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A viscous pressure bulge test for the determination of a plastic hardening curve and equibiaxial material data

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Abstract The importance of an accurate material modeling for the accuracy and reliability of sheet forming simulations has become increasingly evident during the last years. More advanced material models have, however, to be supported by novel methods for material characterization. The recent eight parameter yield functions Yld2000-2d and BBC2003 demand, besides data from the ordinary uniaxial tensile tests, also equibiaxial data. In the present paper a Viscous Pressure Bulge (VPB) test is described. The test yields the equibiaxial stress point and r-value, as well as a plastic hardening curve for large values of plastic strain. The test setup is based on an ARGUSS™ optical measuring system, and provides the desired result data in a very smooth and easy way. In order to verify the results from the current test,

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comparisons have been made with compression tests performed at Corus RD&T and hydraulic bulging tests performed at RWTH in Aachen. A discussion on how to determine the equibiaxial yield stress and how to transform the biaxial stress-strain curve to an effective stress-strain curve is included in the paper.

Keywords Viscous pressure bulge test . Equibiaxial yield stress . Stress-strain curve . Sheet metal forming

Introduction

Background

For some years a project has been going on at Volvo Cars, aiming at improving accuracy and reliability of industrial sheet forming simulations by using improved material modeling. Up to now, these efforts have mainly concerned the yield condition and the plastic hardening curve, of which both have been shown to be of vital importance for the accuracy of sheet forming simulations.

Ordinary uniaxial tensile tests can only provide plastic hardening curves up to the point of diffuse necking, which normally occur for logarithmic strains in the range 0.15– 0.25. However, in ordinary sheet forming operations the magnitude of the plastic strains can reach far beyond this range. The traditional way to get around this problem has been to extrapolate the hardening curve up to the levels of the real plastic strains. It has, however (see e.g. Ref. [\[1](#page-7-0)]), been demonstrated that such a procedure can lead to quite erroneous results. To get reliable stress-strain data up to higher levels of plastic strain, one has to rely on tests involving deformation modes in which necking and fracture appears at higher levels of strain than in the uniaxial tensile

test. Examples of such tests are the hydraulic bulging test, the Viscous Pressure Bulge (VPB) test, the compression test, and the shear test.

Another ingredient of the plastic material model, which have a great influence on the accuracy of the forming simulations, is the shape of the yield locus. Two of the current authors have performed a thorough evaluation (Refs. [\[1](#page-7-0)–[4](#page-7-0)]) of existing yield loci models, in order to identify those models, which best satisfy the, sometimes, conflicting demands for accuracy on one hand, and numerical efficiency and easy identification of material parameters on the other. The eight parameter models by Banabic et al. [[5\]](#page-7-0) (BBC2003) and Barlat et al. [[6\]](#page-7-0) (Yld2000-2d) have been found to well fulfill these demands. Besides from three yield stresses and three rvalues from uniaxial tensile tests in three directions, these models also require information about the equibiaxial yield stress and the equibiaxial r-value. The probably most accurate phenomenological model for the yield locus is the Corus-Vegter model, Vegter et al. [\[7](#page-7-0)]. This model involves 17 parameters in the plane stress case and pure shear stress state, which, thus, have to be determined from 9 tests. Among the required test data the equibiaxial yield stress and r-value are included.

There exists, thus, a strong demand for a test method, which is simple and efficient enough to be used as a standard procedure in an industrial environment, and which can provide the desired material data.

The bulge test

One of the oldest and most well known experimental methods for obtaining flow curves for sheet metals at high values of plastic strain is the hydraulic bulge test. In such a test a circular sheet specimen is clamped around its periphery and is subjected to a hydraulic pressure. Knowing the pressure and the curvature at the top of the dome, the equibiaxial stress can be calculated according to

$$
\sigma_{\rm b} = \frac{p R_{\rm b}}{2 t_{\rm b}} \tag{1}
$$

where p is the pressure, R_b is the radius of curvature, and t_b is the thickness at the top of the dome. The strains are normally evaluated using a circular grid method.

