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Towards modelling of Free Form Extrusion: analytical solution of transient heat transfer

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ABSTRACT: The performance of parts produced by Free Form Extrusion (FFE) may be limited by poor mechanical properties, due to poor bonding between the individual extruded filaments. In this work, an analytical solution is proposed for the transient heat transfer during filament deposition, taking into account contacts between filaments. The solution is inserted in a code developed using Matlab®. This tool allows the study of the influence of the main process variables during filament deposition and may assist process optimization.

Key words: Free Form Extrusion, Heat Transfer, Modelling

1 INTRODUCTION

Free form extrusion (FFE) is a 3D fabrication process that evolved from rapid prototyping technology. It consists in the deposition of an extruded polymeric filament following a specific trajectory in the X–Y plane (according to the geometry of the part to be manufactured), the process being repeated for the required number of layers (implying a displacement of the extruder or support table in the Z-direction). FFE exhibits interesting possibilities, such as the production of relatively simple parts without use of moulds, and if necessary using advanced polymer systems or composites.

Effective bonding between adjacent filaments is mandatory for making parts exhibiting adequate mechanical performance. Thus, each filament must be sufficiently hot when deposited (which is facilitated by keeping the global system in a heated enclosure), but not too hot, otherwise deformation due to gravity may be excessive.

Therefore, it is important to know the evolution of the temperature of the filaments with time and how this is affected by major process variables (e.g., melt temperature at the die outlet, extrusion rate, surrounding temperature). Some work has been done previously on this topic. Rodriguez [1] studied cooling of five elliptical filaments deposited on top of each other via finite element methods (FEM) and later found a 2D analytical solution for rectangular cross-sections [2]. Yardimci *et al.* [3] developed a more general 2D heat transfer analysis model also using FEM. Li and co-workers [4, 5] developed an analytical 1D transient heat transfer analysis of a single filament, using the Lumped Capacity method. Though good agreement with experimental results was reported, the model cannot be used for a sequence of filaments, as thermal contacts are ignored.

The present work expands the above efforts, by aiming at developing a transient heat transfer analysis of filament deposition taking into account the effect of the main FFE variables, particularly that of the physical contact between any filament and its neighbours or supporting table.

2 HEAT TRANSFER ANALYSIS

Regardless of the shape and deposition sequence, during the creation of a part by FFE each of its individual filaments is subjected to the same heat transfer mechanism, but the boundary conditions vary. As shown in the example of Figure 1, one may have to consider heat flux with the support, with the surrounding environment, with colder filaments (deposited before) and with younger hotter filaments (deposited afterwards).

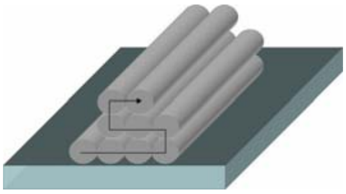

Fig. 1. Possible sequence of filaments deposition.

The energy balance for an element dx of a filament writes as:

$$\left\{\begin{array}{l}\textit{Energy out of one face} - \textit{Loss of heat by convection with environment} \\ - \textit{Loss of heat by conduction with the adjacent filaments or with the support}\end{array}\right. \\ = \textit{Change in internal energy} + \textit{Energy in the opposite face}$$

Given the low thermal conductivity of polymers and the small filament radius, both radial and axial heat conduction are negligible [4, 5]. Therefore, the energy equation becomes:

$$\rho ALC\frac{\partial T}{\partial t} = -h_{conv}A_{conv}\left(T(t)-T_E\right) - \phi\, h_{conv}A\left(T(t)-T_E\right) - \\ -\sum_{i=1}^{5} h_i A_i \left(T(t)-T_i\right) \qquad (1)$$

where ρ is density, A is filament cross-section area, L is filament length, C is heat capacity, h_{conv} is heat transfer coefficient, h_i is thermal contact conductance for contact i $(i \in \{1,2,3,4,5\})$, T_E is environment temperature, T_i is temperature of the adjacent filament at contact i, ϕ is defined by:

$$\phi = \begin{cases} 1 & \textit{if we consider the first filament} \\ 0 & \textit{otherwise} \end{cases} \qquad (2)$$

A_{conv} is area exposed to environment and A_i is area of contact i (Figure 2), given by:

$$A_i = \lambda_i \times P \times dx, \qquad A_{conv} = \left(1-\sum_{i=1}^{5}\lambda_i\right)P\,dx \qquad (3)$$

In this expression, P is the filament perimeter and λ_i is the fraction of P that is in contact with another, or with the support.

