

The Vegter Lite material model: simplifying advanced material modelling

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Abstract The need for cost reduction and increased fuel efficiency in the automotive industry has led to an increased importance of accurate simulations and the need for more advanced material models to achieve highly reliable forming and springback predictions. Conventional material models implemented in FE-codes are in general not capable of describing the plastic material behaviour during monotonic strain paths with sufficient accuracy. This applies to the strain hardening model, the influence of strain rate and the description of the yield locus in these models. This led to the development of the Vegter yield criterion. For the determination of parameters, mechanical tests at different strain paths (uniaxial, plane strain, equibiaxial and shear) are required. Until now the high accuracy of the Vegter yield criterion was only available if all parameters were determined. However, due to the limited availability of the large number of tests methods needed for this material model not all parameters are always determined. For maintaining high accuracy with a reduced set of parameters, the Vegter Lite model was developed.

Keywords Material modelling · Yield locus · Simulation

Introduction

The need for cost reduction and increased fuel efficiency in the automotive industry has led to an increased

importance of accurate simulations and the need for more advanced material models [1–4]. Conventional material models implemented in FE-codes are in general not capable of describing the plastic material behaviour with sufficient accuracy. Therefore, the Vegter yield criterion was developed to obtain a high accuracy combined with a simple mathematical description and a large flexibility. For this model, no complex parameter determination is required; however, the number of tests required for the determination of parameters is relatively high. In order to reduce the number of parameters in the model, a new variant was developed, the Vegter Lite model. Where the original model uses experiments from nine tests, the Lite version only uses four tests of which three are tensile tests. As the Lite version does not take as much experimental data into account, it is not expected to have the ultimate accuracy of the original model, but it provides better results compared to conventional models given the same test data. This article explains the differences between the original model and the Lite version. Moreover, a method for the determination of the model parameters from the mechanical tests is explained. As a validation case, differences in the simulation results for a semi-industrial product are shown using various material models. In order to explain where the Vegter Lite model is different from the original version, first a brief description of the original model is given.

Description of the Vegter Yield criterion

The Vegter yield criterion [2, 5] is based on the measurement of the uni-axial, plane strain, shear and equi-biaxial points, as shown in the yield locus at 0° in Fig. 1.

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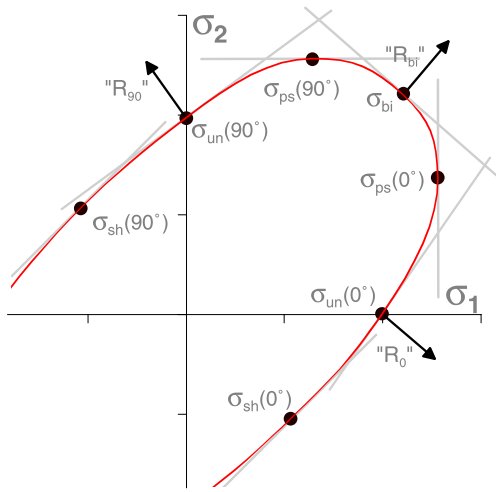


Fig. 1 Vegter Yield Locus: Bezier interpolation between points

Beside the stress values in these points, also the strain vectors are taken into account to construct the tangent to the yield locus. Between the reference points, a 2nd order Bezier interpolation is used where the hinge points are defined by the tangents of the two reference points in question. The Bezier interpolation function is described in Eq. 1.

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = (1 - \lambda)^2 \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^r + 2 \cdot \lambda \cdot (1 - \lambda) \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^h + \lambda^2 \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_{i+1}^r \quad (1)$$

Where λ is the parameter for Bezier interpolation, subscript ‘i’ refers to the first reference point, superscripts ‘r’ and ‘h’ refer to a reference point and hinge point, respectively.

