping I Hydropower Station.

Combined with Equations (1) and (6), the performance function of this plane failure can be established as follows:

$$g(\sigma_t, \varphi, c) = \frac{50c + (17420 \times \cos 32° + 35\sigma_t \times \cos 40°) \times \tan\varphi}{17420 \times \sin 32° - 35\sigma_t \times \sin 40°} - 1. \quad (27)$$

The reliability index can be determined by the method presented in Equation (27). The distribution forms of the three random variables σ_t, φ, c must be known when calculating reliability index β. A normal distribution is assumed for these three random variables. Through iterative calculations, the limit points of the three random variables can be determined, where σ_t^*, φ^*, and c^* are 3.0 kPa, 28.5° and 21.3 kPa, respectively.

Using the characteristic parameters of the average value and the standard deviation of the above three random variables, reliability index β and failure probability P_f of this plane failure can be calculated. Several sets of random mechanical parameters

Table 2. Calculation parameters used for the reliability analysis of plane failure at the Jinping I Hydropower Station

A_1 (m)	A_2 (m)	α (°)	β (°)	γ (kN/m^3)	W (kN/m)
35	50	40	32	26	17420
$\mu_{\sigma t}$ (kPa)	μ_φ (°)	μ_c (kPa)	$\sigma_{\sigma t}$ (kPa)	σ_φ (°)	σ_c (kPa)
3.05	28.45	21.52	0.394	1.92	1.97