From Eq. (1) it is obvious that the radius of curvature has a strong influence on the calculated stress. This quantity can be measured more or less accurate. Many times just the dome height h_d is measured during the lapse of the test. Assuming that the bulge is part of a sphere, the radius of curvature can then be approximated as

$$
R_b \approx \frac{d^2 + 4h_d^2}{8h_d} \tag{2}
$$

where d is the diameter of the periphery of the blank. A more precise measurement of the curvature is obtained, if the radius is determined by means of a three-point coordinate measuring machine in the vicinity of the pole. A procedure like this was used in a recent paper by Lee et al. [[8\]](#page-7-0). At the RWTH, Aachen, two profiles are measured by a laser scan over time, in the rolling direction of the material and perpendicular to it, [[10\]](#page-7-0). The advantage of this method is that the domain of the area in the pole for the determination of the radius of curvature is flexible over time, and that more than three points are involved by using complete sections.

At the institute ERC/NSM in Columbus, Ohio a similar test to the above one, called "the Viscous Pressure Bulge (VPB) test ", has been introduced, Gutscher et al. [\[9](#page-7-0)]. In this test the hydraulic pressure in the hydraulic bulge test is replaced by the pressure from a semi-solid viscous polyurethane punch. The advantage of this method is that the test setup is considerably simplified, and use can be made of an ordinary hydraulic press.

It seems as if the authors of Ref. [\[9](#page-7-0)] have made a great effort to simplify the evaluation of the experimental results, sometimes so much that accuracy is put in the second place. During the test the pressure in the viscous medium and the dome height are recorded. The radius of curvature and the sheet thickness at the pole are determined from a data base created by means of numerical Finite Element simulations. In these simulations an isotropic material model is used, and the hardening curve is assumed to obey a power law ($\sigma = K \varepsilon^n$).

The final results of the evaluation process are a bit confusing. The authors show that the flow curves from uniaxial tension tests and from the VPB tests differ considerably for several different materials. In their conclusions they state: "The study has also shown that the flow stress curves from the tensile and VPB tests can differ considerably …". In reality, these curves should, of course, coincide, and the reason for that this not being the case, could possibly be attributed to a misunderstanding of the concepts of "effective stress" and "effective plastic strain" by the authors.

In the current work the idea of using a viscous pressure medium instead of a hydraulic pressure has been adopted. The measuring technique, and the way to evaluate the experimental results are, however, completely different from the method described above. In the current test setup use is made of a modern optical measuring system, which is described in Chapt. 2. This facilitates a highly accurate and very convenient evaluation of the desired material parameters.

In Chapt. 3 methods for transforming flow curves from various biaxial tests into hardening curves expressed in terms of effective stress and strain variables are discussed. In order to verify the results from the current test, comparisons have been made with results from other

laboratories using other test procedures. Results from these comparisons are accounted for and discussed in Chapt. 4.

Test setup for a viscous pressure bulge test

Introduction

The viscous pressure bulge test is performed in a single action Forming Limit Curve (FLC) testing die. The only modifications of the original die are that a silicone punch and two draw-rings with smaller die openings are used. The die is displayed in Fig. 1 and two silicone punches are shown in Fig. 2. The test sample has a diameter of 200 mm and its periphery is locked with a draw bead. The viscous pressure bulge test can be performed either by repeated tests with gradually increasing bulge height, or by only performing one test to the final height. In both set-ups, the pressure in the silicone punch is measured continuously during the test.

Repeated tests with gradually increasing bulge height

In this set-up the strains and the curvature of the sample are determined after each test. In order to get a number of stressstrain measurements, several tests must therefore be performed with gradually increasing bulge height. In the present study, an optical strain measurement system that measures both the geometry of sample and the strains is used.

Using measured pressures, curvatures and strains, the equibiaxial stress-strain curve can be calculated based on Eq. [\(1](#page-1-0)) after all tests have been performed. The pressure value is the maximum pressure obtained in each test. The radius of curvature and thickness are determined by using measured data inside a sphere with 20 mm diameter, and

Fig. 2 Silicone punches

with its center at the top of the bulge. The radius of curvature, R_b , is determined by least square fitting a second sphere to the chosen measurements. The thickness is determined according to the following steps:

- 1. The average strain on the surface, denoted ε_b , is calculated.
- 2. The average strain in the sheet middle surface is given by

$$
\varepsilon_0 = \left(1 - \frac{t_b}{2R_b}\right) \varepsilon_b \tag{3}
$$

3. Denoting the initial thickness t_0 , we can calculate the current thickness t_b from Eq. (3) and the following relation:

$$
\varepsilon_{t} = -2\varepsilon_{0} = \ln \frac{t_{b}}{t_{0}} \tag{4}
$$

The equibiaxial stress is then calculated according to Eq. [\(1\)](#page-1-0), and the corresponding strain is the absolute value of the logarithmic thickness strain.

