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STOCHASTIC ANALYSIS OF A DEEP DRAWING PROCESS USING FINITE ELEMENT SIMULATIONS

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ABSTRACT: A deep drawing process optimization procedure using finite element simulations is described in this paper. The cross tool geometry is used to demonstrate the procedure and its application to robust design. Taguchi´s fractional factorial design of experiments is applied to plan for the numerical simulations. Stochastic variations in three process parameters were considered and its impact on the part quality is determined using analysis of variance (ANOVA). The relative contribution of each process parameter is determined by using ANOVA. Contact friction conditions have a major role in the deep drawing of cross geometry compared to blank holder force and blank shape. The outcome of the process for a combination of the process parameters can be found from the reliability optimization procedure.

KEYWORDS: Deep drawing, Robust design, Design of experiments, FEM.

1. INTRODUCTION

Generally, the process optimization results in a combination of process parameters that produces parts at the brink of the safe domain. Any small variation in the process parameters can have negative impact on the part quality. With the advent of newer materials, this problem has increased multi-fold due to lack of comprehensive knowledge on the material behavior. Therefore, numerical simulation studies in combination with statistical analysis tools have become the choice of engineers to design a process in order to manufacture reliable products [1, 2]. Taguchi method utilizes an orthogonal array that is a form of fractional factorial design containing representative set of all possible combinations of experimental conditions [3]. This significantly reduces the computational cost as only 9 experiments are required to represent a three factor-three level experimental design which normally needs 27 experiments. The outcome of this design of experiments can be analyzed using statistical tools to determine the contribution of each process parameter to the product quality [4]. The reliability of the process outcome for a process parameter combination can be found from reliability assessment procedure [5]. A robust design procedure using numerical simulation and statistical analysis tools is described in this paper. The study focuses on the optimization problem including three process parameters namely, the blank shape, the blank holder force and the contact friction condition between the tools and the blank. The reliability optimization is carried out on a cross tool using thickness distribution. The objective is to determine the domain at which the thickness variation can be minimized to predict the reliability of the process with that combination of process parameters.

2. DEEP DRAWING SIMULATIONS

Numerical simulation studies provide valuable insight on the deformation pattern of the blank under various process conditions. At low CPU cost, a range of forming parameters can be virtually tried and the optimum values can be chosen. In this study, the deep drawing simulations on a cross geometry were performed using DD3IMP, an in-house FEA code [6]. The cross tool geometry, as shown in figure 1, is used because it develops complex stress state as it flows into the die [7].

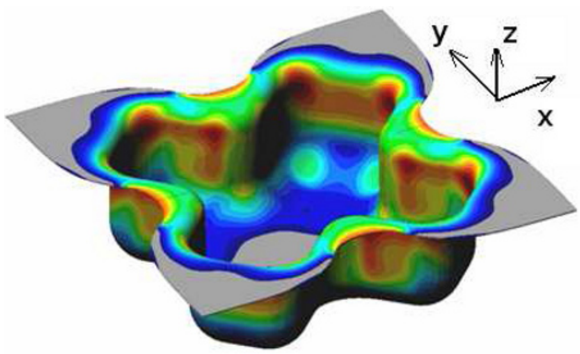

Figure 1: *Cross tool geometry*

Mild steel (DC06) was chosen for this study. The work-hardening behavior is considered isotropic and described by swift power law with the plastic anisotropy described by Hill48 quadratic yield criterion [8]. The elastic properties are: Young's modulus E = 210 GPa, Yield

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stress Y = 123.6 MPa, and Poisson ratio ν = 0.3. The hardening parameters are: strength coefficient K = 529.5 MPa, and strain hardening exponent n = 0.268. Only a quarter of the geometry is considered for the simulations due to geometrical symmetry. A blank of 1 mm thickness was meshed with one layer of 3D solid finite elements with an in-plane mesh size of 2 mm.
The blank shape (BS), the blank holder force (BHF) and the contact friction condition (μ) between the tools and the blank, are the parameters used to optimize the deep drawing process. As shown in table 1, three levels of the three parameters were chosen to capture non-linearity. Adequate care should be taken while choosing the range of process parameters. The lower level of the blank holder force is chosen based on the wrinkling threshold, while friction coefficient is chosen between the limits used for numerical simulations and that generally observed in industrial lubrication condition.

Table 1: *Process parameters and their chosen levels*

Level \ Parameter	BS	BHF	μ
1	S 1	60 kN	0.04
2	S 2	72.5 kN	0.08
3	S 3	85 kN	0.12

Three blank shapes were used in the study to determine its influence on the thickness distribution. A square blank of size 125x125 mm (S1), a blank to achieve a flange width of 15 mm after deep drawing (S2) and a third blank to achieve square flange with rounded corner after deep drawing (S3) as shown in figure 2. These three blank shapes are significantly different and cause different flow pattern during the deep drawing process. However, the objective of this study is to minimize the thickness variation without particular implication on the flange contour.

