Optimization based integrated control of building HVAC system

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Abstract

Improving the control strategy of building HVAC (heating, ventilation, and air-conditioning) systems can lead to significant energy savings while preserving human comfort requirements. This paper focuses on the analysis of the optimal control strategy of the whole HVAC system itself (such as set point value curves for different parts, number control curves of different components) and the followed operating curves of each equipment and device. In order to have a better understanding of the optimal control strategy, performances of the conventional control strategies widely used in China are also shown in this paper.

Keywords

HVAC system, control strategy, optimization, energy savings, building energy systems

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1 Introduction

The energy consumed by heating, ventilation, and airconditioning (HVAC) systems accounts for approximately 40% of the domestic energy consumption in US and 15% in China (Liang and Du 2008). It is well known that the improvement of control strategies of HVAC systems can significantly reduce the building energy consumption. However, in most cases, current strategies are not optimal. For example, the control strategies of HVAC systems in most of the 430 000 commercial and institutional buildings in Canada are inefficiently designed, resulting in energy losses of 15% to 30% (Nassif et al. 2005). Therefore, the potential to reduce these losses provides compelling reason to improve the control strategies of building HVAC systems.

Actually, the energy performance of the HAVC system control mainly depends on the control performance which is decided by the control system hardware and the control algorithm and the strategy performance. To study the optimal control strategy, two types of methods have been implemented.

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(1) Methods which focus on the online performance of supervisory and local control strategies. The properties of control process and control algorithm, e.g. the stability, accuracy and speediness of controllers, are considered in these methods to simulate real-life conditions. These methods aim at developing the optimization tool which can be used for application directly since the details of HVAC system and control process are considered (Liu et al. 2013; Sun et al. 2013; Ma and Wang 2009; Liang and Du 2008; Jin et al. 2005; Nassif et al. 2005; Massie 2002; Mathews et al. 2001; Felsmann and Knabe 2001; Wang 1999; Flake 1998; Zheng and Zaheer-Uddin 1996; Rink and Li 1995; Hyvärinen and Kärki 1996). For example, to optimize the VAV static pressure set point, the dynamic models are needed to simulate the thermal, hydraulic, environmental and mechanic characteristics, energy performance of a building and VAV air-conditioning system. And the control process of PIDcontrol is also considered in order to control the variables to their set points (Wang 1999). To reduce the probability of the large variation or even oscillations occurring during the control process, some optimization methods

were developed. For example, a multiplexed optimization scheme was proposed to guarantee system stability (Sun et al. 2013). However, the control strategies obtained by these methods are usually not the "real" optimal ones. The capability of controllers limited the application of some control strategies. As the control property of controller improves, the control strategy obtained by these methods will also improve. At the same time, these methods are not suitable when the problem scale is large. The model will be complex and the run time will be long due to the complexity of the model and the simulation time step considering the local controllers.

(2) Methods focus on the optimal control strategy itself (such as set point value curves for all parts of the HVAC system, number control curves of components) and the followed operating curves of each equipment and device. These methods assume the control process and control algorithm are ideal. Once the set points of control variables and the number of working components are given, it is assumed that the control algorithm can control these variables

to their set points within a required time scale. They cannot be applied to practice directly since they do not consider the capability of controllers, e.g. the stability, accuracy and speediness of controller. However, these methods can obtain the optimal control strategy, and gain knowledge about what situation can be observed when optimizing the control strategy for the HVAC system for minimum energy consumption. In addition, the ideal situation of HVAC system operation and the lower bound of energy consumption by HVAC system control strategy optimization can be obtained. Great research efforts have been made on this type of methods. These research presented the optimal control strategies of whole HVAC system (Yao and Chen 2010; Lu et al. 2005a,b), chilled water system (Congradac and Kulic 2009; Kusiak et al. 2011), chiller (Chow et al. 2002), air handling unit (AHU) (Kusiak et al. 2010; Kusiak and Li 2010; Fong et al. 2006), and thermal storage (Braun 1990; Kintner-Meyer and Emery 1995). However, most of the HVAC system models used in these researches are simplified. For example, the hydraulic distribution are

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usually simplified or even not considered. And most of the research efforts on the supervisory control focus on the optimization of a specific aspect of an HVAC system, such as the optimization of chiller, AHU, and thermal storage.