The advantages of this approach are that few modifications of the die are needed, and that measuring of the strains and the geometry are easily accomplished. It is not necessary to determine the radius of the bulge with data from the strain measurement system, e.g. the radius could be measured with a Coordinate Measuring Machine (CMM). The major draw-back of the method is that many tests are needed in order to get a sufficient number of stressstrain values.

Continuous strain and geometry measurements

With two cameras mounted on top of the die, see Fig. 1, and the ARAMIS™ strain measurement system, the strains and the geometry of the sample can be measured continuously during the test. After the test the equibiaxial stress-strain curve is determined by a program being a part of the ARAMIS™ system.

The advantages of this set-up are an increased accuracy Fig. 1 The die for the viscous bulge test compared to the method described in "Repeated tests with

[gradually increasing bulge height](#page-2-0)", as well as the fact that only one test is needed to determine the complete equibiaxial stress-strain curve. The draw-backs are that a more complicated die and a more advanced strain measurement system are needed.

Transformation of biaxial stress-strain data into effective values

Introduction

The concept of a yield "yield surface" implies that every point on the surface corresponds to the same value of effective stress and effective plastic strain. The true stresseffective plastic strain relation is usually prescribed to be equal to the uniaxial stress-strain relation in one particular direction, usually the rolling direction. However, a flow curve, determined in any other multi-axial stress/strain state, have to be transformed to an effective stress–plastic strain curve. Theoretically, two flow curves, representing two arbitrary states of stress, should coincide exactly, when both are transformed to effective stress-strain curves. However, in practice there never is such an exact correspondence.

As we are dealing with sheet metal, we will in the following assume that the state of stress is two-dimensional. If a flow curve, representing a biaxial state of stress, is going to be transformed into an effective stress-strain curve, a practical approach is to transform the biaxial stress and strain values in such a way that a "good" correspondence with a uniaxial stress-strain curve is obtained. In fact, we end up in some kind of curve fitting procedure. This curve fitting is of course not unique, and depending on the choice of method, slightly different hardening curves will result.

In the current chapter the background for a theoretically sound transformation of biaxial stress-strain values to effective ones will be outlined.

Transformation of equibiaxial stress-strain data to effective ones

In the case of equibiaxial stress, the rate of plastic work is given by

$$
\dot{W} = \sigma_x \dot{\varepsilon}_x^p + \sigma_y \dot{\varepsilon}_y^p = \sigma_b \left(\dot{\varepsilon}_x^p + \dot{\varepsilon}_y^p \right) = -\sigma_b \dot{\varepsilon}_z^p = \overline{\sigma} \dot{\overline{\varepsilon}}^p \qquad (5)
$$

where σ_b is the equibiaxial yield stress, $\bar{\sigma}$ is the effective stress, and $\dot{\bar{\epsilon}}^{\rm p}$ is the effective plastic strain rate. In Eq. (5) plastic incompressibility has been assumed. Generally, the effective stress can be expressed as

 $\overline{\sigma} = k_b \sigma_b$ (6)

where k_b is a constant scaling parameter. In view of Eqs. (5) and (6) the effective plastic strain rate is found to be

$$
\dot{\bar{\varepsilon}}^{\mathrm{p}} = -\frac{1}{\mathrm{k}_b} \dot{\varepsilon}_z^{\mathrm{p}} \tag{7}
$$

where $\dot{\varepsilon}_z^{\rm p}$ is assumed to be negative (thinning). Integrating Eq. (7) over time, and assuming a proportional strain path, we find

$$
\overline{\varepsilon}^{\mathbf{p}} = -\frac{1}{\mathbf{k}_{\mathbf{b}}} \varepsilon_{\mathbf{z}}^{\mathbf{p}} \tag{8}
$$

