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ROBUST DESIGN OPTIMIZATION OF THE DYNAMIC RESPONSES OF A TRACKED VEHICLE SYSTEM

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ABSTRACT−This paper presents the robust design optimization of the dynamic responses of a heavy military tracked vehicle system. The tracked vehicle model addressed in this study has 954 degrees of freedom and consists of 189 bodies in total: 37 bodies for the chassis, such as sprockets, road wheels, road arms, etc.; 76 track link bodies for each track subsystem; 36 revolute joints; and 152 bushing elements. The design objectives were to minimize the maximum vertical acceleration of the hull and its variance while satisfying the wheel travel constraints for torsion bars and the hydro-pneumatic suspension units within $\pm 1\sigma$ ranges. To avoid the difficulty of the design sensitivity analysis and to overcome the numerical noise, a progressive meta-model technique was employed in the optimization process. First, space-filling methods were used to determine the minimum number of sample points. Second, the simultaneous kriging method was used to construct the initial meta-models, and the augmented Lagrange multiplier (ALM) method was then used to solve the robust design problems of the meta-models. Third, the new design results were added to the analysis results for the initial sample points, and the meta-models were updated automatically. Next, the optimizer resolved the robust design problems of the updated meta-models. These processes were repeated until the convergence tolerances were satisfied. The robust design optimization of the tracked vehicle system, with 11 random design variables, was solved in only 26 analyses, including 12 analyses for the initial meta-models and 14 analyses added during the iterative optimization process.

KEY WORDS : Optimization, Progressive meta-model, Tracked vehicle

1. INTRODUCTION

Most of the studies of tracked vehicle systems conducted to date have involved modeling suspension systems and track systems in terms of multi-body system dynamics (McCullough and Haug, 1986; Nakanishi and Shabana, 1998; Choi *et al*., 1998; Lee *et al*., 1998; Ryu *et al*., 2000, 2006). Recently, however, a design study was required for a tracked vehicle system to improve its ride characteristics. This design study was conducted using the well-developed CAE software for tracked vehicle systems.

Although the analysis process for tracked vehicle systems is well developed, optimization of tracked vehicle system design is still not straightforward because the analytical design sensitivity process is very difficult with multi-body dynamics. In addition, low-pass filters are frequently used to signify the dynamic responses, and these make an analytical approach to design sensitivity.

sTherefore, we introduce in this paper a meta-model-based design optimization method for dynamic response optimization that avoids a design sensitivity analysis and overcomes numerical noise. Using this method, robust design optimization can be implemented easily using the gradient information from meta-models. Section 2 presents a review of the literature on modeling track systems and analyzing tracked vehicle systems. Section 3 presents the proposed optimization strategy for a tracked vehicle system. Section 4 demonstrates the robust optimization of a tracked vehicle system. Section 5 presents the conclusions of this study.

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2. REVIEW OF TRACKED VEHICLE MODELING AND ANALYSIS

In the early 1980's, several dynamic modeling techniques for track systems were developed at universities, research institutes and companies. McCullough and Haug (McCullough and Haug, 1986) designed a "super element" that represented the spatial dynamics of high-mobility tracked vehicle suspension systems. Their track was modeled as an internal force element that acted in the ground, the wheels and the chassis of the vehicle. Track tension was computed using a relaxed catenary relationship. Nakanish and Shabana (Nakanish and Shabana, 1994) introduced a contact search approach for a planar rigid-body track system. This was extended to spatial dynamic analysis by Choi, Lee and Shabana (Choi *et al*., 1998; Lee *et al*., 1998). In these approaches, the track link was modeled as a rigid body and connected by a one-degree-of-freedom pin joint and bushing force element. To overcome the numerical difficulty encountered by Choi *et al*., Ryu *et al*. (Ryu *et al*., 2000) extended the track system modeling techniques to high-mobility military tracked vehicles in the context of a G-Alpha numerical integrator. Recently, Ryu *et al* (Ryu *et al*., 2006) proposed a nonlinear dynamic model for tracked vehicles and validated the model by comparing its results with test results.

3. META-MODEL-BASED OPTIMIZATION

3.1 Simultaneous Kriging Model

Meta-models such as RSM, kriging and radial basis function (RBF) models are increasingly used to approximate responses in engineering fields that would be expensive to measure. RSM was introduced in the classical DOE, which used a polynomial-type regression model. Hence, it required rotatable characteristics for the sampling points, such as CCD and SCD. However, the kriging (Farhang-Meher and Azarm, 2005) and radial basis function (Wang and Liu, 2002) models are Bayesian models. Hence, they use a space filled by sampling-point methods such as a Latin hypercube or descriptive designs (Kim, 2006).