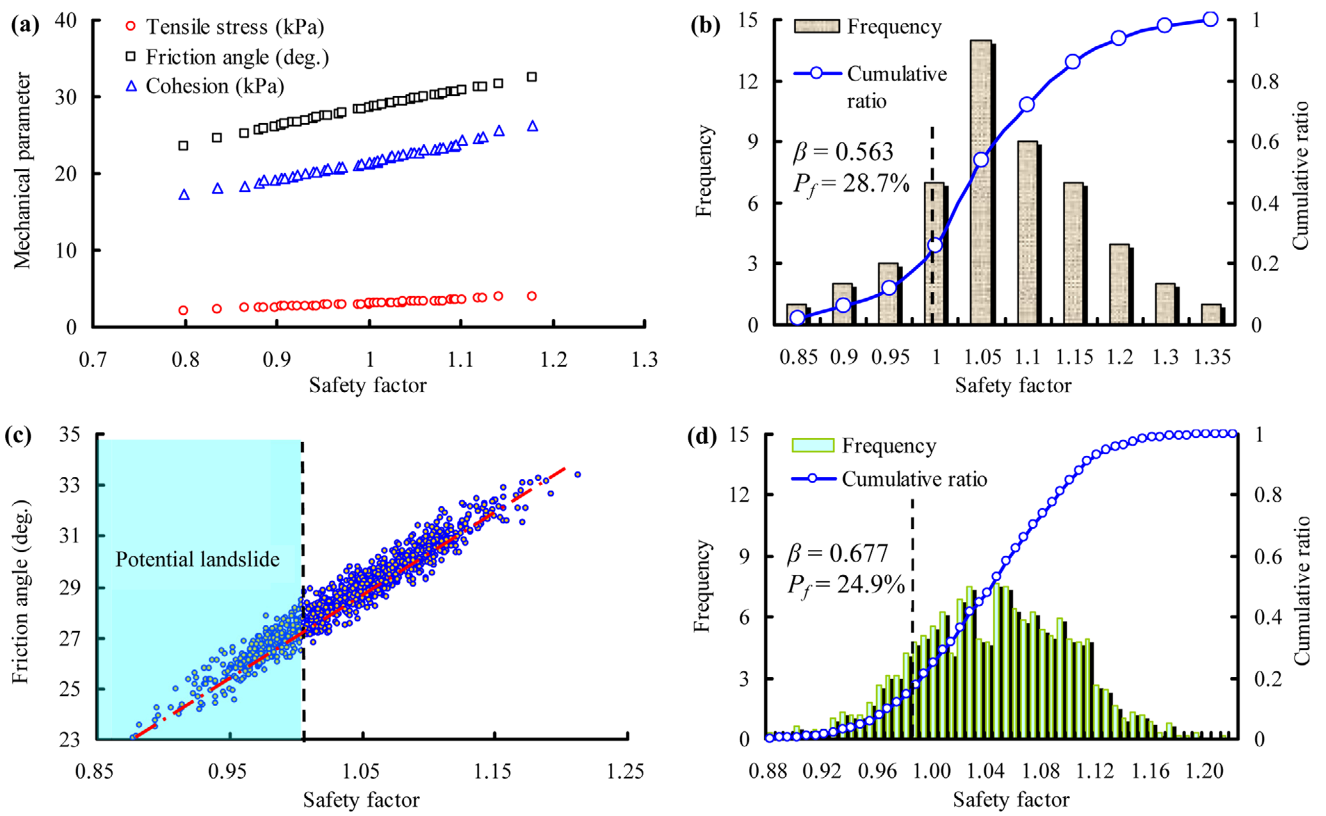

Fig. 7. Reliability analysis results of the plane failure: (a) effect of mechanical parameters on the safety factor, (b) frequency distribution of the safety factor using a set of random parameters, (c) effect of friction angle on the safety factor using the Monte Carlo method, and (d) results of the reliability index and failure probability using the Monte Carlo method.

are generated according to the normal distribution function, and the effect of mechanical parameters on the safety factor of this plane failure is shown in Figure 7a. As shown in Figure 7a, the safety factor of the plane failure increases with the mechanical parameters (including the residual tensile strength of fault fLL1 and the cohesion and friction angle of fault XL21). Using the presented method, the results show that the reliability index of this plane failure is 0.563, and the corresponding failure probability is 28.7%. Although the reliability index and failure probability can be determined directly by the presented method, the distribution characteristics of the safety factor using the generated random parameters are shown in Figure 7b.

Furthermore, two other methods are also used to compute the reliability index of this plane failure. For the Monte Carlo method, one thousand random parameters sets are generated first. We find that the safety factor of the plane failure increases with the mechanical parameters, as shown in Figure 7c. Figure 7d shows the computed results of the reliability index and failure probability using the Monte Carlo method. Compared with Figures 7b and d, there are some similarities between the presented method and Monte Carlo method for the distribution characteristics of the safety factor. Table 3 summarizes the reliability analysis results for the plane failure using different methods. As shown in Table 3, by using the Monte Carlo method, the reliability index of this plane failure is 0.677, and the corresponding failure probability is 24.9%. However, for the FOSM method, the reliability index of this plane failure is −0.025, and the corresponding failure probability is 51.0%. The results of the reliability index and the failure probability determined by the presented method are close to those of the Monte Carlo method, but the results determined by the FOSM method have certain differences from the Monte Carlo method and the presented method.

4.3. Wedge Failure

A typical wedge failure of the left bank slope at the Jinping I Hydropower Station is selected as another case study example, and the geological condition of this wedge failure is shown in Figure 8a. As shown in Figure 8a, the main types of rock masses are sandy slate, marble and greenschist, and the maximum elevation of the dam is 1885 m. The wedge failure is cut by two faults in a vertical plane, fault f42-9 and lamprophyre X, as shown in Figure 8b. Figure 9 shows the three-dimensional geometry information of the wedge failure. As shown in Figure 9a, the slope excavation surface is with the strike of N25°E, dip direction of SE and the dip angle of 63°. Fault F4-29 is slip surface A with the strike of EW, dip direction of S and the dip angle of 50°. Fault SL44-1 is slip surface B with the strike of N20°W, dip direction of NE and the dip angle of 62°. Lamprophyre X is the rear edge surface with the strike of N65°E, dip direction of SE and the dip angle of 75°. The stereographic projection analysis of the wedge failure is shown in Figure 9b. After space geometry analysis, the space information of the intersection line (sliding direction) can be determined, with a dip of 133.41° and a dip angle of 39.82°.

Through geometric analyses and mechanical calculations, the basic parameters for the reliability analysis are determined; for instance, the total mass weight is 995,733.7 kN, and the areas of A_1 and A_2 are 3,866.7 m^2 and 1,595.3 m^2, respectively. The residual tensile strength of the rear edge surface is not considered here, and the shear strength parameters of faults SL44-1 and f42-9 use different values.

