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ORIGINAL RESEARCH

Flatness defects after bridle rolls: a numerical analysis of leveling

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Abstract Leveling is a forming process used in the aluminum industry in order to correct flatness defects and minimize residual stresses in strips thanks to bending under tension. This paper introduces a 3D finite element model to simulate the sheet threading in an elementary part of a leveler called *bridle rolls*. It can compute plastic strains and residual stresses through width and thickness, but predict the deformed strip after springback and potential buckling phenomena as well. The influence of geometric and mechanical parameters (like friction or rolls profile) on final flatness are also investigated. Finally initial defects are taken into account and the model shows how they are modified.

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Keywords Leveling process · Flatness defects · Residual stresses · 3D finite element method · Shell elements

Nomenclature

x	Coordinate in longitudinal direction
y	Coordinate in transversal direction
z	Coordinate in normal direction
R	Radius of rolls (mm)
d	Distance between rolls' centers (mm)
t	Thickness of aluminum strip (mm)
b	Width of aluminum strip (mm)
E	Young's modulus of alumuminum (MPa)
ν	Poisson's ratio of aluminum
σ_0	Yield stress of aluminum (MPa)
IU^{geom}	Geometric flatness defect expressed in *International Units* (IU)
ΔL	Length difference between long and short longitudinal fibers (mm)
L	Mean length of fibers (mm)
A	Amplitude of flatness defect considered as a sine curve (mm)
IU^{pl}	Numerical flatness defect expressed in *International Units* (IU)
ϵ_{xx}^{pl}	Longitudinal plastic strain
$\langle\epsilon_{xx}^{pl}\rangle$	Mean value of longitudinal plastic strain along the strip width
σ_{xx}	Longitudinal stress (MPa)
$S4R$	Linear shell element in *Abaqus*
$S8R$	Quadratic shell element in *Abaqus*

Introduction

Aluminum strip is a product used in various domains like packaging, automotive or aeronautics. Coils of thin metal sheets are produced from a thick ingot by several rolling passes. During this process, stresses involved in the thickness reduction and bending deformation of the work rolls [9] induce heterogeneous plastic strains in the strip width and thickness. After applied tension release or final size cutting, this heterogeneous distribution may lead to flatness defects. These can be of two types:

- The fiber defects like long middle or long edge. For instance, in a strip with a long middle defect, the longitudinal fibers are longer in the central area than near the edge. As all the longitudinal fibers have the same length, there are compressive stresses in the middle whereas edges are in tension. Thus waves may appear in the compressive zone due to buckling phenomenon (it is well known that a compressive membrane stress induces buckling [17]).
- The curvature defects like coil set or crossbow. This kind of defect is due to a length difference through the thickness between either the longitudinal fibers for coil set or the transversal fibers for crossbow (Fig. 1).

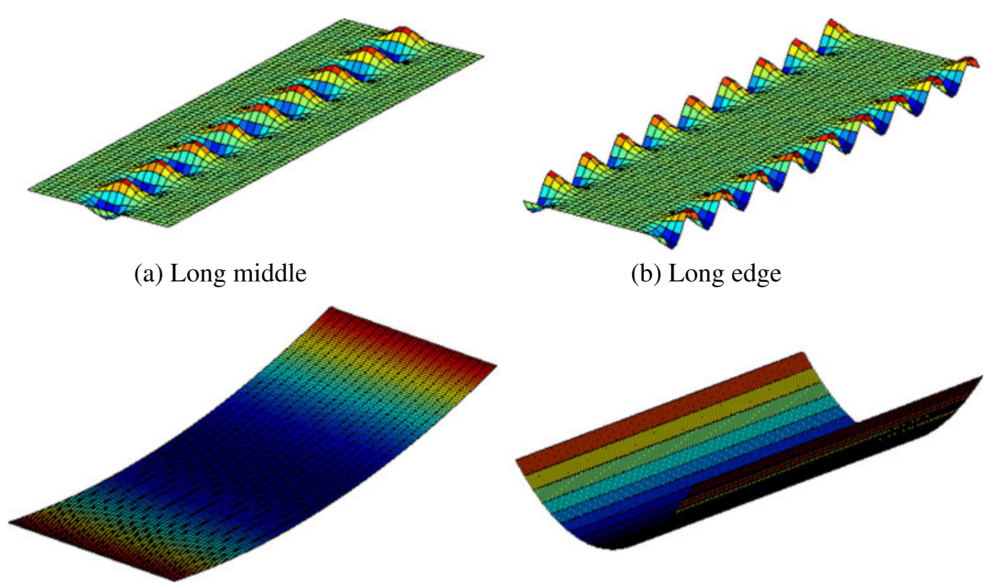
However customers have more and more stringent flatness requirements. Therefore industrialists have to add a leveling operation after rolling to satisfy their criteria. Levelers are constituted by a connection of rolls with different diameters which cancel the undesirable flatness defects thanks to tension (stretcher), bending (*Alligator* multi-roll leveler) or a combination of both (tension leveler) (Fig. 2).

The leveling process consists of plastically elongating the strip in order to bring all the longitudinal and transversal fibers to the same length [16]. Consequently the manifested flatness defects are corrected and the residual stresses through width and thickness are reduced.

Several papers studied the leveling process with analytical or numerical approaches to understand the different parameters influence on flatness and residual stresses.

