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Bending of Work Hardening Sheet Metals subjected to Tension

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ABSTRACT: Most sheet metal operations involve some kind of bending. To avoid unwanted spring back after bending, which causes undesirable effects in final product, as in stretch forming, tension is applied simultaneously with bending. Since the tension plays major role and strongly affects the bending moment, analysis of sheet metal bending under tension is important. In this study, an analytical model has been developed for sheet metal subjected to plane strain bending under tension. The model has been used to describe the effect of tension force on the bending moment in a linear elastic-work hardening sheet material. Then, the bending moment versus tension has been plotted. Finally influence of punch radius and material parameter on the shape of moment-tension curve will be discussed.

Key words: Sheet Metal Bending, Bending Moment, Tension, Work-Hardening Material

1 INTRODUCTION

Sheet metal bending is one of the most widely-used methods in sheet forming operations to produce frames, channels, and other non-symmetrical sheet metal parts [1]. After bending, some elastic spring back occurs which causes undesirable effects in final product. For solving this problem, as in stretch forming, tension is applied simultaneously with bending. Therefore sheet bending without or under tension have been the subject of many researches which mostly focused on simple cases of material models.

One of the first mathematical descriptions of plastic sheet bending was published by Ludwik, a century ago [2]. The theory of plane strain pure bending for rigid-perfectly plastic materials was formulated by Hill [3]. Dadras and Majlessi [4] studied bending of rigid-work hardening materials in a cylindrical pure bending. Duncan and Bird [5] presented a model for sheet stretch forming. Hosford and Caddell [6] presented a simple derivation for bending, and then developed further by superimposed tension. Calculation of strains and stresses when a rigid plastic sheet is bent and stretched under plane strain conditions was carried out by Pourbograt and Chu [7]. Lazim [1] analyzed the draw-bending of work-hardening materials. Marciniak *et al* [8] described mechanics of sheet metal forming. They presented also the moment-tension curve for elastic-perfectly plastic sheet.

In this paper, a model for bending under tension of elasto-plastic sheet materials with work-hardening characteristic has been proposed and effect of tension force on moment has been investigated.

2 THEORY

Figure 1 shows a unit width of sheet in which a cylindrical bent region with radius of curvature ρ is flanked by flat sheet. A moment M, and a tension T are applied at the middle surface of the sheet

Since the width of sheet is much larger than the thickness, t, it can be assumed that plane strain condition, i.e. $\varepsilon_3=0$. For simplicity, cylindrical bending is assumed and bauschinger effect, strain rate and friction are neglected. At present, the behavior of an element through the cross section will be taken to model the entire specimen.

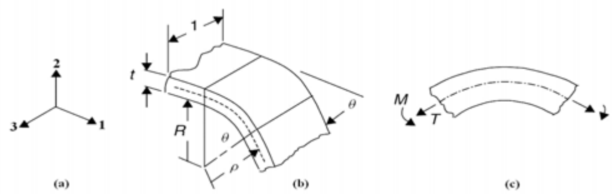

Fig. 1. Coordinate system for analysis of bending (a). A unit length of a sheet bent along a line (b). Transverse section of curved sheet under simultaneous moment and tension (c) [8].

2.1 Strain-stress relationship

If a sheet is bent and simultaneously subjected to tension as showing in figure 2(a), the strain distribution shown in figure 2(b). It can be assumed that the tangential strain ε_1 is linear sum of bending strain ($\varepsilon_b = y/\rho$) and tension strain (ε_T), i.e.

$$\varepsilon_1 = \frac{y}{\rho} + \varepsilon_T \tag{1}$$

Where y takes values between *-0.5t* and *0.5t.* ε_T denotes strain coming from superimposed tension force which depends on the material behavior and value of tension force. The position of the neutral axis, y_0, depends on the tension force or the tensile strain ε_T, which is expressed as equation (2) [9].

$$y_0 = -\rho.\varepsilon_T \tag{2}$$

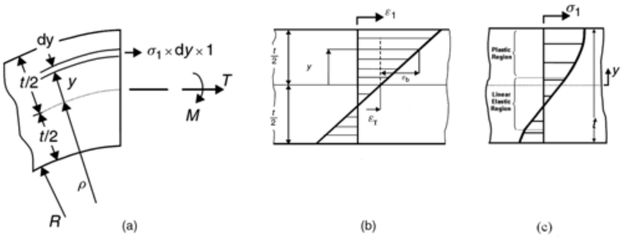

Fig. 2. Equilibrium diagram for a section through a unit width of sheet (a), strain distribution through the thickness (b) and stress distribution of elastic-work hardening sheet (c).