By measuring the reference points at different angle to the rolling direction (e.g. 0°, 45° and 90°), a fully anisotropic yield locus for sheet metal forming is determined. In order to obtain the yield loci at arbitrary angles to

Fig. 2 Vegter Lite Model, NURB interpolation using weight factors

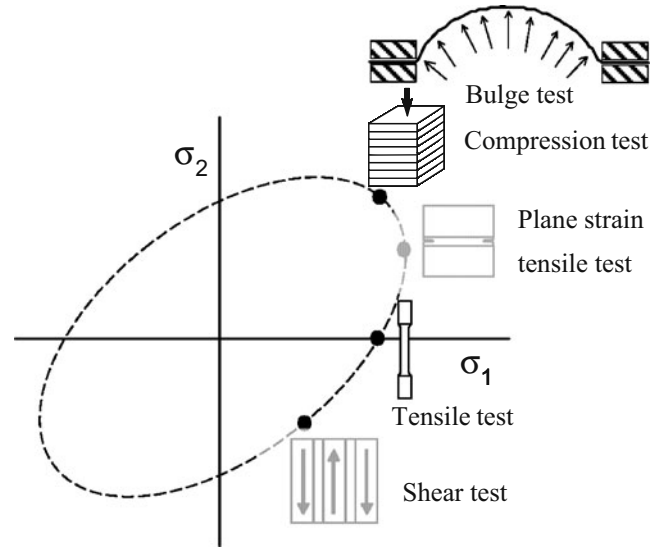
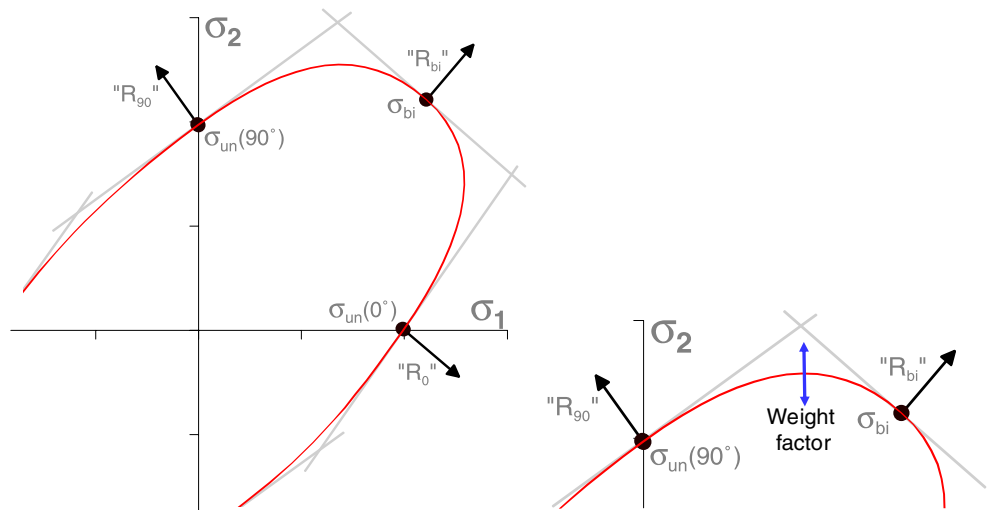


Fig. 3 Mechanical test for deriving the material data for the Vegter model; Light gray test are skipped for deriving data for the Vegter Lite criterion

the rolling direction, a cosine interpolation of the reference points and the strain vectors is used.

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^r = \sum_{j=0}^{m \cos} \begin{pmatrix} a_1^j \\ a_2^j \end{pmatrix}_i^r \cdot \cos(2 \cdot j \cdot \varphi) \quad (2)$$

$$\rho_i(\varphi) = \sum_{j=0}^{m \cos} b_i^j \cdot \cos(2 \cdot j \cdot \varphi)$$

Where $m \cos$ is the number of cosine terms used for the expansion of the reference points and/or strain vectors, a_1^j and a_2^j are parameters for the cosine interpolation to be determined for the reference points, φ is the angle between the principal axes of plane stress and the principal axes of anisotropy, $\rho_i (= \delta \epsilon_2 / \delta \epsilon_1)$ is the strain vector in the reference points and b_i^j are parameters