A kriging model can be defined as a combination of a regression model and a departure term, as shown in the following equation:

$$y = \mathbf{X}\beta + z(\mathbf{x}), \tag{1}$$

where y is the approximate model, $\mathbf{X}\beta$ is a polynomial-type regression model, and $z(\mathbf{x})$ is a Gaussian random process with $N(0, \sigma^2)$. If the regression model ($\mathbf{X}\beta$) globally approximates the design space, the departure term $z(\mathbf{x})$ represents the localized deviations, so that the kriging model interpolates the n_s sampled points. Based on our observations, the regression model plays an important role in design optimization, especially in the case of an insufficient number of sampling points. The covariance matrix of $z(\mathbf{x})$ is given by the following equation:

$$Cov[z(\mathbf{x}_i)z(\mathbf{x}_j)] = \sigma^2\mathbf{R}[R(\mathbf{x}_i, \mathbf{x}_j)], \tag{2}$$

where $\mathbf{R}$ is the correlation matrix and $R(\mathbf{x}_i, \mathbf{x}_j)$ is the correlation function between any two of the n_s sampled points. Hence, $\mathbf{R}$ is an $n_s \times n_s$ symmetric matrix with ones in the diagonal terms. There are many types of $R(\mathbf{x}_i, \mathbf{x}_j)$ correlation functions. Among these, the Gaussian-type correlation function, given by the following equation, is widely used:

$$R(\mathbf{x}_i, \mathbf{x}_j) = exp\left[\sum_{l=1}^{k} \theta_l |\mathbf{x}_i^l - \mathbf{x}_j^2\right], \tag{3}$$

where θ_l are the unknown correlation parameters to fit the model. The estimates $\tilde{y}(\mathbf{x})$ of the response $y(\mathbf{x})$ at the untried values of $\mathbf{x}$ are given by the following equation:

$$y(\mathbf{x}) = \mathbf{X}(\mathbf{x})\tilde{\beta} + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(\mathbf{y} - \mathbf{X}(\mathbf{x})\tilde{\beta}). \tag{4}$$

The correlation vector between $\mathbf{x}$ and the sampled points $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_s}\}$ is given by the following equation

$$\mathbf{r}(\mathbf{x})^T = [R(\mathbf{x}, \mathbf{x}_1), R(\mathbf{x}, \mathbf{x}_2), \ldots, R(\mathbf{x}, \mathbf{x}_{n_s})]^T \tag{5}$$

The unknown coefficients of the regression model are determined as follows:

$$\tilde{\beta} = (\mathbf{X}^T\mathbf{R}^{-1}\mathbf{X})^{-1}\{\mathbf{X}^T\mathbf{R}^{-1}\mathbf{y}\}. \tag{6}$$

To determine the value of the unknown correlation parameter θ_l, the estimate of the variance $\tilde{\sigma}^2$ (not the variance in the observed data) is introduced. Hence, the value of the correction parameter θ_l is determined by solving the following minimization problem:

$$\min_{\theta>0}(\det\mathbf{R}(\theta))^{1/n_s}\tilde{\sigma}(\theta) \tag{7}$$

While any values for θ create an interpolation model, the best kriging model is found by solving the k-dimensional unconstrained optimization problems described above.

From the viewpoint of numerical optimization, equation (7) can be non-smooth because the correlation matrix $\mathbf{R}(\theta)$ is frequently singular during the optimization process. Hence, some special techniques are required to avoid the singular phenomena and non-linearity of it. Hence, we use a singular value decomposition (SVD) and normalization and scaling techniques. A multi-objective formulation is introduced in equation (7) to solve the multiple kriging models simultaneously. This approach uses only one correlation matrix $\mathbf{R}(\theta)$ even for constructing multiple kriging models (Kim, 2006; Kang *et al*., 2010, 2012).