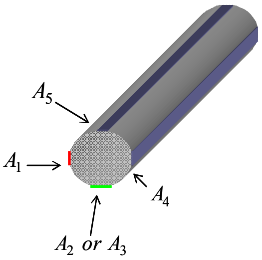

Fig. 2. Possible contact areas of one filament. A_1, A_3, A_4, A_5, areas of contacts 1,3,4,5 with adjacent filaments; A_2 area of contact 2 with the supporting table

Equation (1) can be solved analytically using the characteristic polynomial method to yield the evolution in time of the filament temperature:

$$T(t) = C_1 e^{\left(\frac{-P\,b(a_1,\ldots,a_5)}{\rho AC} - \frac{h_{conv}\phi}{\rho LC}\right)(t-t_0)} + Q(a_1,\ldots,a_5) + \\ + \frac{h_{conv}\phi\ A\ T_E}{PL\ b(a_1,\ldots,a_5)} \qquad (4)$$

T_0 is filament temperature at t_0 and C_1 is defined as:

$$C_1 = T_0 - Q(a_1,\ldots,a_5) - \frac{h_{conv}\phi\ A\ T_E}{PL\ b(a_1,\ldots,a_5)} \qquad (5)$$

Functions $b(a_1,\ldots,a_5)$ and $Q(a_1,\ldots,a_5)$ depend on contacts:

$$b(a_1,\ldots,a_5) = h_{conv}\left(1-\sum_{i=1}^{5}a_i\lambda_i\right) + \sum_{i=1}^{5}a_i h_i \lambda_i \qquad (6)$$

$$Q(a_1,\ldots,a_5) = \frac{h_{conv}\left(1-\sum_{i=1}^{5}a_i\lambda_i\right)T_E + \sum_{i=1}^{5}a_i h_i \lambda_i T_i}{b(a_1,\ldots,a_5)} \qquad (7)$$

where:

$$a_i = \begin{cases} 1 & \textit{if contact i exists} \\ 0 & \textit{otherwise} \end{cases}, \forall i = \{1,\ldots,5\} \qquad (8)$$

Equation (4) is valid if the Biot number, $B_i < 0.1$, with:

$$B_i = \frac{A}{P}\frac{b(a_1,a_2,a_3,a_4,a_5)}{k} \qquad (9)$$

The effect of position x on temperature can be included if t_0 in equation 4 is replaced by extrusion velocity, v, and filament length, L.

When the filament under study contacts filament m, instant t_0 is defined by:

$$t_0(m)=\begin{cases}(mL-x)/v, & \text{if } m \text{ is odd}\\ ((m-1)L+x)/v, & \text{if } m \text{ is even}\end{cases}, m\in\{1,...,n_T\} \quad (10)$$

where n_T is total number of filaments. Time t_0 is up-dated every time a new contact occurs at x. To compute temperature evolution at this point, equation 4 must be rewritten for each filament. Consequently, one needs to know the number of filaments at each layer, as well as the deposition sequence, as they define the succession of contacts for each filament.