The research in this paper aims at using the second type of methods to obtain the optimal control strategy and the followed operating daily curves of each equipment and device of a whole HVAC system. An HVAC system model with the scale as the real system in building (include suitable building model, component models of HVAC system, hydraulic model for the water loop system) is built. And then the optimization is done in a meaningful time scale (say 5 minutes in this work) based on the model. Daily curves for different parts of the whole system, number control curves of different components, and the followed operating curves of each equipment and device can be obtained based on optimization. In order to have a better understanding of the optimal control strategy, the performance of conventional control strategies widely used in China is also obtained to compare with that of the optimal control strategy. Thus, both the performance of optimal control and conventional strategies can be seen in this paper.

2 Research method

A typical HVAC system is shown in Fig. 1. The HVAC system consists of cooling towers, condensing pumps, chillers, chilled water pumps, and AHUs. It achieves the required indoor air conditions (e.g. temperature, humidity, etc.) by adjusting the outlet water temperature of the cooling tower, flow rate of the chilled/cooling water, chilled water supply temperature, and temperature and flow rate of the AHU's supply air. The values of the above variables are determined by the on/off status and frequencies of fans

Fig. 1 An example of the HVAC system

in cooling towers and AHUs, opening of AHU valves, on/off status and frequencies of pumps, and on/off status and chilled water supply temperature of chillers. The optimization problem of the HVAC system in this paper can be expressed as follows:

$$
\begin{aligned}\n\min_{\mathbf{x}_{all} \in \mathbb{R}^q} & E_{all} \\
\text{subject to} \begin{cases}\n\mathbf{h}_{all}(\mathbf{x}_{all}) = \mathbf{0}; \\
\mathbf{g}_{all}(\mathbf{x}_{all}) \leq \mathbf{0}; \\
N_{\text{chi}}, N_{\text{ct}}, N_{\text{chp}}, N_{\text{cp}} \in \mathbb{N}^+\n\end{cases}\n\end{aligned} \tag{1}
$$

where E_{all} is the total energy consumption of the HVAC system; x_{all} is the vector of decision variables, i.e., the input and output variables of the components of the HVAC system, including the supply air temperature of the AHU, the indoor air temperature, and the differential pressure of pump, etc.; $h_{\text{all}}(x_{\text{all}}) = 0$, $g_{\text{all}}(x_{\text{all}}) \leq 0$ are the constraints which x_{all} has to satisfy.

Three steps are needed to compare the performance of the optimal control strategy and conventional control strategies.

1) Build the simulation model based on DeST

Please note that the HVAC system modeling is not the focus of this paper and was only used as the simulation tool for comparison of the optimal and conventional strategies. The simulation tool was developed based on DeST (2008) with the control strategies inputs and performance of the HVAC system output. The model was built according to the system shown in Fig. 1. All the equipment parameters, building parameters, occupant information and outdoor climate configure the model. This paper mainly introduces the basic simplification and inputs and outputs of the submodels in the simulation tool as follows:

The basic simplification: (a) ignore the heat loss through pipe and duct networks; (b) simplify the calculation of the heat conduction between adjacent rooms through walls (assume the air temperature of adjacent room is constant as the air temperature set point to calculate the heat conduction through walls).

In the simulation tool, the control process and control algorithm are assumed to be ideal. The time scale for simulation is 5 minutes since (1) it's an acceptable time scale for the sub-models selected from DeST; (2) this paper concerns on the daily curves of components' states, and the dynamic process within 5 minutes is not our focus.

The simulation model of the HVAC system includes the models of components, the model of hydraulic distribution, and the model of room. Figure 2 shows the input and output of these sub-models, where variables in blue

fonts are the control variables of the components in the HVAC system.

2) Obtain the performance of conventional control strategies through the simulation model

The conventional control strategies in this paper are obtained from the survey of control strategies widely used in China, the details of the strategies of the conventional control strategies are shown in Section 4.1. The conventional control strategies give the set points of some control variables, as shown in Fig. 3. Figure 3 also presents the connections of the HVAC system models. They are connected based on the temperature of water, and flow rate of water which is described by the hydraulic distribution sub-model.

3) Obtain the optimal control strategy

This step is shown in Fig. 4. To obtain the optimal control strategy, the controlled indoor air temperature with the conventional control strategies should be input into the optimization problem. This will make sure $T_r^{\text{con}} = T_r^{\text{opt}}$. The outputs of the optimization method include the set point of control variables, e.g. frequency of fan and number of working chillers, and the performance of the optimal

Fig. 2 The inputs and outputs of the sub-models in the HVAC system

Fig. 3 Obtain the performance of the conventional control strategies

Fig. 4 Obtain the optimal control strategy and its performance

control strategy. The optimization method allows us to obtain the optimal performance directly.

This paper focuses on the optimal control strategy itself. The detailed simulation optimization shown in Section 3 serves as the basic work for analyzing the optimal control strategy. In Section 4, the results from the proposed optimal control strategy of the overall HVAC system are compared with those from the conventional control strategies and the differences are discussed based on the simulated performance.