Combined with Equations (2)–(4) and (6), the performance

Table 3. Reliability analysis results for the plane failure using different methods

Methods	Monte Carlo method	FOSM method	Presented method
Reliability index β	0.677	−0.025	0.563
Failure probability P_f (%)	24.9	51.0	28.7

Note: The FOSM method is the first-order second-moment method.

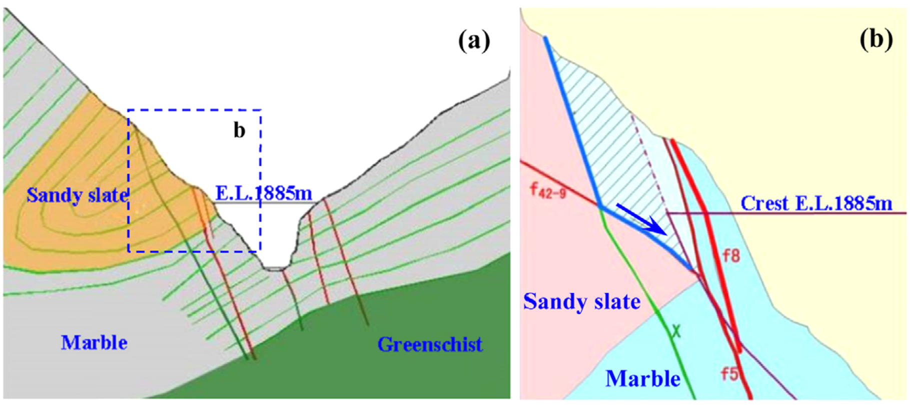

Fig. 8. An example of wedge failure at the Jinping I Hydropower Station: (a) geological condition of the main section, and (b) local amplification of the wedge failure (E.L. is the elevation).

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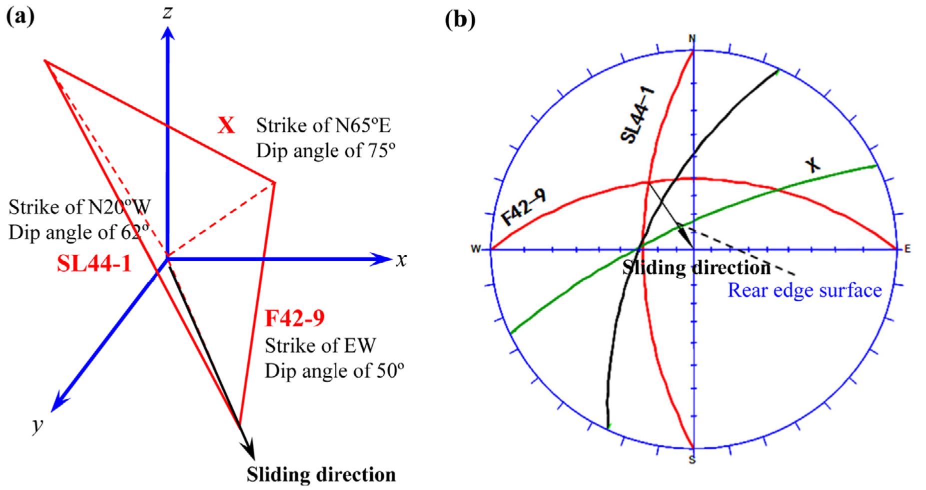

Fig. 9. Three-dimensional geometry information of the wedge failure: (a) wedge failure controlled by several weak structural surfaces, and (b) stereographic projection analysis of the wedge failure.

function of this wedge failure can be established as follows:

$$g(X) = \frac{621667.3\tan\varphi_1 + 403738.8\tan\varphi_2 + 3866.7 \times c_1 + 1595.3 \times c_2}{995733.7 \times \sin 39.82°} - 1. \quad (28)$$

The reliability index can be determined by the method presented in Equation (28). According to the randomness for the shear strength parameters (cohesion c and friction angle φ) of weak structural surfaces, which are sensitivity to the water ratio, here two empirical formulas are introduced to describe the relationship between the shear strength parameters and water ratio (Hu, 2014). The relationship between cohesion and water ratio is as follow:

$$c = f_1(w) = A_1 e^{-B_1 w} - 0.5(C_1 + D_1 w) + E_1, \quad (29)$$

where w is the water ratio, %; c is the cohesion, kPa; A_1, B_1, C_1, D_1 and E_1 are the empirical parameters from several laboratory tests, here are 35, 0.05, 0.3, 0.25 and 35 respectively.

