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ON THE OBJECTIVE FUNCTION EVALUATION IN PARAMETER IDENTIFICATION OF MATERIAL CONSTITUTIVE MODELS - SINGLE-POINT OR FE ANALYSIS

R. de-Carvalho*, **R.A.F. Valente, A. Andrade-Campos**

University of Aveiro, Department of Mechanical Engineering

ABSTRACT: This work deals with the identification of constitutive parameters by inverse methodology. Two different approaches are presented and analysed: the single-point and FE analysis. The use of these two different methodologies for the evaluation of objective functions in the identification process is still an open question and the interest in this field has been increasing among the metal forming community. To discuss this issue, two different constitutive models suitable for metals were used, *i.e.* an elastoplastic hardening model and an elastoplastic model with isotropic and kinematic hardening. The determined material parameters for the two models, the respective objective function values and the CPU time required to perform the simulations are presented and discussed.

KEYWORDS: Inverse problems, Parameters identification, Numerical integration, Finite element analysis (FEA), Single-point analysis.

1 INTRODUCTION

Nowadays, industrial and scientific communities are confronted with extremely complex mechanical engineering problems. These problems can no longer be solved with trial and error methods, mostly for economical reasons. It is in this context that the Finite Element Analysis (FEA) assumes an important role allowing virtual testing. Although FEA is a well established numerical approach in both industry and research, complex techniques are still being developed to simulate with increasing accuracy the behaviour of different materials [1, 2].
In order to obtain accurate stress and strain fields, an effective FEA requires secure input data such as geometry, mesh, non-linear material behaviour laws, loading cases, friction laws, etc. This sort of problems can be defined as direct problems in which the quality of the results relies on the quality of the input data that are not always available. In order to overcome these difficulties a possible approach is the inverse problems that, for instance, refer to the determination of input parameters to be used in geometric or constitutive models, based on experimental observations. Considering the need to evaluate the input data, distinct inverse problems can be formulated. One category of inverse problems is called parameter identification. The aim of these problems, for instance, is to estimate material parameters for constitutive models. This inverse problem is solved with the aid of optimization algorithms [3].
In this work, the use of single-point or metal forming FE analysis for the evaluation of the objective function in the identification process is analysed. This study is conducted considering two different constitutive models and two different steels commonly used in metal forming processes. The models are a non-linear elastoplastic hardening model and a differential elastoplastic model, in which the work-hardening combines isotropic and kinematic contributions. In order to determine the best parameters set, a Levenberg-Marquardt optimization algorithm was applied.

2 PARAMETERS IDENTIFICATION

The determination of the parameters needed for the constitutive models can be accomplished solving an inverse problem, which consists in searching for a set of parameter values for which the experimental reality and the numerical simulation are similar. The comparison between the mathematical model and the experimental data results in the objective function that will be subjected to optimization methods. A correct definition of this function is essential to all the optimization processes and to the efficient determination of the constitutive parameters [1, 2, 4]. One of the most used objective functions consists on the sum of the squares of the stress difference at different strain levels [2]. In the parameters optimization problems, the objective function can be defined as

$$S_{\mathrm{obj}}(\mathbf{x}) = \sum_{k=1}^{n_{\mathrm{tests}}} \sum_{i=1}^{n_{\mathrm{points}}} \left(\frac{\sigma_i^{\mathrm{exp}} - \sigma_i^{\mathrm{num}}(\mathbf{x})}{W_{\mathrm{abs}} + W_{\mathrm{rel}} \sigma_i^{\mathrm{exp}}} \right)^2 \quad (1)$$

The numerator in equation 1 is the difference between the experimental and the numerical stress for the i-th value of strain, and W_{abs} and W_{rel} are weighting factors that

*Corresponding author: Campus Universitário de Santiago, 3810-193 - Aveiro - Portugal Phone: +351-234-370830, Fax: +351-234-270953, raquelsoares@ua.pt.

should be adapted to the optimization problem in study. Analysing the objective function it can be observed that, if the correspondence between the physical experiment and the numerical model is perfect, $S_{\mathrm{obj}}(\mathbf{x})$ will be equal to zero. However this fact never happens and it is only expected that $S_{\mathrm{obj}}(\mathbf{x})$ takes low non-zero value. The existence of local minimums also leads to practical difficulties in the interpretation and selection of the obtained results. Therefore, during the analysis of the results it is necessary to remember that it is possible to obtain several distinct sets of parameters for which the objective function assumes reasonable values. In these cases, it is the duty of the user to evaluate the obtained results considering the physical definition and the meaning of each parameter [4]. In this work, a Levenberg-Marquardt gradient algorithm was used to minimize the objective function.