3 Problem formulation and solving method

3.1 Problem formulation

This part gives the objective and constraints of the optimization problem.

As it mentioned in Section 2, x_{all} is the vector of decision variables, i.e., the input and output variables of the components of the HVAC system. So,

$$
\boldsymbol{x}_{\text{all}}=\begin{bmatrix} \boldsymbol{x}_{\text{chp}} & \boldsymbol{x}_{\text{cp}} & \boldsymbol{x}_{\text{ct}} & \boldsymbol{x}_{\text{fan,ct}} & \boldsymbol{x}_{\text{AHU}} & \boldsymbol{x}_{\text{fan,AHU}} & \boldsymbol{x}_{\text{v}} & \boldsymbol{x}_{\text{r}} & \boldsymbol{x}_{\text{hy}} & \boldsymbol{x}_{\text{chi}} \end{bmatrix}
$$

where the variables of components correspond with the variables shown in Fig. 2. For example, $\mathbf{x}_r = [G_{ar}^{\text{in}} T_{ar}^{\text{in}} T_r]$.

In many real buildings, multiple identical chillers are used, as are pumps and cooling towers. The objective of the optimization problem is then

$$
E_{\text{all}} = E_{\text{chi}} \cdot N_{\text{chi}} + E_{\text{chp}} \cdot N_{\text{chp}} + E_{\text{cp}} \cdot N_{\text{cp}} + E_{\text{fan}, \Delta HUi} + E_{\text{fan}, \Delta HUi} \tag{2}
$$

Equality constraints describe the relationship of input and output variables of sub-models in the HVAC system. As it mentioned in Section 2, the indoor air temperature satisfies $T_r^{\rm con} = T_r^{\rm opt}$. This is also an equality constraint of the problem. Denote the mathematic description of sub-model *i* as $h_i = x_{i,\text{output}} - F_i(x_{i,\text{input}}) = 0$, where the variables with the subscript input and output correspond with the vector of the input/output variables shown in Fig. 2. For example, $\mathbf{x}_{r, \text{input}} = \begin{bmatrix} G_{a,r}^{\text{in}} & T_{a,r}^{\text{in}} \end{bmatrix}$, and $\mathbf{x}_{r, \text{output}} = T_r$. The equality constraints of the optimization problem are shown in Eq. (3).

$$
\boldsymbol{h}_{\text{all}}(\boldsymbol{x}_{\text{all}}) =
$$
\n
$$
\begin{bmatrix} \boldsymbol{h}_{\text{chp}} & \boldsymbol{h}_{\text{cp}} & \boldsymbol{h}_{\text{ct}} & \boldsymbol{h}_{\text{fan,ct}} & \boldsymbol{h}_{\text{AHU}} & \boldsymbol{h}_{\text{v}} & \boldsymbol{h}_{\text{r}} & \boldsymbol{h}_{\text{hy}} & \boldsymbol{h}_{\text{chi}} & T_{\text{r}} - T_{\text{r}}^{\text{con}} \end{bmatrix}^{\text{T}} = 0
$$
\n
$$
(3)
$$

The input and output variables are bound by limited ranges. For example, frequencies of the fan and the pump $(f_{\text{fan}} \text{ and } f_{\text{p}})$, opening value of valve (K_{v}) , flow rate of chilled and cooling water through the chiller ($G_{chw,chi}$ and $G_{cw,chi}$), outlet temperature of chilled water in the chiller (Tout_{chw,chi}), the differential pressure of the pump (H_{chp} and H_{cp}), and the number of chillers, pumps, and cooling towers in operation (N_{chi}, N_{chp}, N_{cp}, and N_{ct}) vary only in limited ranges. In addition, the inlet temperature of cooling water through the chiller ($T_{\text{chw,chi}}^{\text{in}}$) also has a lower bound. These "ranges" form the inequality constraints in Eq. (4).

$$
\mathbf{g}_{\text{all}}(\mathbf{x}_{\text{all}})=\begin{bmatrix} \mathbf{y}_{\text{min}}-\mathbf{y} \\ \mathbf{y}-\mathbf{y}_{\text{max}} \\ T_{\text{chw},\text{chimin}}^{\text{in}}-T_{\text{chw},\text{ch}}^{\text{in}} \end{bmatrix}\leqslant 0
$$
\n(4)

where $y = [f_{\text{fan,AHU}} f_{\text{fan,cf}} f_{\text{chp}} f_{\text{cp}} K_{\text{y}} G_{\text{chw,chi}} G_{\text{cw,chi}} T_{\text{chw,chi}}^{\text{out}} H_{\text{chp}}$ H_{cp} N_{chi} N_{ct} N_{chp} N_{cp} ^T, \boldsymbol{y}_{min} and \boldsymbol{y}_{max} are the lower and upper bounds of *y*.