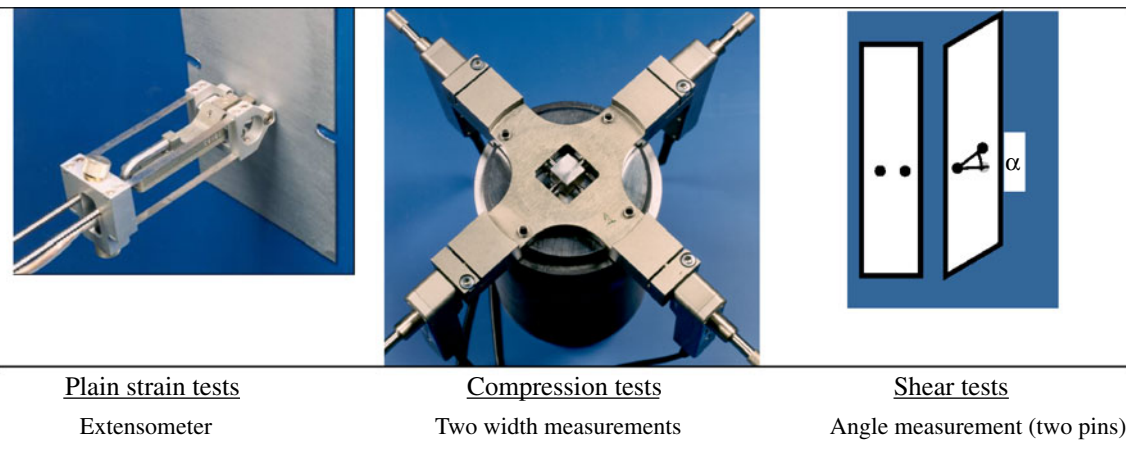


Fig. 4 Principle of strain measurements for the different test methods

for the cosine interpolation to be determined for the strain vectors.

The standard value of the parameter m_{cos} has been set to a value of 2 meaning that data is available for three different angles to the rolling direction commonly at every 45°; this is usually indicated as four earing anisotropy.

The reference points were chosen to have either a fixed stress state or a fixed strain state: the uniaxial and equi-biaxial points have a fixed stress state while the plane strain and shear point have a fixed strain state. This choice resulted in that two or fewer unknowns have to be determined in these points [5]:

- pure shear: $\rho = -1$, the pure shear stresses are needed; for this point symmetry exists for the angles φ and $\varphi + 90^\circ$.
- uniaxial tension: $\sigma_2 = 0$, the uniaxial stress and the R-value are needed; the latter provides the strain vector.
- plane strain: $\rho = 0$, the two plane strain stress components are both required.
- equi-biaxial: $\sigma_2 = \sigma_1$, the equi-biaxial stress and the strain vector are needed which are valid for every direction.

Therefore, in case of a four earing anisotropy this yield criterion needs 17 parameters to be determined by performing 9 different mechanical tests.

Description of the Vegter Lite model

A simplified version is proposed with fewer parameters than in the original version, the Vegter Lite model. A lower amount of reference points is used and the number of parameters is reduced from seventeen to eight. The minimum number of tests required is four; i.e. tensile tests in three directions and an equi-biaxial test (Fig. 3). In this yield criterion only the uni-axial and equi-bi-axial points are used as reference points. In the Vegter Lite model the 2nd order Bezier interpolation is replaced by the 2nd order Nurb

interpolation (Fig. 2). The only difference is the introduction of a weight factor that controls the position of the curve in between the reference points.

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \frac{(1 - \lambda)^2 \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^r + w \cdot 2 \cdot \lambda \cdot (1 - \lambda) \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^h + \lambda^2 \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_{i+1}^r}{(1 - \lambda)^2 + w \cdot 2 \cdot \lambda \cdot (1 - \lambda) + \lambda^2} \tag{3}$$

By variation of this weight factor the position of the plane strain points and pure shear points can be controlled as indicated in Fig. 2. In four earing anisotropy five weight factors have to be known in this yield criterion:

- three weight factors in the plane strain region, w_{ps} , (see Fig. 2) at 0°, 45° and 90°.
- two weight factors in the pure shear region, w_{sh} , at 0°/90° and 45°; for reasons of symmetry of tension and compression in pure shear test w_{sh} , at 0° and 90° have the same value.