3 SINGLE-POINT OR FE ANALYSIS

The use of FE or single-point analysis for the evaluation of objective functions in the identification process is still an open question. By definition a constitutive model is a mathematical representation of the phenomena that take place in an infinitesimal amount of material (according to the continuum mechanics theory). On one hand, the single infinitesimal point evaluation seems to characterize an infinitesimal amount of material subjected to all kind of deformation history. Although it is computationally very inexpensive, it cannot be used to account phenomena such as specimen necking or springback. On the other hand, in all FE analysis codes, the constitutive model is implemented and accounted for each element integration point. Numerical approximations of the FE method include iteratively accumulated errors, and can impair the whole identification process. Nonetheless, FE analyses allow to model the specimen used in the experimental procedure and predict geometrical phenomena such as necking and springback.

The main difference between the two approaches are the time and space integration methods used in each one. This fact influence the obtained results in the evaluation of the objective function and afterwards the optimization process. For the single point analysis the numerical method used to integrate in time was a second order Runge-Kutta explicit method. The Finite Element analysis was performed using the implicit FE code ABAQUS/STANDARD that generally uses Newton's method as a numerical technique for solving the nonlinear equilibrium equations. For each time increment the implicit methods need to converge leading to a large computation time when compared with the explicit methods. However the explicit methods can iteratively accumulate errors.

4 CONSTITUTIVE MODELS

In this section the constitutive models are briefly presented.

4.1 NON-LINEAR ELASTOPLASTIC HARDENING MODEL

The first constitutive model here studied is an elastoplastic model with non-linear hardening for stainless steel AISI 304 [3]. Experimental tests were carried out in proportional loading for uniaxial tension. The hardening law can be described by the following equation [1]:

$$\bar{\sigma}(\bar{\varepsilon}^{\mathrm{pl}}) = \sigma^0 + (\sigma^\infty - \sigma^0)[1 - \exp(-\delta\bar{\varepsilon}^{\mathrm{pl}})] + \zeta\bar{\varepsilon}^{\mathrm{pl}} \quad (2)$$

where $\bar{\varepsilon}^{\mathrm{pl}} = \sqrt{2/3\boldsymbol{\varepsilon}_{ij}^{\mathrm{pl}}\boldsymbol{\varepsilon}_{ij}^{\mathrm{pl}}}$ is the equivalent plastic strain and $\boldsymbol{\varepsilon}_{ij}^{\mathrm{pl}}$ is the plastic strain tensor. Considering a one-dimensional analysis, this elastoplastic model with non-linear hardening contains 4 material parameters to determine: σ^0, σ^∞, δ and ζ. For the elastic part, it was considered E=380 GPa and ν=0.29. This constitutive model leads to feasible stress-strain results only when $\sigma^0 < \boldsymbol{\sigma}^\infty$. Therefore during the formulation of the minimization problem this constraint must be taken into account [3].

4.2 NON-LINEAR ELASTOPLASTIC MODEL WITH ISOTROPIC AND KINEMATIC HARDENING

For this constitutive model the material studied was a mild steel E220BH. The experimental data used was obtained by Thuillier *et al* [5]. This data comprises experimental values of monotonous tensile and shear tests, both carried out at $0°$ to the rolling direction (RD). Additionally three shear tests, in order to highlight the Bauschinger effect and to measure kinematic work-hardening parameter, were performed. The constitutive model analysed is an elasto-plastic model that takes into account the kinematic and the isotropic work-hardening of the material. The yield function considered is given by:

$$\begin{aligned} f(\boldsymbol{\sigma}, \mathbf{X}, R) &= \bar{\sigma} - R \\ &= \sqrt{\frac{3}{2}(\boldsymbol{\sigma}^{\mathrm{d}} - \mathbf{X}) : (\boldsymbol{\sigma}^{\mathrm{d}} - \mathbf{X})} - R \end{aligned} \quad (3)$$

where $\boldsymbol{\sigma}^{\mathrm{d}}$ represents the deviatoric part of $\boldsymbol{\sigma}$ and $\bar{\sigma}$ is the equivalent stress. $\mathbf{X}$ and R represent the back-stress tensor and the isotropic work-hardening respectively. The plastic component of the strain follows a flow rule derived from a plastic potential Ω which is a power function of the yield function (Lemaitre and Chaboche [6]):

$$\Omega(f) = \frac{K^{\mathrm{pl}}}{n^{\mathrm{pl}} + 1}\left(\frac{f^+}{K^{\mathrm{pl}}}\right)^{n^{\mathrm{pl}}+1} \quad (4)$$

where n^{pl} is the strain rate sensitivity coefficient, K^{pl} a weight coefficient of the plastic part of the stress and f^+ the positive part of f. The behaviour is elastic for $f \leq 0$ and plastic for $f > 0$. The plastic strain-rate is written as:

$$\dot{\boldsymbol{\varepsilon}} = \frac{\partial\Omega}{\partial\boldsymbol{\sigma}} = \Omega^{'}(f)\frac{\partial f}{\partial\boldsymbol{\sigma}} \quad (5)$$

The equivalent plastic strain rate $\dot{\bar{\varepsilon}}^{\mathrm{pl}}$ can be defined from the plastic work conservation principle; *i.e.*

$$\dot{\bar{\varepsilon}}^{\mathrm{pl}} = \frac{(\boldsymbol{\sigma}^{\mathrm{d}} - \mathbf{X}) : \dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}}}{\bar{\sigma}}. \tag{6}$$

The work-hardening combines isotropic and kinematic contributions and the evolution of the isotropic work-hardening is related to the cumulated plastic strain following the swift law given as

$$R = K(\bar{\varepsilon}^{\mathrm{pl}} + \varepsilon_0)^n \quad \text{with} \quad \varepsilon_0 = \left(\frac{\sigma_0}{K}\right)^{1/n} \tag{7}$$

where K is a material parameter, n the hardening coefficient and σ_0 is the initial yield stress. The non-linear evolution law of the kinematic work-hardening is based in the additive combination of a purely kinematic term (linear Ziegler hardening law) and a relaxation term (the recall term), which introduces the nonlinearity. This law is expressed as

$$\mathbf{X} = C\frac{1}{\sigma_0}(\boldsymbol{\sigma} - \mathbf{X})\dot{\bar{\varepsilon}}^{\mathrm{pl}} - \gamma\mathbf{X}\dot{\bar{\varepsilon}}^{\mathrm{pl}} \tag{8}$$

where C and γ are material parameters that must be determined. C is the initial kinematic hardening module, and γ determine the rate at which the kinematic hardening module decreases with the increasing of the plastic deformation [7].

5 NUMERICAL RESULTS AND DISCUSSION

The optimization based on the single-point analysis was achieved with the Sdl optimization software [3]. For the FE analysis the ABAQUS code was integrated with the Sdl optimization program. In ABAQUS two models of experimental specimens were modulated in order to simulate the tensile and the shear tests for both constitutive models here studied. For both specimens the nonlinear effects of large deformations were considered. Considering the symmetry inherent of the tensile tests only a quarter of the specimen was modelled for this test. Therefore, the dimensions considered were $7 \times 30 \times 1$ mm and an equal spaced mesh of 20×50 elements was applied. The specimen subject to the shear test has 4.5x50x1 mm and an equal spaced mesh of 20×200 was applied. For both specimens a 4-node bilinear plane stress quadrilateral element with reduced integration and hourglass control was applied. In Figure 1 it is possible to observe the final meshes for one of the tensile and the shear tests performed. During the optimization process a weighting absolute factor equal to 1 and 2 was defined for the objective function evaluation.The optimization process stops if from one iteration to another the relative decrease of the objective function is less than 1×10^{-30} or if the maximum admissible iteration number, 200, is reached. In table 1, the results obtained with the single point and the FE analysis for both models are presented, respectively. The results presented in table 1 allow to conclude that for the elastic-plastic hardening model the single point analysis leads to more satisfactory results in terms of objective function value. Also considering the CPU time, also the single point leads to better results. For this model it is

