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A SHEAR FRACTURE CRITERION TO PREDICT LIMIT STRAINS IN SHEET METAL FORMING

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ABSTRACT: Present work examines a mathematical model to predict the onset of shear fracture in the industrial processes of sheet metal forming such as biaxial stretching. Historically, sheet metal formability has been assessed by simple testing such as the Erichsen test. Lately, the concept of experimental Forming Limit Curve, FLC, was developed to evaluate formability. The Forming Limit Diagram shows the FLC which is the plot of principal strains in the sheet metal surface, ε_1 and ε_2 , occurring at critical points obtained in the laboratory formability tests or in the fabrication process. Two types of undesirable failure mechanisms can occur in sheet metal forming operations: local necking and shear fracture. Therefore, two kinds of limit strain curves can be plotted: the local necking limit curve FLC-N and the shear fracture limit curve FLC-S. The D-Bressan shear instability criterion model proposed for the theoretical prediction of forming limit strain curve owing to the onset of local necking, FLC-N, in sheet metal forming is reformulated to predict the shear fracture strain limit, FLC-S, of sheet metal forming operations. Shear fracture is anticipated to initiate in the direction of pure shear when the shear stress attains some critical value. The Barlat Yld2000-2d anisotropic yield function proposed by Barlat et al. is employed and the material is assumed to display both planar and normal anisotropy. The new approach investigate the influence of mechanical plastic properties such us the plastic anisotropy parameters, pre-strain and work hardening coefficient in sheet metal formability. Some experimental and theoretical results of forming limit curve for aluminium obtained from present model for two values of the power coefficients a =5 and 6 are shown. The relevant issues related with the FLC-S is presented and discussed.

KEYWORDS: Yield criteria, mathematical model, shear fracture, aluminium.

INTRODUCTION

Formability of sheet metals is an important and complex issue related to the optimization and quality control of sheet metal final product. New developments and research have been carried out, aiming at improving sheet materials and fabrication processes such as stamping, stretching, deep drawing and other sheet metal forming processes. Also, it is important to improve equipments in order to increase its productivity, quality and to lower costs through the finished product shape more simple, with zero defects and obtained by less number of operations.

Historically, sheet metal formability has been assessed by simple testing such as the tensile test, the biaxial stretching or Erichsen test and the deep drawing or cup test. Lately, the concept of experimental Forming Limit Curve for strains, FLC, and numerical simulations by finite elements were developed to evaluate sheet metal formability and its forming operations by predicting the onset of local necking and fracture.

Simulation of sheet metal forming is a process which requires a deep understanding and accurate modelling of the mechanics of materials elastic and plastic deformation behavior under small and large strains, strain rate and temperature (macroscopic or micro-mechanics based models). In addition, numerical simulation requires a failure criterion in order to evaluate the critical points in the deformed part.

The concept of formability is based firstly on *rupture* or *local necking* in the sheet metal. This means that a material with good formability characteristics should not fracture or show a visible local necking during the forming operations, but these are not unique factors. Secondly, there are the concepts of rigidity of shape (occurrence of spring back effect or elastic recovery, looseness), surface roughness or texture and the occurrence of wrinkles (or sheet wrinkling).

In summary, the evaluation of sheet metal formability in the press operation is quite sensitive to the material mechanical plastic properties, shape and evolution of yield surface, hardening law, strain path, failure criteria, material friction coefficient, temperature and the FLC.

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The Forming Limit Diagram shows the FLC curve which is the plot of the principal strains in the sheet metal surface, ε_1 and ε_2 , occurring at critical points obtained in laboratory formability tests or in the fabrication process. Two types of undesirable failure mechanisms can occur in sheet metal forming operation: local necking and shear fracture. Therefore, two kinds of limit strain curves can be plotted: the local necking limit curve FLC-N and the shear fracture limit curve FLC-S [1]. Experimental curves and theorethical prediction of local necking and shear fracture limit have been intensively investigated by academic researchers and by industry professionals.