All mathematic description of sub-models used in the optimization are the same as those used in the simulation tool, except for the cooling tower model.

3.1.1 Modified model of cooling tower

The optimization method used in this paper requires that both the objective function and constraints are twice differentiable. However, the model of the cooling tower used for the simulation in this paper has the operator of minimization (Yan et al. 2008; Zhang et al. 2008), which is discontinuous. Therefore, this paper gives an approximate model for the cooling tower. The efficiency of a cooling tower can be described by the following equation:

$$
\eta_{\rm ct} = \frac{T_{\rm w,ct}^{\rm in} - T_{\rm w,ct}^{\rm out}}{T_{\rm w,ct}^{\rm in} - T_{\rm wb,ct}^{\rm in}} \tag{5}
$$

Taking the efficiency of a cooling tower as a horizontal coordinate and the flow rate ratio of air-to-water as a vertical coordinate, the relationship of efficiency and airto-water ratio can be described by a smooth curve when the wet bulb temperature is fixed. Therefore, the efficiency of a cooling tower can be expressed as a polynomial of the flow rate ratio of air-to-water by data fitting when the wet bulb temperature is given as shown in the following equation:

$$
\eta_{\rm ct} = a_0 + a_1 \frac{G_{\rm w,ct}}{G_{\rm a,ct}} + a_2 \left(\frac{G_{\rm w,ct}}{G_{\rm a,ct}}\right)^2 + a_3 \left(\frac{G_{\rm w,ct}}{G_{\rm a,ct}}\right)^3 \tag{6}
$$

where a_0 , a_1 , a_2 , a_3 are determined by the wet bulb temperature and the characteristic of the cooling tower. The cooling tower model in the simulation tool is used to check the acceptability of this data fitting model. The test results show that the difference between the data fitting model and the model in the simulation tool to be roughly 0.5% (acceptable), and the non-differentiable constraints brought by the cooling tower model being made differentiable:

$$
\eta_{\rm ct} = \frac{T_{\rm w,ct}^{\rm in} - T_{\rm w,ct}^{\rm out}}{T_{\rm w,ct}^{\rm in} - T_{\rm w,ct}^{\rm in}} = a_0 + a_1 \frac{G_{\rm w,ct}}{G_{\rm a,ct}} + a_2 \left(\frac{G_{\rm w,ct}}{G_{\rm a,ct}}\right)^2 + a_3 \left(\frac{G_{\rm w,ct}}{G_{\rm a,ct}}\right)^3 \tag{7}
$$

3.2 Solving method

The optimization problem of the HVAC system is

$$
\begin{array}{ll}\n\text{minimize} & E_{\text{all}} \\
\mathbf{x}_{\text{all}} \in \mathbb{R}^q \\
\text{subject to} & \begin{cases} \n\mathbf{h}_{\text{all}}(\mathbf{x}_{\text{all}}) = \mathbf{0}; \\
\mathbf{g}_{\text{all}}(\mathbf{x}_{\text{all}}) \leq \mathbf{0}; \\
N_{\text{chi}} N_{\text{ct}}, N_{\text{chp}}, N_{\text{cp}} \in \mathbb{N}^+\n\end{cases}\n\end{array}\n\tag{8}
$$

For the HVAC system shown in Fig. 1, if there are 3 chillers, 3 chilled water pumps, 3 condensing pumps, 4 cooling towers (one fan for each cooling tower) and 60 AHUs each of which services one room, there will be 996 decision variables, 1159 equality constraints, and 265 inequality constraints in Eq. (8). The problem in this paper includes both continuous and discrete decision variables. The decision variables can be divided into two parts (Sun and Reddy 2005). (1) Deciding on the best operating modes, namely,

(2) Deciding on optimal set points for components such as the frequency of pump and the outlet temperature of chilled water.

determining the number of components in operation.

All the decision variables in (1) are discrete and in (2) are continuous. Therefore, the enumeration method is used to find the best operating mode, i.e. to solve part (1). And the interior point method is used to solve the continuous variables in (2). Firstly, we determine the type of operating mode, namely, determining the number of components, i.e. chillers, cooling towers, pumps, etc. in operation. We then use the interior point method to solve the optimization problem with a fixed number of running components. We enumerate all of the operating modes, and use the operating mode with the minimal optimal objective (energy consumption) as the optimal solution.