If an equibiaxial stress–thickness strain curve from a bulge test is fitted to a uniaxial stress–strain curve using Eqs. (6) and (8), and if the effective stress is defined so that

$$
\overline{\sigma} = \sigma_Y \tag{9}
$$

where σ_Y is the uniaxial yield stress (e.g. in the rolling direction), the following relation holds

$$
\sigma_b = \frac{\sigma_Y}{k_b} \tag{10}
$$

So, to transform an equibiaxial stress-strain curve to an effective stress-strain curve, biaxial stresses are scaled by the parameter k_b according to Eq. (6), and thickness strains are scaled according to Eq. (8). It is also interesting to note that the equibiaxial stress σ_b and the uniaxial stress σ_u are related according to

$$
\frac{\sigma_{b}}{\sigma_{u}} = \frac{1}{k_{b}} \tag{11}
$$

The actual value of the parameter k_b is a result of the procedure of fitting the equibiaxial stress-strain curve to the uniaxial one.

Transformation of stress-strain data from a compression test to effective ones

An alternative method to obtain stress-strain data at high levels of plastic strain is the compression test. In this test stacked specimens are subjected to a through-thickness pressure. The rate of plastic work is in this case given by

$$
\dot{W} = \sigma_z \dot{\varepsilon}_z^{\mathbf{p}} = \overline{\sigma} \dot{\overline{\varepsilon}}^{\mathbf{p}} \tag{12}
$$

The effective stress is assumed to be related to the compressive stress by

$$
\overline{\sigma} = -k_c \sigma_z \tag{13}
$$

were k_c is a constant scaling parameter, and where σ_z is assumed to be negative (compression).

Combining Eqs. ([12\)](#page-3-0) and [\(13](#page-3-0)), we can express the effective plastic strain rate as

$$
\dot{\bar{\varepsilon}}^{\mathrm{p}} = -\frac{1}{\mathrm{k}_{\mathrm{c}}} \dot{\varepsilon}_{\mathrm{z}}^{\mathrm{p}} \tag{14}
$$

or in integrated form

$$
\overline{\varepsilon}^{\mathbf{p}} = -\frac{1}{\mathbf{k}_{\mathbf{c}}} \varepsilon_{\mathbf{z}}^{\mathbf{p}} \tag{15}
$$

From the above equations, and assuming the effective stress to be equal to the uniaxial yield stress, we get the following relation

$$
\sigma_z = -\frac{\sigma_Y}{k_c} \tag{16}
$$

Comparing Eqs. $(8-10)$ $(8-10)$ $(8-10)$ $(8-10)$ with Eqs. $(13-16)$ $(13-16)$, we note that there is a direct correspondence if we put $k_c = k_b$ and σ_b = - σ_z . This means that a stress-strain curve (σ_b - ε_z) from a bulge test can be directly compared with a stress-strain curve $(\sigma_z - \varepsilon_z)$ from a compression test.

Methods for transforming a biaxial stress-strain curve into an effective one

Introduction

As mentioned previously, a practical way to determine an effective stress-strain curve from a biaxial one is to scale and fit the biaxial curve to a uniaxial one. In fact, we are, in some meaning, trying to determine an optimal value of the parameter k_b introduced in the previous chapter. In the current chapter a few different methods for performing this transformation will be described and discussed. As there does not exist any unique "best fit" of the two curves, different methods will yield slightly different results. In the end, the choice of method is mainly a matter of taste.

Methods based on the principle of work equivalence

The plastic work per unit volume can in the case of uniaxial, proportional loading be written

$$
W_u = \int \!\! \sigma_u d\varepsilon_u^p \tag{17}
$$

The plastic work in case of a proportional, balanced biaxial loading is according to Eq. [\(5](#page-3-0)) given by

$$
W_b = -\int \sigma_b d\varepsilon_z^p \tag{18}
$$

When the equality $W_u = W_b$ is prevailing, the two stress/ strain states correspond to the same effective stress and effective strain. This fact forms the basis for some slightly different methods for determining the parameter k_b defined above.