The goal of present work was to examine a mathematical model to predict the onset of shear fracture in industrial processes of sheet metal forming such as biaxial stretching, using the D-Bressan shear instability criterion model and Barlat Yld2000-2d anisotropic yield function.

2 BARLAT YLD2000-2D ANISOTROPIC YIELD FUNCTION

Assuming the anisotropic Yield function Yld2000-2d as the sum of two scalar functions, the plane stress anisotropic yield function is given by [2],

$$\phi = \phi' + \phi''$$
$$= \left| S'_1 - S'_2 \right|^a + \left| 2S''_1 + S''_2 \right|^a + \left| 2S''_2 + S''_1 \right|^a = k\,\overline{\sigma}^a \tag{1}$$

$$\phi = \left| \sigma''_1 - \sigma''_2 \right|^a + \left| \sigma'_1 \right|^a + \left| \sigma'_2 \right|^a = k\,\overline{\sigma}^a \tag{2}$$

where S'_j and S''_k are the principal values of two stress tensors S'_{ij} and S''_{ij} defined by the linear transformation of the stress deviator S_{ij} or the Cauchy stress tensor $\boldsymbol{\sigma_{ij}}$,

$$S'_{ij} = C' S_{ij} = C'\hat{T}\sigma_{ij} = L'_{ij}\,\sigma_{ij}$$
$$= \begin{bmatrix} L'_{11} & L'_{12} & 0 \\ L'_{21} & L'_{22} & 0 \\ 0 & 0 & L'_{66} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} s'_{xx} \\ s'_{yy} \\ s'_{xy} \end{bmatrix} \tag{3}$$

and similar equation for the double prime tensor S''_{ij} .

The principal values of the stress tensor S'_{ij} and S''_{ij} are,

$$S'_1 = \frac{s'_{xx} + s'_{yy}}{2} + \sqrt{\Delta} \tag{4a}$$

$$S'_2 = \frac{s'_{xx} + s'_{yy}}{2} - \sqrt{\Delta} \tag{4b}$$

$$\Delta = \left((s'_{xx} - s'_{yy})/2 \right)^2 + s'^{\,2}_{xy} \tag{4c}$$

with the appropriate indices (prime and double prime) for each case. The principal values of the stress tensor, in equation (1) or equation (2), can be expressed in terms of the Cauchy stress components,

$$\sigma'_1 = 2S''_1 + S''_2 = \frac{\alpha_8\,\sigma_{xx} + \alpha_1\,\sigma_{yy}}{2} + \sqrt{\Delta'} \tag{5a}$$

$$\sigma'_2 = 2S''_2 + S''_1 = \frac{\alpha_8\,\sigma_{xx} + \alpha_1\,\sigma_{yy}}{2} - \sqrt{\Delta'} \tag{5b}$$

$$\sigma''_1 - \sigma''_2 = S'_1 - S'_2 = 2\sqrt{\Delta''} \tag{5c}$$

where,

$$\Delta' = \left(\frac{\alpha_2\,\sigma_{xx} - \alpha_3\,\sigma_{yy}}{2} \right)^2 + \left(\alpha_4 \tau_{xy} \right)^2$$

$$\Delta'' = \left(\frac{\alpha_5\,\sigma_{xx} - \alpha_6\,\sigma_{yy}}{2} \right)^2 + \left(\alpha_7 \tau_{xy} \right)^2$$

α_i are the coefficients of the linear transformation.

Calculating the plasticity modulus dλ :

The loading condition for plastic strain can be written as:

$$f\left(\sigma_{ij}, \overline{\varepsilon}\right) = F(\sigma_{ij}) - \overline{\sigma}\left(\overline{\varepsilon}\right) = 0 \tag{6}$$

where $f\left(\sigma_{ij}, \overline{\varepsilon}\right)$ denotes the yield function, $F(\sigma_{ij})$ is the yield criterion and commonly is a first degree homogeneous function of the Cauchy stress tensor, $\sigma_{ij} = \sigma_{kk}\delta_{ij} + S_{ij}$, and which defines the yield surface shape. $\overline{\sigma}$ is the equivalent stress and σ_ξ is the uniaxial yield stress identified as scalar function of the equivalent plastic strain $\overline{\varepsilon}$. Assuming for the yield criterion a homogeneous stress function of degree 'a', $\phi(\sigma_\xi) = k\,\overline{\sigma}^a$, the uniaxial flow stress is,

$$\sigma_\xi = \left(\frac{k}{\phi(\sigma_\xi)} \right)^{1/a} \overline{\sigma} \tag{7}$$