Doege et al. [5] carried out an analysis of the leveling process based upon an analytical forming model. A one-dimensional model has been developed and the curved metal strip was considered as a beam. They used the bending theory to find the optimal adjustments for rolls to get no curvature and low residual stresses. Their model could even be used in a control system because of a very shorter calculation time than is possible with the finite element method. Industrialists were also interested in leveling understanding to build adjustment tables up for their machines thanks to an analytical model elaborated by Bourgon et al. [3] and Gevers et al. [8]. Strip has been discretized through thickness and width has been cut in a certain number of slices. The authors computed the thickness plastification rate at passing on the rolls, longitudinal and transversal stresses through width and thickness, and also the residual curvatures. Morris et al. [13] conducted experiments

Fig. 1 Flatness defects



(a) Long middle (b) Long edge

(c) Coil set (d) Crossbow

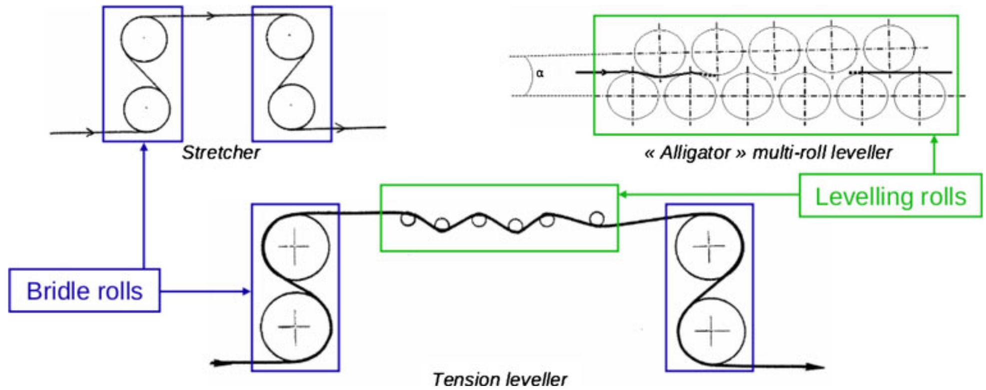


Fig. 2 Three types of levelers

to evaluate the importance of leveling parameters on flatness. The wrap angle on the next to last roll was found to be the most influent, then yield stress and elongation. Yoshida and Urabe [19] performed a computer aided process design for the tension leveling with a finite element analysis taking into account cyclic elasto-plasticity and Bauschinger effect in their own code. They needed less computation time than conventional codes because they did not have to search the contact/detouch with many iterations. They were able to study residual curvature according to roll intermesh. Jamshidian et al. [11] made a theoretical analysis of the process with an incremental theory of plasticity, a combined hardening to evaluate the influence of work curvature and tension. They found that both coil set and crossbow could not be removed simultaneously with the same adjustments. Moreover tension and strip thickness had only a slight effect on residual curvatures compared to bending curvature. On the other hand, Huh et al. [10] used a finite element method to simulate tension leveling process in 2D and to choose the design parameters like tension, roll intermesh, roll pitch, number and diameter of rolls. In the same way Trull [18] developed a 3D finite element model with a strip discretization by linear shell elements. He could obtain plastic strains and residual stresses distributions through thickness and width and observed the influence of profiled rolls or an initial flatness defect. Moreover springback of the slitted metal strip was analyzed.

On the other hand some authors have studied the link between a residual stresses distribution through the strip width and the shape defects. Bush et al. [4] have solved the plate stability equation assuming the sheet deflection as a sine function with separated variables and cutting the width in slices where stresses were homogeneous. Rammerstorfer et al. [15] and Fischer et al. [6] have also discussed this point minimizing the buckling deformation energy with equally a sinusoidal assumption for the strip deflection, or a polynomial one. In particular, Fischer et al. [7] have considered a strip in a pure tension leveler, between two *bridle rolls*, where stretching buckling can be observed. They have predicted the strip shape under tension by solving eigenvalue problems with a Ritz approach. Abdelkhalek et al. [2] have used an Asymptotic Numerical Method to solve the problem equations. Both have predicted the strip deformation according to the residual stresses after rolling.

Thus, Trull [18] is the only reference found in the open literature which treats about a three-dimensional modeling of tension leveling with a flatness study. We propose to focus on a different configuration in which friction between strip and rolls is very important. The aim of the present work is to establish a numerical modeling of leveling in order to evaluate plastic strain and residual stress distributions through the strip width and thickness. Also prediction of the strip flatness at the end of the process according to the different geometric and mechanical parameters has been assessed. To fulfill this purpose, a 3D finite element model has been developed with the commercial software Abaqus [1] and tested on a simple configuration with two rolls called *bridle rolls* (Fig. 2).

Finite element model

Description

The *bridle rolls* configuration is composed of two cylindrical rolls of large diameters (Fig. 3). It allows an increase or a decrease of the strip tension considering contact with friction. So it can be used either in the entrance or in the exit of the machine. The rolls were supposed to be undeformable solids and consequently were modeled as analytical rigid surfaces. Initially the strip was considered as perfectly flat and free of residual stress resulting from a previous forming process. In