Assume that we for a uniaxial tensile test establish a table of values relating the plastic work W_{ν} , according to Eq. (17), and stress σ_u , and for a bulge test establish a table of values relating plastic work W_b and stress σ_b . For a certain value of plastic work we can, thus, determine two corresponding stress values σ_u and σ_b and $k_b = \sigma_u / \sigma_b$. Theoretically, the value of k_b should be constant and independent of the value of the plastic work. However, in practice there is always a certain variation depending on the choice of this value. Below, three methods for determining the parameter k_b are outlined. The main difference between them is the choice of the value of plastic work for which k_b is evaluated.

Method 1

The parameter k_b is evaluated at the point corresponding to the maximum values of stress and strain in the uniaxial tensile test. The method is illustrated graphically in Fig. 3.

Method 2

The parameter k_b is evaluated at an arbitrarily chosen value of plastic work in the range $0 \leq W_u \leq W_u^{max}$ (or in the strain range $0 \leq \varepsilon_{\rm u} \leq \varepsilon_{\rm u}^{\rm max}$).

Fig. 3 Relations between plastic work and stress in the uniaxial case and in the equibiaxial case, respectively

The parameter k_b is evaluated for a large number of points in the W/σ tables, and is calculated as the average value, i.e.

$$
k_b = \frac{1}{n} \sum_{i=1}^{n} \frac{\sigma_{ui}}{\sigma_{bi}}
$$
\n(19)

This approach was used by Lee et al. [\[8](#page-7-0)]

A method based on scaling and fitting of curves

The method that will be discussed here is used at Volvo Cars and have previously been described in Mattiasson and Sigvant [[1\]](#page-7-0).

Method 4

The stress-strain curve from the bulge test is scaled so that the curve in one point coincides with the last point of the uniaxial curve. This fitting of curves is performed in an iterative procedure. The method has a couple of obvious advantages compared to ones based on work equivalence above:

- The hardening curve, calculated from biaxial stressstrain data, will be a direct continuation of the uniaxial curve. In practise, the total hardening curve used in e.g. a FE-simulation will consist of two parts: First the uniaxial curve, and then the hardening curve based on biaxial data.
- The biaxial stress-strain data for low values of plastic strain, which involves a considerable amount of error, are never used.

Fig. 5 Relations between stress and plastic work in a uniaxial tensile test and a bulge test, respectively. Material DX56DZ

A numerical example

The above four methods for determining the parameter k_b will be applied to a mild steel named DX56DZ. The stress-strain curves from a uniaxial tensile test and from a VPB test are shown in Fig. 4. In Fig. 5 the relations between plastic work and uniaxial and equibiaxial stresses, respectively, are displayed, and in Fig. 6 the parameter k_b is displayed for different values of plastic work. In Fig. [7,](#page-6-0) finally, a hardening curve obtained according to Method 4 above is shown. In Table [1](#page-6-0) the resulting values of k_b according to the different methods are shown. As can be seen there is a variation between 0.8260 and 0.8443 for k_b . This corresponds roughly to a variation of $\pm 1\%$ round the mean value.

Fig. 4 Stress-strain relations from a uniaxial tensile test and a bulge test, respectively. Material DX56DZ

Fig. 6 The parameter $k_b = \sigma_u / \sigma_b$ as a function of plastic work.

Material DX56DZ

Table 1 The parameter k_b determined according to different methods. Material DX56DZ

k _b			
Method 1 (ε_u^{\max})	Method 2 ($\varepsilon_{\rm u}$ = 0.10)	Method 3 (average)	Method
0.8260	0.8443	0.8372	0.8269

Fig. 7 The stresses and strains from the bulge test are scaled and fitted to the uniaxial stress-strain curve (method 4 above, k_b =0.8269). Material DX56DZ

Fig. 8 Biaxial and uniaxial stress-strain curves for the DX56DZ material

Fig. 9 Biaxial and uniaxial stress-strain curves for the DP600 material

Fig. 10 Biaxial and uniaxial stress-strain curves for the AA5052 material

Table 2 The parameter k_b , determined according to Method 4 above, for different test methods and materials

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A comparison between different test methods

Introduction

In order to evaluate the quality of the results from the VPB tests, comparisons have been made with two other test methods performed at two other laboratories. The first of these tests is a compression test performed at Corus RD & T in the Netherlands, and the second one is a hydraulic bulge test performed at RWTH in Aachen, Germany. Three different sheet materials have been tested: A mild steel DX56DZ, a dual phase steel DP600, and, finally, an aluminum alloy AA5052.