3.2. Robust Optimization Formulation

Let us consider the general optimization formulation for robust design. Fundamentally, all the functions are composed of meta-models.

$$\text{Minimize } \Psi(\mathbf{x}) = \alpha \cdot f(\mathbf{x}) + k_0 \cdot \sigma_0(\mathbf{x}) \tag{8}$$

$$\text{subject to } h_i(\mathbf{x}) = 0, i = 1, 2, \ldots, l \tag{9}$$

$$g_j(\mathbf{x}) + k_j\sigma_j(\mathbf{x}) \le 0, j = 1, 2, \ldots, m \tag{10}$$

$$\mathbf{x} \pm \sigma \in \Omega \tag{11}$$

where $\sigma_j(\mathbf{x})$, $j = 0,1,\ldots,m$ are the standard deviations of f and g_j evaluated for the meta-models. The values of α and k_i are the alpha weight and the robust index, respectively. If $\alpha = 0$ and $k_0 = 1$, the design objective is a minimization of the variance of $f(\mathbf{x})$. If $k_j = 6$, the inequality constraints become DFSS (design for six sigma) constraints.

3.3. Numerical Optimization Process

The approximate optimization problems, based on meta-modes, are sequentially solved with the augmented Lagrange multiplier method (Kim and Choi, 1998). In the first iteration, the sequential approximate optimization (SAO) process requires the sampling points. RD/AutoDesign (Kim 2006; Kim *et al.*, 2009) provides a discrete Latin hypercube design, incomplete small composite design-I (ISCD-I), incomplete small composite design-II (ISCD-II), generalized small composite design (GSCD) and other classical DOE methods such as orthogonal arrays, CCD and BBD, etc. In this study, the discrete Latin hypercube design was used for the initial sampling. In the subsequent iteration of SAO, a new optimal design was given. Next, an exact analysis was performed for this point. Then, this new information was added to the design database. Hence, the meta-model is newly developed and these processes are repeated until the convergence criteria are satisfied with respect to their tolerance values.

4. ROBUST OPTIMIZATION OF A TRACKED VEHICLE SYSTEM

4.1. System Analysis

Figure 1 shows a high-mobility tracked vehicle model of military tank in the RecurDyn/Track HM, which consists of a chassis and two track systems. The chassis system includes a chassis, sprockets, road wheels, road arms, idlers and tensioners, and the suspension system. The one-side suspension system includes three hydro-pneumatic suspension units (HSU) and three torsion bars. The HSU is installed in the 1st, 2nd and 6th road wheels. The torsion Bar is installed in the 3rd, 4th and 5th road wheels. This high-mobility tracked vehicle encounters a 10-inch (25.4 *cm*) bump at a velocity of 40 (*km/h*). These are the most severe conditions given for the test of this tracked vehicle. The downward deflection of a road wheel is restricted by the track chain. Due to the tension of the track chain and the structure of the chain link, the downward deflection caused by a hole is limited and is not considered.

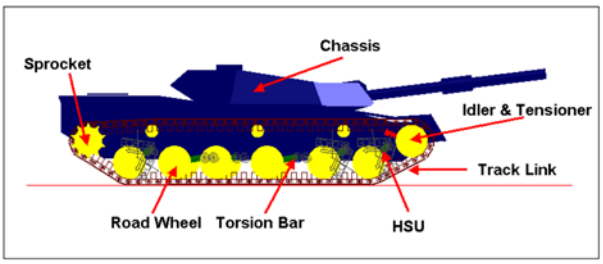

Figure 1. Tracked vehicle model in RecurDyn/Track HM.

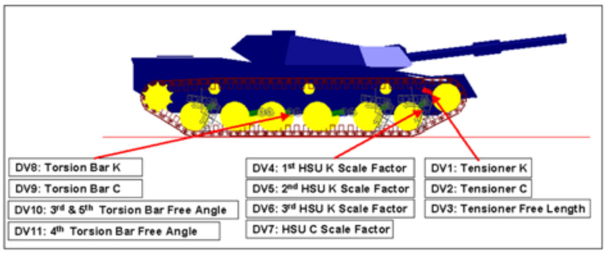

Figure 2. Random design variables.

This tracked vehicle model consists of 189 bodies: 37 bodies for the chassis system components, such as the sprocket, road wheel, road arm, etc.; 76 track link bodies for each track subsystem; 36 revolute joints and 152 bushing elements. Therefore, it has 954 degrees of freedom.

4.2. Random Design Variable Selection

Figure 2 shows the 11 random design variables. The 1st through 3rd design variables are the tensioner stiffness, damping coefficient and the tensioner free length in the idler and tensioner part. The 4th through 7th design variables are the 1st, 2nd, and 6th HSU stiffness scale factors in the HSU part and the HSU damping scale factor. The 8th and 9th design variables are the torsion bar stiffness and the damping coefficient. The 10th and 11th design variables are the free angles of the 3rd (5th) and 4th torsion bars.