For the HVAC system shown in Fig. 1, with 3 chillers, 3 chilled water pumps, 3 condensing pumps, and 4 cooling towers, there are 3×3×3×4=108 types of combinations of these components. The number of combinations is computationally acceptable. And in many real applications with same capacity chillers and pumps, the number of running condensing pumps is the same as that of running chillers, meaning an even smaller number of combinations, e.g. the 108 types of combinations for the HVAC system show in Fig. 1 decreased to 36 types.

Of course, Hybrid method can be used to solve this problem. And it has the elegant mathematical expression. However, it also takes considerable efforts to solve this type of problem. In addition, with the enumeration method, the energy consumption of each combination of working components' numbers can be obtained, which could in turn helps us to analyze the optimal results.

When the number of working components is fixed, the original problem becomes

minimize
$$
E_{\text{all}}
$$

\n
$$
\sum_{\mathbf{x}_{\text{all,p}} \in \mathbb{R}^{q-4}} E_{\text{all}}
$$
\nsubject to
$$
\begin{cases} \n\mathbf{h}_{\text{all}}(\mathbf{x}_{\text{all,ip}}) = \mathbf{0};\\ \n\mathbf{g}_{\text{all}}(\mathbf{x}_{\text{all,ip}}) \leq \mathbf{0} \n\end{cases}
$$
\n(9)

where $x_{\text{all,ip}}$ is the decision variable vector without discrete variables.

The problem can then be solved by the interior point search method. Primal interior point method is used since the problem might be nonconvex. Equation (9) can be further transformed into an approximate problem in Eq. (10) which is easier to solve than the above inequality constrained problem since it is made up by a sequence of equality constraints (Waltz et al. 2006). The primal-dual interior point method can be used to solve Eq. (10) (Bertsekas 1995).

$$
\underset{\mathbf{x}_{\text{allip}} \in \mathbb{R}^{q-4},s}{\text{minimize}} L_{\mu}(\mathbf{x}_{\text{all,ip}}, \mathbf{s}) \equiv E_{\text{all}} - \mu \sum_{i} \ln s_{i}
$$
\n
$$
\text{subject to } \begin{cases} \n\mathbf{h}_{\text{all}}(\mathbf{x}_{\text{all,ip}}) = \mathbf{0};\\ \n\mathbf{g}_{\text{all}}(\mathbf{x}_{\text{all,ip}}) + \mathbf{s} = \mathbf{0};\\ \n\mu > 0 \n\end{cases} \tag{10}
$$

where *L* denotes the function $E_{all} - \mu \sum_{i} \ln s_i$, μ is the barrier parameter, and *s* is a column vector with the slack variable

si as its *i-*th element, and the dimension of *s* is equal to that of $g_{all}(\mathbf{x}_{allin})$. Since there is $\ln s_i$ in the objective function, s_i must be positive. From the equality constraints $g_{\text{all}}(x_{\text{all,ip}})$ + $s = 0$ in Eq. (10), we know that $g_{all}(x_{all,ip}) < 0$. Note that both the functions and variables in Eq. (10) are continuous. So, $g_{\text{all}}(x_{\text{all,ip}})$ infinitely approaches zero. Therefore, $g_{\text{all}}(x_{\text{all,ip}}) + s = 0$ approximately equals to $g_{\text{all}}(x_{\text{all,ip}}) \le 0$. As μ approaches zero, the minimum of $E_{all} - \mu \sum_{i} \ln s_i$

should approach the minimum of *E*all.

4 Simulation results

4.1 Case introduction

The HVAC system shown in Fig. 1 was simulated. For every 5 minutes the decision variables of components can be adjusted. The outdoor climate of a typical summer day (July 1 of Beijing typical year data from DeST) is chosen for our simulation. The parameters of the case are listed in Table 1. The building used for simulation has 60 rooms, belonging to 10 risers. For each riser there are 6 floors and for each floor there is only one room. To simplify, these 10 risers are exactly the same.