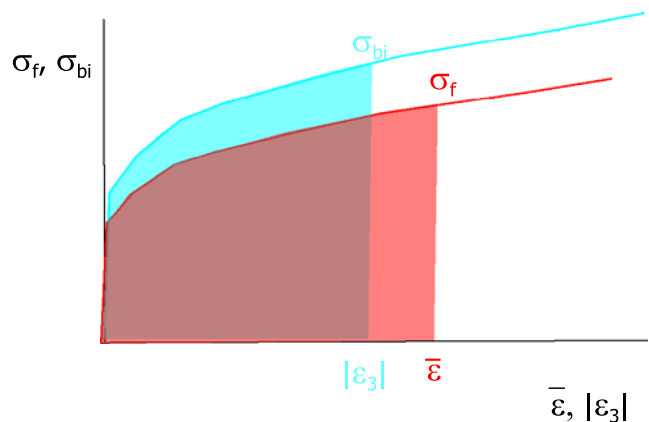


Fig. 5 Looking up points of equal plastic work in the equivalent stress strain curve and the bi-axial stress strain curve

Table 1 The handling of the yield loci data in the tests

Test	reference axis	f_1	f_2	f_3	x_1	x_2	x_3	$\partial\bar{\epsilon}$
Uniaxial	1	$f_{un}(\theta)$	0	0	1	0	0	$f_{un}(\theta) \cdot \partial\epsilon_1$
Plane strain	1	$f_{ps}(\theta)$	n.a. $\rho_2=0$	0	1	n.a.	0	$f_{un}(\theta) \cdot \partial\epsilon_1$
Equi-biaxial or compression	3	f_{bi}	f_{bi}	0	1	1	0	$-f_{bi} \cdot \partial\epsilon_3$
	3	0	0	$-f_{bi}$	0	0	-1	$-f_{bi} \cdot \partial\epsilon_3$
Shear	1	$f_1-f_2=2 \cdot f_{sh}(\theta)$		0	$x_1-x_2=2$		0	$f_{sh}(\theta) \cdot \partial\gamma$, with $\partial\gamma=2 \cdot \partial\epsilon_1$

An advantage with respect to the original Vegter-yield criterion is that values of plane strain and pure shear points are now related to the uni-axial and equi-bi-axial reference points. Based on experience of many tests on different materials fixed values for these weight factors are used for steels and aluminium:

- steels: $w_{ps}=w_{sh}=0.6667$ and
- aluminium: $w_{ps}=0.4125$; $w_{sh}=0.75$

Another possibility is the determination of the value of these weight factors by mechanical tests resulting in five independent weight factors.

The 2nd order Nurb interpolation is used for the determination of the plane strain points and shear points and used as input for the normal Vegter-yield criterion. The 2nd order Bezier interpolation is more simple to handle and more effective compared to the more complex mathematical 2nd order Nurb interpolation.

Workhardening description

Strain hardening is described using an extended Bergström relation [6, 7]. The effect of strain rate and temperature is

included [8]. This results into the following relationship for the flow stress as a function of strain, strain rate and temperature [5]:

$$\sigma_f = \sigma_0 + \Delta\sigma_m \cdot \left[\beta \cdot (\bar{\epsilon} + \epsilon_0) + \left(1 - e^{-\Omega \cdot (\bar{\epsilon} + \epsilon_0)} \right)^{n'} \right] + \sigma_0^* \cdot \left[1 + \frac{k \cdot T}{\Delta G_0} \cdot \ln \left(\frac{\dot{\bar{\epsilon}}}{\dot{\epsilon}_0} \right) \right]^{m'} \quad (4)$$

- σ_0 back stress at zero dislocation density
- $\Delta\sigma_m$ stress increase parameter for strain hardening
- β strain hardening parameter for large strain behaviour
- Ω strain hardening parameter for small strain behaviour
- ϵ_0 pre-deformation parameter
- n' exponent for the strain hardening behaviour
- σ_0^* dynamic stress at zero thermal activation
- ΔG_0 maximum activation enthalpy=0.8 eV
- m' power for the strain rate behaviour
- k Boltzmann-constant=8.617.10⁻⁵ eV/K
- T absolute temperature in (K)
- $\dot{\epsilon}_0$ limit strain rate for thermally activated movement=10⁸ s⁻¹