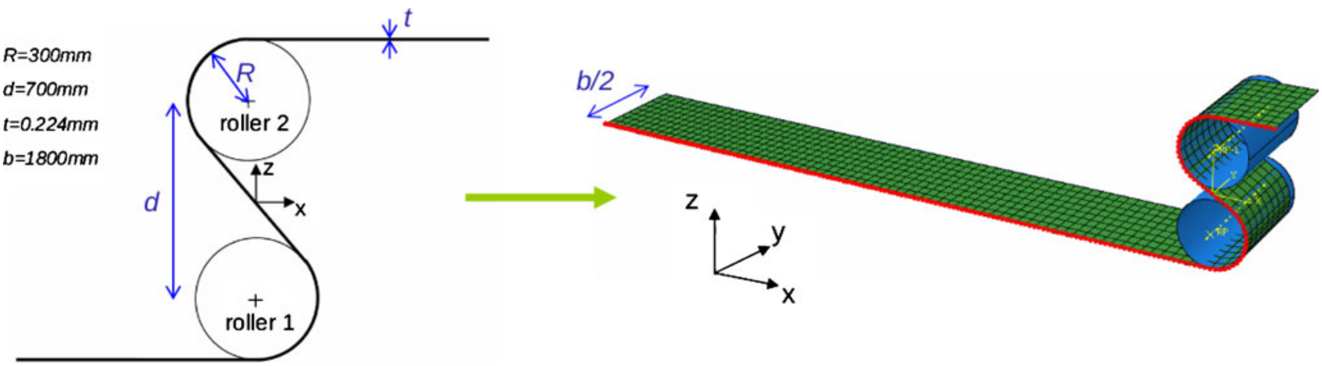


Fig. 3 *Bridle rolls* configuration

a following section, an initial geometric defect will be taken into account.

Due to the aspect ratio of the studied strip ($\frac{\text{width}}{\text{thickness}} >$ 8,000) and the aim to take into account buckling phenomenon after springback, the thin aluminum strip was discretized using shell elements. As well the Simpson integration was adopted with a high number of integration points (15 points) so as to accurately describe the stress and the strain distributions through thickness. Moreover assuming there were no misalignment of the rolls or any other asymmetry in geometry or loads, only half-width of the strip was modeled.

Aluminum was considered with an elastic-plastic behavior($E =$ 70,000 MPa, $\nu = 0.33$, $\sigma_y =$ 235 MPa). Isotropic and non-linear kinematic hardening [12] were compared and results were very close (relative difference <5%) due to major elongation being performed by tension. For a problem with many alternate bendings, like in multi-roll leveling, a greater gap would be noticed and non-linear kinematic hardening would be appropriate. So isotropic hardening was chosen in this study and the hardening law was computed with a piecewise linear function deduced from a traction test (see Fig. 4).

Because of large displacements and large rotations, geometrical nonlinearities were taken into account. The penalty method was used for modeling the normal contact between the rolls and the strip whereas the basic Coulomb friction model was introduced for the tangential contact.

The analysis was performed using three main steps. In the first step, a downstream tension was applied (beyond the yield stress) which represents the machine tension, balanced by an upstream tension (from strip conveying, some tens of *MegaPascal*) and friction, while the rolls were locked in translation and rotation. During the second step, the rotation of the rolls was initiated to ensure the strip conveying (6,000 mm) by friction. In the third and last step, the upstream and downstream applied tensions were released to observe the deformed strip after springback and potential buckling.

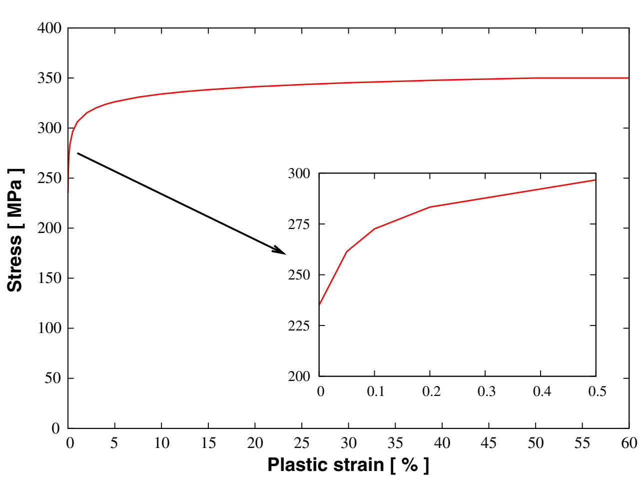


Fig. 4 Hardening law described as a piecewise linear function

Some measurements of the flatness defects

At the end of the simulation, the distributions of the longitudinal plastic strain through width and thickness are plotted as well as the residual longitudinal stress.

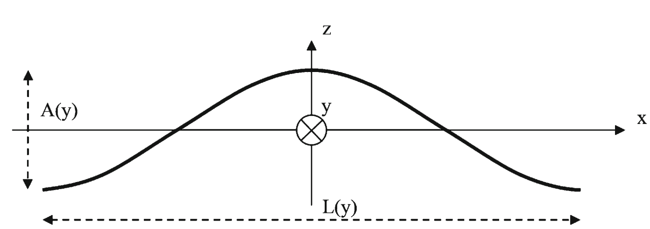


Fig. 5 Flatness defect considered as a sine curve

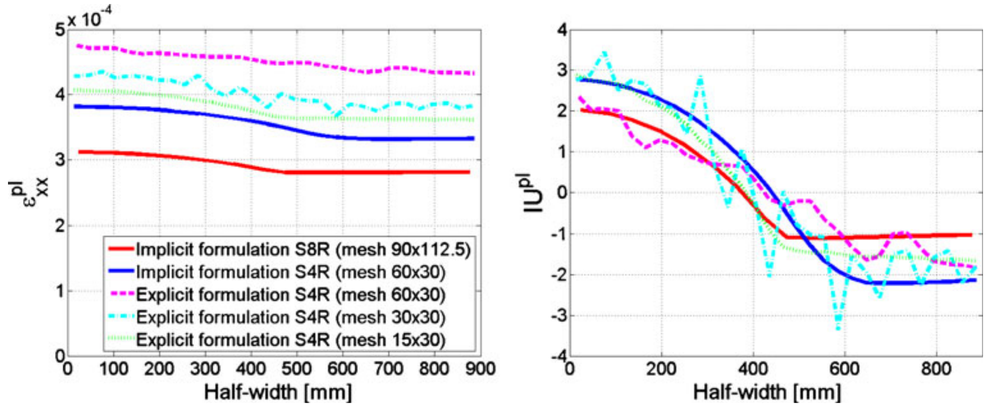


Fig. 6 Strain distribution after *bridle rolls*, Abaqus/Standard vs. Abaqus/Explicit