For example, for the structure and deposition order depicted in Figure 3, the expression for temperature evolution of filament 1, obtained from equations (4) and (10), is:

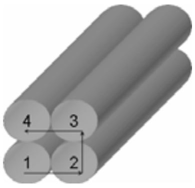

Fig. 3. Example of sequence of deposited filaments.

$$T_1(x,t)=\begin{cases} T_L,\ For\ 0\le t\le \dfrac{L-x}{v} \\ C_1e^{\left(\frac{-P\ b(0,1,0,0,0)}{\rho AC}-\frac{h_{conv}\phi}{\rho LC}\right)\left(t-\frac{L-x}{v}\right)}+Q(0,1,0,0,0)+ \\ +\dfrac{h_{conv}\phi\ A\ T_E}{PL\ b(0,1,0,0,0)}, for\ \dfrac{L-x}{v}<t\le\dfrac{L+x}{v} \\ C_1'e^{\left(\frac{-P\ b(0,1,0,1,0)}{\rho AC}-\frac{h_{conv}\phi}{\rho LC}\right)\left(t-\frac{L+x}{v}\right)}+Q(0,1,0,1,0)+ \\ +\dfrac{h_{conv}\phi\ A\ T_E}{PL\ b(0,1,0,1,0)}, for\ \dfrac{L+x}{v}<t\le\dfrac{3L-x}{v} \\ C_1''e^{\left(\frac{-P\ b(0,1,0,1,1)}{\rho AC}-\frac{h_{conv}\phi}{\rho LC}\right)\left(t-\frac{3L-x}{v}\right)}+Q(0,1,0,1,1)+ \\ +\dfrac{h_{conv}\phi\ A\ T_E}{PL\ b(0,1,0,1,1)}, for\ t>\dfrac{3L-x}{v} \end{cases} \quad (11)$$

Where $\phi=1$ and:

$$C_1=T_L-Q(0,1,0,0,0)-\frac{h_{conv}\phi\ A\ T_E}{PL\ b(0,1,0,0,0)} \quad (12)$$

$$C_1'=C_1e^{\left(\frac{-P\ b(0,1,0,0,0)}{\rho AC}-\frac{h_{conv}\phi}{\rho LC}\right)\left(\frac{L+x}{v}-\frac{L-x}{v}\right)}+Q(0,1,0,0,0)+ \\ +\frac{h_{conv}\phi\ A\ T_E}{PL}\left(\frac{1}{b(0,1,0,0,0)}-\frac{1}{b(0,1,0,1,0)}\right)-Q(0,1,0,1,0) \quad (13)$$

$$C_1''=C_1'e^{\left(\frac{-P\,b(0,1,0,1,0)}{\rho AC}-\frac{h_{conv}\phi}{\rho LC}\right)\left(\frac{3L-x}{v}-\frac{L+x}{v}\right)}+Q(0,1,0,1,0)+ \\ +\frac{h_{conv}\phi\ A\ T_E}{PL}\left(\frac{1}{b(0,1,0,1,0)}-\frac{1}{b(0,1,0,1,1)}\right)-Q(0,1,0,1,1) \quad (14)$$

A computer code for heat transfer in FFE was developed following the aforementioned solution procedures using Matlab® software.

3 RESULTS

Consider, as a case study, the structure and corresponding deposition outline presented in figure 4. The FFE variables and material properties are presented in Table 1.

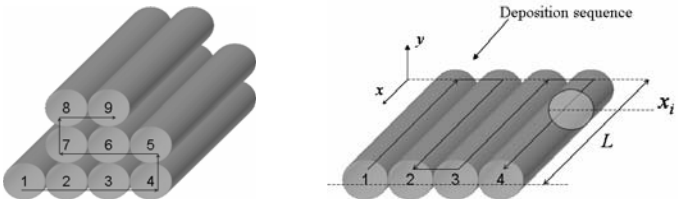

Fig. 4. Example of sequence of filaments deposition

Figure 5 shows the temperature evolution during deposition, at point $x=0.05$ m for all filaments. Every time a new filament is deposited, the temperature of the adjacent filaments increases and so their rate of cooling decreases. Generally, the temperature field depends on the characteristics of the thermal contacts.

The importance of including the contact between any filament and its neighbours during cooling is illustrated in Figure 6. Filament 1 (Figures 4 and 5) was selected for this purpose, giving rise to the curves of Figure 6, where the curve for filament 1 of Figure 5 was reproduced, the second curve being created by ignoring those contacts. As expected, before deposition of the second filament both curves are identical, but then onwards differences are evident, the simpler analysis ignoring the periodic reheating effects. The magnitude of these effects will depend on extrusion rate and part geometry (they influence the length of thermal contacts).