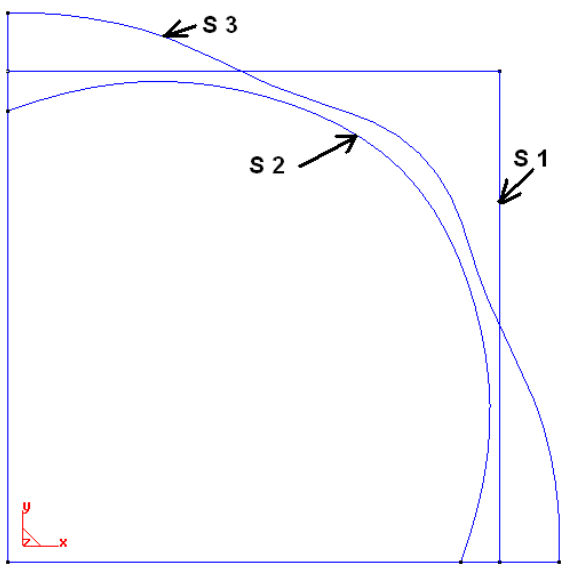

Figure 2: *Blank shapes used in the study*

The information on the main effects can be obtained by running 3^3 = 27 experiments. However, the appropriate Taguchi orthogonal array for the above three factors with three levels is L9, as shown in table 2, to conduct nine simulations. The first column represents the number of simulation and subsequent columns represent the process parameters and the rows represent simulations with the levels of each parameter.

Table 2: *Taguchi experimental design*

Simulation \ Parameter	BS	BHF	μ
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

3. RESULTS

3.1 THICKNESS DISTRIBUTION

Thickness distribution is one of the quality characteristics of a formed part. Minimizing variation in the thickness leads to improved part quality. Figure 3 shows the thickness variation along OX axis in the deep drawn cross geometry. Depending on the combination of process parameters, the material flow into the die varies and consequently leads to thickness variation along OX, as illustrated by the figure. Large thinning occurred at the wall section while the thickness remained constant at the bottom section. Low levels of the process parameters resulted in smooth flow of material into the die cavity and therefore lesser thinning at the wall (case 1). Low contact friction between tools and the blank resulted in lower thinning even with high blank holder force (case 6). On contrary, high friction condition causes large thinning even with an optimal blank shape and low blank holder force (case 5). Maximum thinning was observed in case 3 where high friction condition, blank holder force and a rectangular blank shape was used.

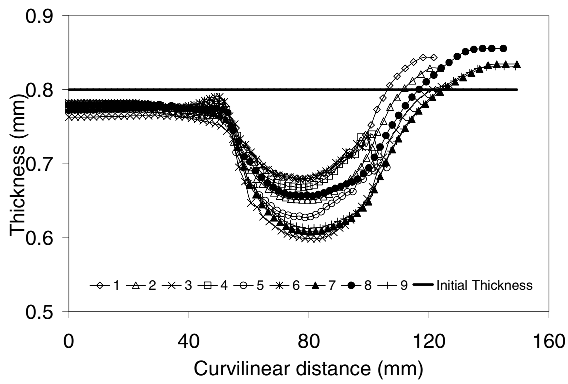

Figure 3: *Thickness distribution along OX axis*

The results presented in figure 3 are local, along OX direction. Considering the complex nature of the cross geometry, the global thickness distribution should be considered to determine the impact of individual process parameter. To establish a robust relationship between these parameters and to determine their influence on the process outcome, the predicted results from the simulations are subjected to statistical analysis. Analysis of variance (ANOVA) is a statistical tool widely used for this type problem.

3.2 ANOVA

Analysis of variance (ANOVA) approach is used to quantify the influence of the process parameters on the product quality characteristics. ANOVA is a statistical tool used to determine the contribution of each parameter on the process outcome. In this study, ANOVA is used to elucidate the parameters that markedly influence the thickness distribution. This will yield information on the impact of each parameter on the results predicted by the numerical simulations. Taguchi recommended a logarithmic transformation of mean square deviation called signal-to-noise ratio (S/N ratio) for analysis of the results.

In this study, the *S/N* ratio is used to measure the thickness deviation. The *S/N* ratio is explained as: −10 log (MS), where

$$MS = \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{Y_i^2} \right) \right) \tag{1}$$

Where n is the number of elements and Y is the value of thicknesses measured at each element. The overall mean *S/N* ratio is expressed as

$$\overline{S/N} = \frac{1}{9} \sum_{i=1}^{9} (S/N)_i \tag{2}$$

The sum of squares due to variation about the overall mean is

$$SS = \sum_{i=1}^{9} \left((S/N)_i - \overline{S/N} \right)^2 \tag{3}$$

For the ith process parameter, the sum of squares due to variation about the mean is:

$$SS_i = \sum_{j=1}^{3} \left((S/N)_{ij} - \overline{S/N} \right)^2 \tag{4}$$

the mean square deviation is calculated from the values of table 3.