An Elastic-plastic sheet material with work-hardening characteristic has been assumed. Principle stress component σ_1 can be calculated using equation (3) with taking into account plane strain condition and elasto-plastic behavior.

$$\sigma_1 = \begin{cases} E'.\varepsilon_1^e & \text{if } \sigma \le \sigma_y \\ \sigma_y + K'.(\varepsilon_1^p)^n & \text{if } \sigma > \sigma_y \end{cases} \tag{3}$$

Where E' and K' are the modulus of elasticity and strength coefficient in plane strain, respectively, and are given by equations (4) and (5).

$$E' = \frac{E}{1-\nu^2} \tag{4}$$

$$K' = K.(4/3)^{\frac{n+1}{2}} \tag{5}$$

Using equation (3), the following elastic strain and the plastic one are obtained.

$$\varepsilon_1^e = \frac{y}{\rho} + \frac{\sigma_T}{E'} \tag{6}$$

$$\varepsilon_1^p = \begin{cases} \frac{y}{\rho} + \frac{\sigma_T}{E'} & \text{if } \sigma_T \le \sigma_y \\ \frac{y}{\rho} + \frac{\sigma_T}{E'} + (\frac{\sigma_T}{K'})^{1/n} & \text{if } \sigma_T > \sigma_y \end{cases} \tag{7}$$

Where tensile stress resulted from superimposed tension force is $\sigma_T = T/t$.

In a sheet bending under tension, with an increasing tensile force, T, the neutral plane shifts towards the inside of the bend and in many operations, this tension is sufficient to move the neutral plane completely out of the sheet so that the entire cross section yields in tension. The strain and stress distribution for such a case are sketched in figure 3.

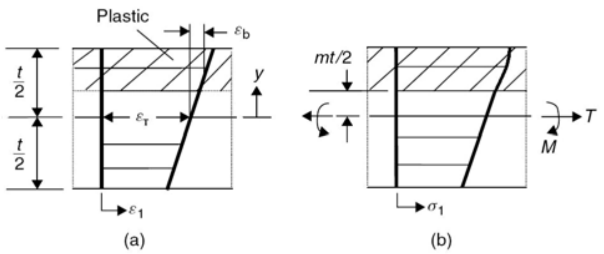

Fig. 3. Distribution of strain (a) and stress (b) in an elastic, work-hardening sheet bent to a gentle curvature and stretched.

2.2 Bending moment

The bending moment at each cross-section can be calculated by integrating principal stresses σ_1 in equation (3) over the current thickness of each element as follows

$$M = \int_{-t/2}^{t/2} \sigma_1 y dy \tag{8}$$

Since the stress-strain relation varies with σ_T, the bending moment M, can be divided M_1 and M_2.

The bending moment in the $\sigma_T \le \sigma_y$ range is

$$M_1 = M_1^e + M_1^p \tag{9}$$

According to equation (8), these moments are given by equations (10) and (11).

$$M_1^e = \int_{-\frac{t}{2}}^{m\frac{t}{2}} E'(\frac{y}{\rho}+\frac{\sigma_T}{E'})\; y\; dy \tag{10}$$

and

$$M_1^p = \int_{m\frac{t}{2}}^{\frac{t}{2}} (\sigma_y + K'(\frac{y}{\rho}+\frac{\sigma_T}{E'})^n)\; y\; dy \tag{11}$$

The elastic-plastic interface is located at a distance $mt/2$ from the middle surface as shown in figure 3, where $-1 < m < 1$. Parameter m is found by inserting equating (6) in equation (3) at elastic-plastic transition point with $y = mt/2$.

$$m = \frac{2\rho}{t}((\frac{K'}{E'})^{\frac{1}{1-n}} - \frac{\sigma_T}{E'}) \quad \text{when } \sigma_T \le \sigma_y \tag{12}$$

The bending moment in the $\sigma_T > \sigma_y$ range is given by equation (13).