And the relationship between cohesion and water ratio is as follow:

$$\varphi = f_2(w) = A_2 e^{0.5B_2} - (C_2 + 0.5D_2)w, \quad (30)$$

where w is the water ratio, %; φ is the friction angle, °; A_1, B_1, C_1, and D_1 are the empirical parameters from several laboratory tests, here are 28, 0.01, 0.05 and 0.04 respectively.

Then the randomness of shear strength parameters is transformed into the random distribution of water ratio. Only two distribution forms of the random variables w_1 and w_2 (water ratios) must be known when calculating reliability index β. A normal distribution is assumed for these two random variables, and the random distribution of cohesion and friction angle can be obtained by using Equations (29) and (30). After iterative calculation, the limit points of the four random variables can be determined, where w_1^* and w_2^* are 65% and 64%, respectively. Table 4 summarizes all calculation parameters used for the reliability analysis of wedge failure at the Jinping I Hydropower Station.

Using the characteristic parameters of the average value and the standard deviation for the above random variables, the reliability index β and the failure probability P_f of this wedge failure can

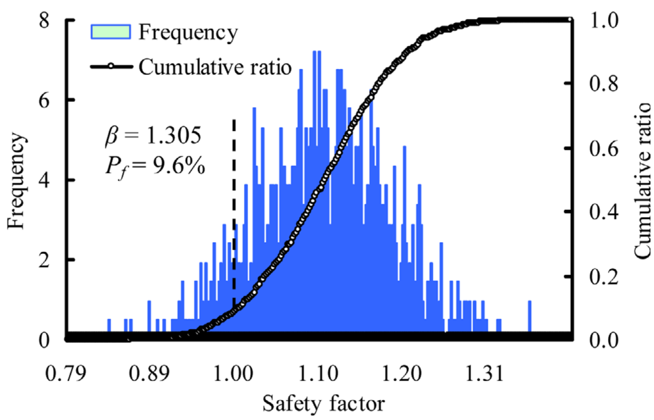

Fig. 10. Reliability analysis result of the reliability index and failure probability using the Monte Carlo method.

Table 4. Calculation parameters of the second rock slope for the wedge sliding

A_1 (m²)	A_2 (m²)	β_s (°)	γ (kN/m³)	W (kN)	N_1 (kN)	N_2 (kN)
3866.7	1595.3	39.82	26	995733.7	621667.3	403738.8
μ_{w1} (%)	μ_{w2} (%)	σ_{w1}	σ_{w2}	w_1^* (%)	w_2^* (%)	r_{12}
44.0	44.5	0.227	0.223	65	64	0.97

Note: r_{12} is the correlation coefficient between w_1 and w_2.

Table 5. Probability analysis results of the wedge by using different methods

Methods	Monte Carlo method	FOSM method	Presented method
Reliability index β	1.305	1.106	1.250
Failure probability P_f (%)	9.6	13.4	10.6

be calculated. The results computed using the presented method show that the reliability index of this plane failure is 1.250, and the corresponding failure probability is 10.6%.

Furthermore, two other different methods are used to compute the reliability index of this wedge failure. Figure 10 shows the computed results of reliability index and failure probability using the Monte Carlo method. Table 5 summarizes the reliability analysis results for wedge failure using different methods. As shown in Table 5, using the Monte Carlo method, the reliability index of this plane failure is 1.305, and the corresponding failure probability is 9.6%. Meanwhile, for the FOSM method, the reliability index of this plane failure is 1.106, and the corresponding failure probability is 13.4%. The difference between the presented method, the FOSM method and the Monte Carlo method is very small for the wedge failure problem, which have some differences between the computational results of plane failure. Tables 4 and 5 indicated that, a relatively large error is appeared for the reliability analysis results of plane failure by using FOSM method, but the presented method (SOSM) can calculate the reliability index and failure probability more reasonable for plane failure and wedge failure.