Fig. 6 An example of the calculation, including extrapolation of the hardening curve

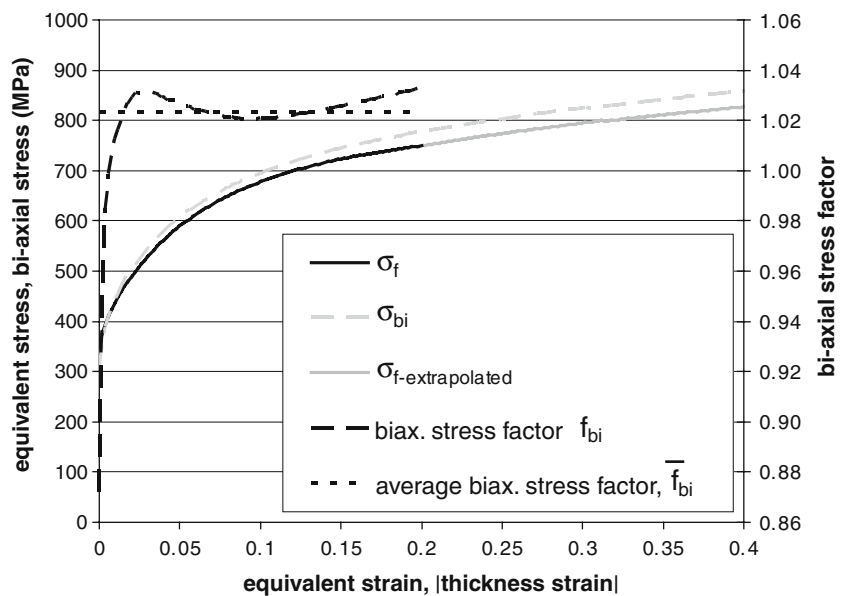
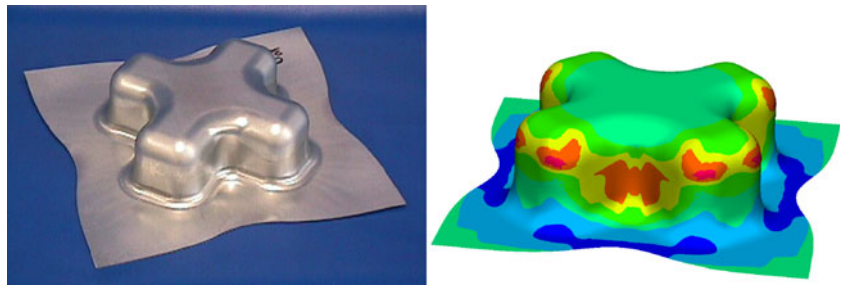


Fig. 7 The Cross-die part and an example of the calculated rupture rise



This model is applied on the tensile test in the rolling direction. The size of the yield locus as a function of strain is prescribed using this reference state assuming isotropic hardening

Determination of material parameters

Mechanical testing

For the determination of the plastic material behaviour, tests are carried out at different strain paths. Four key conditions are important to cover different conditions in sheet metal forming: uni-axial tension, plane strain tension, equi-biaxial tension and pure shear [2, 5] as shown in Fig. 4. At Tata Steel the on plane compression tests is used for determination of the equi-biaxial point of the yield locus which can be made on the same tensile test machine as all other tests [9]. Alternatively, a bulge test can be used for the determination of the equi-biaxial point but in that case an additional testing device has to be applied. In all tests data has to be transformed to true stress—true strain data. The tensile test is the only test where both the experimental procedure and the determination of true stress—true strain curves are standardised according to ISO standards:

- *ISO 6892*—Metallic materials—Tensile testing at ambient temperature: i.e. the experimental procedure is prescribed

- *ISO 10275*, Metallic materials—Sheet and strip—Determination of tensile strain hardening exponent: The procedure for the determination of true plastic strain and true plastic stress is prescribed.
- *ISO 10113*—Metallic materials—Sheet and strip—Determination of plastic strain ratio i.e. the determination of the r-values is prescribed

For the other tests in house procedures are developed [9–12]. For the bulgetest the VDEh has initiated a working group on development of a procedure for a new ISO-standard.