(a) Plastic strain through strip width

(b) Flatness in International Units

In metal industry, a measurement has been established to quantify the strip flatness in International Units according to the following formula 1:

$$IU^{\text{geom}}(y) = \frac{\Delta L(y)}{L(y)} \times 10^5 \tag{1}$$

where ΔL is the length difference between the long and the short longitudinal fibers and L the mean length of the fibers (the strip length). For industrial issues, flatness inferior to a few of International Units is recommended ($\leq 2-3IU$ for aluminum and $\leq 10IU$ for steel).

As an illustration, if the flatness defect is considered as a sine curve (Fig. 5), its measurement in International Units can be calculated with its amplitude A and its wavelength L according to formula 3:

$$z(x;y) = \frac{A(y)}{2} cos\left(\frac{2\pi x}{L(y)}\right) \tag{2}$$

$$IU^{\text{geom}}(y) = \frac{\pi^2}{4}\left(\frac{A(y)}{L(y)}\right)^2 \times 10^5 \tag{3}$$

In the following sections, predictions of the strip flatness will be presented in International Units (IU) in the sense of:

$$IU^{pl}(y) = \left(\epsilon_{xx}^{pl}(y) - \langle\epsilon_{xx}^{pl}\rangle\right) \times 10^5 \tag{4}$$

where $\langle\epsilon_{xx}^{pl}\rangle$ is the mean value of the longitudinal plastic strain along the strip width (y direction).

Then

$$IU^{pl} = \max_{y\in[0;\frac{b}{2}]} IU^{pl}(y) \tag{5}$$

because the long fibers especially cause flatness defect by buckling and the focus is only put on the positive value of $IU^{pl}(y)$.

Implicit formulation versus explicit formulation

A lot of forming processes are simulated with an explicit formulation because of the advantage of a relatively small computation time. But the solution is not sure to converge toward a real and physical solution, contrary to an implicit formulation.

Abaqus offers the possibility to use either an implicit formulation (Abaqus/Standard) or an explicit one (Abaqus/Explicit) [1]. Our model is tested with these two formulations. For the latter a process velocity has to be defined and a conveying speed of 450 m.min^{-1} is elected (levelers speed is between 100 m.min^{-1} and 600 m.min^{-1}). Only shell elements with linear interpolation are available in Abaqus/Explicit.

With an explicit formulation, the two coarsest meshes (8,800 elements—60 × 30 mm and 17,880

Table 1 Computation data, Abaqus/Standard vs. Abaqus/Explicit

Formulation	Element type	Mesh size (mm)	Increments	Computation time (s)
Implicit	Quadratic shell	90 × 112.5	759	13,074
Implicit	Linear shell	60 × 30	677	9,822
Explicit	Linear shell	60 × 30	162,661	4,551
Explicit	Linear shell	30 × 30	168,277	8,306
Explicit	Linear shell	15 × 30	323,542	50,566

Table 2 Parallelization data with an implicit formulation

Processors	CPU time (s)	Computation time (s)
1	12,328	13,074
2	15,641	10,572
4	16,299	8,096
8	16,985	6,166

elements—30 × 30 mm) bring some disrupted results for flatness, with a lot of oscillations from one element to an other. The finest mesh (35,640 elements—15 × 30 mm) provides more stabilized results (Fig. 6). Nevertheless, as there is no solution correction with the explicit formulation, Abaqus/Explicit documentation advocates not to exceed 300,000 increments, which is not the case with this mesh (Table 1). Also computation time is three times superior compared to the one obtained with an implicit formulation.

On the other hand, the implicit formulation of the finite element software needs fewer shell elements to reach the same accuracy as the explicit one. Moreover, Abaqus/Standard permits the use of quadratic shell elements (*S8R*) which better accommodate the contact between metal sheet and rolls with a regular description of the contact pressure. Also the solving algorithm is based on a prediction-correction procedure. So implicit formulation will be used for the next numerical simulations of the proposed study.

As well parallelization helped to decrease the computation time using several processors (Table 2) and only one hundred minutes were necessary to simulate the strip threading under tension and the springback with Abaqus/Standard and eight processors.