Results from different test methods

The biaxial stress-strain curves obtained in the three different tests are shown in Figs. [8](#page-6-0), [9](#page-6-0), [10](#page-6-0) for the DX56DZ, DP600, and AA5052 materials, respectively. The k_b parameter has been calculated according to Method 4 above for every biaxial stress-strain curve. The results are presented in Table [2.](#page-6-0) As can be seen from the figures and from Table [2](#page-6-0) there are some differences in the measured stress-strain curves. However, considering the fact that these curves were obtained with different test methods and at different laboratories the discrepancies are within acceptable limits. Nor is it possible to trace any systematic deviations in the results.

Judging from these results the conclusion can be drawn that the current VPB test procedure is able to produce results with an accuracy, which is comparable with alternative test methods.

Conclusions

In the present paper a VPB test has been described, based on a modern optical measuring system. The current test setup has been shown to able to produce results in a fast and convenient way and with good accuracy. The primary results for this test are a plastic hardening curve for relatively large effective strains, together with an equibiaxial yield stress and an equibiaxial r-value.

Furthermore, it has been demonstrated how biaxial stress-strain data should be transformed to corresponding effective data in a theoretically consistent way. Various practical methods for determining effective stress-strain curves have also been discussed. Finally, results from a comparative study are presented, in which stress-strain curves for three different materials have been determined by three different test methods at three different laboratories. It has been demonstrated that the results from the current VPB test correlate well with the results from the other two test methods. This indicates that the present VPB test is a good alternative to the hydraulic bulge test and the compression test.

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References

- 1. Mattiasson K, Sigvant M (2004) Material characterization and modeling for industrial sheet forming simulations. In: Ghosh, Castro, Lee (eds) Proceedings of NUMIFORM 2004, AIP Conference Proceedings 712
- 2. Mattiasson K, Sigvant M (2005) On The Influence Of The Yield Locus Shape In The Simulation Of Sheet Stretch Forming. In: Smith LM et al. (eds) Proceedings of NUMISHEET 2005, AIP Conference Proceedings 778
- 3. Mattiasson K, Sigvant M (2005) On the choice of yield criterion for industrial sheet forming simulations. Proceedings of IDDRG 2005, Besancon, France
- 4. Mattiasson K, Sigvant M (2006) New non-quadratic yield locus models and their influence on the accuracy of the FEA. In: Hora and Krauer (eds) Proceedings of FLC-Zurich 06, Numerical and experimental methods in prediction of forming limits in sheet forming and tube hydroforming processes, Zurich, Switzerland, pp. 61–63
- 5. Banabic D, Aretz H, Comsa DS, Paraianu L (2005) An improved analytical description of orthotropy in metallic sheets. Int J Plasticity 21:493–512
- 6. Barlat F, Brem JC, Yoon JW, Chung K, Dick RE, Lege DJ, Pourboghrat F, Choi S-H, Chu E (2003) Plane stress yield function for aluminium alloy sheets–Part I: theory. Int J Plasticity 19:1297–1319
- 7. Vegter H, van den Boogaard AH (2006) A plane stress yield function for anisotropic sheet material by interpolation of biaxial stress states. Int J Plasticity 22:557–580
- 8. Lee M-G, Kim D, Kim C, Wenner ML, Wagoner RH, Chung K (2005) Spring-back evaluation of automotive sheets based on isotropic-kinematic hardening laws and non-quadratic anisotropic yield functions – Part II: characterization of material properties. Int. J. Plasticity 21:883-914
- 9. Gutscher G, Wu HC, Ngaile G, Altan T (2004) Determination of flow stress for sheet metal forming using the viscous pressure bulge (VPB) test. J Mater Process Technol 146:1–7
- 10. Blumbach M, Bleck W, Noll R, Vrenegor J (2005) Online Fließkurvenermittlung im hydraulischen Tiefungs-versuch mit Hilfe des Laserlichtschnittverfahrens, 20. Aachener Stahlkolloquium, Aachen (in German)