Calculation of the material parameters

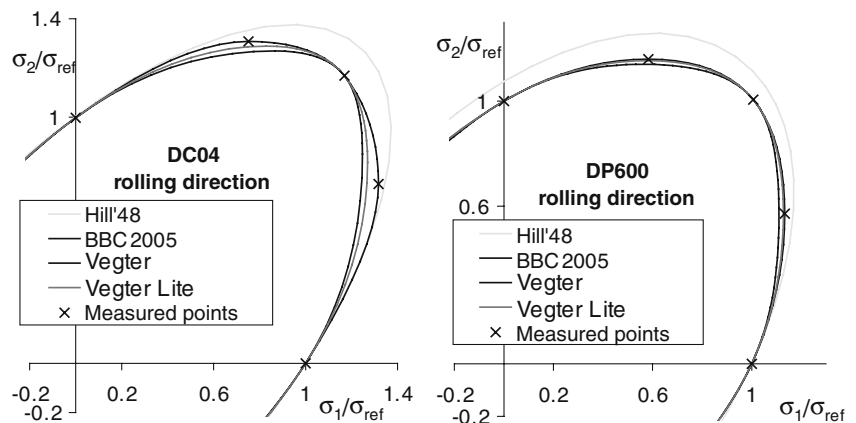
Proportional strain paths and loading conditions are used for the derivation of the constants of the material model (i.e. yield locus and hardening behaviour).

Usually, the strain hardening relation is obtained by fitting of uni-axial tensile test data in RD. This is used as a reference for the determination of the yield locus parameters. By comparing the complete curves of the stress strain data at other stress states with the uni-axial reference curve, one can calculate these parameters.

In the current description the following assumptions are made concerning the material behaviour:

- Isotropic hardening
- The yield locus shape does not change with the strain

Fig. 8 The different yield loci used for DC04 and DP600; Vegter yield criterion matches measured points



- The work hardening is independent from the strain path (loading path)
- An equivalent strain is assumed based on the plastic work principle:

$$\sigma_f \cdot \partial \bar{\varepsilon} = \sigma_1 \cdot \partial \varepsilon_1 + \sigma_2 \cdot \partial \varepsilon_2 + \sigma_3 \cdot \partial \varepsilon_3 \quad (5)$$

$$\partial \bar{\varepsilon} = \frac{\sigma_1}{\sigma_f} \cdot \partial \varepsilon_1 + \frac{\sigma_2}{\sigma_f} \cdot \partial \varepsilon_2 + \frac{\sigma_3}{\sigma_f} \cdot \partial \varepsilon_3$$

Defining a reference axis *i* for the test, with the accompanying stress σ_i and strain increment $\partial \varepsilon_i$ yields:

$$\partial \bar{\varepsilon} = \frac{\sigma_k}{\sigma_f} \cdot \partial \varepsilon_i \cdot \left(\frac{\sigma_1}{\sigma_i} \cdot \frac{\partial \varepsilon_1}{\partial \varepsilon_i} + \frac{\sigma_2}{\sigma_i} \cdot \frac{\partial \varepsilon_2}{\partial \varepsilon_i} + \frac{\sigma_3}{\sigma_i} \cdot \frac{\partial \varepsilon_3}{\partial \varepsilon_i} \right) \quad (6)$$