$$M_2 = M_2^e + M_2^p \tag{13}$$

Where M_2^e and M_2^p are the elastic and plastic portions respectively. These moments are given by

$$M_2^e = \int_{-\frac{t}{2}}^{m\frac{t}{2}} E'(\frac{y}{\rho}+\frac{\sigma_T}{E'}+(\frac{\sigma_T}{K'})^{\frac{1}{n}})\; y\; dy \tag{14}$$

and

$$M_2^p = \int_{m\frac{t}{2}}^{\frac{t}{2}} (\sigma_y + K'(\frac{y}{\rho}+\frac{\sigma_T}{E'}+(\frac{\sigma_T}{K'})^{\frac{1}{n}})^n)\; y\; dy \tag{15}$$

Where m parameter for this case, as obtained before, is calculated by equating (7) to strain ε_1^p in equation (3) at the yield point with $y = mt/2$.

$$m = \frac{2\rho}{t}((\frac{K'}{E'})^{\frac{1}{1-n}} - \frac{\sigma_T}{E'} - (\frac{\sigma_T}{K'})^{\frac{1}{n}}) \quad \text{when } \sigma_T > \sigma_y \tag{16}$$

For the better perception of influence of T on M, the plot of bending moment versus applied tension has received significant importance. Utilizing equations (9) and (13), it becomes possible to plot the moment-tension diagram for a specified sheet material.

3 MATERIALS

To study the influence of material properties on moment-tension diagram of the proposed model, two materials; plane carbon steel, st-14, and aluminum alloy, AA5754 has been selected. The mechanical properties and sheet thicknesses of the two materials are shown in table 1.

Table1. Mechanical properties and thickness of sheet materials used in this investigation [10, 11].

Material	Young's modulus E (GPa)	Yield strength σ_y (MPa)	K (MPa)	n	υ	Thickness t (mm)
St-14	200	220	625	0.27	0.3	1
AA5057	71	136	577	0.359	0.34	1

4 RESULT AND DISCUSSION

Using material properties listed in the table 1, the computed M-T curves for the steel and aluminum alloy sheets are plotted in figures 4 and 5 respectively. The effects of curvature variations from 0.05m to 0.002m are also shown in the mentioned figures.

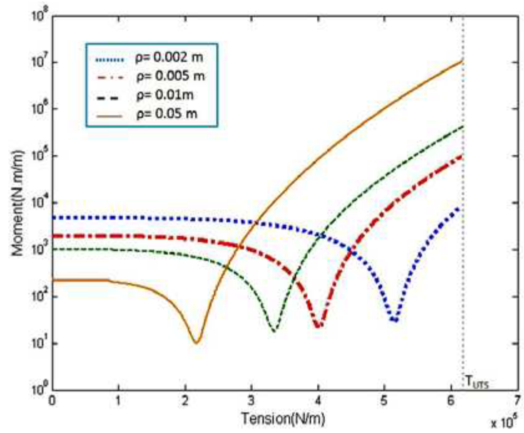

Fig.4. Moment-tension plot for St-14 steel sheet characterised in Table 1.

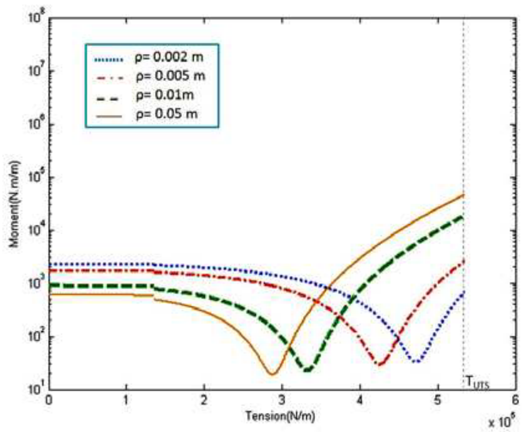

Fig.5. Moment-tension plot for AA5754 sheet characterised in Table 1.

The resulted M-T curves can be divided into three sections. In the first section, when the moment $M > 0$ and $T = 0$, for a given curvature, with increasing T, the moment will be constant until the elastic limit is reached. Also in this section, the neutral axis coincides with the center line.