Applying the Euler's identity theorem for a homogeneous stress function of degree 'a' to the yield criterion function ϕ, then,

$$\sigma_{ij}\frac{\partial \phi}{\partial \sigma_{ij}} = \sigma_{xx}\frac{\partial \phi}{\partial \sigma_{xx}} + \sigma_{yy}\frac{\partial \phi}{\partial \sigma_{yy}} + 2\,\tau_{xy}\frac{\partial \phi}{\partial \tau_{xy}}$$
$$= a\,\phi(\sigma_{ij}) \tag{8}$$

Multiplying by the plasticity modulus $d\lambda$,

$$\sigma_{ij}d\varepsilon_{ij} = \sigma_{xx}d\varepsilon_{xx} + \sigma_{yy}d\varepsilon_{yy} + 2\tau_{xy}d\varepsilon_{xy} = a\,\phi(\sigma_\xi)\,d\lambda \tag{9}$$

Considering that the increment of plastic work is dw = $\sigma_{ij}d\varepsilon_{ij} = \bar{\sigma}\,d\bar{\varepsilon} = a\,\phi(\sigma_\xi)\;d\lambda$, thus,

$$d\lambda = \frac{\bar{\sigma}\,d\bar{\varepsilon}}{a\,\phi(\sigma_\xi)} = \frac{d\bar{\varepsilon}}{a\,k\,\bar{\sigma}^{\,a-1}} \tag{10}$$

Calculating the associated flow rule or the principal strain increments from the normality flow rule :

The normal direction to the yield surface ϕ , needed to calculate the strain increments (or strain rates) through the associated flow rule, is calculated by the gradient of the yield surface. However, the function ϕ depend on four variables S'_1, S'_2, S''_1, S''_2 ; $\phi = \phi(S'_1, S'_2, S''_1, S''_2)$, thus, the potential partial derivative is,

$$\frac{\partial\phi}{\partial\sigma_1} = \frac{\partial\phi}{\partial S'_1}\frac{\partial S'_1}{\partial\sigma_1} + \frac{\partial\phi}{\partial S'_2}\frac{\partial S'_2}{\partial\sigma_1} + \frac{\partial\phi}{\partial S''_1}\frac{\partial S''_1}{\partial\sigma_1} + \frac{\partial\phi}{\partial S''_2}\frac{\partial S''_2}{\partial\sigma_1} \tag{11}$$

Assuming that the plastic potential is identical to the yield function, the principal strain increments can be calculated from the normality flow rule as,

$$\frac{d\varepsilon_1}{\frac{\partial\phi}{\partial\sigma_1}} = \frac{d\varepsilon_2}{\frac{\partial\phi}{\partial\sigma_2}} = \frac{d\varepsilon_3}{\frac{\partial\phi}{\partial\sigma_3}} = \frac{d\bar{\varepsilon}}{a\,k\,\bar{\sigma}^{\,a-1}} = d\lambda \tag{12}$$

The partial derivatives of equation (2) can be calculated,

$$\frac{\partial\phi}{\partial\sigma_1} = \frac{\partial\phi}{\partial\sigma''_1}\frac{\partial\sigma''_1}{\partial\sigma_1} + \frac{\partial\phi}{\partial\sigma''_2}\frac{\partial\sigma''_2}{\partial\sigma_1} + \frac{\partial\phi}{\partial\sigma'_1}\frac{\partial\sigma'_1}{\partial\sigma_1} + \frac{\partial\phi}{\partial\sigma'_2}\frac{\partial\sigma'_2}{\partial\sigma_1} \tag{13}$$