All the design variables are considered random variables. Thus, they have a 1% coefficient of variation (COV), which indicates that the values of the design variables are their mean values and their deviations are ±1% around them. As the design variables are changed, the magnitudes of their deviations will be simultaneously changed according to their COV values.

4.3. Optimization Results

To enhance the ride comfort and vehicle performance of the high-mobility tracked vehicle, when the vehicle goes through the 25.4 (*cm*) hemi-cylinder-type bump at a velocity of 40 (*km/h*), the magnitude of the maximum vertical acceleration $|\ddot{z}_{CG}(\mathbf{b},t)|$ and its standard deviation $\sigma(\max_{t\in\{0,T\}}|\ddot{z}_{CG}(\mathbf{b},t)|)$ should be simultaneously minimized subject to the constraints that the wheel travels of the three torsion bars (θ_3, θ_4, θ_3) and the front road wheel (θ_6) be within $\pm 1\sigma$. Hence, all performance indexes are selected as the maximum value when the vehicle goes through the bump.

$$\text{Minimize } \Psi(\mathbf{b}) = \max_{t\in\{0,T\}}|\ddot{z}_{CG}(\mathbf{b},t)| + \sigma(\max_{t\in\{0,T\}}|\ddot{z}_{CG}(\mathbf{b},t)|) \qquad (12)$$

subject to

$$\max_{t\in\{0,T\}}\theta_i(\mathbf{b},t) + \sigma(\max_{t\in\{0,T\}}\theta_i(\mathbf{b},t)) \le \theta_i^a,\ i = 3, 4, \ldots, 6 \qquad (13)$$

and

$$b_k^L < b_k \pm \sigma_k \le b_k^U, k = 1, 2, \ldots, 11, \quad (14)$$

where the deviation of each random design variable is based on a 1% coefficient of variation (COV). Hence, they are evaluated as $\sigma_k = \pm 0.01 b_k$ in the design process. In practical implementation, a lower-pass filter is used to remove the numerical noise in the time-dependent responses. Hence, all the performance indexes used in the above formulation are evaluated from the filtered results.

In this study, simultaneous kriging models combined with pure quadratic polynomials were employed to construct meta-models. First, the meta-models were constructed from only 12 points corresponding to the current design plus 11 sampling points selected using the discrete Latin hypercube method. The sequential approximate optimization (SAO) process required only 14 iterations to satisfy the convergence tolerances. Consequently, only 26 total analyses were required, even though the robust design problem has 11 random design variables. The convergence criteria selected were the relative change of the objectives between consecutive iterations and the maximum violation of constraints. Their convergence tolerances were set at 0.05 and 0.01, respectively.

To validate the inequality constraints, including the robust design concept, we checked the constraint violation using the sampled variance. To do this, 12 points were sampled using a Latin hypercube method in the neighborhood of the final design ($\mathbf{b}^*$). The sampled range had the same random variable deviation ($\mathbf{b}^* \pm 0.01 \cdot \mathbf{b}^*$). The sampled standard deviation was determined precisely from the final design of this study and the additional 12 analysis results. Table 1 lists the approximate and sampled standard deviations for the wheel travel constraints. The approximate standard deviation values are evaluated at $\mathbf{b}^*$ using a Taylor series.

Although the approximate standard deviation $\tilde{\sigma}$ is used during the sequential approximate optimization process, in the final convergence checking, this approach used σ_s in place of $\tilde{\sigma}$. If this final checking is not satisfied, this approach optionally restarts the optimization process with these additional sampling points that are used to evaluate the sampled variance. However, this optimization result can successfully satisfy the final convergence check even though the approximate values are slightly different from the sampled ones.

Table 1. Comparisons of the approximate and sampled standard deviations.

	Approximate Values ($\tilde{\sigma}$)	Sampled Values (σ_s)
θ_3	0.01546	0.02183
θ_4	0.01951	0.06536
θ_5	0.01587	0.01873
θ_6	0.01684	0.03302

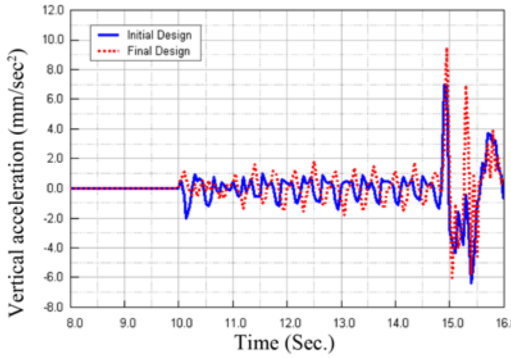

Figure 3. Comparison of the acceleration of the mass center for the initial and final designs.