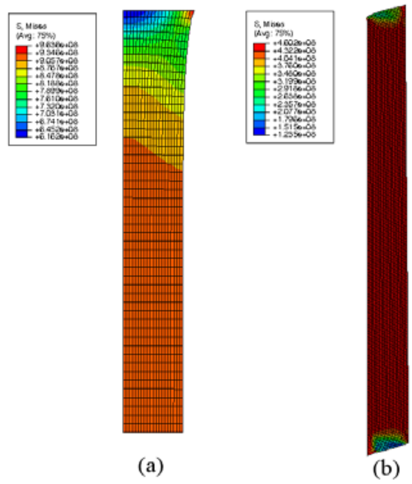

Figure 1: *The finite element mesh and von Mises stress distribution at the end of the (a) tensile and (b) shear test*

Table 1: *Single-point and FE analysis results for the non-linear elastoplastic hardening model.*

Parameters	Starting set	Single-point analysis	FE analysis
$\boldsymbol{\sigma}^0$ [MPa]	310	314.51	320.09
$\boldsymbol{\sigma}^\infty$ [MPa]	700	614.19	680.72
δ	7	7.74	6.71
ζ [MPa]	800	929.84	768.49
$S_{\mathrm{obj}}(\mathbf{x})[\mathrm{MPa}^2]$	-	126.34	141.49
Iterations	-	96	65
CPU [min]	-	0.017	13

Table 2: *Non-linear elastoplastic model with isotropic and kinematic hardening.*

Parameters	Starting set	Single-point analysis	FE analysis
σ_0 [MPa]	180	160.47	188.09
C[MPa]	1000	8613.94	1069.30
γ	28	99.98	28.05
K	550	322.78	547.28
n	0.2	0.20	0.22
$S_{\mathrm{obj1}}(\mathbf{x})[\mathrm{MPa}^2]$	-	1239.41	4877.49
$S_{\mathrm{obj2}}(\mathbf{x})[\mathrm{MPa}^2]$	-	299.94	4744.65
$S_{\mathrm{obj3}}(\mathbf{x})[\mathrm{MPa}^2]$	-	434.91	593.76
$S_{\mathrm{obj4}}(\mathbf{x})[\mathrm{MPa}^2]$	-	539.86	108.24
$S_{\mathrm{obj5}}(\mathbf{x})[\mathrm{MPa}^2]$	-	537.01	1638.35
$\sum S_{\mathrm{obj}}(\mathbf{x})[\mathrm{MPa}^2]$	-	3051.13	11962.49
Iterations	-	189	42
CPU [min]	-	7	480

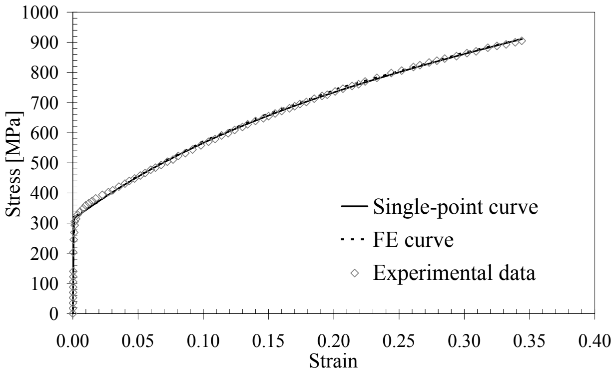

Figure 2: Experimental data and optimized curves for the non-linear elastoplastic hardening model