etc., where,

$$\frac{\partial\phi}{\partial\sigma''_1} = a\left|\sigma''_1 - \sigma''_2\right|^{a-1} \quad ; \quad \frac{\partial\phi}{\partial\sigma''_2} = -a\left|\sigma''_1 - \sigma''_2\right|^{a-1} \tag{14a,b}$$

$$\frac{\partial\phi}{\partial\sigma'_1} = a\left|\sigma'_1\right|^{a-1} \quad ; \quad \frac{\partial\phi}{\partial\sigma'_2} = a\left|\sigma'_2\right|^{a-1} \tag{14c,d}$$

From the Mohr's circle of stresses, Figure 1, the Cauchy stress components as functions of principal stresses are,

$$\sigma_{xx} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta \tag{15a}$$

$$\sigma_{yy} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta \tag{15b}$$

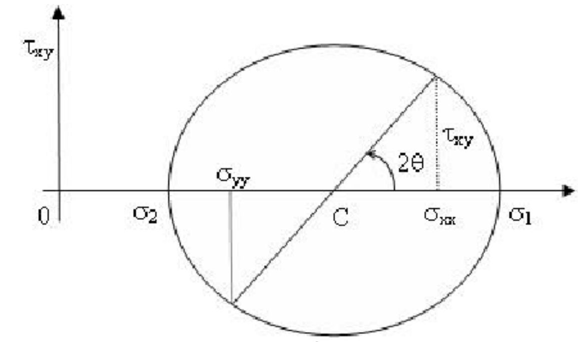

Figure 1: Mohr circle of plane stress state.

$$\tau_{xy} = \frac{(\sigma_1 - \sigma_2)}{2}\,sen\,2\theta \tag{15c}$$

Introducing equations (15) into equations (5),

$$\sigma'_1 = \frac{\left(\alpha_8\cos^2\theta + \alpha_1 sen^2\theta\right)\sigma_1 + \left(\alpha_8 sen^2\theta + \alpha_1\cos^2\theta\right)\sigma_2}{2} + \sqrt{\Delta'} \tag{16a}$$

$$\sigma'_2 = \frac{\left(\alpha_8\cos^2\theta + \alpha_1 sen^2\theta\right)\sigma_1 + \left(\alpha_8 sen^2\theta + \alpha_1\cos^2\theta\right)\sigma_2}{2} - \sqrt{\Delta'} \tag{16b}$$

$$\sigma''_1 - \sigma''_2 = 2\sqrt{\Delta''} \tag{16c}$$

where,

$$\Delta' = \left\{\left(\frac{\alpha_2}{2}\cos^2\theta - \frac{\alpha_3}{2}sen^2\theta\right)\sigma_1 + \left(\frac{\alpha_2}{2}sen^2\theta - \frac{\alpha_3}{2}\cos^2\theta\right)\sigma_2\right\}^2 + \frac{\alpha_4^2}{4}(\sigma_1 - \sigma_2)^2 sen^2\,2\theta$$

$$\Delta'' = \left\{\left(\frac{\alpha_5}{2}\cos^2\theta - \frac{\alpha_6}{2}sen^2\theta\right)\sigma_1 + \left(\frac{\alpha_5}{2}sen^2\theta - \frac{\alpha_6}{2}\cos^2\theta\right)\sigma_2\right\}^2 + \frac{\alpha_7^2}{4}(\sigma_1 - \sigma_2)^2 sen^2\,2\theta$$

The derivatives $\partial\sigma'_i/\partial\sigma_i$ and $\partial\sigma''_i/\partial\sigma_i$ necessary to solve equation (13) are obtained from equations (16a) to (16c).

3 SHEAR FRACTURE MODELLING

Present approach analyses a thin sheet metal with strain hardening behavior which constitutive equation for flow stress is described by,

$$\bar{\sigma} = K\left(\varepsilon_o + \bar{\varepsilon}\right)^n \tag{17}$$

where K is the strength coefficient, $\bar{\varepsilon}$ is the equivalent true strain, ε_o is the prestrain and n represents the strain hardening coefficient.