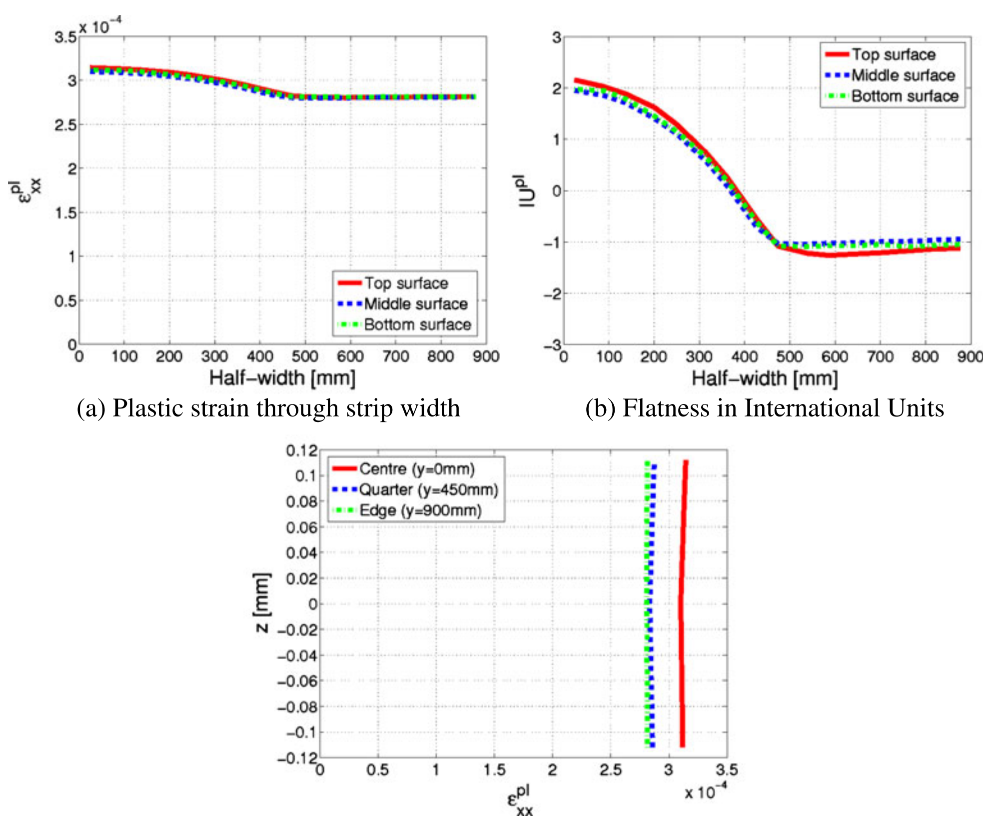
Simulations with a perfect initial strip

Basic mechanisms

Attention was paid to plastic strain and residual stress distributions in a transversal section of elements having passed through the two rolls.

Figure 7a and c show that the longitudinal plastic strain increases approaching the strip center whereas it is almost constant in the thickness direction. That is to say that the longitudinal fibers are more plastically

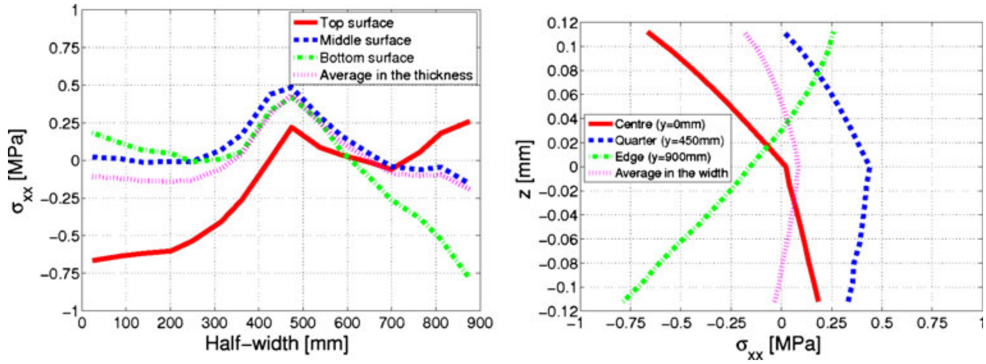
Fig. 7 Strain distribution after *bridle rolls*



(a) Plastic strain through strip width

(b) Flatness in International Units

(c) Plastic strain through strip thickness



(a) Residual stress through strip width (b) Residual stress through strip thickness

Fig. 8 Stress distribution after *bridle rolls*

elongated in the middle than near the edge and that stretching is more important than bending for deformation. The *bridle rolls* configuration seems to produce a long middle defect in a strip which is initially perfectly flat. Even if the amplitude of the defect is very low ($2IU$, Fig. 7b) in our example, this phenomenon is sometimes observed in pure tension lines [14].

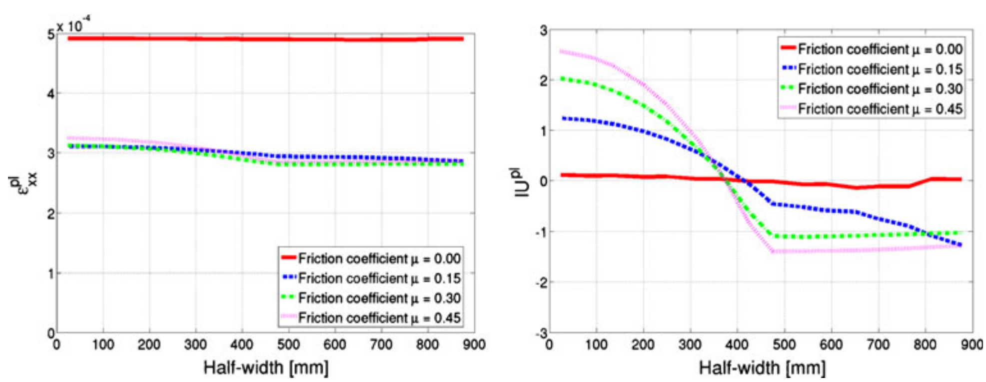
The residual stress distribution through the strip width (Fig. 8a) shows there are two areas, in the middle and next to the edge, of compressive stresses where buckling phenomenon may appear. The distribution through the strip thickness (Fig. 8b) confirms this point and precises that in the center of the strip, the top surface is in compression whereas in the edge, the bottom surface is in compression.

Therefore the *bridle rolls* configuration introduces more plasticity in the longitudinal fibers in the center than in the edge. This difference can be explained taking into account friction in the tangential contact between the aluminum strip and the rolls. The more the friction coefficient increases the more the produced long middle defect amplitude grows. In ideal conditions with no friction the plastic strain is homogeneous in the width direction (Fig. 9) and no flatness defect is generated.