Table1. FFE variables and material properties:

Property	**Value**
Extrusion temperature (ºC)	270
Environment temperature (ºC)	70
Extrusion velocity (m/s)	0.025
Filament length (m)	0.1
Cross section diameter (m)	0.000359
Fraction of perimeter for contact at right side	0.2
Fraction of perimeter for remaining contacts.	0.1
Heat transfer coefficient (convection) (W/m^2ºC)	70
Thermal contact conductance with filaments (W/m^2ºC)	300
Thermal contact conductance with support (W/m^2ºC)	70
Thermal conductivity (W/mºC)	0.1768
Specific heat (J/kgºC)	2019.7
Density	1.05

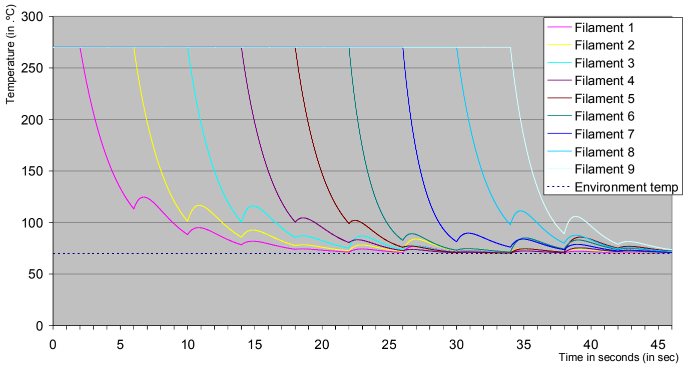

Fig. 5. Temperature evolution along deposition time.

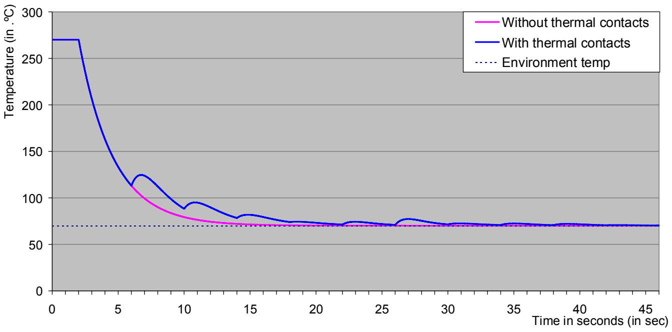

Fig. 6. Prediction of the cooling of one filament with and without thermal contacts

4 CONCLUSIONS

This work proposed an analytical solution for the problem of modelling transient heat transfer during filament deposition in free form extrusion. The code developed considers the effects of the main process parameters, such as extrusion velocity, filament dimensions and material, sequence of deposition, environment temperature.

The preliminary results show that contact characteristics between adjacent filaments have a significant impact on temperature evolution and, consequently, on bonding. Therefore, use of the code can support the definition of the most adequate operating parameters (namely, deposition sequence) for the manufacture of a specific part.

REFERENCES

1. J. F. Rodriguez, Modelling the mechanical behaviour of fused deposition acrylonitrile-butadiene-styrene polymer components, *Ph.D. Dissertation*, Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, USA (1999).
2. J.P. Thomas and J.F. Rodríguez, Modeling the fracture strength between fused deposition extruded roads, *Solid Freeform Fabrication Symposium Proceeding*, Austin, TX (2000).
3. M. A. Yardimci and S.I. Guceri, Conceptual framework for the thermal process modelling of fused deposition, *Rapid Prototyping Journal*, 2 (1996), 26-31.
4. L. Li, Q. Sun, C. Bellehumeur, and P. Gu., Modeling of bond formation in FDM process, *Trans. NAMRI/SME*, 8, 613-620 (2003).
5. Q. Sun, G. Rizvi, C. T. Bellehumeur, and P. Gu, Experimental study and modeling of bond formation between ABS filaments in the FDM process, *Proc. SPE ANTEC'2004* (2004).