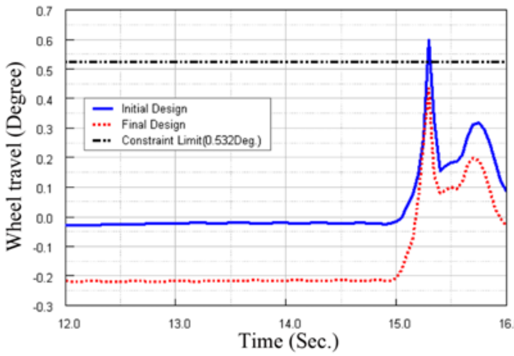

Figure 4. Comparison of wheel travels (θ_6) between the initial and final designs.

Figure 3 compares the acceleration of the mass center for the initial and final designs. Figure 4 shows the wheel travel results for the final design. The maximum acceleration is slightly increased to satisfy the wheel travel constraints. It is noted that the wheel travels seem to be much smaller than their limit. These margins are created by the second term of equation (14), which represents the role of robust design formulation.

5. CONCLUSION

This paper presents a meta-model-based design strategy for dynamic response optimization. This strategy avoids the difficulty of design sensitivity analysis, especially when a lower-pass filter is employed. Additionally, it shows that the robust design concept can be easily implemented using the approximate variance from meta-models. In a numerical application, the design optimization strategy successfully optimized the design parameters of interest for a tracked vehicle system in only 26 analyses, including the initial samplings. The robust optimization results are validated by the sampled variance.

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REFERENCES

Choi, J. H., Lee, H. C. and Shabana, A. A. (1998). Spatial dynamics of multibody tracked vehicles: Spatial equation of motion. *Int. J. Vehicle Mechanics and Mobility, Vehicle System Dynamics*, **29**, 27–49.

Farhang-Mehr, A. and Azarm, S. (2005). Bayesian meta-modeling of engineering design simulations: A sequential approach with adaptation to irregularities in the response behaviour. *Int. J. Numer. Meth. Engng.,* **62**, 2104–2126.

Kang, D.-O., Heo S.-J., Kim, M.-S., Choi, W.-C. and Kim, I.-H. (2012). Robust design optimization of suspension system by using target cascading method. *Int. J. Automotive Technology* **13**, **1**, 109–122.

Kang, D.-O., Heo, S.-J. and Kim, M.-S. (2010). Robust design optimization of the McPherson suspension system with consideration of a bush compliance uncertainty. *Porc. IMechE Part D: J. Automobile Engineering*, **224**, 705–716.

Kim, M.-S and Choi, D.-H. (1998). Min-max dynamic response optimization of mechanical systems using approximate augmented Lagrangian. *Int. J. Numer. Meth. Engng.,* **43**, 549–564.

Kim, M.-S. (2006). *RecurDyn/AutoDesign: Meta-Model Based Design Optimizer: Theoretical Manual*. FunctionBay.

Kim, M.-S., Kim, C.-W., Kim, J.-H. and Choi, J. H. (2009). Efficient optimization method for noisy responses of mechanical systems. *Proc. IMechE Part C: J. Mechanical Engineering Science*, **222**, 2433–2439.

Lee, H. C., Choi, J. H. and Shabana, A. A. (1998). Spatial dynamics of multibody tracked vehicles: Contact forces and simulation results. *Int. J. Vehicle Mechanics and Mobility, Vehicle System Dynamics*, **29**, 113–137.

McCullough, M. C. and Haug, E. J. (1986). Dynamics of high mobility tracked vehicles. *ASME, J. Mechanisms Transmissions, and Automation in Design*, **108**, 189–196.

Nakanishi, T. and Shabana, A. A. (1994). Contact forces in the nonlinear dynamic analysis of tracked vehicles. *Int. J. Numerical Method in Engineering*, **37**, 1251–1275.

Ryu, H. S., Bae, D. S., Choi, J. H. and Shabana, A. A. (2000). A compliant track model for high speed, high mobility tracked vehicle. *Int. J. Numerical Method in Engineering*, **48**, 1481–1502.

Ryu, H. S., Choi, J. H. and Bae, D. S. (2006). Dynamic modeling and experiment of military tracked vehicle. *SAE Paper No*. 2006-01-0929, SP-2040.

Wang, J. G. and Liu, G. R. (2002). A point interpolation meshless method based on radial basis functions. *Int. J. Numer. Meth. Engng.*, **54**, 1623–1648.