First 9 rows depict results obtained for blank shape, three simulations for each level at three levels, totaling 9 simulations. The second set of 9 rows depicts results obtained for blank holder force, three simulations for each level at three levels, totaling 9 simulations. The results are the same as the first set of 9 rows, but the simulations jumbled such that the levels are split according to the process parameter, blank holder force in this case. The third set corresponds to the contact friction conditions, three experiments at three levels. The calculated value for each process parameter is given in column 2 of table 4. The contribution of each process parameters to the product outcome is listed in column 3. For the cross tool geometry, contact friction condition has pronounced influence compared to blank holder force and blank shape. Optimal combination of these process parameters around case 8 should result in a good quality part.

Table 3: *ANOVA data table*

Simulation	*MS*	$(S/N)_i$	$(S/N)_{ij}$
1	0.773290	2.233152	
2	0.756149	2.427852	2.46974
3	0.728768	2.748214	
4	0.757620	2.410971	
5	0.732858	2.699603	2.483346
6	0.763883	2.339463	
7	0.747990	2.522084	
8	0.772658	2.240254	2.418657
9	0.750444	2.493634	
1	0.773290	2.233152	
4	0.757620	2.410971	2.388736
7	0.747990	2.522084	
2	0.756149	2.427852	
5	0.732858	2.699603	2.455903
8	0.772658	2.240254	
3	0.728768	2.748214	
6	0.763883	2.339463	2.527104
9	0.750444	2.493634	
1	0.773290	2.233152	
6	0.763883	2.339463	2.270956
8	0.772658	2.240254	
2	0.756149	2.427852	
4	0.757620	2.410971	2.444153
9	0.750444	2.493634	
3	0.728768	2.748214	
5	0.732858	2.699603	2.656634
7	0.747990	2.522084	
		$\overline{S/N} = 2.457248$	

Table 4: *Contribution of process parameters*

Process parameter	Sum of Squares (SSi)	Contribution %
Blank shape	0.002326	2.7
Blank holder force	0.009576	11.1
Friction condition	0.074631	86.2

3.3 RELIABILITY OPTIMIZATION

Even after identifying optimal combination of process parameters, the stochastic nature of the problem can lead to failure of the deep drawing process. Some of the parameters, such as contact friction condition, material properties of the blank, etc., describing the deep drawing process may possess variability. The influence of this variability in the parameters can be analyzed using the theory of reliability. The aim of reliability optimization is to find a process solution which minimizes part rejection. This requires the assessment of reliability of a process outcome, which is quantified with mean $(\mu_{r,s})$, variance $(\sigma^2_{X_{r,s}})$, probability of failure, as given by the following equations;

$$\sigma^2_{X_{r,s}} = \frac{1}{(n-1)} \sum \left(X_{r,s} - \mu_{r,s}\right) \qquad (5)$$

Where X_r and X_s are non-negative independent random variables. X_r and X_s denote thinning and thickening in the elements, respectively. X_r and X_s are normal distributions with mean $(\mu_{r,s})$ and standard deviation $(\sigma_{X_{r,s}})$, respectively.

Probability of failure is:

$$P_f = \phi(-\beta) \qquad (6)$$

Where β is the reliability index given by,

$$\beta = \frac{\mu_r - \mu_s}{\sqrt{\sigma^2_{X_r} + \sigma^2_{X_s}}} \qquad (7)$$

The reliability of the proposed solution can be found from;

$$R = 1 - P_f \qquad (8)$$

Thus the reliability of the formed parts can be found out from above expressions. For the parts produced from the 9 simulations of Taguchi experimental design, the calculated probability of failure and consequently the reliability are listed in table 5.

Table 5: *Reliability estimation for the 9 simulations*

Simulation	Reliability
1	0.9330
2	0.8180
3	0.8527
4	0.9413
5	0.9232
6	0.9442
7	0.9362
8	0.9626
9	0.9386

It is evident from the list that combination of the process parameters used in simulations 2 and 3 is likely to produce defective part. The most optimal process parameter combination is in case 8, hence choosing the three process parameters close to this range will yield a reliable product.

4. CONCLUSIONS

A robust design procedure is described in this paper using complex cross geometry. The optimization procedure includes finite element simulations, statistical analysis tool and reliability assessment technique. Taguchi orthogonal experimental design was used to plan for the simulations and the predicted results were analyzed using ANOVA technique. The contribution by each parameter for the process outcome is determined and contact friction condition was found to impact more on the product quality. Blank holder force and blank shape have nominal influence on the process outcome for the cross geometry. The reliability of the process parameter combination was assessed. As a result, case 8 with low friction coefficient, medium blank holder force and an S3 blank shape gives the most reliable deep drawn cross geometry part.

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