The governing equation for the onset of shear fracture mechanism in a plane inclined through the thickness, in which the critical shear direction coincides with the direction of zero extension or direction of pure shear strain in sheet metal stretching processes is [1],

$$\sigma_1 = \left|\frac{2+\beta}{\sqrt{1+\beta}}\right|\tau_{cr} \tag{18}$$

where σ_1 is maximum principal in-plane stress, τ_{cr} is the critical shear stress, $\beta = d\varepsilon_2 / d\varepsilon_1$ is the strain path. Defining the stress ratio $X = \sigma_2/\sigma_1$ and assuming $k = 2$, the equivalent flow stress can be written as,

$$\bar{\sigma} = \left(\frac{1}{2}\right)^{\frac{1}{a}} \left\{\left|\frac{\sigma_1'' - \sigma_2''}{\sigma_1}\right|^a + \left|\frac{\sigma_1'}{\sigma_1}\right|^a + \left|\frac{\sigma_2'}{\sigma_1}\right|^a\right\}^{\frac{1}{a}} \sigma_1 \tag{19}$$

where,

$$\frac{\sigma_1'}{\sigma_1} = \frac{\left(\alpha_8 \cos^2\theta + \alpha_1 sen^2\theta\right) + \left(\alpha_8 sen^2\theta + \alpha_1 \cos^2\theta\right)X}{2} + \sqrt{\frac{\Delta'}{\sigma_1^2}}$$

$$\frac{\sigma_2'}{\sigma_1} = \frac{\left(\alpha_8 \cos^2\theta + \alpha_1 sen^2\theta\right) + \left(\alpha_8 sen^2\theta + \alpha_1 \cos^2\theta\right)X}{2} - \sqrt{\frac{\Delta'}{\sigma_1^2}}$$

$$\frac{\Delta'}{\sigma_1^2} = \left\{\left(\frac{\alpha_2}{2}\cos^2\theta - \frac{\alpha_3}{2} sen^2\theta\right) + \left(\frac{\alpha_2}{2} sen^2\theta - \frac{\alpha_3}{2}\cos^2\theta\right)X\right\}^2 + \frac{\alpha_4^2}{4}(1-X)^2 sen^2 2\theta$$

etc..

Combining equations (17), (18) and (19), the equivalent and principal true strain at the onset of shear fracture,

$$\bar{\varepsilon}^* = \left\{\frac{1}{2}\left(\left|\frac{\sigma_1'' - \sigma_2''}{\sigma_1}\right|^a + \left|\frac{\sigma_1'}{\sigma_1}\right|^a + \left|\frac{\sigma_2'}{\sigma_1}\right|^a\right)\right\}^{\frac{1}{an}} \left|\frac{2+\beta}{\sqrt{1+\beta}}\right|^{\frac{1}{n}} \left(\frac{\tau_{cr}}{K}\right)^{\frac{1}{n}} - \varepsilon_o$$

$$\varepsilon_1^* = \left\{\frac{1}{2}\left[\left|\frac{\sigma_1'' - \sigma_2''}{\sigma_1}\right|^a + \left|\frac{\sigma_1'}{\sigma_1}\right|^a + \left|\frac{\sigma_2'}{\sigma_1}\right|^a\right]\right\}^{\frac{1}{a}} \frac{\bar{\varepsilon}^*}{\{1 + X\beta\}} \tag{20}$$

The relation between β and X are obtained from equation (12) combined with equations (13) and (14),

$$\beta = \frac{\left|\sigma_1'' - \sigma_2''\right|^{a-1} \frac{\partial(\sigma_1'' - \sigma_2'')}{\partial\sigma_1} + \left|\sigma_1'\right|^{a-1} \frac{\partial\sigma_1'}{\partial\sigma_1} + \left|\sigma_2'\right|^{a-1} \frac{\partial\sigma_2'}{\partial\sigma_1}}{\left|\sigma_1'' - \sigma_2''\right|^{a-1} \frac{\partial(\sigma_1'' - \sigma_2'')}{\partial\sigma_2} + \left|\sigma_1'\right|^{a-1} \frac{\partial\sigma_1'}{\partial\sigma_2} + \left|\sigma_2'\right|^{a-1} \frac{\partial\sigma_2'}{\partial\sigma_2}} \tag{21}$$