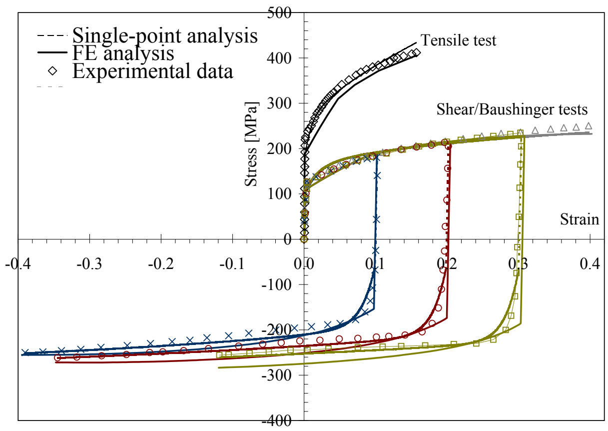

Figure 3: Experimental data and optimized curves for the non-linear elastoplastic model with isotropic and kinematic hardening

considered that the single point analysis is more efficient in terms of objective function value/computational cost relation, but both of the approach leads to good values of the objective function as it is possible to observe in Figure 2. In the case of the elastoplastic model with isotropic and kinematic hardening, it also observed that the single point analysis leads to better results when we consider the error function value and the CPU time. In table 2 is also presented the error function values for each test being, S_{obj1} the error function value for the tensile test, the 2 corresponds to the shear test, 3 the cyclic test with inversion at 0.3 value of deformation, 4 the cyclic test with inversion at 0.2 value of deformation and 5 the cyclic test with inversion at 0.1 value of deformation. In both tests it was for the tensile test that the optimized parameters lead to poor results. Also in this case it is considered that the single point analysis gives good improvements in terms of objective function value when compared with the FE analysis (as it possible to observe in Figure 3) and the CPU time is really advantageous for the single point analysis. The main difference in these analyses is the numerical integration technique. The single point analysis uses a Runge-Kutta explicit method with adaptive step providing error control. The FE analysis applies an Euler implicit method with automatic step, that doesn't guarantee error control. This fact leads to a less objective function sensibility in the FE analysis that takes to a premature stagnation. Table 1 and 2 evidence this fact, being the number of iterations lesser for the FE analysis even with a greater function error.

6 CONCLUSIONS

A comparative study between the use of the single point and the FE analysis in the parameters determination problems was performed. The constitutive models studied were a non-linear elastoplastic hardening model and a non-linear elastoplastic model with isotropic and kinematic hardening. For both constitutive models the single-point analysis was considered more efficient in terms of objective function value/computational cost relation.
It was considered that both of the strategies presented can be applied depending on the studied problem. The FE analysis allows the user to know all the history deformation of a complex geometry and predict geometrical phenomena such as necking, springback and stress concentration. In the problems where the geometrical phenomena doesn't exist the single-point is more appropriate considering the good relation between the CPU time and the objective function values.

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REFERENCES

[1] J. Ponthot, J. Kleinermann, A cascade optimization methodology for automatic parameter identification and shape/process optimization in metal forming simulation, Comp. Methods in Applied Mechanics and Engineering.Vol 195 (41-43), 5472-5508, 2006.

[2] A. Andrade-Campos, P. Pilvin, F. Teixeira-Dias, J. Simões, Determinação dos parâmetros constitutivos atraves de metodologias de optimização em cascata, Proc. of CMNE/CILAMCE, Portugal, a1063, 2007.

[3] R. de-Carvalho, R.A.F. Valente and A. Andrade-Campos, Optimization Strategies for Non-Linear Material Parameters Identification in Metal Forming Problems, Proc. of ESMC2009,7-11 Setembro, IST, Lisbon, 2009.

[4] A. Andrade-Campos, P. Pilvin, J. Simões, F. Teixeira-Dias, Software development for inverse determination of constitutive model parameters, Software Engineering: New Research, Nova Science Publishers, Inc .

[5] S. Thuillier, P. Manach, Comparison of the work-hardening of metallic sheets using tensile and shear strains paths, International Journal of Plasticity, 733-751, 2009.

[6] J. Lemaitre, J. Chaboche, Mécanique des matériaux solides, Editions Dunod, Paris-France, 1996.

[7] Abaqus, 2007. Abaqus v6.7 (2007) Simulia Inc, Dassault Systémes