Case designs of the two types of strategies are described in Table 2 where VSD means variable speed drive. The conventional control strategies in this paper are based on the investigation of more than 100 large-scale public buildings in China (Xue 2007). In the investigation report, for most of the air system in large-scale public buildings in China, the supply air temperature is fixed to 14℃ for cooling season. In the public buildings, the supply chilled water temperature is usually fixed to 7℃. In most of chilled water systems, the differential pressure set point of chilled water pump is fixed. Actually, the set point value is intended

Table 1 The case parameters

	Item	Parameter			
The building	Scale	6 floors \times 10 risers			
	Measure of area $(m2)$	30000			
	Indoor air temperature control point (\mathcal{C})	25			
	Occupied time	9:00am to 8:00pm			
AHU	Water side control	Continuously adjustment			
	Number	One AHU for one floor			
	Rated power (kW)	83 in total			
	Fresh air ratio	Fixed to 0.1			
	Frequency of fan (Hz)	$25 - 50$			
	Number	3			
	Rated power (kW)	234.5			
Chiller	Rated cooling load for each chiller (kW)	1407			
	Chilled water supply temperature $({\degree}C)$	$5 - 12$			
Chilled water pump	Number	3			
	Rated power (kW)	31			
	Frequency of pump (Hz)	$25 - 50$			
Condensing pump	Number	3			
	Rated power (kW)	39			
	Frequency of pump (Hz)	$25 - 50$			
Cooling tower	Number	$\overline{4}$			
	Number of fan	One fan for each cooling tower			
	Rated power (kW)	6.18			
	Frequency of fan (Hz)	$25 - 50$			

Table 2 Case designs of the optimal control strategy and conventional ones

to be adjusted by the operator of HVAC system. However, for most investigated buildings, the differential pressure of chilled water pump keeps its initial setting (decided in design or commissioning). Normally, the requirement value decided in design or commissioning is trying to handle the worst case. Thus, the differential pressure set point of chilled water pump is much higher than the terminal demanded in most of the operation time. The working number of condensing pumps and cooling towers are equal to the number of working chillers in most buildings. The operators change the number of working chillers according to the load ratio of chiller.

Figure 5 shows the simulation result of indoor air temperature for conventional strategies, where the indoor air temperature can be controlled to about 25℃. The optimal control strategy controls the temperature to the same value. As we have mentioned in Section 2, the indoor air temperature does not always equal to the set point value.

4.2 Simulation results

The interior point method can find only a local minimum. However, the optimization problem always desires the global minimum. If the optimization problem is a non-convex problem, it is more likely to find the global minimum by trying different initial points. It is interesting to note that the interior point method always gets the same local optimal solution, x^* , with 10 different initial points, x_{00} , x_{01} , ... x_{09} , which scatter uniformly in the scope of control variable values. More research and analyses are needed to prove the uniqueness of the optimal point.

The overall time required to complete an optimization cycle is about 10 min and the average number of function calls is 2409 based on Intel CoreTM 2 P8600@2.4GHz (2CUPs). As explained in Section 1, the objective of this work is trying to get the detailed operating state of each part in suitable time scale for optimal control strategy analysis. So the approach reported in this paper is not developed for the real case optimization. Therefore, the optimization speed is acceptable.

4.2.1 Terminal system

The simulation results of 15:00 are chosen to introduce the

optimal control strategy in this paper and its comparison with the conventional strategies. Table 3 shows the optimal and conventional control variables' set points of AHUs, where, "opt" represents the optimal strategy while "con" represents the conventional strategies. In order to show it clearly, the simulation results are sorted in the descending order of AHU load ratio. Floor 4 is the terminal with the highest load ratio. We state it the "worst terminal" where its valve is fully opened. As the load ratio decreases, the valve opening and fan frequency decrease. Actually, the optimal water flow rate ratio, supply airflow rate ratio, and cooling load ratio of each floor are close to each other (shown in Fig. 6). Changing of cooling load leads to valve opening changes. However, there is always at least one valve fully opened (shown in Fig. 7) to reduce the resistance of terminal side to its lowest level and the energy consumed by the chilled water pump.

Table 3 Optimal and conventional set points of control variables

	Cooling load (kw)	Load	Valve opening		Frequency of fan(Hz)		Supply air temperature (\mathcal{C})	
		ratio	opt	con	opt	con	opt	con
Floor 4	31.78	0.66	1.00	0.47	0.86	1.00	12.50	14.00
Floor 6	44.80	0.58	0.73	0.21	0.84	1.00	12.02	14.00
Floor ₂	25.32	0.53	0.40	0.27	0.70	0.84	12.29	14.00
Floor 5	23.17	0.48	0.24	0.19	0.67	0.76	12.87	14.00
Floor 1	16.75	0.35	0.16	0.07	0.42	0.53	10.74	14.00
Floor 3	16.17	0.34	0.17	0.08	0.40	0.55	10.50	14.00

Fig. 6 Ratios of optimal water flow rate, supply air flow rate, and the corresponding cooling load