Influence of the rolls profile

An important parameter in forming processes like rolling or leveling is the rolls deformation during strip conveying. Even if rolls are modeled as rigid bodies this phenomenon can be taken into account by considering no flat rolls. Three profiles of rolls—flat, convex shape and concave shape—were tested with our finite element model. For the convex and concave shape rolls, a parabolic profile was adopted in the transversal direction with a radius difference equal to 1 mm between the center and the edge.

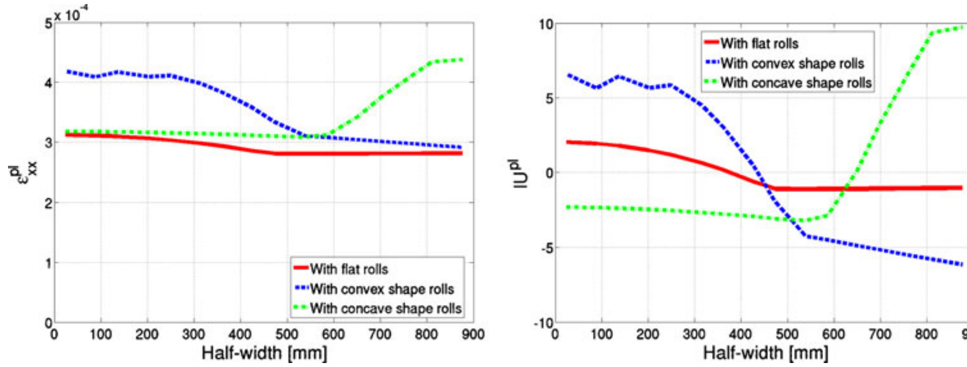
As seen in Fig. 10, the long middle defect naturally produced by a perfect *bridle rolls* configuration is accentuated by the presence of convex shape rolls. In the center of the strip ϵ_{xx}^{pl} is higher than with flat rolls whereas near the edge results are almost identical.



(a) Plastic strain through strip width (b) Flatness in International Units

Fig. 9 Strain distribution according to friction coefficient

Fig. 10 Plastic strain distribution according to rolls profile



(a) Plastic strain through strip width (b) Flatness in International Units

Consequently, the flatness defect is multiplied by 3 ($6.5IU$ against $2IU$ previously) by the change of rolls profile.

On the contrary, with concave shape rolls, the plastic strain distribution is reversed and ϵ_{xx}^{pl} increases toward the edges, that is to say that the longitudinal fibers are more plastically elongated than in the middle. In terms of flatness, a long edge defect of $10IU$ is generated by concave shape rolls.

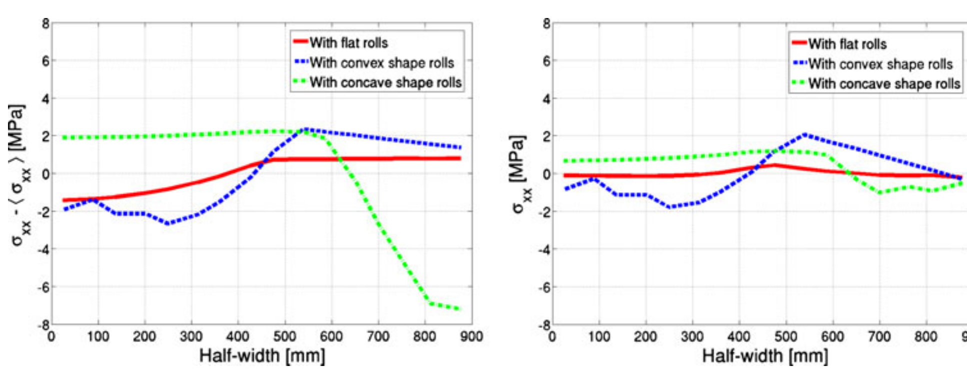
To confirm this, the residual stress distribution (Fig. 11b) shows that compressive stresses in the central area are increased with convex shape rolls in comparison with flat rolls. In contrast, concave shape rolls make negative residual stresses move in the edge. Moreover Fig. 11 permits to understand how buckling decreased the stresses difference in the width with strip shape changes. The residual stress is small (less then 2 MPa with the convex shape rolls, less than 1 MPa with the concave shape rolls), as compared with the stress difference before buckling that can exceed −6 MPa.

Consequently, rolls profile may be used to correct some flatness defects by adapting their geometry—convex or concave profile—to the entering defect and plastically elongating the appropriate short longitudinal fibers. In practice inflatable rolls may be used [14]. Thanks to hydraulic pressure increase, the roll shape can be either concave, cylindrical or convex.

These conclusions are supported by the strip shape at the exit of the *bridle rolls*. Long edge defect produced by concave shape rolls at the end of the simulation is clearly observable in Fig. 12c. Waves are seen on the edge because of buckling of the fibers in compression (Fig. 11b). On the other hand, the long middle defects produced by flat and convex shape rolls are less easily identifiable. A large pocket appears in the central area, whose amplitude and wavelength increase with the roll crown.