4.4. Discussion

The stability of rock slope is usually controlled by different scales of weak structural surfaces, and several uncertainty conditions or parameters exist during the stability evaluation process of rock slope. Deterministic methods are unable to handle the uncertainties during the stability evaluation process of rock slope, and reliability analysis based on probability theory should be introduced to evaluate the stability of rock slope. During the reliability analysis process of a rock slope, three key factors (the distribution of random variables, the foundation of the mechanical model, and the calculation model for the reliability index) have a strong effect on the reliability results (Duzgun et al., 2003; Chu et al., 2015). The distribution of random variables is commonly determined by sampling and fitting distribution models (Li et al., 2011; Jiang et al., 2014). Different sampling methods and distribution models can arrive at considerably different distributions of random variables. Ensuring the randomness of sampling and optimizing the distribution model can make the distribution of random variables closer to the true values and obtain a more reasonable and more accurate result of the reliability index (Gravanis et al., 2014; Li et al., 2014). Closeness to the actual mechanical model is the key to finding the performance function; however, simplified mechanical models are frequently needed because the boundary conditions in rock slope are complicated (Liu and Chen, 2007; Ni et al., 2014). If the simplified mechanical model cannot reflect the stability characteristics of the rock slope as perfectly as possible, it will be difficult to achieve desirable results of the reliability index. Regarding the calculation model for reliability analysis, some integral methods have been shown to be the accurate method to determine the reliability index. However, because of the huge computational work required, an optimized method should be introduced during the reliability analysis process (William et al., 2008; Pourghasemi et al., 2014).

The reasonability and accuracy of the analysis result of the reliability index in rock slope could be improved by optimizing the above three factors. However, it is hard to accurately assess the failure probability of rock slope with the reliability index due to the complicated geological condition and some uncertainty factors. For example, the core of the classical Monte Carlo method is vast samples, but this method usually cannot consider all situations for actual rock slope engineering (Farah et al., 2011; Kourosh et al., 2014). There are some limitations in the sampling in actual rock slope engineering, although some scholars have presented sampling methods to improve the accuracy of the reliability index for rock slope. Because the random variables in rock slope engineering are characterized by discreteness and variability (Ganji and Jowkarshorijeh, 2012; Li et al., 2015a), some methods to calculate the reliability indexes that are fitted in rock slope engineering cannot arrive at the result with considerable accuracy or the reference value in the engineering practice.

Here, the presented method can provide some realistic guidance for the stability evaluation of rock slope, with regard to the random variables in rock slope engineering that have some discreteness and variability. However, the reliability analysis method presented in this paper is still existed some limitations. In this paper, only the randomness of shear strength parameters and the residual tensile strength of the rear edge slip surface are considered, but the randomness and uncertainty of some physical and mechanical parameters are not considered. Furthermore, only the normal distribution is considered for random variables, and the performance functions are derived from the classic limit equilibrium method. These simple assumptions may cause errors for the results of reliability index and failure

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probability. A number of improvements still need to be made in the area of the reliability analysis method in rock slope engineering.

5. CONCLUSIONS

According to the rock slope controlled by weak structural surfaces, two failure modes of plane failure and wedge failure are considered. A second-order second-moment (SOSM) method is presented to calculate the reliability index and the failure probability of rock slope, and performance functions are built up related with the classic limit equilibrium method, which can improve the calculation precision of the reliability index relative to the FOSM method.

Two rock slopes at the Jinping I Hydropower Station are used to verify the reasonability of the presented method. For the plane failure, the results computed using the presented method show that the reliability index of this plane failure is 0.563, and the corresponding failure probability is 28.7%. For the Monte Carlo method, the reliability index of this plane failure is 0.677, and the corresponding failure probability is 24.9%. However, for the FOSM method, the reliability index of this plane failure is −0.025, and the corresponding failure probability is 51.0%. For the wedge failure, the results computed using the presented method show that the reliability index of this wedge failure is 1.250, and the corresponding failure probability is 10.6%. For the Monte Carlo method, the reliability index of this plane failure is 1.305, and the corresponding failure probability is 9.6%. Meanwhile, for the FOSM method, the reliability index of this plane failure is 1.106, and the corresponding failure probability is 13.4%. For both plane failure and wedge failure, the difference between the presented method and the Monte Carlo method is very small, but the failure probability of plane failure determined by FOSM method is larger than that of the other two methods. The above analysis results indicated that, the presented method is a reasonable choice for the reliability assessment of rock slope which is controlled by the weak structural surfaces.

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