Assuming: $f_i = \frac{\sigma_i}{\sigma_f}$; $x_j = \frac{\sigma_j}{\sigma_i}$; $\rho_j = \frac{\partial \varepsilon_j}{\partial \varepsilon_i}$ where *j* is 1, 2 or 3; results into the following expression:

$$\partial \bar{\varepsilon} = f_i \cdot \partial \varepsilon_i \cdot (x_1 \cdot \rho_1 + x_2 \cdot \rho_2 + x_3 \cdot \rho_3) \quad (7)$$

where: $\rho_3 = -\rho_2 - \rho_1$

The total equivalent strain is defined as a state of plastic work and is obtained via integration of Eq. 7

$$\int_0^{\bar{\varepsilon}} \sigma_f \cdot \partial \bar{\varepsilon} = \int_0^{\varepsilon_i} \sigma_i \cdot (x_1 \cdot \rho_1 + x_2 \cdot \rho_2 + x_3 \cdot \rho_3) \cdot \partial \varepsilon_i \quad (8)$$

This expression is simplified because in all test the parameters x_j are 1 or 0, so the integration reduces Eq. 8 to simple expressions for each test:

$$\begin{aligned} \int_0^{\bar{\varepsilon}} \sigma_f \cdot \partial \bar{\varepsilon} &= \int_0^{\varepsilon_1} \sigma_1 \cdot \partial \varepsilon_1 \text{ for uniaxial and plane strain tests} \\ \int_0^{\bar{\varepsilon}} \sigma_f \cdot \partial \bar{\varepsilon} &= \int_0^{\varepsilon_3} \sigma_{bi} \cdot \partial \varepsilon_3 \text{ for bulge tests or compression tests} \\ \int_0^{\bar{\varepsilon}} \sigma_f \cdot \partial \bar{\varepsilon} &= \int_0^{\gamma} \tau_{sh} \cdot \partial \gamma, \text{ with } \tau_{sh} = \frac{\sigma_1 - \sigma_2}{2} \text{ and } \partial \gamma = \frac{\partial \varepsilon_1}{2}, \text{ for shear tests} \end{aligned} \quad (9)$$

The way the data are handled for each test is summarized in Table 1.

The lines of constant equivalent strain are considered to define the yield locus, which is allowed in the framework of the assumptions. This yield stress is obtained by looking up the stress value at the equivalent strain considered. In the derivation of the four stress factors from the four different tests we have to determine these assuming a reference where we derive the strain hardening parameters. Usually we take the uni-axial tensile test at 0°, which makes $f_{un}(0) = 1$. For each point of the stress strain curve at other test the parameters of the stress factors *f* are determined. The procedure is explained for a bulgetest (or compression test) assuming the bi-axial stress-strain curve is available [13]:

1. Calculation of the plastic work of both curves (uni-axial and bi-axial) by numerical integration (Fig. 6)

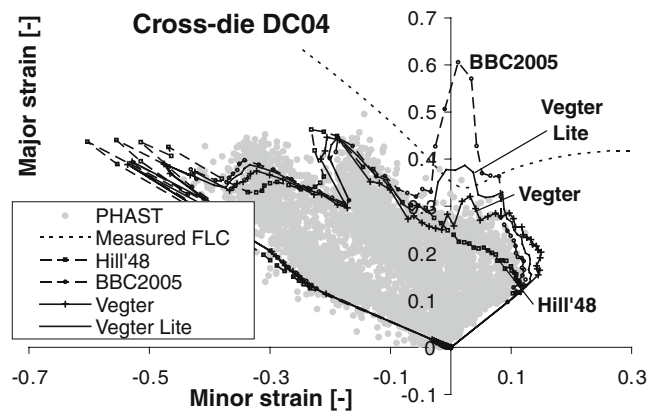


Fig. 9 The forming limit diagram for the Cross-Die using DC04 steel

2. Looking up σ_{bi} and σ_f at the same value of plastic work, the strain in the σ_f - ε -diagram is equal to the equivalent strain
3. Calculation of $f_{bi} = \sigma_{bi} / \sigma_f$ at each strain (Fig. 6)
4. Calculation of an average value over a certain strain area. One has to remain outside the strain area of a yield point elongation.

Additionally there is a possibility to extrapolate the hardening curve using the remaining data of the bi-axial stress strain curve (Fig. 6). The biaxial stress parameter at the end of the uni-axial deformation is used for this operation [13].

For derivation of the stress parameters of other tests than the biaxial tests, the same procedure as described in points 1–4 can be used.