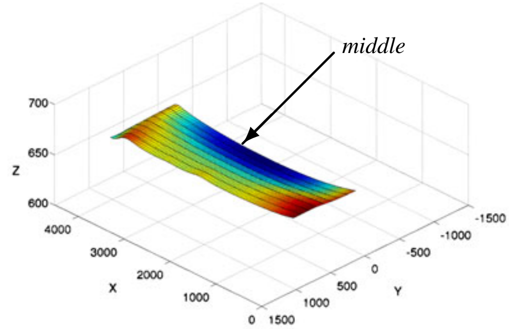
In the case of *bridle rolls* with concave shape rolls which produces a long edge, a comparison is possible between the flatness calculated with the longitudinal plastic strain distribution through width, using

Fig. 11 Stress distribution according to rolls profile

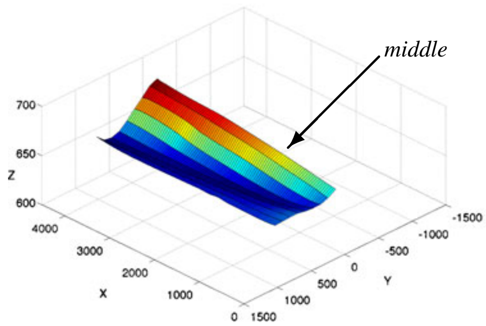


(a) Stress through strip width before relaxation (b) Residual stress through strip width

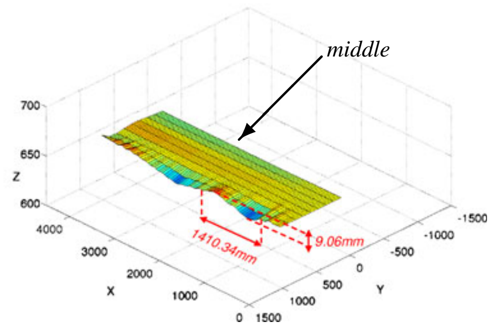
Fig. 12 Strip deformation according to rolls profile



(a) Strip deformation after flat rolls

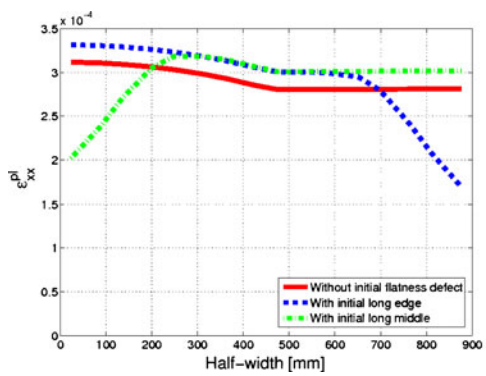


(b) Strip deformation after convex shape rolls

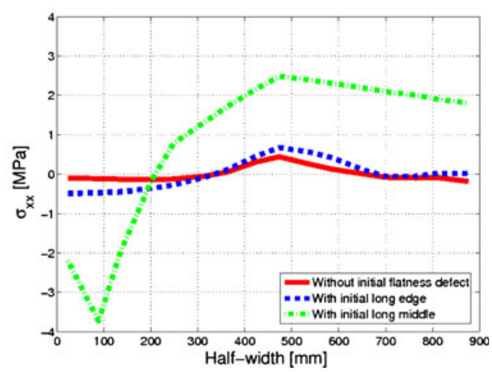


(c) Strip deformation after concave shape rolls

Fig. 13 Plastic strain and residual stress distribution with an initial flatness defect

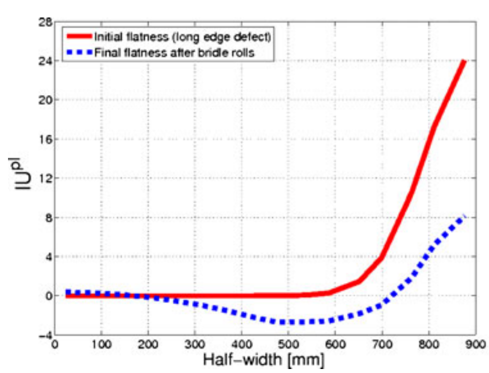


(a) Plastic strain through strip width

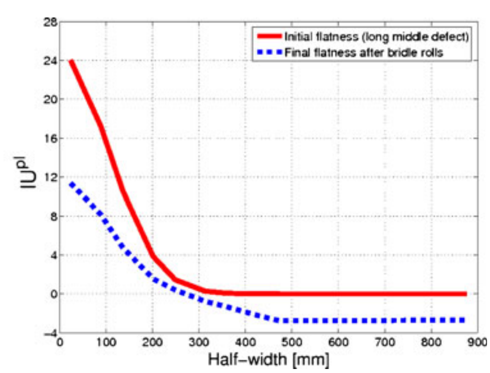


(b) Residual stress through strip width

Fig. 14 Comparison of initial and final flatness

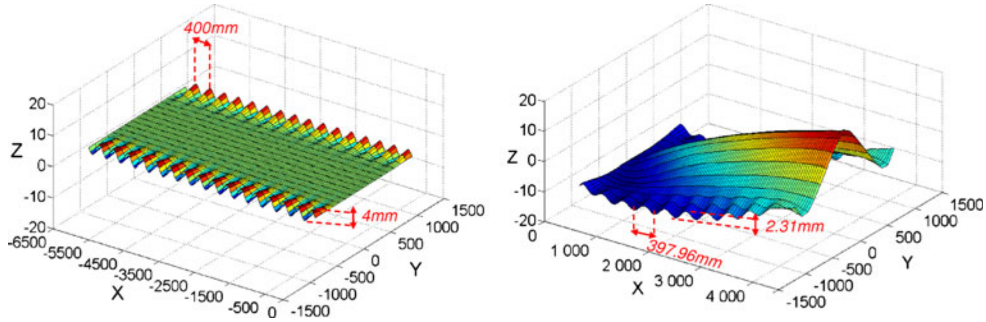


(a) Flatness of a strip with initial long edge



(b) Flatness of a strip with initial long middle

Fig. 15 Strip with initial long edge before and after *bridle rolls*



(a) Strip with initial long edge (b) Strip deformation after *bridle rolls*

formulae 4, 5 and on the other side, the manifested flatness seen in Fig. 12c:

- in Fig. 10b flatness issued from plastic strain distribution is $IU^{pl} = 9.74IU$,
- interpolating flatness defect by means of a sine curve (amplitude $A = 9.06$ and wavelength $L = 1410.34$) and applying formula 3, strip flatness is calculated

$$IU^{\text{geom}} = \frac{\pi^2}{4}\left(\frac{A}{L}\right)^2 \times 10^5 = 10.18IU$$

So the simulation of *bridle rolls* configuration with concave shape rolls provides relatively consistent results between the plastic strain distribution in strip width and the manifested flatness, observable on the deformed geometry. In the case of long middle defects produced by flat or convex shape rolls, this approach is not applicable and the detected central pocket seems to be a global buckling phenomenon, not a local wave.

Simulations with an initial flatness defect

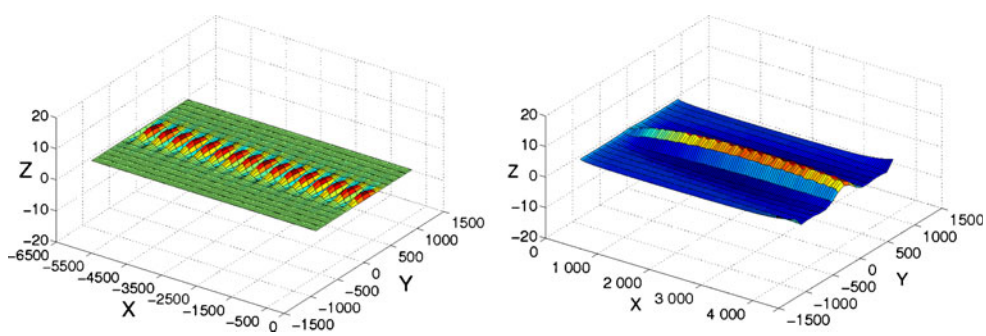
The objective of the leveling process being to correct flatness defects, our finite element model was applied to a strip presenting initial defects. Flatness geometric defects were introduced whereas the incoming strip was free of stress. Some analytical expressions of geometric fibers defects were chosen under the following form:

$$z(x, y) = -\left(\frac{d}{2} + R + \frac{t}{2}\right) + \frac{A}{2}\sin\left(2\pi\frac{x}{L} + \pi\right) \times \frac{1}{2}\left[1 + \cos(\pi Y)\right] \times \exp\left(-16Y^2\right)$$

where $Y = \frac{2y}{b}$ for long middle, $Y = 1 - \frac{2y}{b}$ for long edge, $A = 4$ mm is the amplitude of the flatness defect and $L = 400$ mm its wavelength. According to formula 3, initial defects equal $24.67IU$.

Figure 13a emphasizes that longest fibers of a strip with an initial geometric flatness defect are less stretched out than the shortest ones, going through the

Fig. 16 Strip with initial long middle before and after *bridle rolls*



(a) Strip with initial long middle (b) Strip deformation after *bridle rolls* *bridle rolls* configuration. It seems that the initial length difference between fibers in strip width is reduced.

In terms of flatness, initial and final ones are compared (Fig. 14). Unlike to the previous sections, the translation of plastic strain in flatness in International Units needs here to take the initial length of each longitudinal fibers in the width into account to apply formula 1.

It can be noticed in Fig. 14 that the initial flatness defects were well corrected by *bridle rolls* configuration. As the long fibers of the strip are less stretched, there is a rebalancing of the fibers length. The length difference decreases as well as the strip flatness. From an initial long edge of $24.67IU$, strip flatness is improved with a final long edge of $8.18IU$. Also initial long middle of $24.67IU$ is reduced to $11.36IU$. As the studied configuration is only a part of a real leveler, it cannot perfectly correct the initial flatness defects of strips but it is able to substantially reduce the length differences in transversal direction.

The observation of the deformed strip after the two opposite bendings in *bridle rolls* confirms these statements (Figs. 15 and 16).

The waves of the initial long edge are reduced (amplitude decreased while wavelength was not modified). A comparison can also be made for flatness prediction from plastic strain distribution and from deformed strip, $IU^{pl} = 8.18IU$ and $IU^{\mathrm{geom}} = 8.33IU$. So plastic strain distribution through the strip width is well translated in a geometric flatness defect.

Conclusion

This paper presents a finite element simulation of the leveling process (especially *bridle rolls* configuration) used in the metal industry to correct flatness defects. It is based on an implicit formulation, a discretization of the strip by quadratic shell elements and a threading of enough aluminum metal to reach a steady-state regime.

A simple configuration of two rolls has been studied. The model is able to accurately compute the plastic strains and the residual stresses distribution through width and thickness. Buckling phenomena after springback is also predicted. Furthermore initial geometric defects can be introduced in the incoming sheet.

The analysis has shown that the *bridle rolls* configuration naturally produces a long middle defect in an initially flat strip, due to the friction between strip and rolls. A solution (which is effectively used in practice) to correct more efficiently flatness defects is to modify the roll profile and to adapt their shape in order to lengthen the short longitudinal fibers. The computations have shown that some defects may be reduced in this way.

The study of the influence of those geometric and mechanical parameters on final flatness can help finding optimal adjustments for levelers in order to improve flatness and to considerably decrease residual stresses.

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