Applying tension results in a shift in the position of the neutral axis, y_0 , as expressed in equation (2), and leads to enlarging the zone which is subject to tensile strains and stresses (figure 1). By increasing the tension above the elastic limit, bent sheet entered into the section two of M-T curves in figures 4 and 5 where, y_0 grows, M decreases, and the portion of the cross section deformed elastically shifts towards the inner surface. In the other word, at section two of M-T curves, as shown in the figures, for a given curvature an increase in tension significantly reduces bending moment during bending process until the neutral plane completely go out of cross section.

At the third section of M-T curves, by increasing tension, the moment rises up suddenly, and a minimum is created. In this region, the neutral axis completely exited from sheet section. The growth of bending moment at large strains can be attributed to increasing resistance of sheet to bending under high stretching. In addition, the sheet will be work hardened with increasing strains, so in the higher stretches, larger loads required to bend sheet, i.e. increasing tension lead to increasing moment until ultimate tensile strain reached. Naturally with declining in the rate of work-hardening at high strains, the slop of M-T curves decreases.

As observed in the figures 4 and 5, the pattern of bending moment variations with tension is greatly affected by the bending radius and material properties. In the M-T curves before minimum, at a constant tension, with increasing radius of curvature ρ, the moment decreases. For the small curvature radiuses, according to equation (1), strain and associated stress is large, therefore a larger load is required to bend sheet in comparison to the large curvature radiuses. In addition, according to equation (2) for small radius of curvatures, exit of the neutral axis from sheet section requires high tensions T. Also, with increase in radius of curvature, the curves and minimum shift left to smaller tensions since lower tension loads need to exit the neutral axis from sheet section.

At the third section in M-T curves, after the minimum point, at constant applied tension, increasing of curvature radius increases the bending moment. It is due to the fact that with increasing curvature radius, the angle between sheet direction and horizontal line reduced. Therefore, for compensating the growing required normal force component, the moment should be enlarged accordingly.

5 CONCLUSIONS

In this paper a theoretical model has been presented for calculating the Stress-strain relationships and bending moment in the bending under tension of work-hardening sheet metals. The effects of radius of curvature and material parameters have been also evaluated. It can be seen that Tension strongly affects the bending moment. In addition, the pattern of bending moment variations with tension is greatly affected by the bending radius and material properties. Obtained results also show that the moment-tension curve can be separated to three portions: constant moment, moment decreasing and eventually ascending moment. In the other hand, by increasing radius of curvature, the curves and minimum shift left to smaller tensions. Besides, in the ascending moment region, an increase in radius of curvature increases bending moment.

REFERENCES

1. D. Lazim, *Springback in draw-bending on aerospace alloys*, M.S. Thesis, McGill University, Canada (2003).
2. B. Heller, M. Kleiner, Semi-analytical process modeling and simulation of air bending, *J. Strain Anal. Eng. Des.*, 41(1), (2006) 57-80.
3. R. Hill, *The mathematical theory of plasticity*, Oxford University Press, London, UK (1950).
4. P. Dadras, S.A. Majlessi, Plastic bending of work hardening materials, *Trans. ASME* (1982) 224-230.
5. J.L. Duncan, J.E. Bird, Die forming approximations for aluminum sheet, *Sheet Metal Industries* (1978) 10-15.
6. W.F. Hosford, R.M. Caddell, *Metal forming, Mechanics and metallurgy*, Prentice-Hall Inc., USA (1983).
7. F. Pourboghrat, E. Chu, Springback in plane strain stretch-draw sheet forming, *Int. J. Mech. Sci.* 36(3), (1995) 327-341.
8. Z. Marciniak, J.L. Duncan, S.J. Hu, *Mechanics of Sheet Metal Forming*, Butterworth-Heinemann, UK (2002).
9. R.H. Wagoner, M. Li, Advances in springback, In: *CP778 (A), Numisheet'05*, eds, L.M. Smith, F. Pourboghrat, J.W. Yoon, T.B. Stoughton, Detroit (2005) 209-305.
10. M.H. Parsa, P. Pournia, Optimization of initial blank shape predicted based on inverse finite element method, *Finite Elem. Anal. Des.* 43 (2007) 218–233.
11. M. Samuel, Experimental and numerical prediction of springback and side wall curl in U-bendings of anisotropic sheet metals, *J. Mater. Process. Technol.* 105 (2000) 382-393.