Using the Vegter criterion or Vegter Lite criterion requires the determination of the biaxial strain ratio r_{bi} . This parameter can be obtained from the on plane compression tests from the measured in plane strain strains: $r_{bi} = \varepsilon_{TD} / \varepsilon_{RD}$ (TD=transverse to rolling direction and RD=rolling direction). This parameter cannot be determined

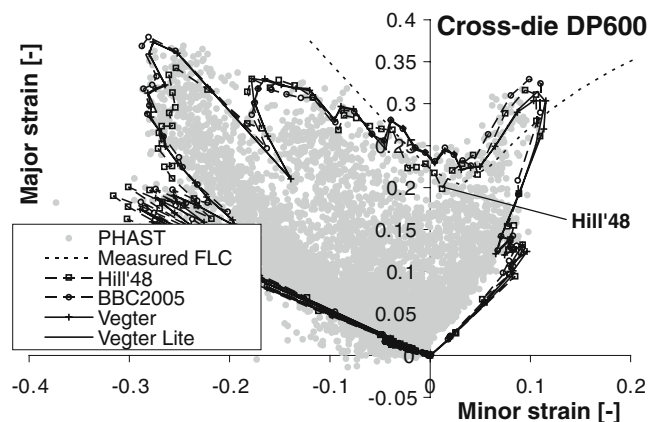


Fig. 10 The forming limit diagram for the Cross-Die using DP600 steel

from a bulgetest. A reasonable estimate of the biaxial strain ratio for this case is the ratio of the r -values in rolling direction (r_{00}) and transverse direction (r_{90}): $r_{bi} = r_{00}/r_{90}$. This last method gives a value for the bi-axial strain ratio according to the Hill48-criterion.

Application in forming

In order to validate that the reducing the number of parameters for the Vegter Lite model has not adversely affected the accuracy, simulations of a semi-industrial part were performed [14, 15]. The part chosen was the Cross-Die as shown in Fig. 7.

Experimental pressings of the cross die with DC04 (formable steel) and DP600 (AHSS) were simulated using different yield loci: Hill'48, BBC2005 [1], Vegter and the Vegter Lite model. Examples of how the yield locus differs for these four models can be seen in Fig. 8. As can be seen in this figure, the Hill'48 model overestimates for both cases the position of the plane strain and biaxial points; this is because it only uses the three r -values to describe the yield locus. The other three yield loci have correct biaxial points as they use it as input. However, the plane strain point shows differences between three models where the Vegter model is the closest to the measured values. The Lite version is the next best but it is closer than the BBC2005 model; that the plane strain point of the Lite version is not perfectly on top of the measurement is a result of the fixed value that is used for the weight factor (it's not perfect for every material).

Using these four yield loci for every material, the cross-die was simulated using PAM-STAMP. As the BBC2005 model is not within the software, the shape of the BBC2005 yield locus was implemented through the Vegter model by setting the parameters such that it perfectly represented the BBC2005 yield locus.

To compare the results of the different yield loci, the strain distributions are plotted for a cross die where the part is critical and is on the verge of necking. The measured and calculated strain distributions can be seen in Figs. 5 and 6 for DC04 and DP600, respectively.

From Fig. 9 it can be clearly seen that the Hill'48 criterion underestimates how critical this part really is. On the other hand, BBC2005 criterion is too conservative as it predicts failure while in reality the part is just on the border of necking. Both the Vegter and Vegter Lite models are quite close the experiment and they are therefore the best models for DC04.

For DP600 the differences between the results for the different yield loci are a lot smaller, as can be seen in Fig. 10. Again Hill'48 predicts strains in the plane strain region that are smaller than in reality. The other three yield

loci are very close; that this would be the case is not surprising as their yield loci (see Fig. 8) are very close together.

Conclusions

- The Vegter Lite model requires fewer parameters compared to Vegter yield criterion and similar accuracy is obtained in simulation results.
- Parameter identification is easy compared to other descriptions using the same number of parameters.
- For a wide range of steels, simulations with the Vegter Lite model are more accurate than those using the Hill'48 criterion

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