Fig. 7 Optimized valve opening of terminal AHUs

In Table 3, the conventional valve opening and fan frequency also decrease with the load ratio decreasing. However, the conventional valve openings are smaller while the fan frequencies are larger than that of the optimal. The above relationship can also be seen in Figs. 8 and 9. Figure 10 shows the comparison of supply air temperature between the optimal control strategy in this paper and other conventional strategies. It is clearly that the supply air temperature and terminal load have an inverse relationship. When the cooling load goes up, the HVAC system has to increase the cooling amount of room to keep the indoor air temperature at the set point. To increase the cooling amount of room, a larger volume and/or a lower temperature of the supply air are/is needed. The lower supply air temperature results in the decrease of the AHU energy consumption, which might also increase the energy consumed by the

Fig. 8 Comparison of valve opening of floor 2

Fig. 9 Comparison of fan frequency of floor 2

Fig. 10 Comparison of supply air temperature

chiller. However, in the conventional control strategies, the temperature stays the same value. The AHU achieves the required indoor air temperature only by adjusting the volume of supply air. The supply air temperature of optimal control strategy is always lower compared with that of the conventional ones. Therefore, the optimized AHU fan frequency is smaller than the conventional one. The optimal valve opening is larger than the conventional one, because of the difference of control strategies in chilled water pump.

4.2.2 Chilled water pump

Figure 11 shows the comparison of the terminal water flow rate and the differential pressure, where "G" is the terminal water flow rate, "delt P " is the differential pressure, "opt" is the optimal control strategy, and "con" is the conventional control strategy. In the conventional control strategies, the differential pressure is consistently around 1.5×10^{5} Pa, which is much larger than the optimal differential pressure. The terminal water flow rates in the two situations are similar. Therefore, the conventional valve opening shown in Fig. 8 is smaller than the optimal one. As mentioned in Section 4.1, in the conventional control strategies, the differential pressure of chilled water pump is much higher than the terminal demanded in most of the operation time. While in the optimal control strategy, the differential pressure just needs to meet the pressure demand in the current condition. As a result, the differential pressure for conventional strategies is consistently more than its optimal counterpart, leading to larger energy consumption for the chilled water pump. Figure 11 also shows that the optimized terminal water flow rate is related mainly to the differential pressure.

Figure 12 shows the comparison of chilled water pump frequency. In the optimal control strategy, the chilled water pump frequency is lower than that in the conventional control strategies.

Fig. 11 Comparison of terminal water flow rate and differential pressure

Fig. 12 Comparison of chilled water pump frequency

4.2.3 Chiller

Figures 13 and 14 shows the simulation results of chiller with the optimal control strategy, where Fig. 13 is the chilled water side and Fig. 14 is the cooling water side. To show it clearly, the differential temperature shown in the two figures is multiplied by 10, the energy consumption of chiller is multiplied by 1/2, and they are represented by "delt_t \times 10" and "*E*/2" respectively. Moreover, the water flow rate through each chiller is represented by "*G*".

Fig. 13 Optimal results of chilled water side of chiller

Fig. 14 Optimal results of cooling water side of chiller

Figure 13 shows that the differential temperature of chilled water and energy consumption of chiller are related mainly to the cooling load, while the flow rate of chilled water stays around its lower bound (the lower bound of chilled water through chiller is 33.76 kg/s). Low chilled water flow leads to high energy consumption of chiller. However, it reduces the energy consumption of chilled water pump.

Figure 15 shows the comparison of the outlet temperature of chilled water. In the conventional control strategies, the outlet temperature of chilled water stays at 7℃, while the optimized outlet temperature decreases when the cooling load goes up, and is consistently lower than 7℃. Low supply chilled water temperature results in high energy consumption of chiller. However, low outlet temperature of chilled water benefits AHUs in terms of energy conservation. Obviously, there is an energy tradeoff between the chiller and AHU. Nevertheless, no matter what the tradeoff is, the energy consumed by chiller increases when the cooling load increases. In addition, there is one point that needs to be emphasized: the optimization does not consider the de-humidification of indoor air.

The optimal results of cooling water side (shown in Fig. 14) are very different from the results of chilled water side. The optimal flow rate of cooling water through chiller is related mainly to the cooling load, while the differential pressure of cooling water does not change very much. It is possible that in the optimal control strategy, the differential temperature of cooling water stays around a constant because of the tradeoff of energy consumption of chiller and cooling tower.