4 THEORETICAL AND EXPERIMENTAL FLC RESULTS

The correlation between the experimental results and the theoretical predictions can be investigated by comparing the FLC of aluminium allow from Tadros and Mellor [3]. Assuming that the material coordinate axis (x,y) coincides with the principal directions (1,2), i.e., $\theta = 0$, equation (21) is reduced to,

$$\beta = \frac{-2^a\alpha_6\left|\alpha_5 - \alpha_6 X\right|^{a-1} + (\alpha_1 - \alpha_3)\left|\alpha_8 + \alpha_2 + (\alpha_1 - \alpha_3)X\right|^{a-1} +}{2^a\alpha_5\left|\alpha_5 - \alpha_6 X\right|^{a-1} + (\alpha_8 + \alpha_2)\left|\alpha_8 + \alpha_2 + (\alpha_1 - \alpha_3)X\right|^{a-1} +}$$
$$\frac{(\alpha_1 + \alpha_3)\left|\alpha_8 - \alpha_2 + (\alpha_1 + \alpha_3)X\right|^{a-1}}{(\alpha_8 - \alpha_2)\left|\alpha_8 - \alpha_2 + (\alpha_1 + \alpha_3)X\right|^{a-1}} \tag{22}$$

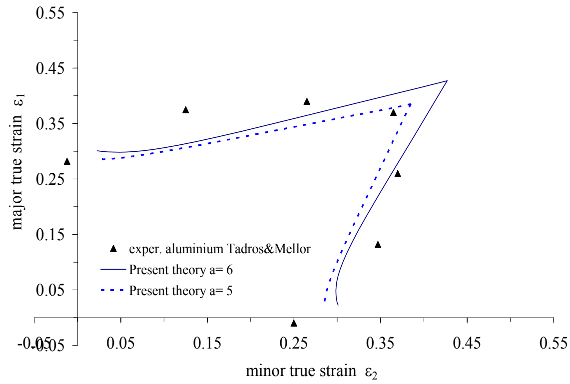

***Figure 2:** Experimental [3] and theoretical results of limit strains for aluminium alloy sheet for two a-values.*

In Figure 2, the predicted FLC for linear strain path for aluminium alloy for two values of parameter a are shown and compared with the experimental results by Tadros and Mellor [3]. The assumed material coefficients α_i are: $\alpha_1 = 0.938$, $\alpha_2 = 1.045$, $\alpha_3 = 0.929$, $\alpha_5 = 0.987$, $\alpha_6 = 1.036$, $\alpha_8 = 1.101$. The other plasticity parameters are: n=0.24, ε_o=0 , τ_{cr}/K = 0.40. The correlation is reasonable good.

5 CONCLUDING REMARKS

The theoretical FLC curves for the shear fracture model depend greatly on the employed yield criteria, anisotropy parameters, critical shear stress and strain path. The FLC is superior for $a = 6$ than the correspondent curve for $a = 5$. However, the exact value has to be obtained from tensile tests comparisons at different tensile direction. The difference between the curves increased largely near balanced biaxial stretching. The comparison with experimental results for aluminium presented by Tadros and Mellor are quite good. However, more experimental data from tensile and biaxial tests are necessary to evaluate and improve present model.

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REFERENCES

[1] J.D. Bressan and J.A. Williams, The Use of a Shear Instability Criterion to Predict Local Necking in Sheet Metal Deformation. Int. J. Mech. Sci., v. 25, pp.155-168, 1983.

[2] F. Barlat, Constitutive Description for Metal Forming Simulations. In: Materials Processing and Design, NUMIFORM '07, pp.3-23, 2007.

[3] A. K. Tadros and P. B. Mellor, *Int. J. Mech.* Sci. , v.20, pp.121, 1978.