4.2.4 Cooling tower

Figure 16 shows the comparison of cooling tower efficiency. High cooling tower efficiency reduces the outlet water temperature of cooling tower and the energy consumption of chiller. In the optimal control strategy, the efficiency of cooling tower keeps around 0.75. Figure 17 shows the relationship of cooling tower efficiency and air-to-water

Fig. 15 Comparison of supply chilled water temperature

ratio of flow rate. In order to make efficiency higher than 0.75, much higher of airflow rate is required and little efficiency increase can be achieved. Therefore, keeping the cooling tower efficiency around 0.75 is the tradeoff of energy consumption of cooling tower and chiller. Equation (5) shows that the cooling tower efficiency is directly related to the differential temperature of water through cooling tower. This is possibly the reason why the optimized differential temperature of cooling water stays around a constant. Figure 18 shows the water temperature through cooling tower and the outdoor air wet bulb temperature, where "*T*" is temperature. It shows that both the inlet and outlet water temperature change with the outdoor air wet bulb temperature, and the differential temperature of cooling water keeps around a constant.

Fig. 16 Comparison of cooling tower efficiency

Fig. 17 The relationship of cooling tower efficiency and flow rate ratio of air to water

Fig. 18 Water temperature through cooling tower and wet bulb temperature of outdoor air

Figure 19 shows that the total airflow rate of optimal and conventional control strategies are the same, where "*f*" is fan frequency and "*N*" is the working cooling tower number. Figure 20 shows the total cooling water through cooling tower. The optimal cooling water flow rate is much higher than the conventional ones. Therefore, the cooling tower efficiency for conventional control strategies is lower than its optimal counterpart (shown in Fig. 16).

Fig. 19 Comparison of fan frequency and number of working cooling towers

Fig. 20 Comparison of cooling water flow rate

4.2.5 Condensing pump

Figure 21 shows the comparison of condensing pump frequency. The optimal frequency of condensing pump is related mainly to the cooling load of HVAC system, while frequency for the conventional control strategies stays around its upper bound. As mentioned in Section 4.2.4, in the optimal control strategy, there is a tradeoff between chiller and condensing pump. While in the conventional control strategies, the condensing pump supplies more cooling water than the optimal one in order to reduce the energy consumption of chiller.

4.2.6 Energy consumption

Figure 22 shows the energy consumption comparison of optimal and conventional control strategies from 9:00 am to 8:00 pm. All components of the HVAC system except

Fig. 21 Comparison of condensing pump frequency

Fig. 22 Energy consumption comparison

the chiller consume less energy with the optimal control strategy compared to the conventional ones. In total, the optimal control strategy can reduce the energy consumption by 26.4%. The condensing pump has the greatest energy conservation potential here because the survey of commercial and institutional buildings in China shows that condensing pumps are usually oversized in order to handle the high water flow rate requirement in extreme conditions. In the case of this paper, the rated power of the condensing pump (39 kW) was nearly 1/6 of the chiller rated power (234.5 kW), which indicates the condensing pump has great energy conservation potential.

The simulation results of the optimal control strategy show that almost all the optimal variables' daily curves have obvious drop or rise at 17:50, 18:10, and 19:45. The above time points are marked with circles in Fig. 23. Figure 23 shows the energy consumption of the entire HVAC system at the situation of "1 working chiller" and "2 working chillers". The optimal number of working chillers changes from 2 to 1 at 17:50 and 18:10 since the "1 working chiller" situation has lower energy consumption. Similarly, the optimal number changes from 1 to 2 at 19:45. The change of optimal number of working chillers leads to the drop or rise of other optimal variables. Actually, the energy consumptions of the two types of operating modes are close to each other when the cooling load of HVAC system

Fig. 23 Total energy consumption comparison of 1 working chiller's situation and 2 working chillers' situation

is lower than the rated cooling load of chiller. Moreover, these changes lead to activities of actuators. Therefore, it is not very necessary to change the number of working chillers at the above moments. The optimal control strategy only supplies the control strategy with which the HVAC system consumes the minimal energy. However, it is not very practical.

5 Conclusions

Knowledge about what situation can be observed when optimizing the control strategy for the HVAC system helps us to understand the control of HVAC system and is very useful for practice. To obtain the optimal control strategy, and gain knowledge about what situation can be observed with optimal control strategy, an HVAC system model with scale as the real system in building (include suitable building model, component models of HVAC system, hydraulic model for the water loop system) is built. And an optimization method is proposed in this paper. Daily curves for different parts of the whole system, number control curves of different components, and the followed operating curves of each equipment and device of both the optimal control strategy and conventional control strategies are shown.

The simulation results show that there are some variables which have not been fully explored in the conventional control strategies. They have big potentials for energy saving, e.g. the number of working cooling towers, supply air temperature of AHU, differential pressure of chilled water pump, and supply chilled water temperature of chiller.

More cases with different HVAC systems and different weather should be studied to find the regular control rules. It will be helpful for application.

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